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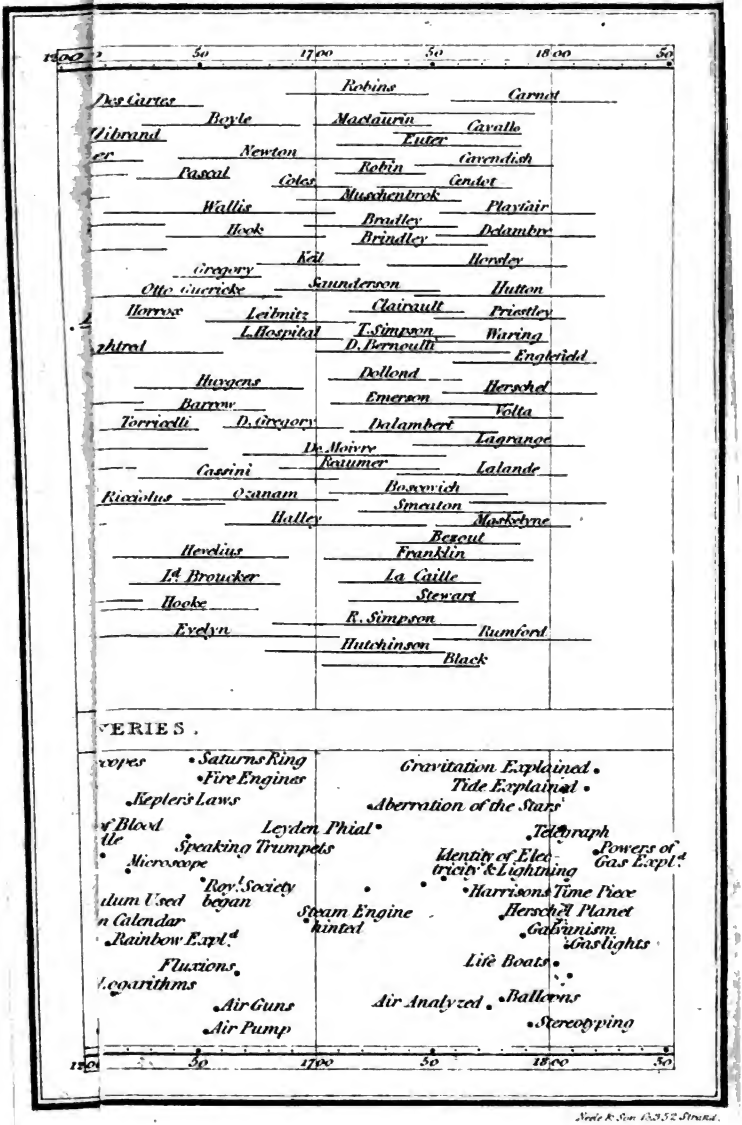

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A

DICTIONARY

OF THE

MATHEMATICAL AND PHYSICAL SCIENCES,

ACCORDING TO

THE LATEST IMPROVEMENTS

AND

DISCOVERIES.

EDITED

BY JAMES MITCHELL, A.M., F.A.S.

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PREFACE.

The Mathematical Sciences are at once the most perfect and the most valuable portion of human knowledge. They are valuable, not only for their own direct applications to the business of life, for the number of sciences to which they are the key, and the number of arts of which they are the foundation, but they have a value, higher and more important, if possible, in the strength which they give to the mind, and the exercise which they afford to its noblest faculties.

There is, indeed, no road to clear, forcible, and connected reasoning, but that which is opened up by the Mathematician; and whatever be a man's profession or station in the world, we are always able, from his mode of stating a proposition, or conducting an argument, to say whether he be or be not a Mathematician. In every other department of knowledge, there is some uncertainty—some hypothesis assumed, of which the foundation is unsearched or inscrutable, or something which hinges upon the undefined and undefinable properties of mind or of matter, or upon the contingency of events. Is language the medium through which one would arrive at logical precision? Then living language is mutable, and dead language is ambiguous. Is it human nature? Then who can gauge the mind, who can number its propensities, or measure its eccentricities? Is it life and manners? Then the modes vary with every individual and with every hour. Is it medicine? Then the life of that man is short indeed who has not seen fifty nostrums and modes of treatment rise into vogue and sink into neglect, without any reason assigned, or assignable, either for their rise or their fall. Is it law? Then who shall number the absurdities of the statute-book, to say nothing about the practice? Chemistry is one tissue of puzzles and disputes; and in pure Physics, it is not much better. In the physical sciences there is indeed a mathematical element; and so far as this goes, they are clear, orderly, and precise; but the moment that it is left out we are in the regions of theory—the wilds of doubt and uncertainty. With the student of most of these, the ground is unstable under his feet, the path by which he advanced has closed upon him, and that by which he has still to advance has not opened: with the mathematician every step is upon sure ground; and he sees how he came and where he is to proceed. Nothing that is contingent or ideal is admitted. He hunts out every lurking sophism, rejects the smallest aberration from truth, and goes on with absolute confidence and absolute certainty.

A system of mental discipline, so valuable for the extensive and positive advantages which it affords, cannot be pressed too earnestly upon the attention of the public, or come before them in too many shapes; and it is to be regretted, that in this age of pretended improvement, the soundness and certainty of mathematical cultivation should have given place to something far more light and flimsy—to the mere blandishments of the *Belles Lettres*, or such simple preparation for a man's individual trade, as may make him a handy tool for carrying on that trade, but nothing else. In proof of this, let any look into the works of those called (not the *Learned*, according to the good old English phrase) but the *Literati*, and he will find, amid all the sparkle of their abundance of words, and all their affectation of sentiment, there is a plenteous lack of depth of thought and soundness of principle.

Let any one turn to the Magazines, those reputed barometers of taste, and he will find that, with one exception, the Monthly Magazine, there is not in any of them a single scrap of science, or a single article aiming at a higher or more durable character, than the idle amusement of an idle hour. Chapters of novels and tales, which have been rejected in the shape of volumes; rhymes, in which there is neither poetry nor point, and slender critiques upon matters still more slender, have come in the place of the profound remarks and masterly dissertations of the Johnsons, Hawksworths, Prices, Priestleys, and Thomsons, who once gave vigour and value to our periodical literature. In the regular books it is not much better; we have tales and travels, accounts of ideal beings who never could exist, and of real ones who will not be remembered; and even in the scientific departments, we have the science of skulls and of shells, instead of the dissection of mind, and the display of the grandeur of nature. The whole structure has, in short, become light and frivolous; and though there be no "royal road to Geometry" now, any more than there was in the time of the Grecian sage, we have found out a bye-path to Fame, such as it is, by which Geometry may be avoided. Under these circumstances, every attempt to bring genuine science within the range of a greater number of readers is praiseworthy.

In the following pages, it has been the object of the compiler to reject every thing trivial, and to present a body of mathematical and physical knowledge as substantial and as complete as it is unobtrusive. He has been sedulous to exclude every thing merely ornamental; and by doing this he hopes that he has squeezed into this little volume all the really useful matter which is to be found in works of far higher price and loftier pretensions. Science is not a thing of fashion, and therefore little novelty can be expected. Discoveries and inventions in pure Mathematics were never abundant at any period, and latterly they have been exceedingly few.

As the compiler has himself had a good deal of practice in communicating mathematical knowledge, he hopes he may be pardoned if he offers a few words on what to him appears to be the most successful way of pursuing the study of them. Number, and form, and quantity, are the three subjects of all mathematical speculation. Number is the most simple, perhaps the most simple of all subjects of correct and rational enquiry; and therefore it is the one with which to begin. The notation, and common operations are easily understood; and these being once managed, a most beautiful field of speculation is opened up. The doctrine of prime and composite numbers, the doctrines of divisors, of fractions, and of ratios, and the doctrine of roots and powers, are all beautiful in themselves, and prepare the student for entering upon the more general subject of Algebra. That science, as far as relates merely to numbers, should immediately follow the study of Arithmetic; and then the mind is prepared for the elements of Geometry. In the study of these, old Euclid never has been, and probably never can be, equalled; and so the first book of his elements may follow the study of the notation, the common rules, and simple equations in Algebra. Before entering upon the second book of the elements, the student may proceed a little way with quadratics, and these and the second book will mutually assist each other.

When quadratics are well understood, the whole remaining books of the elements both of Geometry and of Trigonometry are easily mastered; and then may come the higher equations. After this, the elements of the fluxionary or differential calculus may be studied; and then the doctrine of curves; and the more abstruse parts of the calculus. And, when this has been done, the student is ready for the mixed sciences; and, with such preparation, his progress can hardly fail in being both sure and rapid. Before, however, he proceeds too far in the mechanical sciences, it is advisable that he study the elements of Chemistry, just for the purpose of enabling him to decide what phenomena are mechanical and what are not. Such a course of study as we have been sketching, would, if rightly gone about, not occupy more time than most boys waste upon a few idle, or at least superficial, accomplishments, and it would fill society with men of another description than are now found in them.

To contribute to such a result is one of the objects of this Dictionary; and we shall close this preface by mentioning the names of a few of the books which would be found useful as auxiliaries, or which would furnish those details which cannot be looked for in a work so small and so condensed as this:—For a general synopsis, perhaps no book is better than Nicholson; but Bezout, and some others of the French authors, are more systematic. Of Arithmetic, the books are many; but in English few of them are profound. Barlow's Theory of Numbers is among the best. The practical ones are without number, and many of them without variety. Joyce, Hutton, Bonnycastle, Walkingham, Vyse, and far from the worst, Dr. Hamilton, may be noted. In Algebra, Euler is at once the most simple and the most profound. Then we have Bridge, Wood, Ludlam, Simson, and a hundred others In Elementary Geometry we have Euclid; Playfair's last edition is decidedly the best. Again, in the practical parts, we have Hutton's Course, Crocker's Surveying, Keith's Mensuration, Moore and Mackay's Navigation, and an endless chain. On Trigonometry, we have Wood, Wodehouse, Keith, and many others. On Fluxions and the differential Calculus, we have Simson, Vince, Stone, La Croix, Boucharlat, and many others. In Mechanical Philosophy, we have Blair's Grammar, Young's Lectures, Bridge's Mechanics, Wood and Vince's Course, Playfair's Outlines, part of the same, by Leslie, and a Course by Millington. In Astronomy, we have Vince, Squire, the Wonders of the Heavens, and a variety of others. In short, if we have relaxed in our mathematical and scientific studies, it is not for want of books; for, though they be of lighter fabric, and fewer in proportion to the whole number of books than at some former periods, they are still numerous; and if we do suffer, it is not through want of books, but want of readers.

Every lover and student of the Sciences will duly estimate the value of a portable Dictionary of the Mathematical and Philosophical Sciences. Other Dictionaries of these subjects are, by their high price, placed beyond the reach of the general mass of purchasers, while by their fullness they tend to supersede elementary works, without supplying their places. The great use of a Dictionary is to aid study by convenient reference to particular points of difficulty, and to assist enquiry by an alphabetical arrangement of subjects.

London, Sept. 10, 1823.

A

DICTIONARY

OF

MATHEMATICAL AND PHYSICAL SCIENCE.

ABACUS. A table used before the introduction of the modern or figurate arithmetic, for facilitating the business of calculation. Originally it appears to have been nothing more than a smooth piece of board, covered with sand, and served indifferently for arithmetical computations, or geometrical diagrams. The word *calculate* is derived from the *calculi*, or small pebbles, which were used along with the abacus. These were distributed in rows, each row having a different value, in the same manner as the ranks or places of figures have in the modern scale of numbers. As many rows were required as there were ranks or places in the largest number which entered into the calculation; and one counter less than the root of the scale of arithmetic was required for each row. For instance, if the root of the scale had been 10, nine counters would have been required in each; and to express any particular number, as many would have been required in each row as there were ones in the corresponding word or figure. Thus, 365 would have been expressed by 5 in the right-hand row, 6 in the second, and 3 in the third. It is easy to see how, by the help of such an instrument, the common operations of arithmetic could be performed.

ABERRATION, an apparent motion of the celestial bodies, occasioned by the progressive motion of light, and the earth's motion in its orbit. This apparent motion is so minute, that it could never have been discovered by observations, unless they had been made with extreme care and accuracy. Dr. Bradley, astronomer royal, was led to it accidentally by the result of some careful observations, which he made with a view of determining the annual parallax of the fixed stars. If light be supposed to have a progressive motion, the position of the telescope, through which any celestial object is viewed, must be different from that which it must have been, if light were instantaneous; and, therefore, the place measured in the heavens will be different from the true place.

Clairaut explains the aberration, by supposing drops of rain to fall rapidly after each other from a cloud, under which a person moves with a very narrow tube; in which case it is evident that the tube must have a certain inclination, in order to admit a drop which enters at the top, to fall freely through the axis of the tube, without touching the sides of it; and this inclination must be greater or less, according to the velocity of the drops in respect to that of the tube. In this case, the angle made by the direction of the tube, and that of the falling drops, is the aberration, arising from the combination of these two motions.

To find the Aberration of a Star in Latitude and Longitude.

1. The greatest *aberration in latitude*, is equal to $20''$ multiplied by the sine of the star's latitude.

2. The *aberration in latitude* for any time is equal to $20''$ multiplied by the sine of the star's latitude, and the sine of elongation for the same time.

The aberration is subtractive before opposition, and additive after.

3. The greatest *aberration in longitude* is equal to $20''$ divided by the cosine of the star's latitude; and the aberration for any time is equal to that quotient multiplied by the cosine of the elongation of the star.

This aberration is subtractive in the first and last quadrants of the argument, and additive in the second and fourth quadrants.

EXAMPLE 1. To find the greatest aberration of γ *Ursæ Minoris*, whose latitude is 75° 13′.

Here the sine 75°13′ = ·9669; consequently $20'' \times 9669 = 19.34''$, the greatest *aberration in latitude*. Also cosine 75° 13′ = ·2551; and therefore $\frac{20''}{\cdot 2551} = 78.4''$, the greatest *aberration in longitude*.

2. To find the aberration of the same star in latitude and longitude, when the earth is 30° from syzygies.

Here sine of 30° = ·5; and, therefore, $19.34'' \times \cdot 5 = 9{\cdot}67''$, the *aberration in latitude*. If the earth be 30° beyond conjunction, or before opposition, the latitude is diminished; but if it be 30° before conjunction, or after opposition, the latitude is increased. Again, cosine 30° = ·866; consequently $78{\cdot}4'' \times \cdot 866 = 67.89''$, the *aberration in longitude*. If the earth be 30° from conjunction, the longitude is diminished; but if it be 30° from opposition, it is increased.

To find the Aberration of a Star in Declination and Right Ascension.

1. The greatest *aberration in declination* is $20''$ multiplied by the sine of the angle of position at the star, and divided by the sine of the difference of longitude between the sun and star, when the aberration in declination is nothing.

2. The *aberration in declination* at any other time, will be equal to the greatest aberration multiplied by the sine of the difference, between the sun's place at the given time, and its place when the aberration is nothing.

3. The sine of the latitude of a star : radius = the tangent of the angle of position at the star : the tangent of difference of longitude between the sun and star.

4. The greatest *aberration in right ascension* is equal to $20''$ multiplied by the cosine of the angle of position, and divided by the sine of the difference in longitude between the sun and star, when the aberration in right ascension is nothing.

4. The *aberration in right ascension* at any other time, is equal to the greatest aberration multiplied by the sine of the difference between the sun's place at the given time, and his place when the aberration is nothing. Also the sine of the latitude of the star : the radius the co-tangent of the angle of position at the star : the tangent of the difference of longitude between the sun and star.

ABERRATION of the *Planets* is their geocentric motion, or the space through which they appear to move, as seen from the earth during the time of the light's passing from the planet to the earth.

It is evident that this aberration will be greatest in the longitude, and very small in latitude, because the planets deviate very little from the plane of the ecliptic, so that this aberration is almost insensible and disregarded; the greatest in Mercury being only about $4\frac{1}{3}''$, and much less in the other planets. As to the aberration in right ascension and declination, it must depend upon the place of the planet in the zodiac. The aberration in longitude being equal to the geocentric motion will be greater or less according to this motion: it will be greatest in the superior planets, Mars, Jupiter, Saturn, and Uranus, when they are in opposition to the sun; but in the inferior planets, Mercury and Venus, the aberration is greatest at the time of their superior conjunction. The maxima of aberration for the several planets, when their distance from the sun is the least, are as follow, viz.

1	Moon	0″ $\frac{2}{3}$
2	Mercury	59″ 0
3	Venus	43″ $\frac{1}{5}$
4	Mars	37″ $\frac{4}{5}$
5	Jupiter	29″ $\frac{4}{5}$
6	Saturn	27″ 0
7	Georgian, or Uranus	25″ 0

And between these numbers and nothing the aberration of the planets, in longitude, varies according to their situation. That of the sun, however, is invariable, being constantly 20″; and this may alter his declination, by a quantity which varies from 0 to 8″, being greatest at the equinoxes, and vanishing in the solstices.

ABERRATION, in *Optics*, is that error, or deviation of the rays of light when inflected by a lens, or speculum, whereby they are prevented from meeting or uniting in the same point, called the *geometrical focus*. It is either lateral or longitudinal. The lateral aberration is measured by a perpendicular to the axis of the speculum, produced from the focus, to meet the refracted ray. The longitudinal aberration is the distance of the focus from the point in which the same ray intersects the axis. If the focal distance of any lenses be given, if their aperture be small, and if the incident ray homogeneous and parallel, the longitudinal aberrations will be as the squares, and the lateral aberrations as the cubes of the linear apertures.

There are two species of aberration, distinguished according to their different causes; the one arises from the figures of the speculum, or lens, producing a geometrical dispersion of the rays, when these are perfectly equal in all respects: the other arising from the unequal refrangibility of the rays of light themselves.

In all plano convex lenses, having their plane surfaces exposed to parallel rays, the longitudinal aberration of the extreme ray, or that most remote from the axis, is equal to 4$\frac{1}{6}$ times the thickness of the lens.

In all plano convex lenses, having their convex surfaces exposed to the parallel rays, the longitudinal aberration of the extreme ray is equal to 1$\frac{1}{6}$ of the thickness of the lens.

In all double convex lenses of equal spheres, the aberration of the extreme ray is equal to 1$\frac{2}{3}$ of the thickness of the lens.

In a double convex lens, the radius of whose spheres are as 6 to 1, if the more convex surface be exposed to the parallel rays, the aberration from the figure is less than that of any other spherical lens, being no more than $\frac{15}{14}$ of its thickness.

ABSCISS, Abscisæ, Abscissa, is any part of the diameter or axis of a curve, comprised between any fixed point, where all the abscisses begin, and another line called the ordinate, which is terminated in the curve. Commonly the abscisses are considered as commencing at the vertex of the curve; but this is not necessary, as they may have their origin in any other point; but, generally, when no condition is specified, they are understood as commencing at the vertex. The absciss and corresponding ordinate, considered together, are called co-ordinates, and by means of these the equation of the curve is defined.

ABSOLUTE *Equation*, in *Astronomy*, is the sum of the optic and eccentric equations.

The apparent inequality of a planet's motion, arising from its not being equally distant from the earth at all times, is called its optic equation; and this would subsist if the planet's real motion were uniform. The eccentric inequality is caused by the planet's motion not being uniform. For the illustration of this, conceive the sun to move, or appear to move, in the circumference of a circle, in the centre of which the earth is placed. Then it is manifest, that if the sun move uniformly in this circle, he must appear to move uniformly to a spectator on the earth; and, in this case, there would be no optic or eccentric equation. But suppose the earth to be placed out of the centre of the circle;

and then, though the sun's motion should be really uniform, it would not appear to be so when seen from the earth; and in this case there would be an optic equation, but not an eccentric one. Again, let us imagine the sun's orbit not to be circular, but elliptical, and the earth to be in its focus, then it is evident that the sun cannot appear to have a uniform motion in such ellipse, and, therefore, his motion will be subject to two equations, viz. the Optic and Eccentric Equations, the sum of which is the *Absolute Equation.*

ABSOLUTE *Term*, or *Number*, in *Algebra*, is that which is completely known, and to which all the other parts of the equation is made equal.

ABSTRACT *Mathematics*, or *Pure Mathematics*, is that which treats of the properties of magnitude, figure, or quantity, absolutely and generally considered, without restriction to any species in particular, such as Arithmetic and Geometry. It is thus distinguished from *Mixed Mathematics*, in which simple and abstract quantities, primatively considered in Pure Mathematics, are applied to sensible objects, as in Astronomy, Mechanics, Optics, &c.

ACCELERATION is principally used in Physics, to denote the increasing rapidity of bodies in falling towards the centre of the earth, by a force called gravity, whether a property of matter, or an effect of the earth's motions.

That natural bodies are accelerated in their descent, is evident from various considerations, both *a priori* and *posteriori.* Thus we actually find, that the greater height a body descends from, the more rapidly it descends, the greater impression it makes, and the more intense is the blow with which it strikes the obstacle upon which it impinges.

Some have attributed this acceleration to the pressure of the air; others to an inherent principle in matter, by which all bodies tend to the centre of the earth as their proper seat or element, where they would be at rest; and hence, say they, the nearer that bodies approach to it, the more is their motion accelerated. Another class held, that the earth emitted a sort of attractive effluvia, innumerable threads of which continually ascend and descend, proceeding, like radii, from a common centre, and diverging the more the further they go; so that the nearer a heavy body is to the centre, the more of these magnetic threads it receives; and hence the more its motion is accelerated.

But leaving all such visionary theories, and only admitting the existence of some such force as gravity, inherent in all bodies, without regard to what may be the cause of it, the whole mystery of acceleration will be cleared up, and the theory of it established on the most obvious principles.

Suppose a body let fall from any height,* and that the primary cause of its beginning to descend is the power called gravity; then, when once the descent is commenced, motion becomes, in some measure, natural to the body; so that, if left to itself, it would persevere in it for ever; as we see in a stone cast from the hand, which continues to move after it is left by the cause that first gave it motion; and which motion would continue for ever, was it not destroyed by resistance and gravity, which cause it to fall to the earth. But beside this tendency, which of itself is sufficient to continue the same degree of motion, *in finitum*, there is a constant accession of subsequent efforts of the same principle, which

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* Sir Richard Phillips, in his Essays, maintains that there is no such force inherent in matter as the attraction of gravitation, and that the cause of a body's descending to the earth, as well as all the other phenomena usually ascribed to the action of this force, are the natural and necessary results of the two motions of the earth. (See articles *Attraction* and *Motion.*)—This explanation of the true cause of the phenomena does not, however, alter the law of acceleration, or, indeed, any law of the earth or the planetary system; though it varies our reasoning.

began the motion, and which continues to act on the body already in motion, in the same manner as if it were at rest. Here, then, being two causes of motion, and both acting in the same direction, the motion they jointly produce must necessarily be greater than that of one of them; and the same cause of increase acting still on the body, the descent of it must, of course, be continually accelerated.

For, supposing gravity, or the earth's motion, to act uniformly on all bodies, at equal distances from the earth's centre; and that the time in which a heavy body falls to the earth to be divided into equal parts, indefinitely small. Let this force incline the body towards the earth's centre, while it moves in the first indefinitely small space of time of its descent; if after this the force be supposed to cease, the body would proceed uniformly towards the centre of the earth, with a velocity equal to that which results from the force at the first impression.

But since the action of the force is supposed still to continue, in the second moment of time the body will receive a new impulse downwards, equal to what it received in the first; and thus its velocity will be double of what it was in the first moment: in the third, it will be treble; in the fourth, quadruple; and so on continually: for the impression made in one moment, is not at all altered by what is made in another. The whole are, as it were, united into one sum. Hence the velocity will be proportional to the time in which it is acquired.

Thus, if a body, by means of this constant force, acquire a velocity v, in one second of time, it will, in two seconds, acquire a velocity $2v$; in three seconds, it will acquire a $3v$; and so on: and all bodies, whatever be their quantity of matter, will acquire, by the force of gravity, the same velocity in the same time; that is, supposing no resistance from the atmosphere, or any other obstacle in a perfect vacuum.

Hence, if t be made to represent the time a body has been falling, and v the velocity acquired in one second, then will tv represent the velocity at the end of t seconds.

And now if we represent by ϕ the velocity of a body when it first begins to fall, or its velocity acquired in the first instant of its descent, then will the terms of the series

$$\phi,\ 2\,\phi,\ 3\,\phi,\ 4\,\phi,\ \&c.\ t\,\phi,$$

represent the successive velocities at each successive instant; and, since the velocity v, multiplied by the time in uniform motions is equal to the space, and since we may consider the motions as uniform during any indefinite small instant of time, the above may also be taken to represent the spaces described at each successive instant; and hence the sum of them will be the whole space described in the time t. Now the number of terms in this series being t, and $t\,\phi$ being the final velocity, it may be represented by v, and thus the sum of the series will be expressed by $\frac{1}{2}\,t\,(\phi + v)$. But since ϕ represents the first velocity of the descending body, it is indefinitely small, and may be considered as nothing with regard to v. It may, therefore, be cancelled out of the expression, and consequently the whole space will be represented by $\frac{1}{2}\,t\,v$; or, taking s to represent the space, we shall have $s = \frac{1}{2}\,t\,v$, that is,—the space described by a body uniformly accelerated in any time, is equal to half the space that would be described by the same body, in the same time, with a uniform velocity equal to that last acquired.

It has been found by experiment that a body falling freely in the latitude of London, passes through a space equal to $16\frac{1}{12}$ feet in the first second of time; hence we have, by representing this space by g, $g = \frac{1}{2}\,t\,v$, or $g = \frac{1}{2}\,v$; because $t = 1$, whence $v = 2g$, is the velocity acquired at the end of the first second of time, and, therefore, from what has been above demonstrated, $v = 2g\,t$, will represent the acquired velocity at the end of any time t.

Also $s = \frac{1}{2}\,t\,v$; substituting, therefore, for v, we have $s = \frac{1}{2}\,t\,(2g\,t)$, or $s = t^2 g$; and since g is constant it follows that the spaces described by falling bodies are to each other as the squares of the times of descent, the spaces themselves being accurately expressed by the formula $s = t^2 g$, where g represents $16\frac{1}{12}$ feet, the space a body falls through in the first second of time.

Hence we deduce the following general laws of motions uniformly accelerated, viz.

1. The velocities acquired are constantly proportional to the times.

2. That the spaces are proportional to the squares of the time; so that if a body describe any given space in a given time, it will describe four times that space in a double time, nine times that space in a treble time, and so on. And universally, if the times be in arithmetical proportion, as 1, 2, 3, 4, &c. t, the spaces described will be as 1, 4, 9, 16, &c. t^2. Thus a body, which falls by gravity through $16\frac{1}{12}$ feet in the first second, will fall through $64\frac{1}{3}$ feet in two seconds, and so on. And since the velocities acquired in falling are as the times, the spaces will be as the square of the velocities; and both the times and velocities will be as the square roots of the spaces.

3. The spaces described by a falling body in a series of equal moments, or intervals of time, will be as the odd numbers 1, 3, 5, 7, 9, &c. which are the differences of the squares or whole spaces; that is, a body which falls through $16\frac{1}{12}$ feet in the first second, will fall through $3 \times 16\frac{1}{12}$ in the second, $5 \times 16\frac{1}{12}$ in the third, and so on.

These properties may be otherwise represented, thus:—Let S, V, be put for the space and velocity of a falling body in any time T; and s, v, the same for the time t; then we shall have

$$S : s = T^2 : t^2$$
$$S : s = V^2 : v^2$$
$$: v = \sqrt{S} : \sqrt{s}$$
$$T : t = \sqrt{S} : \sqrt{s}$$
$$S : s = TV : tv$$
$$T : t = \frac{S}{V} : \frac{s}{v}$$
$$T : t = Sv : Vv$$

When the accelerating forces are different, but constant, the spaces will be as the products of the forces into the square of the times; and the times will be as the square roots of the spaces directly, and of the forces inversely. For when the force is given or constant, the velocity (V) is as the time (T); and when the forces are different, but constant, and the time is given, the velocity (v) will be as the force (F). But when neither the force nor the time is given, the velocity (v) will be as the product of the force into the time, that is as (F × T).

Hence, $V : v = F \times T : f \times t$

conseq. $F \times T^2 : f \times t^2 = V \times T : v \times t = S : s$

therefore, $T^2 : t^2 = \frac{S}{F} : \frac{s}{f}$

or, . . . $T : t = \sqrt{\frac{S}{F}} : \sqrt{\frac{s}{f}}$

From the properties above demonstrated, we obtain the following practical theorems: Let g denote the space passed over in the first second of time, by a body urged by any uniform force, denoted by 1; and let t denote the time or number of seconds in which the body passes over any other space s, and v the velocity acquired at the end of that time: then we shall have $v = 2gt$, and $s = gt^2$; and from these two equations we obtain the following general formulæ:

1. . . . $t = \frac{v}{2g} = \frac{2s}{gv} = \sqrt{\frac{s}{g}}$

2. . . . $v = 2gt = 2\sqrt{g\,s}\frac{2s}{t}$

3. . . . $s = gt^2 = \frac{v^2}{4g} = \frac{tv}{2}$

4. . . . $g = \frac{s}{t^2} = \frac{v}{2t} = \frac{v^2}{4s}$

Let us now illustrate these formulæ by a few examples.

Exam. 1. How far will a body fall in 12″ in the latitude of London; and what will be its last velocity?

Here $g = 16\frac{1}{12}$, and $t = 12$; therefore $s = gt^2 = 16\frac{1}{12} \times 144 = 2316$ feet, the space required.

Also, $v = 2gt = 2 \times 16\frac{1}{12} \times 12 =$ 386 feet, the last acquired velocity.

Exam. 2. How long will a body be in falling through a space of 1608 feet?

Here $s = 1608$, and $t = \sqrt{\frac{s}{g}} = \sqrt{\frac{1608}{16\frac{1}{12}}}$ $= 10$ seconds *nearly*.

Exam. 3. How far must a body fall to acquire a velocity of 386 feet per second?

Here $v = 386$, and $t \ \frac{v}{2g} \quad \frac{386}{32\frac{1}{6}} =$ 12 seconds *nearly*.

The same law of acceleration obtains equally in the descents of bodies down inclined planes, except that the force of gravity will in that case vary as the sines of the angles of inclination of the planes, that is, the force down the inclined plane is to the whole force of gravity as the sine of the angle of inclination of the plane to radius. If, therefore, the angle of inclination of the plane be a, the force down the plane will be sin. $a \times g$; and by using this instead of g, the above formulæ will be equally applicable to the descents of heavy bodies down inclined planes.

Of variably accelerated motion. Having illustrated the laws of accelerated motion, when the accelerating forces are constant, and deduced the formulæ for expressing them in final determinate quantities, we subjoin those that belong to the cases of variably accelerated motions.

Here the formulæ will be fluxionary expressions, the fluents of which, adapted to particular cases, will give the relation of time, space, velocity, &c. Let t denote the time of motion; v the velocity generated by any force; s the space passed over; and $2g$ the variable force at any part of the motion, or the velocity that the force would generate in one second of time, if it should continue invariably like the force of gravity during that one second, and the value of this velocity, $2g$, will be in proportion to $32\frac{1}{6}$ feet, as that variable force is to the force of gravity. Then, because the force may be supposed constant during the indefinitely small space of time t, and spaces and velocities in uniform motions being proportional to the times, we shall have these two fundamental proportions, *viz.*

$$v : \dot{s} = 1'' : \dot{t} \qquad \text{or } \dot{s} = \dot{t}\, v$$

$$2g : \dot{v} = 1'' : \dot{t} \qquad \text{or } \dot{v} = 2g\, \dot{t}$$

Whence are deduced the following formulæ, in which the value of each quantity is expressed in terms of the rest:

$$1. \ \cdots\cdots \quad t = \frac{s}{v} = \frac{\dot{v}}{2g}$$

$$2. \ \cdots\cdots \quad \dot{v} = 2g\,\dot{t} = \frac{2gs}{v}$$

$$3. \ \cdots\cdots \quad \dot{s} = v\,\dot{t} = \frac{v\dot{v}}{2g}$$

$$4. \ \cdots\cdots \quad 2g = \frac{\dot{v}}{\dot{t}} = \frac{\dot{r}v}{\dot{s}}$$

These formulæ, like those in the preceding part of this article, are equally applicable to the destruction of motion and velocity, by means of retarding forces, as to the generation of them by means of accelerating forces.

The motion of a body ascending, or impelled upwards, is diminished or retarded from the same principle acting in a contrary direction, in the same manner as a falling body is accelerated.

A body projected upwards, rises till it has lost all its motion; which it does in the same time that a falling body would have acquired a velocity equal to that with which the body was thrown up. Hence the same body projected up, will rise to the same height, from which, if it fell, it would have acquired the velocity with which it was projected upwards. And hence the heights to which bodies thrown upwards with different velocities, ascend, are to each other as the squares of those velocities.

ACCELERATION, in *Astronomy*, is a term applied to the fixed stars. The diurnal acceleration is the time by which the stars, in one diurnal revolution, anticipate the mean diurnal revolution of the sun, which is $3'\ 55\frac{9}{10}''$ of mean time, or nearly $3'\ 56''$; that is to say, a star rises or sets, or passes the meridian, $3' \, 56''$ sooner each day. This apparent acceleration of the stars is owing to the real retardation of the sun, which depends upon his apparent motion towards the east, at the rate of about $59' \, 8\frac{1}{3}''$ of a degree every day. In consequence of this, the star which passed the meridian at the same moment with the sun yesterday, is about $59' \, 8\frac{1}{3}''$ beyond the meridian to the west, when the sun arrives at it; and this distance will require about $3' \, 56''$ of time for the sun to pass over, and therefore the star will anticipate the motion of the sun at this rate every day.

ACCELERATION of a *Planet.* A planet is said to be accelerated in its motion, when its real diurnal motion exceeds its mean diurnal motion; and retarded in its motion, when the mean exceeds the real diurnal motion. This inequality arises from the change in the distance of the planet from the sun, which is continually varying; the planet moving always quicker in its orbit when nearest the sun, and slower when furthest off.

ACCELERATION of the *Moon,* is the increase of the moon's mean motion from the sun, compared with the diurnal motion of the earth; and it appears that, from a certain cause, it is now a little quicker than it was formerly.

La Place has shown this acceleration of the moon's motion to arise from the action of the sun upon the moon, combined with the variation of the eccentricity of the earth's orbit. By the present diminution of the eccentricity, the moon's mean motion is accelerated; but when the eccentricity is arrived at the *minimum,* the acceleration will cease; after this the eccentricity will increase, and the moon's mean motion will be retarded.

ACUTE, or SHARP, a term opposed to obtuse; thus we say, *acute-angled triangle, acute-angled cone,* &c.

ADDITION, one of the fundamental operations in Arithmetic and Algebra, denotes the finding of one number or quantity, equal to two or more given numbers or quantities. The general principle upon which it is founded, is that the whole is equal to all the parts.

Addition is indicated or expressed by writing the sign $+$, which is called the sign of addition between the numbers or quantities; thus $7+5$, and $a+b$, indicate the addition of 7 and 5, and of a and b.

The result of the addition is called the sum. It may be expressed in one number, when the proposed numbers are all of the same kind, and when 1 in any of them is equal to 1 in each of the others. When this is not the case, an alteration must be made in the numbers, and if such an alteration cannot be made, then the sum cannot be obtained in one number. In Algebra, quantities cannot be collected into one quantity, unless they consist of the same power of the same letters.

Addition is variously denominated, according to the nature of the numbers or quantities that are to be added. Thus,

Simple Addition is the finding of the sum of several simple or abstract numbers; and it is thus performed,—

Arrange the numbers so that like places stand under each other. Collect the sum of the units, write below the column what is over tens, and for every ten reckon one in the next column. Do this with all the columns, in order, to the left hand.

EXAMPLES.

6973		69321
4218		1168
9374		600
8231		12345
28796	Sums	83734

Addition may be proved in various ways: One may begin alternately at the top and bottom of the columns, and see if the results correspond. The excess over 9 may be noted in each line, and also in the sum, and if the sum of the former be equal to the latter, or differ from it only by one or more 9's, there can be no error but 9 in the addition: Thus in the first of the preceding examples, the remainder on the lines are 7, 6, 5, 5; the remainder in the sum 5; and the sum of the former (23,) abating two nines, leaves 5 also. Hence the addition is right.

Compound Addition is the collecting several quantities of different denominations into one sum.

Place the same denominations to stand directly under each other. Add up the figures in the lowest denomination, and find how many units of the next higher denomination are contained in their sum. Write down the remainder, and carry these units to the next higher denomination; which add up in the same manner as before: and proceed thus through all the different denominations to the highest.

EXAMPLES.

l.	*s.*	*d.*		cwt.	qr.	lb.
17	18	9		11	3	17
2	11	2½		31	1	18
6	9	3¼		47	2	10
29	18	4¾		38	0	14
49	10	3		26	3	21
106	7	10½	Sums	155	3	24

Addition of *Fractions* is the adding of several fractions into one sum.

1. If the fractions are of different denominations, they must be first reduced to the same denomination. 2. Reduce all mixed numbers to improper fractions; and all fractions, having different denominators, to the same denominator. 3. Add all the numerators into one sum; which, placed over the common denominator, will be the answer required.

EXAMPLES.

1. Add together the fraction $\frac{1}{2}$, $\frac{1}{5}$, and $\frac{1}{7}$.

First, $\frac{1}{2}=\frac{35}{70}$; $\frac{1}{5}=\frac{14}{70}$; $\frac{1}{7}=\frac{10}{70}$

then, $\frac{35}{70}+\frac{14}{70}+\frac{10}{70}=\frac{59}{70}$.

2. Required the sum of $\frac{2}{3}$ of a shilling, $\frac{3}{4}$ of a guinea, and $\frac{7}{8}$ of a pound.

$\frac{3}{4}$ of a guinea $=\frac{63}{4}$ of a shilling,

$\frac{7}{8}$ of a pound $=\frac{35}{2}$ of a shilling;

Therefore,

$$\frac{2}{3}+\frac{63}{4}+\frac{35}{2}=\frac{8}{12}+\frac{189}{12}+\frac{210}{12}$$

$$=\frac{407}{12}=33s.\ 11d.=1l.\ 13s.\ 11d.$$

Addition of *Decimals*, is finding the sum of several numbers, consisting partly of integers and partly of decimals, or of decimals only.

Arrange all the quantities so that the several decimal points may fall in a line directly under each other, and then proceed as in simple addition.

EXAMPLES.

1. Find the sum of 34.17; 19.143; 167.13; and 143.5.

2. Find the sum of .1176; .1344; .746; and .1468.

1st. Exam.		2d. Exam.
34.17		.1176
19.143		.1344
167.13		.746
143.5		.1468
363.943	Sums	1.1448

Addition of *Intermediate Decimals*, is the finding the sum of any number of circulating decimals.

1. Reduce all the decimals to their equivalent fractions; and the sum of these will be the answer required; or,

2. Carry on the repetends till they all begin and end their periods of circulation in the same lines, and let the circulation of each be carried two figures beyond this place; then add them up as in the former rule, observing not to set down any thing in the first two places; only carry the proper number from them to the conterminous period; and the result will give the true period of circulation in the sum required.

EXAMPLE.

Add $3.\dot{6}$; $78.3\dot{4}7\dot{6}$; $42.\dot{8}\dot{4}$; and 15.5 together.

$3.\dot{6}$	=	$3.6\dot{6}66666\dot{6}$	66
$78.3\dot{4}7\dot{6}$	=	$78.3\dot{4}76476\dot{6}$	47
$42.\dot{8}\dot{4}$	=	$42.8\dot{4}8484\dot{8}$	48
15.5	=	$15.500000\dot{0}$	00

The sum . . . $140.3\dot{6}2790\dot{1}$

In this example, the periods of circulation do not commence together till after the seventh place of decimals; they are then carried on two places further, in order to ascertain what ought to be carried to the conterminous period; which in the present case is 1, as will appear from the above operation.

Note. There may arise cases in which it will be necessary to carry the circulation on to three or more places beyond the conterminous period, but in general two or three places are sufficient.

Addition, in *Algebra*, is finding the sum of several algebraical quantities, and connecting those quantities together with their proper signs. And this is generally divided into the following cases.

Case 1. When all the indeterminate letters are the same, and have the same sign.

Add the co-efficients of the several quantities together, and prefix before the sum the proper sign, whether it be *plus* or *minus*.

EXAMPLES.

$$\begin{array}{rr}
+\ 7a + \ 3b \\
+\ 5a + \ 4b \\
+\ 6a + 11b \\
\hline
+\ 18a + 18b \\
\hline
\end{array}
\quad \text{Sums} \quad
\begin{array}{rr}
+\ 5x - 13y \\
+\ 4x - 11y \\
+\ 14x - \ 7y \\
\hline
+\ 23x - 31y \\
\hline
\end{array}$$

Case 2. When the quantities are the same, but have different signs.

Add all the like quantities together that have also the same sign, and thus two separate sums will be obtained; then subtract the less of these from the greater, and prefix before the remained the sign of the greater sum.

EXAMPLES.

$$\begin{array}{rr}
+7ab - \ 3c^2 \\
-6ab + \ 7c^2 \\
+4ab - 13c^2 \\
\hline
+5ab - \ 9c^2 \\
\hline
\end{array}
\quad \text{Sums} \quad
\begin{array}{rr}
-4xy + \ 3z^3 \\
+6xy - 11z^3 \\
+4xy - 14z^3 \\
\hline
+6xy - 22z^3 \\
\hline
\end{array}$$

Note 1. When the leading quantity of any algebraical expression has no sign, it is supposed to be affected with the sign +.

Note 2. Unlike quantities can only be added by means of the sign + placed between them.

In the addition of *Algebraic Fractions*, reduce to a common denominator, and then add the numerators for the sum required.

EXAMPLES.

1. Add together $\frac{5a}{6b^2} + \frac{3a^2}{4b} + \frac{5c}{12b^3}$

$$\frac{5a}{6b^2} = \frac{10ab}{12b^3};\ \frac{3a^2}{4b}\frac{9a^2b^2}{12b^3};\ \frac{5c}{12b^3} = \frac{5c}{12b^3}$$

$$\frac{10ab}{12b^3} + \frac{9a^2b^2}{12b^3} + \frac{5c}{12b^3} = \frac{10ab+9a^2b^2+5c}{12b^3}$$

2. Thus also we find,

$$\frac{7x}{11y} + \frac{4x}{3y} = \frac{21x}{33y} + \frac{44x}{33y} = \frac{65x}{33y}$$

3. And,

$$\frac{7x+4y}{3ab} + \frac{5x-3y}{abc} = \frac{7cx+4cy}{3abc} + \frac{15x-9y}{3abc} = \frac{7cx+15x+4cy-9y}{3abc} = \frac{(7c+15)x+(4c-9)y}{3abc}$$

In the addition of *Surds*, reduce all the given quantities to their most simple form; then add the co-efficients of the radicals which have the same index and the same number.

EXAMPLES.

Thus,

$$\sqrt{8}+\sqrt{18}=2\sqrt{2}+3\sqrt{2}=5\sqrt{2}$$
$$\sqrt{12}+\sqrt{27}=2\sqrt{3}+3\sqrt{3}=5\sqrt{3}$$
$$\sqrt[3]{108a4}+\sqrt[3]{32a}=3a\sqrt[3]{4a}+2\sqrt[3]{4a}$$
$$=(3a+2\sqrt[3]{4a}.$$

Note. When the quantities are reduced to their lowest terms, or simplest form, and have different indices and numbers, they can only be added together by means of the sign + placed between them.

Thus, $\sqrt{18}+\sqrt{108}=3\sqrt{2}+6\sqrt{3}$, cannot be reduced to a simpler form than that above; and the same with various others.

Addition of *Ratios*, is the same as composition of ratios;

thus, if $a : b = c : d$;

then by addition, or composition,

$a + b : a = c + d : c$

or, $a + b : b = c + d : d$.

ADDITIVE, denotes something to be added to another, in contradistinction to something to be taken away, or subtracted. Thus, astronomers speak of additive equations; and geometricians, of additive ratios, &c. &c.

ADFECTED *Equation*, in *Algebra*, is that in which the unknown quantity is found in two or more different degrees or powers; thus, $x^3 - px^2 + qx = a$, is an adfected equation, having three different powers of the unknown quantity x entering into its composition. Such equations are distinguished from *simple*, which involve but one power.

ADHESION, in *Philosophy*, is a species of attraction which takes place between the surfaces of bodies, whether similar or dissimilar, and which, in a certain degree, connects them together; differs from cohesion, which, uniting particle to particle, retains together the component parts of the same mass. The power of adhesion is proportional to the number of touching points, which depends upon the figures of the particles that form the bodies; and in solid bodies, upon the degree in which their surfaces are polished and compressed. The effects of this power are extremely curious, and in many instances astonishing. Musschenbroek relates that two cylinders of glass, whose diameters were not quite two inches, being heated to the same degree as boiling water, and joined together by means of melted tallow lightly put between them, adhered with a force equal to 130 pounds: lead, of the same diameter and in similar circumstances, adhered with a force of 275 pounds; and soft iron with one of 300 pounds.

	Grains.
Gold adheres to mercury, with a force of	446
Silver	429
Tin	418
Lead	397
Bismuth	372
Platina	282
Zinc	204
Copper	242
Antimony	126
Iron	115
Cobalt	8

ÆOLIPILE, Æolipila, an instrument consisting of a hollow metalline ball, with a slender neck or pipe arising from it. This being filled with water, and thus exposed to the fire, produces a vehement blast of wind.

ÆOLUS'S *Harp*, an instrument so named from its producing agreeable harmony, merely by the action of the wind. It is thus constructed: let a box be made of as thin deal as possible, of the exact length answering to the width of the window in which it is intended to be placed, five or six inches deep, and seven or eight inches wide; let there be glued upon it two pieces of wainscot, about half an inch high and a quarter of an inch thick, to serve as bridges for the strings; and within side, at each end, glue two pieces of beech, about an inch square, of length equal to the width of the box, which are to sustain the pegs; into these fix as many pins, such as are used in a harpsichord, as there are to be strings in the instrument, half at one end and half at the other, at equal distances: it now remains to string it with small catgut, or blue first fiddle-strings, fixing one end to a small brass pin, and twisting the other round the opposite pin. When these strings are tuned in unison, and the instrument placed with the strings outward in the window to which it is fitted, it will, provided the air blows on that window, give a sound like a distant choir, increasing or decreasing according to the strength of the wind.

ÆRA, in *Chronology*, is the same as *Epoch*, and means a fixed point of time, from which to begin a computation of the succeeding years.

Æra also means the way or mode of accounting time. Thus, we say, such a year of the Christian *æra*, &c.

Christian Æra. It is generally allowed by chronologers, that the computation of time from the birth of Christ, was only introduced in the sixth century, in the reign of Justinian; and is generally ascribed to Dionysius Exiguus. See *Epoch*.

ÆRIAL, *Perspective*, is that which represents bodies diminished and weakened in proportion to their distance from the eye; but it relates principally to the colours of objects, which are less distinct the greater the distance at which they are viewed.

ÆROGRAPHY, a description of the air or atmosphere, its nature, composition, limits, dimensions, properties, &c.

ÆROLITHS, a name given to those solid semi-metallic substances which fall from the atmosphere. The descent of such bodies had been long reported; but the fact was not confirmed till within a few years. The larger sort of these stones have been seen as luminous bodies, to move with great velocities, descending in oblique directions, and frequently with a loud hissing noise, resembling that of a mortar-shell when projected from a piece of ordnance; they are sometimes surrounded with a blaze or flame, tapering off to a narrow stream at the hinder part, are heard to explode, and seen to fly in pieces. The velocity with which they strike the earth is very great, frequently penetrating to a considerable depth, and when taken up they have been, in some cases, found to be still hot, and bearing evident marks of recent fusion. Sometimes such falls have happened during a storm of thunder and lightning, at others, when the sky has been clear and serene; whence one may infer that these phenomena are unconnected with the state of the atmosphere.

One of the most remarkable and distinguishing characteristics of these stones is, that they perfectly resemble each other; at the same time, that they are totally different from any known terrestrial substance. They present, in all cases, the same appearance of semi-metallic matter, coated on the outside with a black incrustation. The stone which fell at l'Aigle in France, in 1803, was found to contain

54	parts of silica
36	. . . oxyde of iron
9	. . . magnesia
3	. . . oxyde of nickel
2	. . . sulphur
1	. . . lime

And all that have been analized have given nearly the same results.

The specific gravity of these bodies is also found to be nearly the same, being about 3.400, or nearly 3½ times that of common water.

These general and constant characters strongly indicate a common origin, and various hypotheses have been advanced to account for them. Some have attributed them to terrestrial, and others to lunar, volcanoes; they have again been supposed to be concretions actually formed in the regions of our atmosphere; while others have considered them as small planets circulating about the sun or earth, which coming in contact with our atmosphere, take fire from the resistance and friction they experience in passing through it.

With regard to the first supposition, *viz.* that these stones proceed from terrestrial volcanoes, it will be sufficient to observe, that no remarkable eruption has been known to happen at or near the time of their fall, and that such bodies have been found at the distance of some thousand miles from any known volcano; beside the immense force that would be necessary to project bodies, some of them of many hundred weight, to so great a distance, far exceeds any force that we can conceive to arise from volcanic eruptions. The theory, that they proceed from volcanoes in the moon, has certainly a much greater degree of probability. The same force that would project a body from the moon to the earth, would not, if it were exerted at the earth's surface, send the same body to the distance of ten miles; in consequence of the superior mass of our planet, and density of its atmosphere. It is readily computed, that a body projected from a favourable spot on the moon's surface, that is, from the centre of her disc opposite the earth, with a velocity about four times what is commonly given to a cannon-ball, or, about 8220 feet per second, would carry it beyond the centre of attraction, and consequently into the sphere of the earth's activity; whence it must necessarily either fall to our surface, or circulate about us as a satellite. The time that a body, so projected, would be in its passage from the moon to the earth, is also found to be three days; which is not so long but that it might retain its heat, particularly as it is doubtful whether in passing through a vacuum, or very attenuated medium, it would be possible for the caloric to escape, not to say that it might acquire a fresh accumulation of heat, by passing through the denser parts of our atmosphere. Add to this, that eruptions resembling those of our volcanoes have been frequently observed in the moon, and that her atmosphere is extremely rare, and consequently presenting but little resistance to projected bodies; and it will then be seen that this hypothesis, though at first sight apparently extravagant, is notwithstanding much more probable than the one we before examined. It must be acknowledged, however, that the explosions of which we have spoken, are difficult to account for upon this supposition. With regard to these bodies being concretions formed in the air, there is one principal objection, *viz.* that the velocity with which they strike the earth, estimated by the depth to which they have been known to penetrate, is so great as to indicate their having fallen from heights far exceeding the limits of the terrestrial atmosphere.

AEROLOGY, the doctrine or science of the air, and its phenomena, properties, good and bad qualities, &c.

AEROMETRY, the art of measuring powers and properties of the air; including the laws of motion, gravitation, pressure, elasticity, refraction, condensation, &c. of the atmospheric fluid.

AERONAUT, a person who sails or navigates through the air.

AEROSTATION, in the modern application of the term, signifies the art of navigating through the air, both in the principles and practice of it. Hence also the machines, which are employed for this purpose, are called ærostatic machines; and on account of their round figure, *air-balloons*. The person who navigates them is called an *æronaut*.

The fundamental principles of this art have been long known; although the application of them to practice seems to be altogether a modern discovery. These chiefly relate to the weight, pressure, and elasticity of the air, its specific gravity, and that of the other bodies to be raised or floated in it. Any body which is specifically, or bulk for bulk, lighter than the atmospheric air, will be buoyed up by it, and ascend; just as wood, or cork, ascends in water; but as the density of the atmosphere decreases, this body can rise only to a height in which the surrounding air will be of the same specific gravity with itself: in this situation it will either float, or be driven in the direction of the wind or current of air to which it is exposed. An air-balloon is a body of this kind, the whole mass of which, including the covering, contents, and appendages, is of less specific gravity than that of the air through which it rises.

Heat is well known to rarify and expand, and consequently to lessen the specific gravity of the air to which it is applied; and the diminution of its weight is proportional to the heat. According to the scale of Fahrenheit's thermometer, 400, or rather 435, degrees of heat, will just double the bulk of a quantity of air. If, therefore, the air enclosed in any kind of covering be heated, and consequently dilated, to such a degree as that the excess of the weight of an equal bulk of common air above the weight of the heated air, is greater than the weight of the covering and its appendages, this whole mass will ascend in the atmosphere; till by the cooling and condensation of the included air, or the diminished density of the surrounding fluid, it becomes of the same specific gravity with the air in which it floats, and without renewed heat it will then gradually descend. If instead of heating common air, inclosed in any covering, and thus diminishing its gravity, the covering be filled with an elastic fluid lighter than atmospheric air, the whole mass will ascend as in the former case, and continue to rise till it attains an altitude at which the surrounding air is of the same specific gravity with itself.

The idea of flying by means of wings and other contrivances, was certainly entertained by the ancients; and some accounts relate exploits of this kind having been performed; but still there is reason to suppose they are mere fictions, and that no means were ever possessed for accomplishing this undertaking till the invention of balloons, which dates no further back than the conclusion of the last century. Soon after Mr. Cavendish's discovery of the specific gravity of inflammable air, it occurred to the ingenious Dr. Black, that if a bladder sufficiently light and thin were filled with this air, it would form a mass lighter than the same bulk of atmospheric air, and rise in it. This thought was suggested in his lectures, in 1767 and 1768; and he proposed, by means of the allantois of a calf, to try the experiment. Other employments, however, prevented the execution of his design. The possibility of constructing a vessel, which when filled with inflammable air would ascend in the atmosphere, had occurred also to Mr. Cavallo, and to him belongs the honour of having first made experiments on this subject, in the beginning of the year 1782, of which an account was read to the Royal Society, on the 20th of June in that year.

But while aerostation seemed thus on the point of being made known in Britain, it was unexpectedly announced in France by two brothers, Stephen and John Montgolfier, natives of Annonay, and masters of a considerable paper-manufactory there, who had turned their thoughts to this project as early as the middle of the year 1782. Their idea was to form an artificial cloud, by inclosing smoke in a bag, and making it carry up the covering along with it. In that year the experiment was made at Avignon with a fine silk bag; and by applying burning paper to an aperture at the bottom, the air was rarefied, and the bag ascended to the height of 70 feet. Various experiments were now made upon a large scale, which excited the public curiosity very greatly. An immense bag of linen, lined with paper, and containing upwards of 23,000 cubic feet, was found to have a power of lifting about 500 pounds, including its own weight. Burning chopped straw and wool under the aperture of the machine, immediately occasioned it to swell, and afterwards to ascend into the atmosphere. In ten minutes it had risen 6000 feet; and when its force was exhausted, it fell to the ground at the distance of 7668 feet from the place it ascended.—Soon after this one of the brothers, invited by the Academy of Sciences to repeat his experiment at their expense, constructed a large balloon of an elliptical form; and in a preliminary experiment, this machine lifted from the ground eight persons who held it, and would have carried them all off, if more had not quickly come to their assistance. Next day the machine was filled by the combustion of 50 pounds of straw and 12 pounds of wool. The balloon soon swelled and sustained itself in the air, together with the burden of between 400 and 500 pounds weight. It was designed to repeat the experiment before the king at Versailles; but a violent storm of rain and wind happening to damage the machine, it became necessary to prepare a new one; and such expedition was used, that this vast balloon, near 60 feet in height and 43 in diameter, was made, painted within and without, and finely decorated, in the course of four days and four nights. Along with it was sent a wicker cage, containing a sheep, a cock, and a duck, which were the first animals ever sent on such a voyage. The full success of the experiment was, however, prevented by a violent gust of wind, which tore the machine in two places near the top, before it ascended. Still it rose 1440 feet; and after remaining in the air about eight minutes, fell to the ground, at the distance of 10,200 feet from the place of its ascent, with the animals in perfect safety. —As the great powers of those aerostatic machines, and their very gradual descent, showed they were capable of transporting people through the air with all imaginable safety, M. Pilatre de Rozier offered himself to be the first aerial adventurer, in a new machine constructed in a garden in the Fauxbourg of St. Antoine. It was of an oval shape, 48 feet in diameter and 74 in height, elegantly painted with the signs of the zodiac, ciphers of the king's name, and other ornaments. A proper gallery, grate, &c. enabled the person who ascended to supply the fire with fuel, and thus keep up the machine as long as he pleased: the weight of the whole apparatus was upwards of 1600 pounds. On the 15th of October, 1783, M. Pilatre placing himself in the gallery, the machine was inflated and permitted to ascend to the height of 84 feet, where he kept it afloat about four minutes and a half; after which it descended very gently: and such was its tendency to ascend, that it rebounded to a considerable height after touching the ground. On repeating the experiment, he ascended to the height of 210 feet. His next ascent was 262 feet; and in the descent, a gust of wind having blown the machine over some large trees in an adjoining garden, M. Pilatre suddenly extricated himself by throwing straw and wool on the fire, which raised him at once to a sufficient height; and on descending again, he once more raised himself to a proper height by the same means. Some time after he ascended with M. Girond de Vilette to the height of 330 feet; hovering over Paris at least nine minutes in sight of all the inhabitants, and the machine keeping all the while a steady position. These experiments having shown, that aerostatic machines might be raised or lowered at the pleasure of the persons who ascended, M. Pilatre and the Marquis d'Arlandes, on the 21st of November, 1783, undertook an aerial voyage, which lasted about 25 minutes, and during which time they passed over a space of above five miles. From the account given by the marquis, they met with several different currents of air, the effect of which was to give a very sensible shock to the machine, and the directions of the motion seemed to be from the upper part downwards. It appears, also, that they were in some danger of having the balloon burnt altogether; as the marquis observed several round holes made by the fire in the lower part of it, which alarmed him considerably, and indeed not without reason. However, the progress of the fire was easily stopped by the application of a wet sponge, and all appearance of danger ceased.—This voyage of M. Pilatre and the marquis may be said to conclude the history of those aerostatic machines which were elevated by means of fire; these having been soon after superseded by balloons in which inflammable air was enclosed. The first experiment was made by two brothers, the Messrs. Roberts, and M. Charles, a professor of experimental philosophy. The bag was composed of lutestring varnished over with a solution of gum caoutchouc; and was about thirteen English feet in diameter. Many difficulties occurred in filling it with the inflammable air; but being at last set at liberty, after having been well filled, it was thirty-five pounds lighter than an equal bulk of common air. It remained in the atmosphere only three quarters of an hour, during which it traversed fifteen miles. Its sudden descent was supposed to have been owing to a rupture which had taken place when it ascended into the higher regions of the atmosphere. The event of this experiment, and the aerial voyage made by Messrs. Rozier and Arlandes, naturally suggested the idea of undertaking something of the same kind with a balloon filled with inflammable air. The machine used on this occasion was formed of gores of silk, covered with a caoutchouc varnish, of a spherical figure, and measuring 27½ feet in diameter. A net was spread over the upper hemisphere, and fastened to a hoop which passed round the middle of the balloon. To this a sort of car was suspended, a few feet below the lower part of the balloon; and, in order to prevent the bursting of the machine, a valve was placed in it; by opening of which some of the inflammable air might be occasionally let out. The car was of basket-work, covered with linen, and beautifully ornamented; being 8 feet long, 4 broad, and 3½ deep; its weight 130 pounds. Great difficulties again occurred in filling the machine; but these at last being removed, the two adventurers took their seats at three-quarters after one in the afternoon of the 1st of December, 1783. They continued in the air an hour and three-quarters, and alighted at the distance of 27 miles from Paris; having suffered no inconvenience during their voyage, nor experienced any contrary currents of air, as had been felt by Messrs. Pilatre and Arlandes. As the balloon still retained a great quantity of inflammable gas, M. Charles determined to take another voyage by himself. M. Robert accordingly got out of the machine; which now being 130 pounds lighter, arose with such velocity that in twenty minutes he was almost 9000 feet high, and entirely out of sight of terrestrial objects. The globe, which had been rather flaccid, soon began to swell, and the inflammable air escaped in great quantity. He also drew the valve, to prevent the balloon from bursting; and the inflammable gas being considerably warmer than the external air, diffused itself all around, and was felt like a warm atmosphere. In ten minutes, however, the thermometer indicated a great variation of temperature; his fingers were benumbed with cold, and he felt a violent pain in his right ear and jaw, which he ascribed to the expansion of the air in these organs, as well as to the external cold. The beauty of the prospect which he now enjoyed, however, made amends for these inconveniences. At his departure the sun was set on the valleys; but the height to which M. Charles was got in the atmosphere, rendered him again visible, though only for a short time. He saw for a few seconds vapours rising from the valleys and rivers. The clouds seemed to ascend from the earth, and collect one upon the other, still preserving their usual form, only their colour was grey and obscure, for want of sufficient light in the atmosphere. By the light of the moon, he perceived that the machine was turning round with him in the air, and he observed that there were contrary currents, which brought him back again. He observed, also, with surprise, the effects of the wind, and that the streamers of his banners pointed upwards; which, he says, could not be the effect either of his ascent or descent, as he was moving horizontally at the time. At last, recollecting his promise of returning to his friends in half-an-hour, he pulled the valve, and accelerated his descent. When within 200 feet of the earth, he threw out two or three pounds of ballast, which rendered the balloon again stationary; but, in a little time afterwards, he gently alighted in a field about three miles distant from the place whence he set out; though by making allowance for all the turnings and windings of the voyage, he supposes that he had gone through nine miles at least. By the calculations made, it appears that he rose at this time not less than 10,500 feet; a height somewhat greater than that of Mount Ætna. The subsequent aerial voyages differ so little from those above related, that any particular description of them would be superfluous.

The expense in filling a balloon with inflammable gas is very great, and it therefore became an object of anxious inquiry how this might be avoided or reduced. The first attempt of this kind was made by the duke de Chartres: it consisted in having a smaller balloon within the greater one, the smaller being filled with common air, by means of a pair of bellows, when necessary; viz. whenever it was thought proper to descend, it being supposed that the machine would thus become heavier, and the air in the outer balloon condensed, and consequently, that the ascent or descent might be effected at pleasure. Another scheme was to put a small aerostatic machine, with rarified air, under an inflammable air-balloon, but at such a distance, that the inflammable air of the latter might be perfectly out of the reach of the fire used for inflating the former; and thus by increasing or diminishing the fire in the small machine, the absolute weight of the whole would be considerably diminished or augmented. This scheme was unhappily put in execution by the celebrated M. Pilatre de Rozier and M. Romaine. Their inflammable air-balloon was about thirty-seven feet in diameter, and the power of the rarified air one was equivalent to about sixty pounds. They ascended without any accident; but had not been long in the atmosphere, when the machine took fire at the height of about three quarters of a mile from the ground. No explosion was heard; and the silk of the balloon seemed to resist the atmosphere for about a minute, after which it colapsed, and descended along with the two unfortunate travellers so rapidly, that both of them were killed. The first aerial voyage in England was performed on the 15th of September, 1784, by Vincent Lunardi, a native of Italy. His balloon was made of oiled silk, painted in alternate stripes of blue and red: its diameter was thirty-three feet. M. Lunardi departed from the Artillery-ground at two o'clock; and at ten minutes past four, he descended in a meadow near Ware, in Hertfordshire. The only philosophical instrument which he carried with him was a thermometer, which, in the course of his voyage stood as low as 29°, and he observed that the drops of water collected round the balloon were frozen. The second aerial voyage in England was performed by Mr. Blanchard, and Mr. Sheldon, professor of Anatomy to the Royal Academy. They ascended at Chelsea at nine minutes past twelve o'clock. Mr. Blanchard having landed Mr. Sheldon at about 14 miles from Chelsea, re-ascended alone, and finally landed near Rumsey, in Hampshire, about seventy-five miles distant from London, having gone nearly at the rate of twenty miles an hour. The wings used on this occasion, it seems, produced no deviation from the direction of the wind.

Inflammable air for balloons may be obtained in several ways: but the best methods are, by applying acids to certain metals; by exposing animal, vegetable, and some mineral substances, in a close vessel, to a strong fire; or by transmitting the vapour of certain fluids through red-hot tubes. In the first of these methods, iron, zinc, and vitriolic acid, are the materials most commonly used: the vitriolic acid must be diluted with about five or six parts of water. Iron may be expected to yield, in the common way, about 1700 times its own bulk in gas; or 4½ ounces of iron, the like weight of oil of vitriol, and 22½ ounces of water, will produce one cubic foot of inflammable air: six ounces of zinc, and an equal weight of oil of vitriol, and thirty ounces of water, are necessary for producing the same quantity. Inflammable air may also be obtained at a much cheaper rate, by the action of fire on various substances; but the gas thus obtained is not so light as that produced by the effervescence of acids and metals. The substances proper to use for this purpose are,—pit-coal, asphaltum, amber-rock oil, and other minerals; wood, and especially oak; camphor oil, spirits of wine, æther, and animal substances, which yield air of different kinds and of various specific gravities.

AFFECTED is a term made use of by algebraists, when speaking of the signs or co-efficients of a quantity. Thus, if a quantity is preceded by the sign +, it is said to be affected with a positive sign; and if by —, it is affected with a negative sign, &c. Thus also, $2a$, $7a$, are said to be affected with the co-efficients 2, 7, &c.

AFFECTION, in *Geometry* and *Physics*, is a term signifying the same as property.

AFFIRMATIVE *Quantity*, or Positive *Quantity*, in *Algebra*, is one that is to be added or taken effectively, in contradistinction to a negative quantity, which is to be subtracted.

AFFIRMATIVE *Sign*, or sign of addition, is +, and is read *plus*, or *more*, or *added to:* thus $a+b$ signifies that the quantity represented by b, is to be added to the quantity represented by a. When a single letter or quantity is preceded by the sign +, as $+a$, it indicates that it is to be taken positively; and the same is always to be understood if there be no sign prefixed. Before the introduction of this sign into algebra, the word *plus*, or its equivalent, was used to imply the addition of quantities; as was the word *minus*, to indicate the subtraction of them.

AIR, in *Physics*, an elastic, transparent, ponderous, compressible, and dilatable fluid body surrounding the globe to a considerable height. The ancients considered air as an element; but, taken in its popular sense, it is very far from the simplicity of an elementary substance. The most important physical or mechanical properties of the air, are its *fluidity*, *weight*, and *elasticity*.

Of the Fluidity of the Air.—That the air is a fluid is evident from the easy passage it affords to bodies moving through it. That air is a fluid, is also proved from this circumstance, that it is found to exert an equal pressure in all directions; an effect that could not take place otherwise than from its extreme fluidity.

The Weight or Gravity of the Air.—With this property of air the ancients were not totally unacquainted, though their sentiments on the subject were confused and unsatisfactory. The effects which are now known to result from the weight and elasticity of the air, were for a long time attributed to an imaginary principle called *fuga vacui*, or *Nature's abhorrence of a vacuum;* and Galileo himself admitted the principle, although he was under the necessity of assigning a limit to it, corresponding to the weight of a column of water thirty-four feet high. In the year 1643 it occurred to Torricelli, the disciple of Galileo, that whatever might be the cause by which a column of water, thirty-four feet high, is sustained above its level in the tube of a common pump, the same force would sustain a column of any other fluid, which weighed as much as that column of water; and hence he concluded, that quicksilver, being about fourteen times as heavy as water, would not be sustained at a greater height than twenty-nine or thirty inches. He then made the experiment, called after him the Torricellian experiment. He took a glass tube, of several feet in length, and having sealed it hermetically at one end, he filled it with quicksilver; then inverting it, and holding it upright by pressing his finger against the lower or open orifice, he immersed that end in a vessel of quicksilver; then removing his finger, and suffering the fluid to run out, the result verified his conjecture: for, the quicksilver, faithful to the laws of Hydrostatics, descended till the column of it was about thirty inches high above the surface of that in the open vessel: and hence he concluded, that it was no other than the weight of the air incumbent on the external surface of the quicksilver, which counterbalanced the fluid contained in the tube. By this experiment he not only proved, as Galileo had done, that the air had weight, but that its weight was the cause of the suspension of water in pumps, and of quicksilver in the tube; and that the weight of the whole column of it, was equal to that of a like column of quicksilver thirty inches high, or of water thirty-four or thirty-five feet high; but he did not ascertain the weight, or any particular quantity of it, as a gallon, or a cubic foot; nor its specific gravity to water, as had been done, though inaccurately, by Galileo.

Torricelli's experiment soon became very popular. Father Mersenne was apprised of it in 1644, and immediately conveyed an account of it to the philosophers of France; and Pascal, after some hesitation, adopted Torricelli's idea, and devised several experiments for confirming it. One of these was to procure a vacuum above the reservoir of quicksilver, in which case he found the column sunk down to the common level; but this appearing to him not sufficiently powerful to dissipate the prejudices of the philosophers of the old school, he prevailed on M. Perier, his brother-in-law, to execute the famous experiment of the Puy-de-domme, who found that the height of the quicksilver, half way up the mountain, was considerably less than at the foot of it, and still less at the top: by this means the question was set to rest; as no doubt could any longer be entertained, that it was the weight of the atmosphere that counterpoised the column of quicksilver. The medium of all these is about as one to 832, or one to 833; when reduced to the pressure of thirty inches of the barometer, and the mean temperature of fifty-five degrees of the thermometer; or, by adding ⅓ to the last number, the proportion becomes one to 833⅓, or 3 to 2500; whence, upon the whole, we may conclude, that the density of air is to that of water as 3 to 2500; and, consequently, that a cubic foot of air weighs at a mean $1\frac{1}{5}$ ounces avoirdupois; the weight of a cubic foot of water being 1000 ounces, and of a cubic foot of quicksilver 13,600 ounces.

Elasticity of the Air.—It is another quality of this fluid, that it will yield to any pressure, by contracting its dimensions; and that, upon removing or diminishing the pressure, it will again return to its natural state or volume. This elastic force may be accounted the distinguishing property of air, the others which we have mentioned above being common to all fluids.

Various experiments were soon instituted both on the continent and in England, with a view of ascertaining the specific gravity of the air, or the average weight of a given bulk of it compared with that of water. To prove the existence of this property of air, various experiments might be cited; it will, however, be sufficient to mention one; which is, simply inverting an open vessel full of air in water; in which case, the resistance it offers to further immersion, and the height to which the water ascends within it, in proportion as it is further immersed, are proofs of the elasticity of the air contained in it.

With regard to the degree of this elastic force, it has been shown, by the most satisfactory experiments, that with moderate pressure it is always proportional to the density of the air, and that the density is always as the compressing force; whence also the elasticity of air is as the force by which it is compressed. Thus, if air is confined in a bent tube, one end being open and the other hermetically sealed, and quicksilver be poured in at the open end, it will be found that the spaces into which the air is compressed are always inversely as the weights by which it is loaded, and since these weights are the measures of the elasticity, therefore the elasticities are inversely as the spaces which the air occupies.

It has been questioned, amongst philosophers, whether this elastic power of the air is capable of being destroyed or diminished. Mr. Boyle could not discover that any state of rarefaction, which he was capable of producing, was sufficient for destroying this property. Colonel Roy has shown that the particles of air may be so far removed as to lose a very great part of their elastic force: the same experiments show that moist air possesses the greatest elastic force; and that common air, in its natural state, is proportionally more elastic than when its density is considerably augmented by pressure. The elasticity of air may be so effected by a violent pressure as to require some time to return to its natural tone. The weight or pressure of the air, it is obvious, has no dependence on its elasticity; but would be the same whether it possossed that property or not. But being elastic, it is necessarily affected by pressure, which reduces it into such a space, that the elasticity which re-acts against the compressing weight, is equal to that weight.

The elasticity of the air exerts itself equally in all ways; and when it is at liberty, and freed from the cause which compresses it, it expands equally in all directions, in consequence of which it always assumes a spherical figure, in the interstices of the fluid in which it is lodged. This is evident in liquors placed under the receiver of an air-pump; for, by exhausting the air, at first there appears a multitude of exceeding small bubbles, like grains of fine sand, dispersed through the fluid mass, and rising upwards; and as more air is pumped out, they enlarge in size, but still retain their spherical figure.

The expansion of air, by means of its elastic property, when only the compressing force is taken off or diminished, is found to be surprisingly great; it having been known, in certain experiments, to expand itself into 13,679 times its original space, and this solely by its own natural power, without the application of fire.

The elasticity of the air, under the same pressure, is increased by heat and diminished by cold, and that according to several very accurate experiments, at the rate of about one-435th part for every degree of heat of Fahrenheit's thermometer.

AIR-*Gun*, a *pneumatic machine*, which expels bullets or shot with great velocity and effect; its operation depends upon the elastic power of air, which we have seen above increases in proportion to the greater degree of condensation. Now the elastic power of fired gunpowder being equal to the pressure of 1000 atmospheres, or 1000 times greater than that of common air, it follows that in order to produce the same effect with an air-gun as with a musket, the air must be compressed into one-thousandth part of its natural bulk; and for all inferior degrees of condensation, the effect will be proportionally diminished; and as the velocities with which equal balls are impelled, are directly as the square roots of the forces acting upon them, we shall be able always to estimate the effect previous to any explosion taking place: thus, if n be the number of times the air is condensed, then as $\sqrt{1000} : \sqrt{n} =$ the velocity arising from the explosion of gunpowder to that arising from condensation. If, therefore, the air be condensed twenty times, its velocity will be about one seventh of that arising from fired gunpowder. In the air-gun, however, the reservoir of condensed air is commonly very large, in proportion to the tube which contains the ball, and its density will be very little altered by expanding through the narrow tube; and, consequently, the ball will be urged throughout by nearly the same uniform force with that of the first instant, and hence the exploding power of condensed air is much more considerable than appears from the preceding estimation, being little less than that of gunpowder, even under a condensation of ten times, provided the reservoir be of any considerable magnitude.

AIR-*Pipes*, an invention intended for clearing the holds of ships, and other close places, of their foul air.

AIR-*Pump*, a *pneumatic machine*, which is of great use, in explaining and demonstrating the properties of air. It was invented by Otto de Guericke, consul of Magdeburgh, about the year 1654; and though it has since been much improved in form, its principle remains the same. The principal use of the air-pump is to extract the air from a vessel, which in that state is said to be exhausted, and the degree of exhaustion depends upon the goodness of the machine. The vessel, which is called a receiver, is fitted to a plate on the air-pump, having a small orifice in the middle, which communicates by a tube with the barrels of the pump. These barrels are of metal, or of glass, in each of which works an air-tight piston, having a valve, which opens outwards when the piston is forced down towards the bottom of the barrel.

There is another valve to prevent the air from being drawn back into the receiver. The elasticity of the compressed air at the bottom of the barrel being greater than that of the external atmosphere, forces open the valve, and escapes; and when the piston is again raised, air from the receiver escapes into the barrel. The two barrels act thus alternately, till the air in the receiver be so much exhausted as not to have elasticity sufficient for forcing open the valves, and there the degree of exhaustion which the pump can produce terminates.

In whatever manner or form this machine is made, its use and operation are much the same. The air in the receiver is diminished at each stroke of the piston, by the quantity of the barrel or cylinder full, and thus repeating the operation again and again, the air is rarefied to any proposed degree, a mercurial gage showing at any time what the degree of exhaustion is. But, supposing no vapour from moisture, &c. to rise in the receiver, the degree of exhaustion, after any number of strokes of the piston, may be determined by knowing the respective capacities of the barrel and the receiver, including the tube of communication. For as every stroke diminishes the density in a constant proportion, namely, as much as the whole content exceeds that of the cylinder or barrel, the sum of as many diminutions as there are strokes of the piston, will show the whole diminution by all the strokes. Thus if the capacity of the barrel be equal to that of the receiver, then, the barrel being half the sum of the whole content, half the air will be drawn out at one stroke; and consequently the remaining half, being dilated through the whole or first capacity, will be of only half the density of the first; in like manner, after the second stroke, the density of the remaining contents will be only half of that after the first stroke, that is only $\frac{1}{4}$ of the original density: continuing this operation, it follows that the density of the remaining air will be $\frac{1}{8}$ after three strokes of the piston, $\frac{1}{16}$ after four strokes, and so on, according to the powers of the ratio $\frac{1}{2}$; that is, such power of the ratio as is denoted by the number of the strokes. Universally, if s denote the sum of the contents of the receiver and barrel, and r that of the receiver only without the barrel, and n any number of strokes of the piston; then, the original density of the air being 1, the density after n strokes will be $\frac{r^n}{s^n}$, that is, the nth power of the ratio $\frac{r}{s}$.

From the same formulæ, namely $\left(\frac{r}{s}\right)^n = d$ the density, we easily derive a rule for finding the number of strokes of the piston, necessary to rarefy the air any number of times, or to reduce it to a given density d, that of the natural air being 1. For since $\left(\frac{r}{s}\right)^n = d$, by taking the logarithm of this equation, it is $n \times \log.\frac{r}{s} = \log.$ of d; and hence $n = \frac{\log. d}{\log.\frac{r}{s}} = \frac{\log. d.}{\log. r - \log. s}$; that is, divide the log. of the proposed density by the log. of the ratio of the receiver to the sum of the receiver and barrel together, and the quotient will show the number of strokes of the piston requisite to produce the degree of exhaustion required.

Some of the principal effects and phenomena of the air-pump are the following:—That, in the exhausted receiver, heavy and light bodies fall equally swift; so, a piece of metal and a feather fall from the top of a tall receiver to the bottom exactly together. That most animals die in a minute or two: but, however, that vipers and frogs, though they swell much, live an hour or two; and after being seemingly quite dead, come to life again in the open air: that snails survive about ten hours; efts, or slow-worms, two or three days; and leeches five or six. That oysters live for 24 hours. That the heart of an eel taken out of the body continues to beat for good part of an hour, and that more briskly than in the air. That warm blood, milk, gall, &c. undergo a considerable intumescence and ebullition. That a mouse or other animal may be brought, by degrees, to survive longer in a rarefied air, than it does naturally. That air may retain its usual pressure, after it is become unfit for respiration. That the eggs of silk-worms hatch in vacuo. That vegetation stops. That fire extinguishes; the flame of a candle usually going out in one minute; and a charcoal in about five minutes. That red-hot iron, however, seems not to be affected; and yet sulphur or gunpowder are not lighted by it, but only fused. That a match, after lying seemingly extinct a long time, revives again on re-admitting the air. That a flint and steel strike sparks of fire as copiously, and in all directions, as in air. That magnets, and magnetic needles, act the same as in air. That the smoke of an extinguished luminary gradually settles to the bottom in a darkish body, leaving the upper part of the receiver clear and transparent; and that on inclining the vessel sometimes to one side, and sometimes to another, the fume preserves its surface horizontal, after the nature of other fluids. That heat may be produced by attrition. That camphire will not take fire; and that gunpowder, though some of the grains of a heap of it be kindled by a burning-glass, will not give fire to the contiguous grains. That glow-worms lose their light in proportion as the air is exhausted, and, at length, become totally obscure; but on re-admitting the air, they presently recover it again. That a bell, on being struck, is not heard to ring, or very faintly. That water freezes much more rapidly than in the open air, especially if the basin which contains it be floating in a chemical mixture, by which the vapour arising from the water can be absorbed. But that a syphon will not run. That electricity appears like the aurora borealis.

AIR-*Vessel*, is a name given to those metallic cylinders which are placed between the two forcing-pumps in the improved fire-engines. The water is injected by the action of the pistons through two pipes, with valves, into this vessel; the air previously contained in it will be compressed by the water, in proportion to the quantity admitted, and by its spring force the water into a pipe, which will discharge a constant and equal stream; whereas, in the common squirting engine, the stream is discontinued between the several strokes.

ALGEBRA, a general method of solving problems and questions, whether arithmetical or mathematical, by means of symbols which have no fixed or determinate values like the arithmetical figures, and which, by preserving their original form through all the steps of a calculation, however long, enable the calculator to deduce, from the solutions of particular problems general formulæ for the solution of all problems of the same kind.

The origin of algebra, like that of other sciences of ancient date and gradual progress, is not easily ascertainable. The earliest treatise on that part of analytics, which is properly called algebra, now extant, is that of Diophantus, a Greek of Alexandria, who flourished about the year 350, and who wrote thirteen books, though only six "Arithmeticorum" of them are preserved. He is the only Greek author on algebra whose works have been handed down to us, though some traces of it appear in the writings of some authors much more ancient, as Archimedes, Euclid, Apollonius, &c.; and we know that Hypatia wrote a commentary on the work of Diophantus. By what means the Arabs became possessed of this art is not known; but both the name and the science were transmitted to Europe, and particularly to Spain, by the Arabians or Saracens, about the year 1100, or probably a little earlier.

Italy, however, took the lead in the cultivation of the science, after its introduction into Europe. Leonardo Bonacci, of Pisa, has some allusions to it in his arithmetic written after his return from Africa and the Levant, in 1202. From his manuscript, Lucus Paccioli, or de Borgo, derived the rudiments of that knowledge which enabled him to compose his "Summa Arithmeticæ et Geometriæ Proportionumque et Proportionalitatum," which was published in Italian at Venice, in 1494, and again in 1523.

After this, algebra became more generally known and improved, especially by many in Italy; and soon after, the first rule was there found out by Scipio Ferreus, for resolving one case of a compound cubic equation. But this science, as well as other branches of mathematics, was, in an eminent manner, cultivated and improved by Hieronymus Cardan of Bonomia, a very learned author, who, in the year 1545, published a book containing the whole doctrine of cubic equations, which had been in part revealed to him by Nicholas Tartalea.

The chief improvements made by Cardan, as collected from his writings, are stated by Dr. Hutton as follows:

Tartalea communicated to him only the rules for resolving three cases of cubic equations, namely; $x^3 + bx = c$, $x^3 = bx + c$, and $x^3 + c = bx$; and he from thence raised a very large and complete work, laying down rules for all forms and varieties of cubic equations, having all their terms, or wanting any of them, and having all possible varieties of signs; demonstrating all these rules geometrically; and treating very fully of almost all sorts of transformations of equation, in a manner totally new. It appears, also, that he was well acquainted with all the roots of equations, that are real, both positive and negative; or, as he calls them, true and fictitious; and that he made use of them both occasionally. He also shewed that the even roots of positive quantities are either positive or negative; that the odd roots of negative quantities are real and negative; but that the even roots of them are impossible or nothing, as to common use. He was also acquainted with the number and nature of the roots of an equation, and that partly from the signs of the terms, and partly from the magnitude and relation of the co-efficients. He knew that the number of positive roots is equal to the number of changes of the signs of the terms. That the co-efficient of the second term of the equation, is the difference between the positive and negative roots: That when the second term is wanted, the sum of the negative roots is equal to the sum of the positive roots. How to compose equations that shall have given roots. That changing the signs of the even terms, changes the signs of all the roots. That the number of roots failed in pairs; or what we now call impossible roots, were always in pairs. To change the equation from one form to another, by taking away any term out of it. To increase or diminish the roots of a given equation. It appears, also, that he had a rule for extracting the cube root of such binomials as admit of extraction; and that he often used the literal notation *a*, *b*, *c*, *d*, &c. That he gave a rule for biquadratic equations, suiting all their cases; and that in the investigation of that rule, he made use of an assumed indeterminate quantity; and afterwards found its value by the arbitrary assumption of a relation between the terms. Also, that he applied algebra to the resolution of geometrical problems; and was well acquainted with the difficulty of what is called the *Irreducible Case*, in attempting the solution of which he spent a good deal of time.

About the time that Cardan and Tartalea flourished in Italy, the science of algebra was cultivated in Germany by Stifelius and Scheubelius. Stifelius introduced the characters +, —, √, for plus, minus, and root, or radix, as he called it. Also the initials ♃, 𝔷, ♑, for the powers 1, 2, 3, &c. He treated all the higher orders of quadratics by the same general rule; and introduced the numeral exponents of the powers—3,—2,—1, 0, 1, 2, 3, &c. both positive and negative, as far as integral numbers, but not fractional ones; calling them by the name *exponens*, exponent; and taught the general uses of the exponents in the several operations of powers, as we now use them in the logarithms. He likewise used the general literal notation A, B, C, D, &c. for so many different, unknown, or general quantities.

Scheubelius treats pretty largely upon surds, and gives a general rule for extracting the root of any binomial or residual, $a \pm b$, where one or both parts are surds, and a the greater quantity; namely, that the square root of it is

$$\sqrt{\frac{a+\sqrt{a^2-b^2}}{2}} \pm \sqrt{\frac{a-\sqrt{a^2-b^2}}{2}}$$

which he illustrates by various examples.

A few years after the appearance of these treatises in Italy and Germany, Robert Recorde, a celebrated mathematician and physician, born in Wales, published "The Whetstone of Witte, which is the seconde part of Arithmetike: containing the Extraction of Rootes; the Cossike Practice, with the Rule of Equation; and the Works of Surde Nombers." He introduced the extraction of the roots of compound algebraic quantities; the use of the terms binomial and residual; the use of the sign of equality, or =.

Peletarius, in 1558, shewed that the root of an equation is one of the divisors of the last, or absolute term. He taught how to reduce trinomials to simple terms, by multiplying them by compound factors. He taught curious precepts and properties concerning square and cube numbers, and the method of constructing a series of each by addition only; namely, by adding successively their several orders of differences.

The science received further improvements up to the year 1600, from Ramus, Bombelli, Steven, and others; and a few years after, Schooten published the whole mathematical works of Vieta. Vieta introduced the general use of the letters of the alphabet to denote indefinite given quantities. He uses the vowels A, E, I, O, Y, for the unknown quantities, and the consonants, B, C, D, &c. for known ones. He invented and introduced many expressions or terms, several of which are in use to this day: such as co-efficient, affirmative and negative, pure and adfected, or affected, unciæ, homogeneum, &c. and the line, or vinculum, over compound quantities, thus, $\overline{A+B}$.

Albert Girard was the first person who understood the general doctrine of the formation of the co-efficients of the powers, from the sums of their roots, and their products, &c. He was the first who understood the use of negative roots, in the solution of geometrical problems; and was the first who spoke of the imaginary roots, and understood that every equation might have as many roots, real and imaginary, and no more, as there are units in the index of the highest power.

Thomas Harriot flourished about the year 1610. He introduced the uniform use of the small letters *a*, *b*, *c*, *d*, &c.; viz. the vowels *a*, *e*, and *o*, for unknown quantities, and the consonants, *b*, *c*, *d*, *f*, &c. for the known ones; which he joins together like the letters of a word, to represent the multiplication or product of any number of these literal quantities, and prefixing the numeral co-efficient as we do at present, except being separated by a point, thus *5.bbc*. For a root he sets the index of the root after the mark √; as √3 for the cube root. He also introduced the characters > and <, for greater and less; and in the reduction of equations he arranged the operations in separate steps, or lines, setting the explanations in the margin on the left-hand for line. He may be considered as the introducer of the modern state of algebra. He also showed the universal generation of all the compound or adfected equations, by the continual multiplication of so many binomial roots.

Oughtred's Clavis appeared in 1631, the same year in which Harriot's Algebra was published. He was the first, as far as we can learn, who set down the decimals without their denominator; separating them thus, 21(56. In algebraic multiplication, he either joins the letters which represent the factors together like a word, or connects them by the mark ×, which is the first introduction of this character. He also seems to be the first who used points to denote proportions, thus 7 . 9 :: 28 . 36; and for continued proportion he has this mark ∺. In his work we likewise meet with the first instance of applying algebra to investigate new geometrical properties.

Descartes' Geometry, first published in 1637, was rather an application of algebra to geometry, than either algebra or geometry separately considered. Still he made improvements in both. His inventions and discoveries in algebra may be comprehended in the application of algebra to the geometry of curved lines; the construction of equations of the higher orders; a rule for resolving biquadratic equations by means of a cubic and two quadratics; and his method of *maxima et minima*.

In 1655, Wallis published his "Arithmetica Infinitorum," which in a great measure led the way to infinite series, the universal application of the binomial theorem, and the method of fluxions. He first gave an expression for the quadrature of the circle by an infinite series; and it was he who first substituted the fractional exponents in the place of radical signs, by which the operations are in many cases much facilitated and abridged.

In 1707, Whiston published the first edition of Sir Isaac Newton's "Arithmetico Universalis." This was the text-book used by Newton, while he was professor of Mathematics in the university of Cambridge: and although it was never intended for publication, it contains many and great improvements in analytics; particularly in the nature and transmutation of equations; the limits of the roots of equations; the number of impossible roots; the invention of divisors, both surd and rational; the resolution of problems, arithmetical and geometrical; the linear construction of equations; approximating to the roots of all equations. To the later editions of this work is commonly subjoined, Dr. Halley's method of finding the roots of equations. Commentaries upon this work, by several persons, as P. Gravesande, Castilion, Wilden, and others, and many of Newton's algebraical discoveries, were farther developed and explained by Halley, Maclaurin, Nicoles, Stirling, Euler, Clairaut, &c.

Algebra is sometimes divided into numeral, and specious or literal.

Numeral ALGEBRA, is that in which all the given quantities are expressed by numbers.

Specious or Literal ALGEBRA, is that in which all quantities, whether known or unknown, are expressed by general characters, as letters, &c. in consequence of which general designation, all the conclusions become universal theorems.

In algebraical inquiries, some quantities are assumed as known or given; and others are unknown and to be found out: the former are commonly represented by the leading letters of the alphabet, a, b, c, d, &c.; the latter by the final letters, w, x, y, z. Though it often tends to relieve the memory, if the initial letter of the subject under consideration be made use of, whether that be known or unknown: thus r may denote a radius, b a base, p a perpendicular, s a side, d density, m mass, &c.

The characters used to denote operations and relations are principally as follow:

$=$ denotes the equality of the quantities between which it is placed: thus, $a=b$, denotes that a is equal to b.

$>$ denotes the inequality of the quantities between which it is placed, the point being turned toward the less: thus, $a>b$, means that a is greater than b.

$+$ *(plus)* denotes the addition of

the quantities between which it is placed: thus, $a+b$ means that a and b are to be added.

$\backsim$ means the difference of the quantities between which it is placed, without indicating what the difference is, or which is the greater: thus, $a \backsim b$ means a subtracted from b, or b subtracted from a, according as the one or the other is the greater.

$-$ *(minus)* means that the quantity *after* it is to be subtracted: thus, $a+b-c$ means that c is to be subtracted from the sum of a and b. A quantity having this sign is called a negative quantity, in opposition to one which has no sign, or the sign $+$, which is called an affirmative quantity.

$\times$ or . denotes the product of the quantities between which it is placed: thus, $a \times b$, or $a \,.\, b$, means the product of a and b. When each quantity is expressed by a single letter, the product is also denoted by writing the letters without any sign.

$\div$ denotes that the quantity before it is to be divided by the quantity after; and the same thing is indicated by writing the divisor before the dividend: thus, $a \div b$, or, $\frac{a}{b}$ means a divided by b.

When two or more quantities having the signs $+$, or $-$, are to be considered as one quantity, or *vinculum*, ——, is drawn over them, or they are inclosed in parenthesis: thus, $\overline{a+b}$, or $(a+b)$, means that the two quantities are to be taken as one whole.

: means the proportion or ratio of one quantity to another; and when two quantities have the same ratio as other two, the sign of equality is introduced between the pairs: thus, $a : b = c : d$ means that a has the same ratio to b, that c has to d.

A power of any quantity is denoted by writing the exponent over it; and a root by the radical sign $\sqrt{}$ with the exponent when different from 2, or else with a fractional exponent: thus a^2 means the square of a; and $\sqrt{a}$, or $a^{\frac{1}{2}}$, means the square root of a.

$5a$ denotes that the quantity a is to be taken 5 times, and $7,(b+c)$ is 7 times $b+c$. And these numbers, 5 or 7, shewing how often the quantities are to be taken, or multiplied, are called co-efficients.

Like quantities, are those which consist of the same letters and powers. As a and $3a$; or $2ab$ and $4ab$; or $3a^2bc$ and $-5a^2bc$.

Unlike quantities are those which consist of different letters or different powers. As a and b; or $2a$ and a^2; or $3ab^2$ and $3abc$.

Simple quantities, or monomials, are those which consist of one term only. As $3a$, or $5ab$, or $6abc^2$.

Compound quantities, are those which consist of two or more terms. As $a+b$, or $a+2b-3c$.

And when the compound quantity consists of two terms, it is called a binomial; when of three terms, it is a trinomial; when of four terms, a quadrinomial; more than four terms a multinomial or polynomial.

Positive or affirmative quantities, are those which are to be added, or have the sign $+$. As a, or $+a$, or ab; for when a quantity is found without a sign, it is understood to be positive, or to have the sign $+$ prefixed.

Negative quantities, are those which are to be subtracted. As $-a$, or $-2ab$, or $-3ab^2$.

Like signs, are either all positive $(+)$, or all negative $(-)$.

Unlike signs, are when some are positive $(+)$, and others negative $(-)$.

A residual quantity, is a binomial having one of the terms negative. As $a-2b$.

The power of a quantity (a), is its square (a^2), or cube (a^3), or biquadrate (a^4), &c.; called also the 2d power, or 3d power, or 4th power, &c.

The index or exponent, is the number which denotes the power or root of a quantity. So 2 is the exponent of the square or 2d power a^2; and 3 is the index of the cube or 3d power; and $\frac{1}{2}$ is the index of the square root $a^{\frac{1}{2}}$ or $\sqrt{a}$; and $\frac{1}{3}$ is the index of the cube root $a^{\frac{1}{3}}$ or $\sqrt[3]{a}$.

A rational quantity, is that which has no radical sign or index annexed to it. As a, or $3ab$.

An irrational quantity, or surd, is that which has not an exact root, or is expressed by means of the radical sign $\sqrt{}$. As $\sqrt{2}$, or $\sqrt{a}$, or $\sqrt[3]{a^2}$, or $ab^{\frac{1}{2}}$.

The reciprocal of any quantity, is that quantity inverted, or unity divided by it. So, the reciprocal of a, or $\frac{a}{1}$ is $\frac{1}{a}$, and the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

For the several operations in Algebra, see the corresponding articles.

ALGEBRAICAL, something relating to Algebra.

ALGEBRAICAL *Curve*, is a curve the nature of which may be expressed by an equation.

ALGORITHM, or ALGORISM, an Arabic term expressive of numerical computation; or the common rules of computing in any branch of analysis; as the algorithm of numbers, of surds, of imaginary quantities, &c.

ALIQUOT *Part*, is such a part of a number as is contained in it a certain number of times exactly.

ALLIGATION, one of the rules in *Arithmetic*, relating generally to the compounding or mixing together of various ingredients, of which the prices or qualities are given, so that the compound may be of a certain value or quality. This rule is usually divided into two distinct cases, viz. *Alligation Medial*, and *Alligation Alternate*.

ALLIGATION *Medial*, is the method of finding the rate or quality of the composition, from having the rates or qualities of the simple ingredients given.

Rule. Multiply each quantity by its rate, and divide the sum of all these products by the sum of the quantities, and the quotient will be the price or quality required.

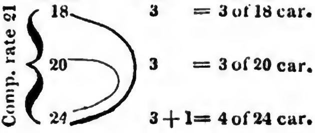
Exam. Require the value per gallon of brandy, made up of the following composition; viz. 12 gallons at 24 shillings per gallon, 10 at 30 shillings per gallon, and 20 at 36 shillings per gallon. By the rule,

```
Gal. Price. Products.
 12 × 24 = 288
 10 × 30 = 300
 20 × 36 = 720
 —         ———
 42       )1308 (31⅐ shillings the
           126           answer.
          ————
            48
            42
            —
             6
            —
```

ALLIGATION *Alternate*, is the method of finding the quantities of ingredients necessary to form a compound of a given rate.

Rule. Place the given rates of the ingredients in a line under each other; noting which rates are less and which greater than the proposed compound. Then connect or link with a crooked line, each rate which is less than the proposed compound with one or any number of those that are greater than the same; and every one which is greater with one or any number of those that are less. Take the difference between the given compound rate, and that of each simple rate; and set this difference opposite every rate with which that one is linked. Then if only one difference stand opposite any rate, it will be the quantity belonging to that rate: but when there are several differences to any one, their sum will be quantity.

Exam. Let it be required to mix together gold of various degrees of fineness; viz. of 18, 20, and 24 carats fine, so that the mixture may be of 21 carats fine.



Comp. rate 21	Rates.	diff.	results.
	18	3	= 3 of 18 car.
	20	3	= 3 of 20 car.
	24	3 + 1	= 4 of 24 car.

Note. If the compound is to consist of a certain quantity, then take the sum of the above results, and say, as that quantity or sum is to the proposed quantity, so is each of the above results to the quantity of each required.

Questions of this kind are better solved by Algebra, in which they form a species of indeterminate problems.

ALMANAC, a calendar or table, in which are noted down all the most remarkable phenomena of the heavenly bodies; such as eclipses, occultations, the conjunctions and oppositions of the planets, the risings and settings of the sun and moon, &c. &c.

Nautical **ALMANAC** and *Astronomical Ephemeris*, is a kind of national Almanac, begun in 1767. This Almanac is generally computed a few years forward, for the convenience of ships going out on long voyages, for which it is highly useful. Besides most things essential to general use, and which are found in other Almanacs, it contains many important matters, particularly the distance of the moon from the sun and fixed stars, computed to the meridian of Greenwich, for every three hours of time, which is of great use in computing the longitude at sea.

ALMUCANTARS, or ALMACANTARS, circles parallel to the horizon, passing through every degree of the meridian; and, having the zenith for their common centre. They are the same as parallels of altitude.

ALTITUDE, in *Geometry*, the third dimension of a body, considered with regard to its elevation above the plane of its base.

ALTITUDE of a *Figure*, is the distance of its vertex from its base, or the length of a perpendicular let fall from the vertex to the base.

Altitudes are divided into *accessible* and *inaccessible*.

Accessible Altitude, of an object, is that to whose base we can have access, so as to measure the distance between it and the station from which the measure is to be taken.

Inaccessible Altitude, is when the base of the object cannot be approached.

There are several methods of measuring the height or altitude of bodies, viz. by *Geometry, Trigonometry*, by *Optical Reflection*, by means of the *Barometer*, &c.

The instruments commonly used in measuring altitudes, are, the *Geometrical Square, Quadrant*, and *Theodolite;* a description of each of which will be found under the respective articles.

To measure an accessible or inaccessible Altitude geometrically.—Under this head are included all those cases in which the calculation depends upon pure geometrical principles, and particularly on the similarity of triangles, of which we propose to give an illustration in the following examples: let A B (Plate I. fig. 1.) represent an object of which the altitude is required. Being provided with two rods or staves of different lengths, plant the longest of them, as F C, at a certain measured distance from the base of the object. Then, at a farther distance, plant the second or shorter staff E D, in such a manner that the tops of the two, E and F, may be in a line with the top of the tower B. Then having measured the distance I D, as also the length E D, we shall have by similar triangles, as

$$\mathrm{ID} : \mathrm{ED} = \mathrm{IA} : \mathrm{AB};$$

that is, by multiplying the second and third terms, and dividing by the first, we shall have the whole altitude of the tower BA: that is,

$$\mathrm{BA} = \frac{\mathrm{IA} \times \mathrm{ED}}{\mathrm{ID}} = 8 \text{ feet.}$$

For example, suppose

$$\mathrm{IA} = 100 \text{ feet}, \ \mathrm{ID} = 8 \text{ feet, and}$$
$$\mathrm{ED} = 4 \text{ feet},$$

being the height of the staff;

$$\text{then } \mathrm{BA} = \frac{100 \times 4}{8} = 50 \text{ feet},$$

the altitude of the tower.

If the object AB were inaccessible, two such operations as the above must be made, in order to ascertain the altitude required: thus,

Let $\mathrm{ID} = a$, and $\mathrm{ED} = d$; also the unknown distance $\mathrm{IA} = x$, and the required altitude of the object $= y$; then, in a second operation, in which both the staves must be replanted, make the new distance $\mathrm{ID} = a'$, and the second unknown

ALTITUDES.

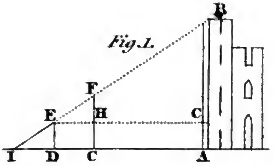


Fig. 1.

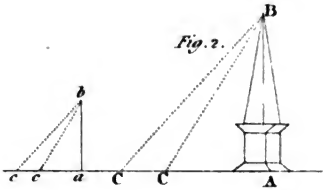


Fig. 2.

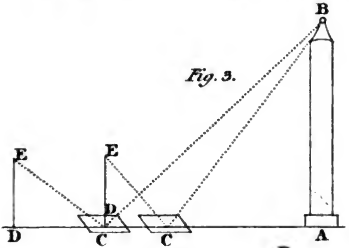


Fig. 3.

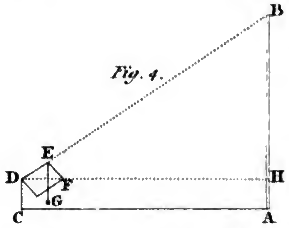


Fig. 4.

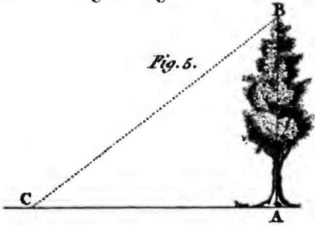


Fig. 5.

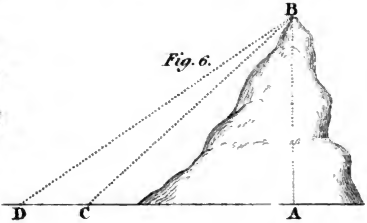
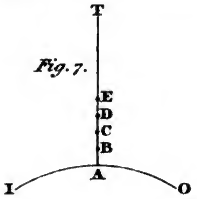


Fig. 6.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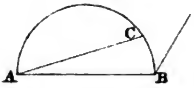
Fig. 8.

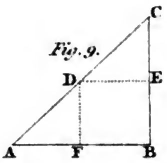
Fig. 9.

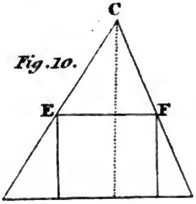
Fig. 10.

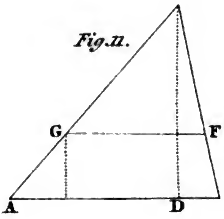
Fig. 11.

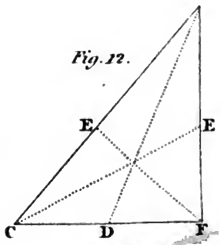
Fig. 12.

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ASTOR, LENOX AND
TILDEN FOUNDATIONS.

distance $= x \pm c$, c being the distance between the two stations of the shorter staff ED; also the lengths of the staves remaining still the same, ED will be represented by d, as above.

Now, from the preceding proportion we shall have, by substituting for ID, ED, IA, and AB, the above letters:

1st operation $a : d = x : y$
2d operation $a' : d = x \pm c : y$
Whence, by subtraction, $a' \backsim a : d = c : y$

that is, $y = \text{AB} = \dfrac{dc}{a' \backsim a}$ the height of the tower required.

For the method of determining the distance IA, see the conclusion of problem 4.

To measure accessible or inaccessible Altitudes by means of Shadows. (Plate I. fig. 2.)

At any time when the sun shines, plant a rod ab perpendicularly at a, and measure the length of its shadow, and, immediately after, measure the shadow of the proposed object AB. Then by similar triangles,

$$ca : ba = \text{CA} : \frac{ba \times \text{CA}}{ca} = \text{AB},$$

the altitude required.

If the object be inaccessible, but still such that the difference of the lengths of its shadows, taken at two different times, can be ascertained, the altitude may be found nearly the same as in the last example. Thus, make $ab = a$, $ac = b$, and the unknown length of the shadow $= x$; let the second shadow of the rod $= b'$, and the second shadow of the tower $= x \pm d$, the height of the tower $= y$; then, by the preceding proportion,

1st operation $b : a = x : y$
2d operation $b : a = x \pm d : y$
Whence, by subtraction, $b \backsim b' : a = d : y$

that is, $y = \dfrac{ad}{b \backsim b'}$, the altitude required.

To measure accessible or inaccessible Objects by means of Optical Reflection. (Plate I. fig. 3.)

Place a mirror, or other reflecting surface, horizontally in the plane of the figure's base, as at C, and when the object is accessible, measure the distance CA. Now retire back in the direction AC to D, till the eye observes the top of the object exactly in the centre of the mirror, which, for the greater degree of accuracy, may be marked by a line across it. Then having measured the distance DC, and ascertained the height of the eye of the observer, it will be, from the known laws of reflection, as

$$\text{DC} : \text{DE} = \text{CA} : \frac{\text{DE} \times \text{CA}}{\text{DC}} = \text{AB},$$

the altitude of the object required.

When the object is inaccessible, that is, when the distance CA cannot be measured, two such operations as that above must be employed. Thus,

Let $\text{ED} = a$, $\text{DC} = d$, and the unknown distance $\text{CA} = x$; also let d' represent the analogous distance D′C′ in the second operation, and $x \pm c$ the second distance C′A, that is, making $c =$ the distances between the two situations of the mirror; and let the required height of the object $= y$. Then substituting the above letters in the preceding proportion, we have,

1st operation $d : a = x : y$
2d operation $d' : a = x \pm c : y$
Whence, by subtraction, $d' \backsim d : a = c : y$

and consequently, $y = \dfrac{ac}{d' \backsim d}$, the altitude sought.

To measure an accessible or inaccessible Object, by the Geometrical Square. (Plate I. fig. 4.)

Having fixed the instrument at any place C, turn the square about the centre of motion D, till the top of the object B is perceived in the direction of the sights placed on the side of the square DE, and note the number of divisions; cut off the other side by the plumb-line EG; then, having measured the distance CA, we have by similar triangles, as

$$\text{EF} : \text{FG} = \text{CA or DH} : \frac{\text{CA} \times \text{FG}}{\text{EF}} = \text{BH};$$

to which adding the height of the observer's eye, DC, we shall have

$\frac{\text{CA} \times \text{FG}}{\text{EF}} + \text{DC} = \text{AB}$, the altitude sought.

In the case of an inaccessible object, two observations must be made similar to that above, in which the only variable lines will be GF and CA. Let, therefore, the side of the square $= s$, and the variable part GF $= a$, in the first observation, and a' in the second; also put the unknown distance $= x$ in the first case, and $x \pm c$, in the second, so that c will be the distance of the observer's two stations, and make the required height of the object $= y$. Then on the same principle as those above,

1st operation $\quad s : a = x : y$
2d operation $\quad s : a' = x \pm c : y$

Whence, by equality,

$$ax = a'x = a'c, \text{ or } x = \frac{a'c}{a \backsim d};$$

but by the first $y = \frac{ax}{s}$,

therefore $y = \frac{aa'c}{s(a \backsim a'}$

the altitude required.

It is obvious, that the methods made use of in all these problems, for inaccessible objects, will give the distance of the objects as well as their altitudes. Thus,

Prob.

1. The distance

$$\text{EG} = x = \frac{ay}{a} = \frac{adc}{a(a \backsim a)}$$

2. The distance

$$\text{CA} = x = \frac{by}{a} = \frac{bad}{a(b \backsim b)}$$

3. The distance

$$\text{CA} = x = \frac{dy}{a} = \frac{acd}{a(d' \backsim d)}$$

4. The distance

$$\text{CA} = x = \frac{a'c}{a \backsim a'}.$$

It is singular that the above methods for inaccessible altitudes and distance, being so very simple, should never have been mentioned by any author; at least they are not mentioned by any one that we are acquainted with.

To measure an accessible Object Trigonometrically. (Plate I. fig. 5.) Let AB be the object of which the altitude is required.

At any convenient station C with a quadrant, theodolite, or other graduated instrument, measure the angle of elevation ACB; then, having also measured the distance CA, we have, from the elementary principles of trigonometry, as

Rad. : AC = tan. ∠ACB : AB, the altitude required.

Or by logarithms, log. AB = log. AC + log. tan. ACB—log. rad.

Suppose, for example, the distance AC = 340 feet, and the angle of elevation ACB = 34° 30′; then, by the above formula,

Log. AC or log. 340 = 2·5314789
Log. tan. 34° 30′ . . = 9·8371343

12·3686132
Log. radius . . . . . . 10·0000000

Log. BA = 233·67 . . 2.3686132

To measure an inaccessible Object by two Stations. (Plate I. fig. 6.)

Let AB be the object of which the altitude is required.

Take the angles of elevation at the two stations D and C, and measure the distance between them. The angle DBC = the difference of the two measured angles BCA and BDA, (Euc. prop. 16, book 1.), which angle therefore becomes known. And by trigonometry, as

$$\text{sin. DBC} : \text{DC} = \text{sin. CDB} : \frac{\text{DC} \times \text{sin. CDB}}{\text{sin. DBC}} = \text{CB}.$$

Again, as radius : CB = sin. ACB :

$$\frac{\text{CB} \times \text{sin. ACB}}{\text{Rad.}} = \text{BA},$$

the altitude required.

Or substituting, in the second expression, the value of CB, we have

$$\text{BA} = \frac{\text{DC. sin. CDB. sin. ACB.}}{\text{Rad. sin. DBC.}}$$

Or by Logarithms.—From the sum of the logarithms of the terms in the numerator subtract the sum of those in the denominator, and the remainder will be the logarithm of the required altitude BA.

To measure Altitudes by the Barometer.—The application of this instrument to the measuring of altitudes, suggested itself to mathematicians, soon after the experi ment of the Puy de Domme, which was executed in order to confirm the Torricellian experiment.

Let EAQ, (Plate I. fig. 7.) represent the surface of the earth, and A, B, C, D, &c. a column of the atmosphere. Conceive this to be divided into a number of equal and indefinitely small parts, AB, BC, CD, &c. in each of which we may suppose the density to be uniform, because they are indefinitely small. Now since the density of the air is always directly as the compressing force; and the compressing force being in this case the pressure of the superior strata, we shall have the density of the air in any of those parts, as the weight of the column of atmosphere above; that is, if P represent generally the pressure, and D the density at any place, and P' and D' the pressure and density at any other place whatever, we shall have

$$P : D = P' : D';$$

and, consequently, the pressure is to the density in a constant ratio, which may be represented by $n : 1$; that is, $P : D = P' : D' = n : 1$;

hence, $D = \frac{1}{n} P$, $D' = \frac{1}{n} P'$, &c.

or, the density at any place is, ${}_{n}$th of the pressure of the superior strata at that place.

If we make P represent the pressure at the surface A, and call each of the parts AB, BC, CD, &c. equal to 1, then will also $\frac{1}{n}$ P represent the weight or pressure of the part AB, and, consequently,

$P - \frac{1}{n} P = \frac{n-1}{n}$ P will be the pressure at B;

and $\frac{n-1}{n^2}$ P the density or weight of BC.

In the same way $\frac{(n-1)^2}{n^2}$ P will be the pressure at C.

$\frac{(n-1)^3}{n^3}$ P the pressure at D, &c. &c.

So that the pressures, and consequently the densities, will decrease in geometrical progression, as the altitudes increase in arithmetical progression; hence, calling the density at the surface equal to d^n, and the several altitudes 1, 2, 3, &c. we shall have

$$\begin{array}{ll} \text{Alt.} & 0, \quad 1, \quad 2, \quad 3, \quad 4. \\ \left.\begin{array}{l}\text{cors.}\\ \text{dens.}\end{array}\right\} & d^n, d^{n-1}, d^{n-2}, d^{n-3}, d^{n-4}, \&c. \end{array}$$

Or, dividing the latter series by d^n, we have

$$\begin{array}{ll} \text{Alt.} & 0, \quad 1, \quad 2, \quad 3, \quad 4, \ \&c. \\ \left.\begin{array}{l}\text{recip.}\\ \text{dens.}\end{array}\right\} & 1, d^{-1}, d^{-2}, d^{-3}, d^{-4}, \&c. \end{array}$$

which is strictly analogous to the property of logarithms. In fact, the several altitudes form a peculiar system of logarithms, of which the reciprocals of the corresponding densities are the natural numbers, and which may, therefore, properly enough, be denominated atmospheric logarithms, on the same ground as the Napierian are termed hyperbolic logarithms. We shall consider them in this way, and shall denote them by $\lambda o\gamma$. to distinguish them from the common logarithms, which are generally marked log.; and from the hyperbolic, which are denoted by h. log.

Hence, if a and A represent any two altitudes, and d and D the corresponding densities, we shall have

$$A = -\lambda o\gamma.\, d$$
$$a = -\lambda o\gamma.\, D$$

whence $A - a = -\lambda o\gamma.\, d - \lambda o\gamma.\, D = \lambda o\gamma.\, \frac{d}{D}$.

Now it is well-known, that all the various systems of logarithms are convertible from one system to the other by a constant multiplier; and hence, in this case, we have only to determine the constant multiplier, that shall convert the common logarithm of a number into its atmospheric logarithm.

Let x denote this multiplier,

and make $\lambda o\gamma.\, \frac{d}{D} = x \log.\, \frac{d}{D}$

so that

$$A - a = x \log.\, \frac{d}{D}.$$

Then if d to represent the density of the atmosphere at the surface of the earth, we shall have

$a = o$, and $A = x \log. \dfrac{d}{D}$.

Again, to find x, let us take the density at the surface, and at one foot above it, which, when the thermometer stands at 31°, and barometer at $29\frac{1}{2}$ inches, are found to be

$$a = 0; \quad d = 26057$$
$$A = 1; \quad D = 26056$$

whence,

$$A - a = 1 = x \log. \frac{26057}{26056}$$

or, making

$26057 = n$, we have $1 = x \log. \dfrac{n}{n-1}$.

But log.

$$\frac{n}{n-1} = M \left\{ \frac{1}{n} + \frac{1}{2n^2} + \frac{}{3\,n^3} \&c. \right\}$$

And as in the present case, n is a great number, all the terms past the first may be neglected; and since also $M = \cdot 43429448$ in the common system, we have

$$1 = x \times \frac{\cdot 43429448}{26057};$$

whence

$$x = \frac{26057}{\cdot 43429448} = 60000, \textit{ very nearly:}$$

therefore,

$$A = 60000 \times \log. \frac{d}{D} \textit{ feet.}$$

Or, since the height of the mercury in the barometer is always proportional to the density, the fraction $\dfrac{d}{D} = \dfrac{m}{M}$, m and M being the height of the mercury at the earth's surface, and at the altitude A; also, since six feet is equal one fathom, the same formula may be otherwise expressed thus:

$$A = 10000 \log. \frac{m}{M} \textit{ fathoms;}$$

which is the general formula for measuring altitudes by the barometer.

Hence we have, according to the preceding principles, the following rule for measuring altitudes by the barometer, *viz*

Observe the height of the mercury at the bottom of the object to be measured, and again at its top; as also the degree of the thermometer in both these situations; and half the sum of these two last may be accounted the mean temperature. Then multiply the difference of the logarithms of the two heights of the barometer by 10000, and correct the result by adding or subtracting so many times its 435th part, (the proportion in which air expends for every degree of heat), as the degrees of the mean temperature are more or less than 31°; and the last number will be the altitude in fathoms.

Exam.—If the heights of the barometer at the bottom and top of a hill are 29·37 and 26·59 inches respectively, and the mean temperature 26°, what is the height?

$$\begin{array}{rr} \text{Log. } 29\cdot37 = & 1\cdot467904 \\ \text{Log. } 26\cdot59 = & 1\cdot424718 \\ \hline \text{Diff. of Logs.} = & 0\cdot043186 \\ \text{Mult. by} & 10000 \\ \hline & 431\cdot86 \\ \hline \end{array}$$

Now $31° - 26° = 5°$ temp. below 31°;
therefore $\frac{5}{435}$ of $431\cdot86 = 4\cdot96$;
conseq. $431\cdot86 - 4\cdot96 = 426\cdot90$ fathoms, the altitude of the hill.

Later observations have shown, that this requires certain modifications, on account of the difference in low and elevated situations, the expansion of the column of mercury, and other circumstances.

These, according to General Roy and Sir G. Shuckburgh, are as follow:

ROY.

$$\left\{10000\, l \pm \cdot 468\, d \right\} \times$$
$$\left\{1 + (f - 32°)\, \cdot 90245 \right\}$$

SHUCKBURGH.

$$\left\{10000\, l \pm .440\, d \right\} \times$$
$$\left\{1 + (f - 32°)\, \cdot 00243 \right\}$$

Where l = diff. of logs. of the heights of barom.
d = diff. of degrees Fahrenheit's therm.
f = mean of the two temp. shown by the detached thermometer exposed for a few minutes to the open air, in the shade of the two stations.

The sign — takes place when the attached thermometer is highest at the lower station, and the sign + when it is lowest at that station.

Exam.—Find the height of a mountain, from the following observations taken at the foot and summit:

	Bar.	At. ther.	De. th.
Low. stat.	29·862 ...	68° ...	71°
Hig. stat.	26·137 ...	63° ...	55°

Here we have, $d = 5°$, diff. detach. thermometer.

And .... $f = 63°$ mean of detach. thermometer.

Log. 29·862 = 1·475119
Log. 26·137 = 1·417256

Diff. of Logs. = 0·057863 = l.

Then, by the first formula,
$f - 32° = 31°$, and $1 + (31 \times ·00245) = 1.07595$
$10000\,l = 100000 \times ·057863 = 578·63$
$·468\,d = ·468 \times 5$ 2.34

576·29
Multiply by 1·07595

620 fathoms

the altitude sought; the decimals being rejected as unimportant.

Altitude, in *Astronomy*, is the arch measuring the height of a celestial object above the horizon. It is either true or apparent. The apparent altitude is that which appears by observations made on the surface of the earth. And the true is that which results by correcting the apparent, on account of refraction and parallax.

The quantity of the refraction is different at different altitudes; and the quantity of the parallax is different according to the distance of the different luminaries: in the fixed stars this is too small to be observed; in the sun it is only about 8¾ seconds; but in the moon it is at a mean about 58 minutes.

The altitude of a celestial object may be very accurately determined, by measuring the arc of an oblique great circle intercepted between the star and the horizon, and the inclination of the same great circle to the horizon.

***Meridian* Altitude of *any celestial object*,** is an arch of the meridian intercepted between the horizon and the centre of the object upon the meridian. The altitude of a celestial boy is greatest when it comes to the meridian of any place (the poles of the earth excepted, for there the altitude of a fixed body is subject to no variation;) and the altitude of any star which sets not, is least, and the depression of any star which does set, is greatest when in the opposite part of the meridian.

Altitude of the *Pole*, is an arch of the meridian intercepted between the horizon and the pole: it is equal to the latitude.

Altitude of the *Equinoxial*, is the elevation of that circle above the horizon, and is equal to the complement of the latitude.

Refraction of Altitude, is an arch of a verticle circle, whereby the altitude of an heavenly body is increased by refraction. And ***Parallax of Altitude***, is an arch of a verticle circle, whereby the altitude is decreased by parallax.

Altitude *Instrument*, or ***Equal Altitude Instrument***, one used to observe a celestial object when it has the same altitude on the east and west sides of the meridian.

Observations of this kind are made for the purpose of obtaining the true time of the sun's passing the meridian: various modes of calculation have been recommended at different times; but we know of none preferable to the following, invented by Dr. Rittenhouse, the American astronomer.

Suppose there are four sets of altitudes obtained on two successive days, (*viz.* one set in the morning, and one in the afternoon, of each day,) the instrument being kept at exactly the same height both days; then take the difference in time between the forenoon observations of the two days, and also between the afternoon observations.

Call half the diff. of the two differences, X;
And half the sum of the two differences, Y;
Let the half interval between the two observations of the same day, be Z.

Then, if the times of the altitudes observed on the second day be both nearer 12, or both farther from 12 per clock than on the first day,—X will be the daily variation of the clock, from apparent time, and Y will be the daily difference in time, of the sun's coming to the same altitude, arising from the change of declination. And the proposition will be,—$24^h : Y = Z : E$, the equation sought; which will be found the same (without any sensible difference) as the equation obtained from the tables.

But if one of the observations on the second day be nearer 12, and the other more remote from 12, than on the first day,...then Y will become the daily variation of the clock from apparent time, and X will be the daily difference in time of the sun's being at the same altitude. And the proportion will be....$24^h : X = Z : E$.

The equation E, thus obtained, is to be subtracted from the mean noon, if the sun's meridian altitude be daily increasing; but to be added, if it be daily decreasing.

AMBIGUOUS *Case*, in *Trigonometry*, is that which arises in the solution of a problem, in which an acute angle and its opposite side are two of the given parts, and one of the sides about the given angle is the third part.

With these data, the angle opposite the unknown side may be either obtuse or acute, because every sine answers to two angles which are supplements to each other.

AMBIGUOUS *Sign*, in *Algebra*, is that in which both *plus* and *minus* enter; being written thus, ±, and is read *plus* or *minus*.

AMPLITUDE, in *Astronomy*, is an arch of the horizon, intercepted between the true east and west points, and the centre of the sun or a star at its rising or setting; so the amplitude is of two kinds; eastern and western. These are also called northern or southern, as they fall in the northern or southern quarters of the horizon; and the compliment of the amplitude, or the distance of the point of rising or setting from the N. or S. point of the horizon, is called the *azimuth*.

To find the sun's amplitude, having the latitude and the sun's declination given.

Say, as the cosine of the latitude is to radius, so is the sine of the sun's or star's declination to the sine of the amplitude.

EXAM. Let it be required to find the amplitude of the sun in latitude 51° 32′; the declination being 23° 28′.

As cosine 51° 32′ . .	9·7938317
is to radius . . .	10·0000000
so is sin. dec. 23° 28′	9·6001181
To amplitude 39° 48′.	9·8062864

And this is of the same name with the declination: *viz.* north, when the declination is north; and south, when that is south.

AMPLITUDE, in *Projectiles*, the right line upon the ground, subtending the curvelinear path in which a projectile moves.

AMPLITUDE, *Magnetical*, is an arch of the horizon, contained between the sun or a star at its rising or setting, and the magnetical east or west point of the horizon indicated by the magnetical compass, or the amplitude or azimuth compass; or it is the difference of the rising or setting of the sun from the east or west points of the compass.

In order to ascertain this amplitude, place the compass on a steady place from which the horizon may be clearly seen, and looking through the sight vanes of the compass, turn the instrument round till the centre of the sun or star may be seen through the narrow slit, which is one of the sight vanes, exactly in the thread which bisects the aperture in the other sight vane; and when the centre of the celestial object, whether rising or setting, is just in the horizon, push the stop in the side of the box, so as to stop the card, and then read the degree of the card which stands just against the siducial line in the box; and this gives the amplitude required. Then subtracting from this amplitude the known or true amplitude, and the difference will be the variation of the magnetic needle.

ANALEMMA, a projection of the sphere, on the plane of the meridian, orthographically made by perpendiculars from every point of that plane; the eye being supposed to be at an infinite distance, and in the E. or W. point of the horizon.

ANALYSIS is, generally, the method of resolving mathematical problems, by reducing them to equations; and may be divided into *ancient* and *modern*.

The *ancient* analysis is the method of proceeding from the thing sought taken for granted, through its consequences, to something that is really granted or known; in which sense it is opposed to *synthesis*, or composition, which commences with the last step of the analysis, and traces the several steps backwards, making that in this case antecedent, which in the other was consequent, till we arrive at the thing sought, which was assumed in the first step of the analysis.

Montucla, in his Histoire des Math. has given an example illustrating the method pursued by the ancients in their analysis, as follows:

Prob. From the extremities of the base A and B, of a given segment of a circle, it is required to draw two lines AC, BC, meeting at the point C, in the circumference, so that they shall have a given ratio to each other, suppose that of F to G.

ANALYSIS.

Suppose the thing done, viz. that (Plate I. fig. 8.) AC : BC = F : G, and let there be drawn BH, making the angle ABH equal to the angle ACB, and meeting AC produced in H. Then the angle A being also common, the two triangles ABC, ABH, are equiangular; and, therefore,

$$AC : BC = AB : BH$$

in the given ratio; also AB being given, BH is given in position and magnitude.

SYNTHESIS.

Construction. Draw BH, making the angle ABH, equal to that which may be contained in the given segment, and take AB to BH, in the given ratio of F to G. Draw ACH and BC.

Demonstration. The triangles ABC, ABH, are equi-angular, therefore

$$AC : CB = AB : BH$$

which is the given ratio by construction.

Modern Analysis comprehends algebra, arithmetic of infinites, infinite series, increments, fluxions, &c.; for an account and illustration of each of which see the respective articles.

ANEMOMETER, in *Mechanics*, a machine for measuring the force of the wind.

ANEMOSCOPE, a term sometimes given to a wind-dial, which points out the course of the wind by an index that is connected by the spindle on which the vane works.

ANGLE, in *Geometry*, the opening between two lines which meet at a point, or the inclination of two lines which would meet if produced. When an angle is spoken of without reference to the lines by which it is formed, it is named by a single letter at the point; and when the lines are referred to by a letter in the one line, a letter at the point, and a letter on the other line.

An Angle is measured by an arch of a circle, the point being the centre, and the containing lines the extremities.

Angles are of several kinds or denominations, as *rectilinear*, *curvilinear*, *spherical*, *mixed*, *solid*, &c.

A *Rectilinear* Angle is formed by the meeting of two right lines.

A *Curvilinear* Angle is formed by the meeting of two curve lines.

A *Mixtilinear* Angle is formed by the meeting of a right line and curve.

A *Spherical* Angle is that which is formed on the surface of a sphere, by the intersection of two great circles. See Sphere, and Spherical *Trigonometry*.

A *Solid* Angle is formed by the mutual inclination of more than two plane angles, the sum of which is less than four right angles. meeting in a common point.

A *Salient* Angle has its point outwards; a *Re-entering* Angle has its point inwards; a *Direct* Angle measures less than half a circle, and a *Retroflected* Angle more.

A *Supplemental* Angle is the

difference between any angle and a semicircle.

A *Right* ANGLE is equal to its supplement; an *Obtuse* ANGLE is greater than its supplement; and an *Acute* ANGLE is less. Hence, all right angles are equal, and each is measured by a quadrant, or the fourth part of a circle.

Vertically opposite ANGLES are formed by two lines which cross each other. At each crossing there are two pairs of such angles; those of each pair are equal, and the supplements of the other pair.

Alternate ANGLES, are those made on the opposite sides of a line cutting two other lines; and if these two lines are parallel, the alternate angles are equal.

Exterior ANGLES, are those formed by the sides of any right-lined figure, and the adjacent sides produced; they are the supplements of the interior angles. Hence the exterior and interior angles are equal to twice as many right-angles as the figure has sides; and it can be shewn that all the exterior angles of any figure are equal to four right-angles. Hence, again, if any figure have its angles all equal, the measure of one of its angles, calling n the number, and R, a right-angle, will be

$$\frac{2n - 4R}{n} = \frac{(n-2)\,R}{\frac{n}{2}} = R\,\frac{n-2}{\frac{n}{2}}$$

EXAMPLES.

1. Required the angle of a square?

Here $n = 4$, $\frac{n}{2} = 2$. Hence $90° \times \frac{2}{2} = 90° =$ angle.

2. Required the angle of a pentagon?

Here $n = 5$, $\frac{n}{2} = 2.5$ Hence $90° \times \frac{3}{25} = 90° \times 1\frac{1}{5} = 108° =$ angle required.

The sum of all the external angles of any figure is equal to four right angles.

Internal ANGLES, are the angles within a figure, formed by the meeting of each two adjacent sides; as the angles *a*, *b*, *c*, &c.

The sum of all the inward angles of any right-lined figure, is equal to twice as many right angles, wanting four, as the figure has sides.

An ANGLE *at the Centre of a Circle*, is that whose angular point is at the centre.

An ANGLE *at the Circumference*, is that whose angular point is in any part of the circumference.

An angle at the centre is double an angle at the circumference, when both stand on the same arc.

An ANGLE *in a Semi-Circle*, is an angle at the circumference contained in a semi-circle, or standing upon a semi-circle or diameter.

An angle in a semi-circle, is a right-angle.

An angle in a segment, greater than a semi-circle, is less than a right-angle.

An angle in a segment, less than a semi-circle, is greater than a right-angle.

ANGULAR, something relating to, or having angles.

ANGULAR *Motion*, is that which is performed by an oscillating or vibrating body, as referred to the angle which it describes or passes over in a given time, the vertex of which is the point of suspension, or centre of motion. Hence all points in a pendulum have the same angular motion, although their absolute motions are different from each other, being greater or less, according to their distance from the centre of suspension.

ANGULAR *Motion* is also sometimes used to denote a motion which is partly curvilinear, and partly rectilinear; as the motion of a coach-wheel on a plane.

ANGULAR *Sections*, is a term used by Vieta to denote a species of analytical trigonometry, relating to the law of increase and decrease of the sines and chords of multiple arcs.

Vieta demonstrated that if a semi-circle be divided into equal arcs, and the radius be taken equal to unity, and the first chord $= x$; then will the chords drawn from the one extremity, beginning with the diameter respectively, be

Dia $= 2$
1st $= x$
2nd $= x^2 - 2$
3rd $= x^3 - 3x$

$$4\text{th} = x^4 - 4x^2 + 2$$
$$5\text{th} = x^5 - 5x^3 + 5x$$
$$6\text{th} = x^6 - 6x^4 + 9x^2 - 2$$
&c. &c.

Also, if the supplemental chord be taken equal to y, then will the supplemental chords be

$$1\text{st} = y$$
$$2\text{d} = 2 - y^2$$
$$3\text{d} = 3 + y^3$$
$$4\text{th} = 2 - 4y^2 + y^5$$
$$5\text{th} = 5y - 5y^3 + y^5$$
&c. &c.

ANIMATED *Needle*, a needle touched with a magnet.

ANNUAL, in *Astronomy*, any thing which relates to the year, or which returns yearly.

ANNUITIES, signify any interest of money, rents, or pensions, payable from time to time, at particular periods.

The most general division of annuities is into *annuities certain*, and *annuities contingent;* the payment of the latter depending upon some contingency; such, in particular, as the continuance of a life.

Annuities have also been divided into *annuities* in *possession*, and *annuities* in *reversion;* the former meaning such as have commenced, or are to commence *immediately;* and the latter, such as will not commence till some particular future event has happened, or till some given period of time has expired.

Annuities may be farther considered as being payable *yearly*, *half-yearly*, or *quarterly*.

The present value of an Annuity is that sum which being improved at compound interest, will be sufficient to pay the annuity.

The present value of an *Annuity certain*, payable yearly, and the first payment of which is to be made at the end of a year, is computed as follows:

Let the annuity be supposed £100; the present value of the first payment of it, or of a hundred pounds to be received a year hence, is that sum in hand, which being put to interest will amount to £100 in a year. In like manner, the present value of the second payment, or of £100 to be received two years hence, is that sum which being put to interest immediately will amount to £100 in two years; and so on for any number of years or payment; and the sum of the values of all the payments will be the present value of the *annuity*.

Let the interest be supposed to be 4 *per cent*. The sum which improved at 4 *per cent.* interest for the year, will produce £100 at the end of the year, is that sum which has the same proportion to 100 as 100 has to 104; the sum of the interest and principal. Say therefore, as

$$104 : 100 = 100 : \frac{100 \times 100}{104} = 96{\cdot}15,$$

value of the first payment.

And that sum which in two years will amount to £100, at the same rate of interest, is evidently that which in one year will amount to £96·15. because we have seen that this sum will, in a year, amount to £100; we have, therefore, on the same principle as above.

$$104 : 100 = 96{\cdot}15 : \frac{96{\cdot}15}{104} = 92{\cdot}45$$

value of 2d payment,

$$104 : 100 = 92{\cdot}45 : \frac{92{\cdot}45}{104} = 85{\cdot}48$$

value of 3d payment.

$$104 : 100 = 85{\cdot}48 : \frac{85{\cdot}48}{104} = 82{\cdot}19$$

value of 4th payment.

and so on.

But since the interest of money bears a constant ratio to principal, we may represent these proportions more conveniently by assuming generally £1 as the yearly payment, and r as the amount of £1 for one year; and whatever results are thus produced, being multiplied by the yearly payment, will be the value of the annuity in such case. By this means the above proportions become

$$r : 1 = 1 : \frac{1}{r} \text{ value 1st payment}$$
$$r : 1 = \frac{1}{r} : \frac{1}{r^2} \text{ . . . 2d payment}$$
$$r : 1 = \frac{1}{r^2} : \frac{1}{r^3} \text{ . . . 3d payment}$$

and so on.

The value, therefore, of an annuity of £1 for n years, is equal to the sum of the series.

$$\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \dots \frac{1}{r^n}$$

and therefore when the annuity is perpetual, or in *perpetuity*, the value of it is equal to the sum of the above series continued *in infinitum*.

Now, in order to find the sum of this series, let us make it equal to v; then since

$$v = \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \&c.$$

$$\frac{1}{r^{n-1}}\ \frac{1}{r^n}\quad vr = 1 + \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3}$$

$$+ \&c.\ \frac{1}{r^{n-1}}$$

whence, $(r-1)\,v = 1 - \frac{1}{r^n}n$

or, ... $v = \dfrac{r^n - 1}{r - 1} \times \dfrac{1}{r^n}$;

which is a general formula for the present value of an annuity of £1 for n years. And since an annuity of a pounds per year will be worth a times as much; we have generally

$$v = \frac{r^n - 1}{r - 1} \times \frac{a}{r^n}$$

for the value of an annuity of a pounds per year, for n years.

If the annuity is to be at s equal distant times in the year, then r instead of representing the amount of £1 for a year, must be taken for the amount of £1 for the time of the first payment; also n, which is the number of payments in the above, will become $n\,s$, and a will become $\frac{a}{ns}$; whence the more general formula will be

$$v = \frac{r^{mn} - 1}{r - 1} \times \frac{a}{mr^n} = \frac{r^{\prime n \prime} - 1}{r^{\prime} - 1}$$

$$\times \frac{a^{\prime}}{r^{\prime n \prime}}$$

Substituting therefore as above,

a = the annuity, or yearly rent;
n = the number of years, or payments;
r = the amount of 1*l.* for a year, or for one payment;
v = the present value of the annuity;
m = the whole amount;

we shall have the several cases of annuities expressed by the following formulæ:

1. . . $v = \dfrac{r^n - 1}{r - 1} \times \dfrac{a}{r^n}$

2. . . $a = \dfrac{r - 1}{r^n - 1} \times vr^n$

3. . . $m = \dfrac{r^n - 1}{r - 1} \times a$

4. . . $n = \dfrac{\log. m - \log. v}{\log. r}$

5. $\log. r = \dfrac{\log. m - \log. v}{}$

6. . $R = \left\{\dfrac{1}{r^n} - \dfrac{1}{r^p}\right\} \times \dfrac{a}{r - 1}$

In this last theorem, R denotes the present value in reversion, after p years, or not commencing till after the first p years, being found by taking the difference between the two values.

$$\frac{r^n - 1}{r - 1} \times \frac{a}{r^n} \text{ and } \frac{r^p - 1}{r - 1} \times \frac{a}{r^p}$$

Let us now illustrate these formulæ by a few examples.

Required the present worth of an annuity of 100*l.* per annum for 5 years, at yearly payments, half-yearly payments, and quarterly payments; the rate of interest being 4 per cent. in each case.

In the first case, $r = 1{\cdot}04$, $n = 5$, and $a = 100$; therefore, by formula I, we have

$$v = \frac{1{\cdot}04^5 - 1}{1{\cdot}04 - 1} \times \frac{100}{1{\cdot}04^5} = £445{\cdot}18.$$

In the second case, $r = 1{\cdot}02$, $n = 10$, and $a = 50$; therefore

$$v = \frac{1{\cdot}02^{10} - 1}{1{\cdot}02 - 1} \times \frac{50}{1{\cdot}02^{10}} = £449{\cdot}13.$$

In the third case, $r = 1{\cdot}01$, $n = 20$, and $a = 25$; therefore

$$v = \frac{1{\cdot}01^{20} - 1}{1{\cdot}01 - 1} \times \frac{25}{1{\cdot}01^{20}} = £454{\cdot}15.$$

Hence, it is evident that the greater the number of payments, the greater is the present value of the annuity.

What annuity, payable yearly, may be purchased for 5 years with 2226*l.*; taking the rate of interest at 4 per cent.?

Here we have $v = 2226$, $r =$

$1{\cdot}04$, $n = 5$; whence, by formula 2, we obtain

$$a = \frac{1{\cdot}04 - 1}{1{\cdot}04^{n} - 1} \times 2226 \times 1{\cdot}04^{5} =$$

£500 per annum, nearly.

Required the whole amount of an annuity of 100*l*. per annum, for 20 years, at 5 per cent. per ann. yearly payments.

Here we have $r = 1{\cdot}05$, $n = 20$, $a = 100$.

$$m = \frac{r^{n} - 1}{r - 1} \times a.$$

$$m = \frac{1{\cdot}05^{20} - 1}{1{\cdot}05 - 1} \times 100 = 3306l\ \ 12s.$$

Required the amount of the same annuity, at half-yearly payments.

Here $r = 1{\cdot}025$, $n = 40$, $a = 50$.

$$m = \frac{r^{n} - 1}{r - 1} \times a.$$

$$m = \frac{1{\cdot}025^{40} - 1}{1{\cdot}025 - 1} \times 50 = 3370l\ 0s\ 9\tfrac{1}{2}d$$

Required the amount of the same at quarterly payments.

Here $r = 1\ 0125$, $n = 80$ $a = 25$.

therefore $m = \dfrac{1\ 0125^{80} - 1}{1{\cdot}0125 - 1} \times 25$

$= 3402l.\ 16s.$

Whence again it follows, that the greater the number of payments, the greater will be the amount of the annuity.

To find the present value of an annuity by the following table, we have only to find the amount for 1*l*. at the given rate of interest, and for the given time; which multiplied by the given annuity, or payment, will be the present worth.

Exam. What is the present value of an annuity of 40*l*. per ann. to continue 20 years, at the rate of 4 per cent.?

By the table, the amount of 1*l*. for 20 years, at 4 per cent. is 13.590326; therefore

$13{\cdot}590326 \times 40 = 543l.\ 12s.$ very nearly.

For what relates to life annuities, see Life *Annuities* and *Insurances*.

TABLE,

Showing the present Value of an Annuity of £1. per Ann. for any number of Years not exceeding 60, at any rate of Compound Interest from 3 to 6 per Cent.

Yrs.	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
1	.970874	.966184	.961538	.956938	9.52381	.943396
2	1.913470	1.899694	1.086095	1.872668	1.859410	1.833393
3	2.828611	2.801637	2.775091	2.748964	2.723248	2.673012
4	3.716098	3.673079	3.629895	3.587526	3.545950	3.465106
5	4.579708	4.515052	4.451822	4.389977	4.329477	4.212364
6	5.417191	5.328553	5.242137	5.157872	5.075692	4.917324
7	6.230283	6.114544	6.002055	5.892701	5 786373	5.582381
8	7.019692	6.873956	6.732745	6.595886	6 463213	6.209794
9	7.786109	7.607687	7.435332	7.265790	7.107822	6 801692
10	8.530203	8.316605	8.110896	7.912718	7 721735	7.360087
11	9.252624	9.001551	8.760477	8.528917	8.306414	7.886875
12	9.951004	9.663334	9.385074	9.118581	8.863252	8 383844
13	10.634955	10.302738	9.985648	9.682852	9 393573	8 852683
14	11.296073	19.920520	10.563123	10.222825	9 898641	9.294984
15	11 937935	11.517411	11.118387	10 739546	10.379658	9.712249
16	12.561102	12 094117	11.652296	11.234015	10.837770	10.105895
17	13.166118	12.651321	12.165669	11.707191	11.274066	10.477260
18	13.753513	13.189682	12 659297	12.159902	11 689587	10.827603
19	14.323799	13.709837	13.133839	12.593294	12.085321	11.158116
20	14.877475	14.212403	13.590326	13.007936	12.462210	11.469921
21	15.415024	14.697974	14.029160	13.404724	12.821153	11.764077
22	15.936917	15.167125	14.451115	13.784425	13.163003	12.041582
23	16.443608	15.602410	14.856842	14.147775	13.488574	12.303379

TABLE,

Showing the present Value of an Annuity of £1. per Ann. for any number of Years not exceeding 60, at any rate of Compound Interest from 3 to 6 per Cent.

Yrs.	3 per Ct.	3½ perCt.	4 per Ct.	4½ perCt.	5 per Ct.	6 per Ct.
24	16.935542	16.058368	15.246963	14.495478	13.798642	12.550358
25	17.413148	16.481515	15.622080	14.828209	14 093945	12.783356
26	17.876842	16.890352	15.982769	15.146611	14.375165	13.003166
27	18.327031	17.285364	16.329580	15.451303	14.643034	13.210534
28	18.764108	17.667019	16.663063	15.742874	14.898127	13.406164
29	19.188455	18.035767	16.983715	16.021889	15.141074	13.590721
30	19.600441	18.392045	17.292033	16.288889	15.372451	13.764831
31	20.000428	18.736276	17.588494	16.544391	15.592810	13 929086
32	20.388705	19.068865	17.873551	16.788891	15.802677	14.084043
33	20.765792	19.390208	18.146674	17.022862	16 002549	14.230230
34	21.131837	19.700684	18.411198	17.246758	16.192904	14.368141
35	21.487220	20.000661	18.664613	17.461012	16.374194	14.498246
36	21.832252	20.290494	18.908282	17.666040	16.546852	14.620986
37	22.167235	20.570525	19.142579	17.862240	16.711287	14.736780
38	22.492462	20.841087	19.367864	18.049990	16.867893	14.846019
39	22.808215	21.102500	19.584485	18.229656	17.017041	14.949075
40	23.114772	21.355072	19.792774	18.401584	17.159086	15.046297
41	23.412400	21.599104	19.993052	18 566109	17.294368	15.138016
42	23.701359	21.834882	20.185627	18.723550	17.423208	15.224543
43	23.981902	22.062689	20.370795	18.874210	17.545912	15.306173
44	24.254274	22.282791	20.548841	19.018383	17.662773	15.383182
45	24.518713	22.495450	20.720040	19.156343	17.774070	15.455832
46	24.775449	22.700918	20.884652	19.288371	17.880066	15.524370
47	25.024708	22.899438	21.042936	19.414709	17.981016	15.589028
48	25.266707	23.091244	21.195131	19.535607	18.077158	15.650027
49	25.501657	23.276564	21.341472	19.651298	18.168722	15 707572
50	25.729764	23.455618	21.482185	19.762008	18.255925	15.761861
51	25.951227	23.628616	21.617485	19.867950	18.338977	15.813076
52	26.166240	23.795765	21.747582	19.969330	18.418673	15.861393
53	26.374990	23.957260	21.872675	20.066345	18.493405	15.906974
54	26.577660	24.113295	21.992957	20.159181	18.165146	15.949976
55	26.774428	24.264053	22.108612	20.248021	18.633472	15.990543
56	26.965464	24.409713	22.219819	20.333034	18.698515	16.028814
57	27.150936	24.550448	22.326749	20.414387	18.760519	16.064919
58	27.331005	24.686423	22.429567	20.492236	18.819542	16.098980
59	27.505831	24.817800	22.528430	20.566733	18.875754	16.131113
60	27.675564	24.944734	22.623490	20.638022	18.929290	16.161428

ANNULAR *Eclipse*, is an eclipse of the sun, in which the moon appearing less than the sun, leaves a bright ring round the sun's disc.

ANOMALISTICAL *Year*, in *Astronomy*, called also *Periodical Year*, is the space of time in which the earth, or other planet, passes through its orbit; which is longer than the tropical year, by reason of the precession of the equinox. For example, the tropical revolution of the sun, with respect to the equinox, is . . . . $365^d\ 5^h\ 48^m\ 45^s$
but the siderial, or return to the same star . . . . } *365d 6h 9m 11s*
and the anomalistic revolution is } 365 6 15 20
because the sun's apogee advances every year 65½″ with respect to the equinoxes, and the sun cannot arrive at the apogee till he has passed over the 65½″ more than the revolution of the year answering to the equinoxes.

ANOMALY, in *Astronomy*, is an irregularity in the motion of a planet, by which it deviates from the aphelion, or apogee; or it is the angular distance of a planet from the aphelion, or apogee; that is, the angle formed by the line of the apsides, and another drawn through the planet.

Kepler distinguishes three kinds of anomaly; *mean*, *eccentric*, and *true*.

Mean, or *Simple* ANOMALY, in the ancient astronomy, is the distance of a planet's mean place from the apogee. But in the modern astronomy, in which a planet is considered as revolving about the sun, in an elliptic orbit, it is the time in which a planet moves from its aphelion to the mean place or point of its orbit.

Hence, as the elliptical area is proportional to the time in which the planet describes the arc bounding that area, that area may represent the mean anomaly.

Eccentric ANOMALY, or *of the Centre*, is the arc, intercepted between the apsis, and the point determined by the perpendicular to the line of apsides, drawn through the place of the planet; or it is the angle at the centre of the cirle.

True, or *Equated* ANOMALY, is the angle at the sun, which the planet's distance from the aphelion appears under; or the angle formed by the radius vector, drawn from the sun to the planet, with the line of the apsides.

The finding of the true anomaly, when the mean anomaly is given, is a problem which has engaged the attention of many able astronomers. Dr. Wallis gave the first geometrical solution of it, by means of the protracted cycloid; and Newton did the same at prop. 31. lib. 1. *Principia*.

ANTARCTIC *Circle*, is a small circle parallel to the equator, at the distance of 23° 28′ from the antarctic or southern pole.

ANTARCTIC *Pole*, is the southern pole of the earth's axis.

ANTECEDENT of *a Ratio*, denotes the first of the two terms of the ratio; thus in the proportion $a : b = c : d$, a and c are the two *antecedents*, and b and d the two consequents.

ANTECEDENTAL *Calculus*, a branch of analysis invented by J. Glenie, esq. and published by him in 1793. The author professes to employ it, with advantage, instead of fluxions; but it has not been much attended to by other mathematicians.

ANTECEDENTIA, a term used by astronomers to denote a planet moving westward, or contrary to the order of the signs. When its motion is eastward, it is said to move *in consequentia*.

ANTILOGARITHMS, the complement of the logarithmic sine, tangent, &c. of an angle; being the difference between them and radius.

ANTIPARALLELS, in *Geometry*, are those lines which make equal angles with two other lines, but in contrary order; that is, calling the former pair the first and second lines, and the latter pair the third and fourth, if the angle made by the first and third lines be equal to the angles made by the second and fourth; and, on the contrary, the angle made by the first and fourth be equal to the angles made by the second and third; then each pair of lines are antiparallels to each other; *viz.* the first and second, and the third and fourth.

It has been commonly asserted of these lines, that each pair cuts the other into proportional segments, taking them alternately; but this, upon examination, will be found erroneous.

ANTIPODES, in *Geography*, are the inhabitants of two places on the earth diametrically opposite to each other, and who therefore walk feet to feet.

It is obvious that antipodes must have the same degree of latitude, but in a different hemisphere; and the difference in longitude is 180°. It is therefore night with one, when it is day with the other; and summer with one, when it is winter with the other.

APERTURE, in *Hydraulics*, is the hole through which a spouting fluid passes.

APERTURE, in *Optics*, is the hole next the object-glass of a telescope, or microscope, through which the light and the image of the object come into the tube, and are thence conveyed to the eye.

APEX, the vertex, top, or summit of any thing.

APHELION, or APHELIUM, that point in the orbit of a planet, in which it is at its greatest distance from the sun: that is, at the extremity of the transverse diameter of the elliptic orbit, which is farthest distant from that focus in which the sun is placed.

The aphelia of the planets are not fixed; for their mutual actions upon each other keep those points of their orbits in a continual motion, which is greater or less in the different planets.

APOGEE, in the *ancient Astronomy*, that point in the orbit of the sun, or a planet, which is farthest distant from the earth; and which corresponds with the *aphelion* of modern astronomers. It is still used to denote the greatest distance of a body from the earth. The moon, for example, is in its apogee when farthest from the earth; and its aphelion, when farthest from the sun.

APOTOME, the remainder or difference between two lines or quantities, which are only commensurable in power. Such as the difference between 1 and $\sqrt{2}$, or the difference between the side and diagonal of a square.

Euclid, in his tenth book, distinguishes these quantities, under six heads, *viz.*

APOTOME *Prima*, when the greater term is rational, and the difference of the squares of the two is a square number; as, $3-\sqrt{5}$.

APOTOME *Secunda*. When the less term is rational, and the square root of the difference of the squares of the two terms has to the greater term a ratio expressible in numbers; such is $\sqrt{18}-4$, because the difference of the squares 18 and 16 is 2, and $\sqrt{2}$ is to $\sqrt{18}$ as $\sqrt{1}$ to $\sqrt{9}$, or as 1 to 3.

APOTOME *Tertia*. When both the terms are irrational, and, as in the second, the square root of the difference of their squares has to the greater term a rational ratio; as $\sqrt{24}-\sqrt{18}$, for the difference of their squares 24 and 18 is 6, and $\sqrt{6}$ is to $\sqrt{24}$ as $\sqrt{1}$ to $\sqrt{4}$ or as 1 to 2.

APOTOME *Quarta*. When the greater term is a rational number, and the square root of the difference of the squares of the two terms has not a rational ratio to it; as, $4-\sqrt{3}$, where the difference of the squares 16 and 3 is 13, and $\sqrt{13}$ has not a ratio in numbers to 4.

APOTOME *Quinta*, when the less term is a rational number, and the square root of the difference of the squares of the two has not a rational ratio to the greater; as $\sqrt{6}-2$, where the difference of the squares 6 and 4 is 2, and $\sqrt{2}$ to $\sqrt{6}$, or $\sqrt{1}$ to $\sqrt{3}$, or 1 to $\sqrt{3}$, is not a rational ratio.

APOTOME *Sexta*, where both terms are irrational, and the square root of the difference of their squares has not a rational ratio to the greater; as $\sqrt{6}-\sqrt{2}$, where the difference of the squares 6 and 2 is 4, and $\sqrt{4}$ to $\sqrt{6}$, or 2 to $\sqrt{6}$, is not a rational ratio.

APPARATUS, the appendages or utensils belonging to machines; as the apparatus of an air-pump, electrical machine, &c.; meaning the various detached parts which are necessary for putting the machinery in action, and for performing experiments, &c.

APPARENT, in *Mathematics* and *Astronomy*, is used to signify things as they appear to us, in contradistinction from real or true; and in this respect the apparent state of things is often very different from their real state: as is the case of distance, magnitude, &c.

APPARENT *Conjunction of the Planets*, is when a right line, supposed to be drawn through their centres, passes through the eye of the spectator, and not through the centre of the earth.

APPARENT *Diameter of an Object*, is not the real length of that diameter, but the angle which it subtends at the eye, or under which it appears. This angle diminishes as the distance increases; so that a small object at a small distance may have the same apparent diameter as a much larger object at a greater distance. If the objects are parallel to each other, their real diameters are, in this case, proportional to their distances. The apparent diameter also varies with the position of the object and of equal objects at equal distances, those which stand in a position most nearly perpendicular to the line of their direction from the observer, will appear to have the greatest diameter: our idea of the apparent magnitude generally, varying nearly as the optic angle.

But although the optic angle be the usual or sensible measure of the apparent magnitude of an object, yet habit, and the frequent experience of looking at distant objects, by which we know that they are larger than they appear, has so far prevailed upon the imagination and judgment, as to cause this likewise to have some share in our estimation of apparent magnitudes; so that these will be judged to be more than in the ratio of the optic angles.

APPARENT *Distance.* See DISTANCE.

APPARENT *Altitude of celestial Objects*, is effected chiefly by refraction and parallax; and that of terrestrial objects, by refraction. See those words.

APPARENT *Figure*, is the figure or shape which an object appears under when viewed at a distance; and is often very different from the true figure. For a straight line, viewed at a distance, may appear but as a point; a surface, as a line; and a solid, as a surface. Also these may appear of different magnitudes, and the surface and solid of different figures, according to their situation with respect to the eye; thus the arch of a circle may appear a straight line; a square, a trapezium, or even a triangle; a circle, an ellipsis; angular magnitudes, round; and a sphere, a circle. Also all objects have a tendency to roundness and smoothness, or appear less angular, as their distance is greater: for, as the distance is increased, the smaller angles and asperities first disappear; after these, the next larger; and so on, as the distance is more and more increased, the object seeming still more and more round and smooth.

APPARENT *Motion*, is either that motion which we perceive in a distant body that moves, the eye at the same time being either in motion or at rest; or that motion which an object at rest seems to have, while the eye itself only is in motion.

The motions of bodies at a great distance, though really moving equally, or passing over equal spaces in equal times, may appear to be very unequal and irregular to the eye, which can only judge of them by the mutation of the angle at the eye. And motions, to be equally visible, or appear equal, must be directly proportional to the distances of the objects moving. Again, very swift motions, as, those of the luminaries, may not appear to be motions at all, like that of the hour-hand of a clock, on account of the great distance of the objects: and this will always happen, when the space actually passed over in one second of time, is less than about the 14000th part of its distance from the eye. On the other hand, it is possible for the motion of a body to be so swift, as not to appear any motion at all; as when through the whole space it describes, there constantly appears a continued surface or solid as it were generated by the motion of the object, as is the case when any thing is whirled very swiftly round, describing a ring, &c.

Also the more oblique the eye is to the line which a distant body moves in, the more will the apparent motion differ from the true one.

If an eye move directly forwards in one direction, any remote object at rest will appear to move in a parallel line the contrary way. But if the object move the same way, and with equal velocity, it will seem to be at rest. If it move the same way, with less velocity, it will appear to move backwards, with the difference of the velocities: if it move with greater velocity, it will appear to move forwards with the difference of the velocities. And when the object has a real motion contrary to that of the eye, it appears to move backwards with the sum of the velocities. The truth of all this is experienced by persons in a boat moving on water, or in a moving carriage, making observations on distant objects in motion, or at rest.

APPARENT *Place of an Object*, in *Optics*, is that in which it appears, when seen in or through glass, water, or other reflecting or refracting media. In most cases, it differs much from the true place.

APPARENT *Station*, in *Astronomy*, the position or appearance of a planet, or comet, in the same point of the zodiac for several days.

APPARITION, in *Astronomy*, denotes a star or other luminary's becoming visible, which before was hid: in which sense it stands opposed to *Occultation*.

APPLICATE, *Ordinate* APPLICATE, in *Geometry*, is a right line drawn across a curve, so as to be bisected by the diameter of it; being what we commonly call a *double ordinate*.

APPLICATION, in *Arithmetic*, is sometimes used to signify division.

APPLICATION, in *Geometry*, generally means the placing of one line, angle, or surface upon another, with a view to prove their equality. This method of proof is also called proof by *supraposition*.

APPLICATION of one *Science* to another, signifies the use that is made of the principles of the one, for extending and perfecting the other.

APPLICATION of *Algebra* to *Geometry*, is of two kinds; viz. to plane or common geometry, and to curve lines. The first of these is concerned in the algebraical solution of geometrical problems, the investigation of geometrical figures, &c. This method of resolving geometrical problems is, in many cases, more direct and easy than that of geometrical analysis; but the latter method by synthesis, or construction, and demonstration, is the most elegant. The algebraical solution generally succeeds best in such problems as respect the sides and other lines of geometrical figures; and those geometrical problems in which angles are concerned, are best resolved by the geometrical analysis.

In the solution of problems of this kind, principles can be laid down which are applicable in all cases, and much must necessarily be left to the skill and ingenuity of the analyst.

When any geometrical problem is proposed for algebraical solution, one must, in the first place, draw a figure that shall represent the parts or conditions of the problem, and regard that figure as the true one; then, having considered the nature of the problem, the figure must be prepared for solution, by producing or drawing such lines as may be thought necessary. When this is done, let the unknown line or lines, which appear to be the most easily found, and any of the known ones that may be requisite, be denoted by proper symbols; then proceed to the operation, by observing the relation that the several parts have to each other. No rules can be given for drawing or producing the lines, as mentioned above, and it would therefore be useless to attempt any directions on that head; it is practice only which will render a student ready in the solution of problems of this kind; but some idea of the method may be collected from the following examples:

Prob. 1. Given the base, and the sum of the hypothenuse and perpendicular of a right-angled triangle, to find the sides severally.

Let A B C (plate I. fig. 9,) represent the right-angled triangle, of which the base A B is given, and put A B $= b$, A C $= x$, C B $= y$, the sum of A C and B C $= s$; then we readily obtain the two following equations; viz.

1st. $x^2 - y^2 = b^2$ (Euclid 1—47.)

2d. $x + y = s \cdot \cdot \cdot \cdot$ by the quest.

$$x - y = \frac{b^2}{s} = \cdot \cdot \cdot \text{ by divis.}$$

$$2x = s + \frac{b^2}{s} = \frac{s^2 - b^2}{s} \text{ by addit.}$$

$$2y = s - \frac{b^2}{s} = \frac{s^2 - b^2}{s} \text{ by subtr.}$$

Whence,

$$x=\frac{s^2+b^2}{2s}=AC,$$

and $y=\frac{s^2-b^2}{2s}=CB$, as required.

Prob. 2. It is required to determine the side of a square inscribed in a given triangle.

Let A B C (plate I. fig. 10,) represent the given triangle, and E F G H its inscribed square. Put the base $AB=b$, the perpendicular $CD=a$, and the side of the square GF or $GH=x$; then will $CI=CD-DI=a-x$,

And, because of the similar triangles ABC, GFC, it will be, as

$$AB:CD=GF:CI;\ \text{or,}$$
$$b\ :\ a=x\ :a-x$$

Hence multiplying extremes and means: we have

$$ax\quad=ba-bx;$$

whence $ax+bx=\quad ba$,

or . . . $x\quad=\quad\frac{ba}{a+b}$, the side of the square required.

Cor. Hence it is obvious, that in all triangles whose bases and perpendiculars are constant, the side of the inscribed square will be constant also.

Prob. 3. Giving the area or space of a rectangle, inscribed in a given triangle, to determine the sides of the rectangle.

Let ABC (fig. 11,) represent the given triangle, and EFGH the inscribed rectangle, the area of which is given, and which let be represented by a; make $CD=p$, $AB=b$, $EF=x$, and $ID=y$; then will $CI=p-y$; and by similar triangles we shall have

$$AB:CD=EF:CI,\ \text{or}$$
$$b\ :\ p\ =\ x\ :p-y;$$

whence $px=bp-by$,

and . . . $xy=a$, the arc a.

The first gives $x=\frac{bp-by}{p}$;

the second .. $x=\frac{a}{y}$.

Whence again $\frac{bp-by}{p}=\frac{a}{y}$,

or . . . . $bpy-by^2=pa$;

or . . . . $y^2-py=-\frac{pa}{b}$.

The root of which is $y=\frac{p}{2}\pm\surd\left(\frac{p^2}{4}-\frac{pa}{b}\right)$, the breadth required.

Having thus found the breadth, the length may be obtained by dividing the area by the breadth; or otherwise we have, by finding the values of y instead of x,

$$y\quad=\frac{bp-px}{b}$$

$$y\quad=\frac{a}{x};$$

whence $\frac{bp-px}{b}\quad=\frac{a}{x}$,

or . . . . $bpx-px^2\quad=\quad ba$:

which gives $x^2-bx\quad=-\frac{ba}{p}$;

whence $x=\frac{b}{2}\pm\surd\left(\frac{b}{4}-\frac{ba}{p}\right)$.

Prob. 4. In a right-angled triangle, having given the lengths of two lines drawn from the acute angles to the middle of the opposite sides, to find the sides of the triangle.

Let ABC (Fig. 12) represent the proposed triangle, of which the lines AD, CE, are given. Make $AD=a$, $CE=b$, $AB=2x$, $CB=2y$, $AC=2z$. Then we have

$$\left.\begin{aligned}4x^2+\ y^2&=a^2\\ x^2+4y^2&=b^2\end{aligned}\right\}\text{Euc. 47. 1.}$$

Now, . . $4x^2+16y^2=4b^2$

again . . $4x^2+\quad y^2=a^2$;

therefore $15y^2=4b^2-a^2$

and consq. $2y=2\surd\left(\frac{4b^2-a^2}{15}\right)=CB$.

And in the same manner we find

$$2x=2\surd\left(\frac{4a^2-b^2}{15}\right)=AB.$$

conseq. $2z=2\surd\left(\frac{3a^2+3b^2}{15}\right)=AC$.

Prob. 5. In a right-angled triangle, having given the hypothenuse and side of the inscribed square, to find the base and perpendicular.

Let ABC (Fig. 9) be the proposed triangle, AC the given hypothenuse, and BD the given square.

Make $AC = h$, $FD = DE = s$, $AB = x$, $BC = y$.

Then we have

$$x^2 + y^2 = h^2 \text{ (Eucl. 47, 1.)}$$

and $\cdot \cdot (x - s) : s = s : (y - s)$ by sim. trians.

Whence $xy - sx - sy + s^2 = s^2$;

or $\cdot \cdot \cdot$ $xy = s\,(x + y)$.

By adding double this to the first equation, we obtain

$$x^2 + 2xy + y^2 = h^2 + 2s\,(x + y),$$

or $(x + y)^2 - 2s\,(x + y) = h^2$;

whence $x + y = s \pm \sqrt{(h^2 + s^2)}$.

Now $x + y$ being known, make it $= n$, then we have

$$x^2 + y^2 = h^2$$
$$x + y \;= n$$

Square the second, and subtract it from double the first, and we obtain

$$x^2 - 2xy + y^2 = 2h^2 - n^2.$$

By extracting, $x - y = \sqrt{(2h^2 - n^2)}$.

Again $\cdot \cdot \cdot \cdot$ $x + y = n$;

therefore,

$$x = \frac{n + \sqrt{(2h^2 - n^2)}}{2} = AB;$$

and $$y = \frac{n - \sqrt{(2h^2 - n^2)}}{2} = BC,$$

as required.

Prob. 6.—Having given the sum of the three sides of a right-angled triangle, and the perpendicular, let fall from the right-angle upon the hypothenuse, to find the three sides of the triangle.

Let ABC (fig. 12,) be the proposed triangle, of which the sum of the sides and perpendicular BD are given. Make the sum of the sides $= s$, the perpendicular $= p$, $AB = x$, $BC = y$, $AC = z$.

Then we have $\quad z + y + z = s$,
by the question $\quad x^2 + y^2 = z^2$
(Eucl. 47, 1.) $x : z = p : y$, by sim. trians.; or $\quad xy = pz$.

Now, add double this last equation to the second, then

$$x^2 + 2xy + y^2 = z^2 + 2pz$$
$$x^2 + 2xy + y^2 = s^2 - 2z + z^2,$$

by transposing z in the first equation and squaring.

Whence, $z^2 + 2pz = s^2 - 2sz + z^2$,

or, $$z = \frac{s^2}{2p + 2s} = AC.$$

Now z being known, make it $= n$; then the second and third equations become

$$x^2 + y^2 = n^2$$
$$xy = pn.$$

Add and subtract double the last equation from the first, and we have

$$x^2 + 2xy + y^2 = n^2 + 2pn$$
$$x^2 - 2xy + y^2 = n^2 - 2pn,$$

the roots of which are,

$$x + y = \sqrt{(n^2 + 2pn)}$$

and $$x - y = \sqrt{(n^2 - 2pn)}.$$

Whence again, $x = \frac{1}{2}\sqrt{(n^2 + 2pn)} + \frac{1}{2}\sqrt{(n^2 - 2pn} = AB$,

and $\cdot \cdot \cdot$ $y = \frac{1}{2}\sqrt{(n^2 + 2pn)} - \frac{1}{2}\sqrt{(n^2 - 2pn)} = BC$.

Wherefore the three sides AB, BC, AC, are determined.

In the second branch of the *Application of Algebra to Geometry*, or that which respects the higher geometry, or the nature and properties of curve lines, the nature of the curve is expressed, or denoted by an algebraical equation, which is formed as follows: a line is conceived to be drawn to represent the diameter, or some other principal line of the curve; and upon this line, at any indefinite points, are erected perpendiculars, which are called ordinates; and the parts of the first line cut off by them are termed abscisses. Calling the abscis x, and its corresponding ordinate y, the known nature of the curve, or the mutual relation of the other lines in it, will furnish an equation, involving x and y, with some other letter or letters which are known. And as x and y are common to every point in the primary line, the equation, derived in this manner, will belong to every position or value of the absciss and ordinate; and may be properly considered as expressing the nature of the curve in all points of it, and is usually called *the equation of the curve*. Hence every particular curve will appear to have an appropriate equation, differing from that of every other; either as to the number of the terms, the powers of the unknown quantities x and y, or the signs of the coefficients of the terms of the equation.

APPLICATION of *Geometry* to *Algebra*, is the converse of the first of the two preceding cases. It relates principally to the finding the roots of an equation by a geometrical construction.

APPLICATION of *Algebra* and *Geometry* to *Mechanics.* This is found on the same principles as the Application of Algebra to Geometry; and consists principally in representing, by equations, the curves described by bodies in motion; as in the theory of Projectiles, &c.

APPLICATION of *Mechanics* to *Geometry,* consists chiefly in the use that is sometimes made of the centre of gravity of figures, for determining the contents of solids described by those figures.

APPLICATION of *Geometry* and *Astronomy* to *Geography,* principally consists in the three following articles; viz. in determining by geometrical and astronomical operations the figure of the terrestrial globe; in finding the positions of places by their observed latitude and longitude; and in determining, by geometrical operations, the positions of places that are not very remote from one another. Astronomy and geography are again applicable to the theory of navigation.

APPLICATION of *Geometry* and *Algebra* to *Physics* or *Natural Philosophy.* For this application we are indebted to Sir Isaac Newton, whose philosophy may, therefore, be called the geometrical or mathematical philosophy; and upon this application are founded all the phyico-mathematical sciences.—Hence a single observation or experiment will often produce a whole science. Having ascertained, by experience, that the rays of light, by reflection, make the angle of incidence equal to that of reflection, we hence deduce the whole science of Captoptics. The case is also the same in many other sciences.

APPROACH. The *Curve of equable Approach,* is of such a nature, that a body descending by the sole power of its own gravity approaches the horizon equally in equal times. This curve has been found, by Bernoulli, Varignon, Maupertuis, and others, to be the second cubical parabola, so placed that its point of regression, or vertex, is uppermost, and the descending body must commence its motion in it with a certain determinate velocity. Varignon rendered the question more general by investigating the curve which a body might describe *in vacuo,* so as to approach through a given point through equal spaces in equal times, according to any law of gravity. Maupertuis also resolved the same problem, in the case of a body descending in a medium, the resistance of which is proportionate to the square of the velocity.

Method of APPROACHES, a term used by Dr. Wallis, in his Algebra, to denote a method of resolving certain problems, relating to square numbers, &c.; which is done by first assigning certain limits to the quantities required, and then approaching nearer and nearer till a coincidence is obtained. This method was invented by Dr. Pell, for the solution of equations of the form $x^2 - ay^2 = 1$; which problem was proposed by Fermat, as a challenge to all the English mathematicians of his time; viz. to find rational and integral values of x and y, in the above equation, for every value of a, except when it is a complete square.

APPROXIMATION, in *Algebra* and *Arithmetic,* is the method of approaching nearer and nearer to the quantity sought, when there is no method of obtaining the exact value.

APPROXIMATION *to the Roots of Equations.* As there is no direct method of determining the roots of equations beyond those of the fourth degree, and even in those of the third and fourth degree being very laborious by the direct rules, mathematicians have endeavoured to find methods of approximating the roots: of these, Newton's rule is the most popular, and is founded on the following principles:

If any two numbers, being substituted for the unknown quantity in an equation, give results with opposite signs, an odd number of roots must be between these numbers.

This appears from the property of the absolute term, and from this obvious maxim, that if a number of quantities be multiplied together, and the sign of an odd number of them be changed, the sign of the product is changed. For when a positive number is substituted for x, the result is the absolute term of an equation, whose roots are less than the roots of the given equation by that quantity. If the result has the same sign as the given absolute term, then from the property of this term, either none, or an even number only, have had their signs changed by the transformation; but if the result has an opposite sign to that of the given absolute term, the sign of an odd number of the positive roots must have been changed.

In the first case, then, the quantity substituted must have been either greater than each of an even number of the positive roots of the given equation, or less than any of them; in the second case, it must have been greater than each of an odd number of the positive roots. An odd number of the positive roots must, therefore, lie between them, when they give results with opposite signs. The same observation is to be extended to the substitution of negative quantities and the negative roots.

Let it, for example, be proposed to find an approximate value of x in the equation

$$x^3-2x-5=0.$$

Here we easily see, that one root is between 2 and 3; for these numbers being substituted for x, will give, the one a positive, and the other a negative result. The root, therefore, is greater than 2 and less than 3, and we may, therefore, write $x=2+f$, or $x=3-f$; and by finding f, or an approximate value of it, in either case, will obviously also give an approximate value of x, by adding the value of f to 2, in the first case, and subtracting it from 3 in the second case.

Suppose $x=2+f$, and subtituting this value for x in the proposed equation, we have,

$$\begin{array}{rl} x^3 & =8+12f+6f^2+f^3 \\ -2x & =-4-2f \\ -5 & =-5 \\ \hline x^3-2x-5 & =-1+10f+6f^2+f^3=0; \end{array}$$

therefore, to find f we have the equation

$$f^3+f^2+10f-1=0,$$

which is still a cubic; but since, from the nature of our substitution, f must be less than 1, the cube of it may be omitted as inconsiderable, and this will give

$$f^2+\tfrac{5}{3}f-\tfrac{1}{6}=0, \text{ or}$$

$$f=\frac{-5}{6}\pm\sqrt{\left(\frac{25}{36}+\frac{1}{6}\right)}= \frac{-5\pm\sqrt{31}}{6};$$

which value of f, being added to 2, will give an approximate value of x.

But generally, when f is less than unity, all the higher powers of f may be omitted, without any very sensible error; which being done in the present instance, will give $10f-1=0$, or $f=0{\cdot}1$, and therefore $x=2{\cdot}1$ nearly.

Now as $f=0{\cdot}1$ nearly,

let $f={\cdot}1+g$,

and substituting this value for f in the preceding equation, we have

$$\begin{array}{rl} f^3 & =0{\cdot}001+0{\cdot}03g+0{\cdot}3g^2+g^3 \\ 6f^2 & =0{\cdot}06+1{\cdot}2g+6g^2 \\ 10f & =1.+10g \\ -1 & =1 \\ \hline f^3+6f^2+10f-1 & = \\ \hline 0{\cdot}061+11{\cdot}23g+6{\cdot}3g^2+g^3 & =0. \end{array}$$

And neglecting g^3 and g^4, for the reasons above stated, this equation becomes

$$0{\cdot}061+11{\cdot}23g=0,$$

$$\text{or } g=\frac{-0{\cdot}061}{11{\cdot}23}=-{\cdot}0054.$$

Hence, then,

$$=f={\cdot}1+g={\cdot}1-{\cdot}0054={\cdot}0946,$$

nearly; and, consequently,

$$x=2+f=2+{\cdot}0946=2{\cdot}0946.$$

This operation may be extended to any length at pleasure, by supposing again $g=-{\cdot}0054+h$, and thence finding a value of h, and so on: this last substitution, if worked out, will give

$$x=2{\cdot}09455147.$$

Another method of approximating towards the roots of equations in general, is by the following formula, first published by Barlow:

Let $x^n + ax^{n-1} + bx^{n-2} + cx^{n-3} +$ &c. $= w$, by any equation; and $x'^n + ax'^{n-1} + bx'^{n-2} + cx'^{n-3} +$ &c. $= v$, x' being an approximate value of x; then

$$x = x' + \frac{(w-v)2x'}{(n-1)w + (n+1)x^n + (n-1)ax^{n-1} + (n-3)bx^{n-2} + \&c.};$$

$$\text{or, } x = x' + \frac{(w-v)2x'}{(n-1)v + (n+1)x'^n + (n-1)ax'^{n-1} + (n-3)bx'^{n-2} + \&c.}$$

The first formula being applicable to the cases in which x' is greater than unity, and the second when x' is less than unity.

These formulæ are general for equations of all dimensions, but when reduced to particular cases, they become much more simple: thus, reducing them both into one, we have for

(1.) *Equations of the third Degree,*

$$x = x' + \frac{(w-v)x'}{w \text{ or } v + 2x'^3 + ax'^2};$$

(2.) *Equations of the fourth Degree,*

$$x = x' + \frac{(w-v)2x'}{3w \text{ or } 3v + 5x'^4 + 3ax'^3 + bx'^2 - cx};$$

(3.) *Equations of the fifth Degree,*

$$x = x' + \frac{(w-v)x'}{2w \text{ or } 2v + 3x'^5 + 2ax'^4 + bx^3 - dx}.$$

And in the same way we might find the particular formula answering to an equation of any other degree.

Application of the preceding Formulæ.—In order to this it may be observed, that x' is an approximate value of x, found first by trial, as near the true root as possible, which in all cases ought to be true to the nearest integer. Then substituting this value of x', a nearer approximate value of x will be obtained, which will in all cases double the number of figures. Then, considering this as a new value of x', another still nearer value of x will be determined; and so on.

In stating the degree of approximation to be that of doubling the number of figures in the assumed root, we are rather under than over the real state of convergency; for generally, if the first supposition be a simple integer, the first approximation may be obtained to three places, which, though it be not always correct in the last place, yet by employing it as a new approximation, the next value of x will, in most cases, be found true to six or seven figures, which is a degree of convergency that cannot be obtained by any other rule that we are acquainted with, at the same time that the operation is much more simple than by any other rule whatever.

An example will sufficiently illustrate the preceding remarks.

Example.—Find the value of x in the cubic equation

$$x^3 + 3x^2 + 3x = 130;$$

Assume $x' = 4$, then by formula (1)

$x'^3 = 64$ $\quad 2x'^3 = 128$
$3x'^2 = 48$ $\quad ax'^2 = 48$
$3x' = 12$ $\quad w = 130$

$v = 124$ $\quad$ 306 denominator
$w = 130$

$w - v = 6$
$x' = 4$

306) 24 (4·08 nearly.

Therefore now $x' = 4·08$.

Hence, adopting this new value, we have

$x'^3 = 67·9173$ $\quad 2x'^3 = 135·8346$
$3x'^2 = 49·9392$ $\quad ax'^2 = 49·9392$
$3x' = 12·24$ $\quad w = 130$

$v = 130·0965$ $\quad$ 315·7738
$w = 130$
$w - v = ·0965$
$x' = 4·08$

315·77) ·39372 (·001247.

Whence, 4·08 — ·001247 = 4·078753 Answer.

This root is true in the seventh figure; and, in the same manner, the root of any other equation may be determined.

APSES, in *Astronomy*, are the two points in the orbits of the planets where they are at their greatest and less distance from the sun or earth; the former being called the *higher* apsis, and the latter the *lower* apsis: but the higher apsis is more commonly called the *aphelion*, and the lower apsis the *perihelion;* or, according to the ancient astronomy, the *apogee* and *perigee*. The diameter which joins these two points is called the ***line of the apsides***, and is supposed to pass through the centre of the orbit of the planet, and the centre of the sun or earth; in the modern astronomy, this line makes the longest or transverse axis of the elliptical orbit of the planet; and in its line is counted the eccentricity of the orbit.

According to the above definition, the lines of the greatest and least distance are supposed to lie in the same straight line; which is not always precisely the case, as the two frequently make an angle with each other; and what this angle differs from 180°, is called the motion of the line of the apsides; and when this is less than 180°, the motion is said to be contrary to the order of the signs; and when it is greater than 180°, the motion is said to be according to the order of the signs.

AQUARIUS, in *Astronómy*, the 11th sign of the zodiac, beginning from Aries; its character is ♒.

AQUEDUCT, a *conduit of water*, in *Architecture* and *Hydraulics*, is a construction of stone and timber built on uneven ground, to preserve the level of water, and to conduct it through canals from one place to another.

ARCH, or ARC, in *Geometry*, a part of a curve line: as of a circle, ellipse, &c.

Circular Arc, is any part of the arc of a circle, and by which the magnitudes of angles are compared; an angle being said to contain so many degrees, minutes, &c. as are contained in the arc which subtends it.

Co-centric Arcs, are such as have the same centre.

Equal Arcs, are such arcs of the same circle, or of equal circles, which have the same measure, or the same number of degrees, minutes, &c.

Similar Arcs, are those which have the same measure, but belonging to different circles. The lengths of similar arcs are to each other as the radii of the corresponding circles.

The length of circular arcs may be expressed in terms of the sine, cosine, tangent, &c. in the following manner:

Let a represent any arc to radius 1.; s its sine, c its cosine, t its tangent, $'s$ the secant, and v its versed sine; then will

$$a = t - \tfrac{1}{3}t^3 + \tfrac{1}{5}t^5 - \tfrac{1}{7}t^7 + \tfrac{1}{9}t^9 - \&c.$$

$$a = \frac{'s}{c} - \frac{'s^3}{3c^3} + \frac{'s^5}{5c^5} - \frac{'s^7}{7c^7} + \frac{'s^9}{9c^9} - \&c.$$

$$a = s + \frac{1.s^3}{2.3} + \frac{1.3.s^5}{2.4.5} + \frac{1.3.5.s^7}{2.4.6.7} + \&c.$$

$$a = \sqrt{2v}\,.\left\{1 + \frac{v}{2^2.3} + \frac{1\,3v^2}{2^4.4.5} + \frac{1.3.5v^3}{2^4.46.7}, \&c.\right\}$$

$$a = \text{sin. } a, \text{ sec. } \tfrac{1}{2}a, \text{ sec. } \tfrac{1}{4}a, \text{ sec. } \tfrac{1}{8}a, \&c.$$

$$a = \frac{3.1416}{180}.\ d = \cdot01745329, \ \&c. \times d;$$ where d represents the number of degrees, &c. contained in the given arc.

$$a = \frac{8c - C}{3}, \textit{ nearly}.$$ Where C and c are the chords of the arc and half arc.

As since similar arcs are to each other as their radii, it is obvious, that having the length of any arc given to radius 1, the length of a similar arc may be found for any other radius, by multiplying the length of the first arc by the given radius. Or, since ·01745329 is the length of an arc of 1°, to radius 1; the length of any arc, of which the measure is given, will be found by multiplying the number of degrees by ·01745329: and that product again by the given radius.

ARCH in *Astronomy*, has various denominations, according to the circle of which it is a part.

Diurnal Arch of the *Sun*, is part of a circle described by the sun in his course between rising and setting. His nocturnal arch is that described between setting and rising.

The latitude and elevation of the

pole are measured by arcs of the meridian, and the longitude by an arch of a parallel circle.

Arch of *Progression*, or *Direction*, is an arch of the ecliptic, which a planet seems to pass over, when its motion is direct, or according to the order of the signs.

Arch of *Retrogradation*, is an arch of the ecliptic described when a planet is retrograde, or moves contrary to the order of the signs.

Arch of *Position*, or Angle of Position, is the same with the horary angle.

Arch of *Vision*, is the sun's depth below the horizon at which a planet or star, before hid in his rays, begins to appear. This arch is different for different planets; being for Mercury 10°, Venus 5°, Mars 11¼°, Jupiter 10°, Saturn 11°; a star of the 1st magnitude 12°, 2d magnitude 13°, &c. This angle is not, however, constant in all cases for the same planet, but varies a little with the latitude and declination, &c. With respect to Venus, it is sometimes reduced to 0, as she is at times visible when the sun is some degrees above the horizon.

Arch of *Equilibrium*, in the *Theory of Bridges*, is that which is in equilibrio in all its parts, and therefore equally strong throughout, having no tendency to break in one part more than another. It is not of any determinate curve, but varies according to the figure of the extrados; every different extrados requiring a particular intrados, so that the thickness in every part may be proportional to the pressure. If the arch were equally thick throughout, the catenary curve would be the arch of equilibration; but as this can seldom or never happen, it is a mistaken idea to suppose this curve the best in all cases. It therefore appears, that when the upper side of the wall is a straight horizontal line, the equation of the curve is thus expressed:

$$y = h \times \frac{\log. \dfrac{a+x+\sqrt{(2ax+x^2)}}{a}}{\log. \dfrac{a+r+\sqrt{(2ar+r^2)}}{a}}:$$

where h = half the span, r = the height of the arch, y = the ordinate, x = the abscissa, and a = the thickness at the keystone; hence, when a, h, and r, are any given numbers, a table is formed for the corresponding values of x and y, by means of which the curve is constructed for any particular occasion.

And in a similar manner, if the curve of the intrados, and the depth of the key-stone be given, the equation of the extrados may be computed. Thus, for example, in the case where the intrados is a circle, the equation, readily deduced from the above construction, is

$$x = \frac{y\sqrt{(a^2+b^2-y^2)}}{\sqrt{(y^2-b^2)}};$$

where a = the height; and (putting n = half the span, and m = the height of the keystone) $\sqrt{(a+m)^2-n^2}=b$; the distance of the point from the centre of the arch $= x$, and its height above the centre $= y$. It will be found that a line at right angles to the perpendicular, and meeting it at the point where $\sqrt{(a+m)^2-n^2}$ falls, will be an assymptote to the extrados. Hence the extrados, in the case of a circular arch, is a curve of the fourth order, very much resembling the conchoid of Nicomedes, and that it coincides very nearly with the curve, in which a road is usually carried over a bridge. This holds good, of whatever portion of the circle the arch is supposed to consist.

ARCTIC *Circle*, in *Astronomy*, a small circle of the sphere parallel to the equator, and distant 23° 28′ from the arctic or northern pole.

Arctic *Pole*, the northern pole of the world.

AREA, in *Geometry*, is the superficial measure or surface of any figure. The areas of similar plane figures are to each other as the square of their like sides, or other lineal dimensions.

AREOMETER, an instrument for measuring the density or gravity of fluids. It is now commonly made of glass; consisting of a round hollow ball, which terminates in a long slender neck, hermetically sealed at top; there being first as much mercury put into it, as will serve to balance or keep it swimming in an erect position. The stem or neck is divided into degrees or parts, which are numbered, to show the specific gravity by the depth of its descent.

ARGUMENT, in *Astronomy*, is in general a quantity upon which another quantity or equation depends; or it is an arch, whereby we seek another unknown arch, bearing some proportion to the first: hence

Argument of *Inclination*, or Argument of *Latitude*, of any planet, is an arch of a planet's orbit, intercepted between the ascending node and the place of the planet from the sun, numbered according to the succession of the signs.

Menstrual Argument of *Latitude*, is the distance of the moon's true place from the sun's true place; by which is found the quantity of the real obscuration in eclipses.

Annual Argument of the *Moon's Apogee*, or simply *Annual Argument*, is the distance of the sun's place from the place of the moon's apogee; that is, the arc of the ecliptic comprised between those two places.

Argument of the *Parallax*, denotes the effect it produces on an observation, and which serves for determining the true quantity of the horizontal parallax.

Argument of the *Equation* of the *Centre*, is the anomaly, or distance, from the apogee or aphelion; because this equation is calculated in an elliptic orbit for every degree of anomaly, and varies according to the variation of the anomaly.

ARIES, or the *Ram*, in *Astronomy*, the first of the old 12 signs of the zodiac: it is marked ♈, in imitation of a ram's head. The sun enters this sign generally about the 20th of March.

ARITHMETIC, that part of mathematical science which treats of the nature and properties of numbers, the representing of them by proper symbols, and the application of them to the business of calculation. This science has undergone various improvements, and has partaken of the genius and language of the different nations among whom it has been used. Its origin, like that of all sciences, is involved in obscurity; nor is it fully ascertained to whom we are indebted for the improved system now in use.

Arithmetic is divided into various kinds, according to the nature of the numbers that form the subject of it; but the most simple form, and that which is the root or foundation of all the others, is the arithmetic of abstract or simple numbers. A number is called abstract when it merely answers to the question, "how many?" and has no allusion to the value of the said many, and no reference to things of any kind, but which admits of having the name and value of any kind of things whatever applied to it. In this sense, *one* is the smallest and simplest number that can be mentioned; and the other numbers proceed by constant additions of *one*, without limit. This being the case, the first thing requisite for the formation of an arithmetical language, is the invention of a limited number of original names, which shall comprehend a great and almost endless variety of numbers. For this purpose, what is usually called a "scale of numbers," becomes necessary, and the limits of this scale are the powers of any number, m which is taken as the modulus or root. Thus m^0, m^1, m^2, m^3, m^4, will form five ranks or places in the scale, any number whatever being taken for m; and the scale will receive its name according to the number so taken: thus,

If $m = 2$, the scale is *binary.*
If $m = 3$, the scale is *ternary.*
If $m = 7$, the scale is *senary.*
If $m = 10$, the scale is *denary.*
If $m = 12$, the scale is *duodenary.*

When the lower or any of the intermediate places is blank, it is supplied by a character (0) having no separate value, and another power of the modulus is given by adding this character.

Whatever may be the value of m, m^0 is always $= 1$. Hence, the first five places in each of the above scales, expressed in common numbers, or according to the *denary* scale, will be,

Denary	1,	2,	4,	8,	16
Ternary	1,	3,	9,	27,	81
Senary	1,	7,	49,	343,	2401
Denary	1,	10,	100,	1000,	10000
Duodenary	1,	12,	144,	1728,	20736

If, however, any of the scale were adopted, all these expressions would be the same; and each would require one character less than the number of *ones* in *m*, thus,

Denary	1,
Ternary	1, 2,
Senary	1, 2, 3, 4, 5, 6,
Denary	1, 2, 3, 4, 5, 6, 7, 8, 9,
Duodenary	1, 2, 3, 4, 5, 6, 7, 8, 9, φ, ϰ,

In which last φ stands for 10, and ϰ for 11. For the method of changing a number from one scale to another, see *Notation*.

Arithmetic receives various denominations, according to the nature of the numbers treated of: thus,

Simple or *Integral* arithmetic is the method of calculating by simple or abstract numbers.

Fractional arithmetic is the method of calculating by numbers which are less than the number 1, and is of two kinds, *vulgar* and *decimal*. For these, as well as for the arithmetic of *exponents, logarithms, sines*, &c. see those articles.

Whatever may be the nature of the numbers which form the subject of arithmetical computation, there are but two changes, of which any number is susceptible, namely, being made greater, and being made less; and, generally speaking, each of these may be performed in two different ways. When a number is increased by adding to it any number or numbers whatever, the process is called *addition*, and when it is increased by repeating itself any number of times, the process is called *multiplication*. When a number is diminished by taking away from it any number or numbers whatever, the process is called *subtraction*; and when it is diminished by taking away one or more numbers equal to that which is left, or when (which is the same thing,) the number left, if repeated a certain number of times, would produce the given number, the process is called *division*. These four processes are called the four fundamental operations, and the methods of performing them the four fundamental rules of arithmetic; and each of the latter is the converse of the corresponding one of the former. Subtraction is the converse of addition, and division the converse of multiplication. For the method of performing each of these, see the articles *Addition, Subtraction*, &c.

ARITHMETICAL, any thing relating to arithmetic.

ARITHMETICAL *complement of a Logarithm*. See *Anti*-LOGARITHM and COMPLEMENT.

ARITHMETICAL *Mean*, is the middle term of three quantities in arithmetical progression, or half the sum of any two proposed numbers.

ARITHMETICAL *Progression* and *Proportion*. See PROGRESSION and PROPORTION.

ARITHMETICAL *Ratio*, is the difference between any two adjacent terms in arithmetical progression.

ARITHMETICAL *Scales*. See NOTATION.

ARITHMETICAL *Triangle*. See TRIANGLE.

ARMILLARY *Sphere*, a name given to an artificial sphere, which represents the several circles of the system of the world, put together in their natural order.

The armillary sphere revolves upon an axis, within a horizon, which is divided into degrees, and moveable every way upon a brass supporter. The other parts are the equinoctial, zodiac, meridian, the two tropics, and the two polar circles.

ARTIFICIAL *Numbers*, a term sometimes used for logarithms, logarithmic sines, tangents, &c.

ASCENDING, in *Astronomy*, is understood of those stars, &c. which are rising above the horizon, in any parallel of the equator.

ASCENDING *Latitude*, is the latitude of a planet when going towards the north pole.

ASCENDING *Node*, is that point of a planet's orbit where it passes the ecliptic to proceed northward.

ASCENSION, in *Astronomy*, is either right or oblique.

Right ASCENSION *of the Sun or of a Star*, is that degree of the equinoctial (counted from the beginning of Aries,) which rises with the sun or star in a right sphere. Or, it is that degree and minute of the equinoctial, counted as before, which comes to the meridian with the sun or star.

The reason of thus referring it to the meridian is, because that is always at right angles to the equinoctial, whereas the horizon is only so in a right or direct sphere. The right ascension stands opposed to the right descension, and corresponds to the longitude of places on the earth. Two fixed stars, that have the same right ascension, that is, which are at the same distance from the first point of Aries, or which is still the same, are in the same meridian, rise at the same time in a right sphere, or with respect to persons living under the equator. If they be not in the same meridian, the difference between their rising, or coming to the meridian, is the precise difference of their right ascension. But in an oblique sphere, where the horizon cuts all the meridians obliquely, different points of the meridian never rise or set together; so that two stars, on the same meridian, never rise or set at the same time; and the more oblique the sphere is, the greater is the interval of time between them.

To find the right Ascension of the Sun or a Star.

Say, for the sun,
As radius
Is to the cosine of the sun's greatest declination, or obliquity of the ecliptic,
So is the tangent of the sun's longitude
To the tangent of the right ascension.

If the obliquity of the ecliptic, and the sun's declination were given, the proportion for the right ascension would be:

As radius
Is to the cotangent of the obliquity of the ecliptic,
So is the tangent of the sun's declination
To the sine of the right ascension.

The sun's right ascension in time is useful to the practical astronomer in regular observations, who adjusts his clock by sidereal time. It serves also for converting apparent into siderial time; as, for instance, that of an eclipse of Jupiter's satellites, in order to know at what time it may be expected to happen by the clock. For this purpose, the sun's right ascension for the preceding noon, together with the increase of right ascension from noon, must be added to the apparent time of a known star passing the meridian: then subtract the sun's right ascension in time at noon, from the star's right ascension in time, the remainder is the apparent time of the stars passing the meridian nearly: from which the proportional part of the daily increase of the same right ascension from his apparent time from noon being subtracted, leaves the correct time of the star's passing the meridian. The sun's right ascension in time is also useful for computing the time of the moon, or a planet's, passing the meridian.

The *Arch of Right* ASCENSION, is that portion of the equator, intercepted between the beginning of Aries and the point of the equator, which is the meridian: or it is the number of degrees contained in it.

Oblique ASCENSION, is an arch of the equator, intercepted between the first point of Aries, and that point of the equator which rises, together with the star, &c. in an oblique sphere. The oblique ascension is counted from west to east; and is greater or less, according to the different obliquities of the sphere.

Arch of Oblique ASCENSION, is the arch on which the oblique ascension is measured.

ASCENSIONAL *Difference*, is the difference between the right and oblique ascension.

To find the ascensional Difference of the Sun, having the Sun's Declination and the Latitude of the Place.

Say, As radius
Is to the tangent of the latitude,

So is the tangent of the sun's declination
To the sine of the ascensional difference.

When the latitude and declination have the same name, the difference between the right ascension, and the ascensional difference, is the oblique ascension; and their sum is the oblique descension; but when they are of contrary names, the sun is the oblique ascension, and the difference is the oblique descension.

ASCII, are those inhabitants of the globe, who, at certain times of the year, have no shadow; such are all those who inhabit the torrid zone.

ASPECT, in *Astronomy*, is the situation of the stars and planets with regard to each other. There are five principal aspects; which, with their respective characters, are as follows: viz.

☌, *Conjunction*, when the angle contained between any two planets is . . .	0
⚹, *Sextile*, when the angle is	60°
□, *Quartile*, when the angle is	90°
△, *Trine*, when the angle is	120°
☍, *Opposition*, when the angle becomes	180°

When the planets have exactly the distances described above, they are called *partile aspects;* and when the distances have not precisely these measures, they are called *platic aspects.*

ASPERITY, the roughness or inequality in the surface of bodies.

ASSURANCE *on Lives.* See Lives.

ASTEROIDS, in *Astronomy*, a name given by Dr. Herschel to the four new planets discovered by the foreign astronomers Piazzi, Olbers, and Harding.

ASTERISM, in *Astronomy*, an ancient term, signifying the same as Constellation.

ASTRÆA, in *Astronomy*, a name given by some authors to the sign Virgo.

ASTRAL, depending or belonging to the stars; as Astral *year*, &c.

ASTRODICTICUM, an astronomical instrument invented by Mr. Weighel, by means of which several persons may view the same star at the same time.

ASTROGNOSIA, signifies a knowledge of the fixed stars, their names, ranks, situations, &c.

ASTROLABE, the name of an ancient astronomical instrument, very much resembling our armillary sphere.

It is likewise the name of an instrument formerly much used at sea for ascertaining the altitude of the sun, stars, &c. which consisted of a brass ring about fifteen inches in diameter, graduated into degrees and minutes, and fitted with an index moveable about its centre, and carrying two sights; the whole being attached to a small brass ring for suspending the instrument at the time of observation.

Modern astronomers use the term Astrolabe, to denote a stereographic projection of the sphere, either upon the plane of the equator, the eye being supposed to be in the pole of the world; or upon the plane of the meridian, when the eye is supposed in the point of the intersection of the equinoctial and horizon.

ASTRONOMICAL, any thing relating to astronomy.

ASTRONOMY, a mixed mathematical science, which treats of the heavenly bodies, their motions, periods, eclipses, magnitudes, &c. and of the causes on which they depend. That part of the science which relates to the motions, magnitudes, and periods of revolution, is called *Pure* or *Plain Astronomy:* and that which investigates the causes and laws by which these motions are regulated, is called *Physical Astronomy.*

History of Astronomy.—The early history of this science, like that of all others of ancient date, is too much disfigured by fabulous and allegorical representations, to admit of any regular or satisfactory elucidation. It is probable, however, that some knowledge of this kind must have been nearly coeval with the formation of society.

Many traces of it have been found amongst various nations, which show that several of the most remarkable celestial phenomena must have been observed, and a knowledge of them disseminated, at a very remote period. But in what age or country the science first originated, or by whom it was gradually methodised and improved, is extremely uncertain.

M. Bailly, in his elaborate history of ancient and modern astronomy, endeavours to trace the origin of this science among the Chaldeans, Egyptians, Persians, Indians, and Chinese, to a very early period. And thence, he maintains, that it was cultivated in Egypt and Chaldea 2800 years before Christ; in Persia, 3209; in India, 3101; and in China, 2952 years before that æra. He also apprehends, that astronomy had been studied even long before this distant period, and that we are only to date its revival from this time.

In investigating the antiquity and progress of astronomy among the Indians, M. Bailly examines and compares four different sets of astronomical tables of the Indian philosophers; namely, that of the Siamese, explained by M. Cassini in 1689; that brought from India by M. le Gentil, of the Academy of Sciences; and two other manuscript tables, found among the papers of the late M. de Lisle; all of which he found to accord together, and all referring to the meridian of Benares. It appears that the fundamental epoch of the Indian astronomy, is a remarkable conjunction of the sun and moon, which took place at the distance of 3102 years before Christ: and M. Bailly informs us, that by our most accurate astronomical tables, such a conjunotion did really happen at that time. He farther observes, that at present the Indians calculate eclipses by the mean motions of the sun and moon commencing at a period 5000 years distant.

The solar year of the Brahmins of Tervalore is divided into twelve unequal months, each being equal to the time the sun occupies in moving through a sign; and in their calculations for a day, they employ the time the sun moves 1° in the ecliptic. Their sidereal year consists of 365^d, 6^h, 12^m, 30^s; and the tropical, of 365^d, 5^h, 50^m, 35^s. They assign inequalities to the motions of the planets, answering very well to the annual parallax, and the equation of the centre.

Most authors, however, fix the origin of astronomy and astrology either in Chaldea, or in Egypt; and, accordingly, among the ancients, we find the word Chaldean often used for astronomer, or astrologer. Both of these nations pretended to a very high antiquity, and claimed the honour of producing the first cultivators of this science. The Chaldeans boasted of their temple, or Tower of Belus, and of Zoroaster, whom they placed 5000 years before the destruction of Troy: while the Egyptians spoke with equal pride of their colleges of priests, where astronomy was taught; and of the monument of Osymandyas, in which, it is said, there was a golden circle of 365 cubits in circumference, and one cubit thick, divided into 365 equal parts, answering to the days of the year, &c.

From Chaldea and Egypt, the science of astronomy passed into Phenicia, and was by that people applied to the purposes of navigation, whence they became masters of the sea, and of almost all the commerce in the world. The Greeks, it is probable, derived their astronomical knowledge chiefly from the Egyptians and Phenicians, by means of several of their countrymen who visited those nations for the purpose of learning the different sciences.

Several of the constellations are mentioned by Hesiod and Homer, who lived 870 years before Christ. The knowledge of the Greeks in this science, was greatly improved by Thales the Milesian, and others, who travelled into Egypt. Thales was born about 640 years before Christ; and was the first among the Greeks who observed stars, the solstices, the eclipses of the sun and moon, and predicted an eclipse of the sun. The science was farther cultivated and extended by his successors, Anaximander, Anaximenes, and Anaxagoras; but most especially by Pythagoras, who having resided a long time in Egypt, &c. brought thence the learning of the Egyptians, taught the same in Greece and Italy, and founded the sect of the Pythagoreans. He taught that the sun was in the centre of the universe; that the earth was round, and people had antipodes; that the moon reflected the rays of the sun, and was inhabited like the earth; that comets were a kind of wandering stars, disappearing in the farther parts of their orbits; that the white colour of the milky way was owing to the united brightness of a great multitude of small stars; and he supposed that the distances of the moon and planets from the earth, were in certain harmonic proportions to one another.

Philolaus, a Pythagorean, who flourished about 450 years before Christ, asserted the annual motion of the earth about the sun; and not long after, the diurnal motion of the earth, on its own axis, was taught by Hicetus a Syracusian. About the same time, Meton and Euctemon flourished at Athens, where they observed the summer solstice 432 years before Christ; and observed the risings and settings of the stars, and to what seasons they answered. Meton also invented the cycle of nineteen years, which still bears his name.

Eratosthenes, who was born at Cyrene in the year 271 before Christ, measured the circumference of the earth; and being invited from Athens to Alexandria, by Ptolemy Euergetes, and made keeper of the royal library there, he set up for that prince those armillary spheres, which Hipparchus and Ptolemy afterwards employed so successfully in observing the heavens. He also determined the distance between the tropics to be $\frac{11}{83}$ of the whole meridian circle, which makes the obliquity of the ecliptic, in his time, to be $23^\circ\ 51\frac{1}{3}'$. The celebrated Archimedes also cultivated astronomy, as well as geometry and mechanics; and constructed a kind of planetarium, or orrery, to represent the phenomena and motions of the heavenly bodies.

Hipparchus, who flourished about 140 years before Christ, was the first who applied himself to the study of every part of this science; and, as we are informed by Ptolemy, made great improvements in it: he discovered that the orbits of the planets are eccentric, that the moon moved slower in the apogee than in her perigee, and that there was a motion of anticipation of the moon's nodes; he constructed tables of the motions of the sun and moon, collected accounts of such eclipses, &c. as had been made by the Egyptians and Chaldeans, and calculated all that were to happen for 600 years to come: he discovered that the fixed stars changed their places, having a slow motion of their own from west to east: he corrected the Calippic period, and pointed out some errors in the method of Erastosthenes for measuring the circumference of the earth; he computed the sun's distance more accurately than any of his predecessors: but his chief work is a catalogue which he made of the fixed stars, to the number of 1022, with their longitudes and latitudes, and apparent magnitudes; which, with most of his other observations, are preserved by Ptolemy in his Almagest.

But little progress was made in astronomy from the time of Hipparchus to that of Ptolemy, who was born at Pelusium in Egypt, in the first century of the Christian era, and who made the greatest part of his observations at the celebrated school of Alexandria in that country. Profiting by the observations of Hipparchus, and other ancient astronomers, he formed a system of his own, which, though erroneous, was followed for many ages by all nations: he compiled the Almagest, which contained the observations and collections of Hipparchus, and others of his predecessors in astronomy; a performance which will ever be valuable to the professors of that science. This work was preserved from the conflagration of the Alexandrian Library, and translated out of Greek into Arabic, in the year 827, and again into Latin in 1230.

From the year 800 till the beginning of the fourteenth century, the western parts of Europe were immersed in ignorance and barbarity, while the Arabians, profiting by the books they had preserved from the wreck of the Alexandrian Library, cultivated and improved all the sciences, and particularly astronomy, in which they had many able professors and authors; the caliph, Al Manor, first introduced a taste for the sciences into his empire; and his grandson, Al Mamon, who ascended the throne in 814, was a great encourager and improver of the sciences, and especially of astronomy. Having constructed proper instruments, he made many observations; determined the obliquity of the ecliptic to be 23° 35′; and under his auspices, a degree of the circle of the earth was measured a second time, in the plain of Singar, on the border of the Red Sea.

The settlement of the Moors in Spain introduced the sciences into Europe; from which time they have continued to improve, and to be communicated from one people to another, to the present time, when astronomy and all the sciences have arrived at a very eminent degree of perfection. The Emperor Frederick II. about 1230, first began to encourage learning, by restoring some decayed universities and founding a new one in Vienna: he also caused the works of Aristotle, and Ptolemy's Almagest, to be translated into Latin; and from the translation of this work we may date the revival of astronomy in Europe. Two years after this, John de Sacro Bosco, or John of Halifax, compiled from Ptolemy, Albategnius, Alferganus, and other Arabic astronomers, his work "De Sphæra," which was held in the greatest estimation for 300 years after, and was honoured with commentaries by Clavius and other learned men. In 1240, Alphonso, King of Castile, not only cultivated astronomy himself, but greatly encouraged others; and by the assistance of several learned men he corrected the tables of Ptolemy, and composed those which were denominated from him the Alphonsine Tables. About the same time also Roger Bacon, an English monk, wrote several tracts relative to astronomy, particularly of the lunar aspects, the solar rays, and the places of the fixed stars. And about the year 1270, Vitello, a Polander, composed a treatise on optics, in which he showed the use of refraction in astronomy.

Little other improvement was made in this science till the time of Purbach, who was born in 1423. He composed new tables of sines for every ten minutes, making the radius sixty, with four ciphers annexed. He constructed spheres and globes, and wrote several astronomical tracts, as a commentary on Ptolemy's Almagest; some treatises on arithmetic and dialling, with tables for various climates; new tables of the fixed stars, reduced to the middle of that century; and he corrected the tables of the planets, making new equations to them where the Alphonsine tables were erroneous. He had just finished a theory of the planets, when he died in 1462, being only thirty-nine years of age.

After Purbach, the subject of astronomy was cultivated by John Muller, commonly called Regiomontanus; by Bernard Walther, and John Werner; Copernicus was the next who made any considerable figure in astronomy. He very early conceived doubts of the Ptolemaic system, and entertained notions about the true one, which he gradually improved by a series of astronomical observations, and the contemplation of former authors. By these he formed new tables, and completed his work in the year 1530, containing these, and a renovation of the true system of the universe, in which all the planets are considered as revolving about the sun, as their common centre.

After the death of Copernicus, the science and practice of astronomy were greatly improved by Schoner, Nonius, Appian, Gemma Frisius, Byrgius, &c.; and about the year 1561, William IV. Landgrave of Hesse-Cassel, made a great number of observations, published by Snelius in 1618, and preferred by Hevelius to those of Tycho Brahe. From these observations he formed a catalogue of 400 stars, with their latitudes and longitudes, and adapted them to the beginning of the year 1593.

Tycho Brahe, a Dane, began his observations about the same time with the Landgrave of Hesse, and observed the great conjunction of Jupiter and Saturn: but finding the usual instruments very inaccurate, he constructed many others, much larger and more exact. In 1571, he discovered a new star in the chair of Cassiopeia; which induced him, like Hipparchus on a similar occasion, to make a new catalogue of the stars; which he composed to the number of 777, and adapted their places to the year 1600. Tycho invented a system to account for the planetary motions; but he is more to be noted on account of his accurate observations, which tended much to the discovery of the real nature of the planetary orbits.

While Tycho resided at Prague, with the emperor, he prevailed on Kepler to leave the University of Glatz and to come to him; and Tycho dying in 1601, Kepler enjoyed the title of mathematician to the emperor; who ordered him to finish the tables of Tycho Brahe, which he published in 1627, under the title of Rodolphine. He died about the year 1630, at Ratisbon, where he was soliciting the arrears of his pension. From his own observations, and those of Tycho, Kepler discovered several of the true laws of nature, by which the motions of the celestial bodies are regulated. He discovered that all the planets revolve about the sun, not in circular but in elliptical orbits, having the sun in one of the foci of the ellipse; that their motions are not equable, but varying quicker or slower, as they are near to the sun, or farther from him; that the areas described by the variable line drawn from the planet to the sun are equal in equal times, and always proportional to the times of describing them. He also discovered, by trials, that the cubes of the distances of the planets from the sun were in the same proportion as the squares of their periodical times of revolution. By observations also on comets, he concluded that they are freely carried about among the orbits of the planets, in paths that are nearly rectilinear; but which he could not then determine. See Dr. Small on the discoveries of Kepler.

About this time much was done by Wright, Napier, Bayer, Mercator, Maurolycus, Magnius, Homelius, Schulter, Steven, Galileo, Thomas and Leonard Digges, John Dee, Robert Hood, Harriot, &c.

The beginning of the seventeenth century was particularly distinguished by the invention of telescopes, and the application of them to the purposes of astronomy.

Hevelius, from his own curious observations, furnished a catalogue of fixed stars, much more complete than Tycho's. Huygens and Cassini discovered the satellites of Saturn and his ring. And Gassendus, Horrox, Bullialdus, Ward, Ricciolus, Gascoign, &c. each contributed very considerably to the improvement of astronomy.

Newton demonstrated, from physical consideration, the great law that regulates all the heavenly motions, sets bounds to the planetary orbs, determined their greatest excursions from the sun, and their nearest approaches to him. It was he who first discovered whence arose that constant and regular proportion, observed by both primary and secondary planets, in their circulation round their central bodies; and their distances compared with their periods. He also gave a new theory of the moon, which accounts for all her inequalities from the laws of gravity and mechanics.

Mr. Flamstead was appointed the first astronomer royal at Greenwich in 1675. He observed, for forty-four years, all the celestial phenomena, the sun, moon, planets, and fixed stars; of all which he gave an improved theory and tables. Cassini also, the first French astronomer royal, very much distinguished himself, making many observations on the sun, moon, planets, and comets, and greatly improved the elements of their motions. He also erected the gnomon, and drew the celebrated meridian line in the church of Petronia, at Bologna.

In 1719, Mr. Flamstead was succeeded by Dr. Halley, who had been sent, at the early age of 21, to the island of St. Helena, to observe the southern stars, a catalogue of which he published in 1679; and, a few years afterward, he gave to the public his "Synopsis Astronomiæ Cometicæ," in which he ventured to predict the return of a comet in 1758, or 1759. He was the first who discovered the acceleration of the moon's mean motion; and is the author of a very ingenious method for finding her parallax, by three observed places of a solar eclipse: he also showed the use that might be made of the approaching transit of Venus, in 1761, in determining the distance of the sun from the earth; and recommended the method of determining the longitude by the moon's distance from the sun and certain fixed stars, which has since been carried into execution at the instance of the late Astronomer Royal. Dr. Halley also composed tables of the sun, moon, and planets, with which he compared the observations he made of the moon at Greenwich, amounting to near 1500, and noticed the differences. About this time, an attempt was made in France to measure a degree of the earth, which was the occasion of a warm dispute concerning its figure. M. Cassini concluded, from the measurement of Picard, that it was an oblong spheroid; but Newton, from a consideration of the laws of gravity, and the diurnal motion of the earth, had determined its figure to be that of an oblate spheroid, flattened at the poles, and protuberant at the equator. To determine this point, Louis XV. ordered two degrees of the meridian to be measured; one under or near the equator, and the other as near as possible to the pole: the result of this arduous undertaking was a confirmation of Newton's investigation. It was found, by those who went to the south, that the attraction of the mountains of Peru had a sensible effect on the plumb-lines of their large instruments, which afforded an experimental proof of the Newtonian doctrine of gravitation, that has since been completely verified by the observations of Dr. Maskelyne, made on the mountain Schehallien, in Scotland. On the death of Dr. Halley, in 1742, he was succeeded by Dr. Bradley, who has rendered himself celebrated by two of the finest discoveries that have ever been made in astronomy,—the aberration of light, and the nutation of the earth's axis. Among other things he also formed new and accurate tables of the motions of Jupiter's satellites, as well as the most correct table of refractions yet extant. Also, with a large transit instrument, and a new mural quadrant of eight feet radius, constructed by Bird, in 1750 he made an immense number of observations for settling the places of all the stars in the British catalogue, together with near 150 places of the moon, the greater part of which he compared with Mayer's tables. Dr. Bradley was succeeded in 1762, in his office of astronomer royal, by Mr. Bliss; but who died in 1765, and was succeeded by Nevil Maskelyne, D. D. who rendered very considerable services to the science, by his publication of the "Nautical Almanac," the "Requisite Tables," &c.; and more particularly by his great assiduity and zeal, in bringing the lunar method of determining the longitude at sea into general practice.

In the mean time, many other eminent mathematicians were assiduously employed in endeavouring to promote the science of astronomy. Amongst these may be particularly distinguished, Clairaut, d'Alembert, Euler, Simpson, Walmsley, Mayer, de la Caille, Manfredi, Lambert, &c.

Such was the state of astronomy when Dr. Herschel, by augmenting the powers of telescopes beyond the most sanguine expectations, opened a scene altogether unlooked for, by the discovery of a new planet, 12th March, 1781, the *Georgium Sidus*, or *Uranus*.

On the 1st of January, 1801, another new planet was discovered between Mars and Jupiter, by M. Piazzi, of Palermo, which is named Ceres; and since this time three others have been observed, revolving also between Mars and Jupiter, *viz.* Pallas, discovered by Dr. Olbers, March 28, 1802; Juno, first observed by Mr. Harding, at the observatory at Lilienthal, near Bremen, Sep. 1, 1804; and Vester, discovered by Dr. Olbers, 29th of March, 1807, being the second that we owe to this eminent astronomer. For the particular elements of these new planets, see the respective articles URANUS CERES, PALLAS, JUNO, and VESTER.

Hence it appears, that within a few years the number of planets in our system have been nearly doubled, and many other important and interesting discoveries have been made during the same period: yet it must be acknowledged, that we are still unacquainted with many particulars, and which therefore still remain to exercise the talents of modern astronomers. We have not yet determined the times of rotation, and the proper figures of some of the planets and their satellites; nor do we know, with sufficient precision, the masses of those bodies. The theory of their motions also consists in a series of approximations, of which the convergence depends both upon the perfection of the instruments and the progress of analysis, and which for that reason ought to acquire continually new degrees of exactness.

ASTROSCOPE, an astronomical instrument, composed of two cones, on whose surfaces are exhibited the stars and constellations, by means of which they are both easily found in the heavens.

ASYMPTOTE, a right line to which some curve continually approaches nearer, in such sort, that when they are both indefinitely produced, they are nearer together than by any assignable finite distance; or it may otherwise be considered as a tangent to the curve, when infinitely produced, or at an infinite distance. Two curves are also said to be asymptotical, when they thus continually approach indefinitely to a coincidence: thus two parabolas, placed with their axis in the same right line, are assymptotes to each other.

Of lines of the second kind, or curves of the first kind, that is, the conic sections, only the hyperbola has asymptotes, which are two in number. All curves of the second kind have at least one asymptote, but they may have three; and all curves of the third kind may have four asymptotes, and so on. The conchoid, cissoid, and logarithmic curve, though not geometrical curves, have each one asymptote; and the branch or leg of the curve that has an asymptote, is said to be of the hyperbolic kind.

The nature of an asymptote is very difficult to be conceived, by persons who are not acquainted with the higher geometry: they cannot comprehend how two lines should always continually approach each other, without the possibility of touching or coinciding; this mystery, however, may be elucidated, and the nature of these lines readily comprehended, by considering the generation of the conchoid of Nicomedes. *See Conchoid.*

ASYMPTOTES, by some are distinguished into various orders. The asymptote is said to be of the first order, when it coincides with the base of the curvilinear figure; of the 2d order, when it is a right line parallel to the base; of the 3d order, when it is a right line oblique to the base; of the 4th order, when it is the common parabola, having its axis perpendicular to the base; and, in general, of the $n+2$ order, when it is a parabola whose ordinate is always as the n power of the base. The asymptote is oblique to the base, when the ratio of the first fluxion of the ordinate to the fluxion of the base, approaches to an assignable ratio, as its limit; but it is parallel to the base, or coincides with it, when this limit is not assignable.

The areas bounded by curves and their asymptotes, though indefinitely extended, have sometimes limits to which they may approach indefinitely near: and this happens in hyperbolas of all kinds, except the first or Apollonian, and in the logarithmic curve. But in the common hyperbola, and many other curves, the asymptotical area has no such limit, but is infinitely great. Solids, too, generated by hyperbolic areas, revolving about their asymptotes, have sometimes their limits; and sometimes they may be produced till they exceed and given solid. Also the surface of such solid, when supposed to be infinitely produced, is either finite or infinite, according as the area of the generating figure is finite or infinite.

The way of discovering whether any proposed curves have asymptotes, and the manner of drawing them when they are inclined to the axis, may be easily derived from the method of tangents.

ATMOSPHERE, that gaseous or aeriform fluid which every where invests the surface of the terraqueous globe; and partakes of all its motions, both annual and diurnal.

We have already considered the mechanical properties of this fluid, under the article AIR; and it therefore only remains to treat of it as forming one body, *viz.* its figure, pressure, altitude, &c.

Figure of the Atmosphere.—As the atmosphere envelopes all parts of the surface of the earth, if both the one and the other were perfectly at rest, and were not endowed with a diurnal motion round their axis, the atmosphere would be exactly spherical, according to the universal laws of gravity; for the parts of the surface of a fluid in a state of rest, must be equally remote from its centre. But the earth and the ambient atmosphere are invested with a diurnal motion, which carries them round their common axis of rotation; and the different parts of both having a centrifugal force, the tendency of which is more considerable, and that of the centripetal force less, as the parts are more remote from the axis; and, consequently, the figure of the atmosphere must become that of an oblate spheroid, because the parts that correspond to the equator have a greater centrifugal force than the parts which correspond to the poles. Besides, the figure of the atmosphere must represent such a spheroid, in consequence of the sun striking the equatorial regions more directly than those about the poles: whence it follows, that the mass of air, or part of the atmosphere, about the polar regions, being less heated, cannot expand so much, nor reach so high; nevertheless, as the same force which contributes to elevate air, diminishes the pressure on the surface of the earth, higher columns of it at or near the equator, all other circumstances being the same, will not be heavier than those of the lower belonging to the poles; but, on the contrary, without some compensation they would be lighter, in consequence of the diminished gravity of the upper strata.

Weight or Pressure of the Atmosphere.—Torricelli found that the pressure of the atmosphere sustains a column of quicksilver, of an equal base and 39 inches height; and as a cubical inch of quicksilver is found to weigh near half a pound avoirdupoise, therefore the whole 30 inches, or the weight of the atmosphere on every square inch, is nearly equal to 15lb. Again, it has been found that the pressure of the atmosphere balances, in the case of pumps, &c. a column of water of about 34½ feet high; and, the cubical foot of water weighing just 1000 ounces, or 62½lb. 34½ times 62½, or 2158lb. will be the weight of the column of water, or of the atmosphere on a base of a square foot; and, consequently, the 144th part of this, or 15lb. is the weight of the atmosphere on a square inch; the same as before. Hence the pressure of this ambient fluid on the whole surface of the earth, is equivalent to that of a globe of lead of 60 miles in diameter. And hence also it appears, that the pressure upon the human body must be very considerable; for, admitting the surface of a man's body to be about 15 square feet, and the pressure about 15lb. on a square inch, he must sustain 32,400lb. or nearly 14½ tons weight for his ordinary load. And it might be easily shown, that the difference in the weight of air sustained by our bodies in different states of the atmosphere, is often near a ton and a half.

Height and Density of the Atmosphere.—The densities of the air decrease in geometrical progression, as the altitudes increase in arithmetical progression; and, therefore, if no other cause existed, it would follow that the atmosphere was of indefinite height. But this cannot be, in consequence of the other planetary bodies: our atmosphere, for instance, cannot extend beyond the common centre of attraction of the earth and moon; for if in the first instance we conceive it to surpass this limit, it is obvious, that as the earth revolves on its axis, and thereby turns all its parts successively towards the moon, this body, in consequence of its superior attraction beyond that point, would draw that part of our atmosphere towards her own centre, and either leave a vacuum between the terrestrial and lunar atmospheres, or the limits of both would be the common centre of attraction of the two bodies. Another cause, *viz.* the centrifugal force, would also operate against an indefinitely extended atmosphere; for as this fluid partakes of the diurnal motion of the earth, it is obvious, that beyond that point where the centrifugal force is equal to the force of gravity, the fluid would be thrown off by the rotatory motion of the body, and the limits of the atmosphere terminated in that point.

If the air was every where of the same uniform density as at the earth's surface, where its specific gravity to that of water is about as 3 to 2500; or where a cubic foot weighs 1⅕ ounces, it would follow, that its altitude would be about 5½ miles: for the whole atmospheric pressure is equal to about 33 or 34 feet of water; and the density of this latter fluid being about 833⅓ times greater than that of air, we should have $833\frac{1}{3} \times 33 = 27500$ feet, for the height of a uniform column; which is a little more than 5¼ miles. But the density decreases as the altitudes increase, the former in geometrical, and the latter in arithmetical progression. We have also, under the article **Altitude**, shown that the general formula for ascertaining altitudes above the earth's surface, at the temperature of 31°, is $A = 10000 \times \log. \frac{m}{M}$; A being the altitude, and m and M the heights of the barometer, the former at the lower place and the latter at the top of the eminence, which are also as the densities of the air at those places, and therefore conversely to find the density of the air corresponding to any particular altitudes, we may change the formula into $A = 10000 \log. m - 10000 \log. M$; whence

$$\log. m. = \frac{A + 10000 \log. M.}{10000}$$

From which formula is deduced the following table, which exhibits the comparative density of the air at the several corresponding heights, *viz.*

Height in Miles.	No. Times rarer.
0	1
3½	2
7	4
14	16
21	64
28	256
35	1024
42	4096
49	16384
56	65536
63	262144
70	1048576

And by pursuing the calculation in this table, it might easily be shown, that a cubic inch of the air we breathe would be so much rarefied at the height of 500 miles, that it would fill a sphere equal in diameter to the orbit of Saturn.

With regard to the extent of the atmosphere, from the principles upon which our calculation is founded it is indefinite; but it must of necessity have a certain limit. For one of the principal effects of the atmosphere being the refraction of light, the particles of which are the smallest of any we know of in nature, it is reasonable to fix the boundary of the atmosphere where it begins to have the effect of bending the rays of light. Now Kepler, and after him La Hire, computed the height of the sensible atmosphere from the duration of the twilight, and from the magnitude of the terrestrial shadow in lunar eclipses, and found that it was sufficiently dense at a height of between 40 and 50 miles, to reflect and intercept the light of the sun. So far, therefore, we may be certain that the atmosphere extends; and at that altitude we may collect, from what has been already said, that the air is more than 10,000 times rarer than at the earth's surface; but how much farther it may be extended, is totally unknown.

Refractive and Reflective Powers of the Atmosphere.—The atmosphere has a refractive power, which is the cause of various phenomena. This power is ascertained by the production of twilight, and by many other facts and experiments. It has also a reflective power, and this power is the cause of objects being so uniformly enlightened on all sides. Were it not for this, the shadows of objects would be so dark, and their enlightened sides so very bright, that probably we should only be able to see those parts of them which were absolutely exposed to the sun's rays, if indeed the extreme light in this case did not even render them too powerful for the delicacy of the optic nerve.

Temperature of the Atmosphere.—The temperature of the atmosphere diminishes, as the distance from the earth increases, though apparently in a less ratio. M. de Saussure found, that by ascending from Geneva to Chamouni, a height of 347 toises, Reaumer's thermometer fell 4° 2′; and that, on ascending from thence to the top of Mont Blanc, 1911 toises, it fell 20° 7′: this gives 221 feet English for a diminution of 1° of Fahrenheit, in the first case, and 268 in the second. Nevertheless, from the accuracy which the rule for barometical measurement possesses, it may be inferred, that the decrease of heat for the greatest heights which we can reach, is not far from uniform; but that the rate for any particular case must be determined by observation, though the average in our climate may be stated at 1° for 270 feet of perpendicular ascent.

Professor Leslie has given a formula for determining the temperature of any stratum of air when the height of the mercury in the barometer is given. The column of mercury at the lower of two stations being b, and at the upper β, and the diminution of heat, in degrees of the centigrade thermometer, is $2.5\left(\frac{b}{\beta}-\frac{\beta}{b}\right)$, which seems to agree well with observation.

The mean temperature of the atmosphere in any parallel of latitude remains nearly constant, but it decreases from the equator to either pole; and if t be made to represent the mean temperature of any parallel of which the latitude is L, M the mean temperature in the latitude of 45°, and M + E the mean temperature at the equator; then is

$$t = M + E \,.\, \cos.\, 2\,L;$$

whence the mean temperature in any latitude is readily ascertained. The mean temperature in latitude 45° is 58° = M, at the equator it is 85°, whence $85° - 58° = 27° = E$; therefore $t = 58° + 27° \times \cos.\, 2\,L$, which, when $2\,L > 90$, the cosine being negative, is less than 58°. But if the place is at any height above the surface, then the formula becomes

$$t = M - \frac{H}{270} + E \,.\, \cos.\, 2\,L;$$

M and E being still the same as above, and H the height of the place in English feet.

On ascending into the atmosphere, at a certain height in every latitude a point is found where it always freezes, or where it freezes more than it thaws, so that the mean temperature does not exceed 32°, and the curve joining or passing through all those points is called the *curve of perpetual congelation*. The equation to which is found by making

$$32 = M - \frac{H}{n} + E \,.\, \cos.\, 2\, L,$$

and this line, at the equator, is elevated 15577 feet above the level of the sea.

$$\text{Whence } n = \frac{H}{E \,.\, \cos.\, 2\, L + M - 32}$$

$$= \frac{15577}{27 + 58 - 32} = 294,$$

and conseq. $H = 7642 + 7933 \,.\, \cos.\, 2\, L$. Which formula seems to agree very nearly with actual observation.

See Playfair's "Outlines of Natural Phil." p. 285; see also a different formula for expressing the line of perpetual congelation, Leslie, "Elements of Geometry," 2d edition, p. 495.

ATMOSPHERE of the *Planets*.—Since the planets and their satellites are allowed to be bodies of a nature similar to the earth we inhabit, there are few who attempt to deny that the planets are surrounded with atmospheres analogous, in most respects, to that whose properties have been explained in the preceding articles. These atmospheres are flattened towards the poles, and protuberant at the equator. But this oblateness has its limits; and in the case where it is greatest, the ratio of the polar and equatorial diameter is as 2 to 3. The atmosphere cannot extend itself at the equator to a greater distance than to the place where the centrifugal force is exactly equal to the force of gravity. With regard to the sun, this point is remote from its centre to a distance measuring the radius of the orbit of a planet which would make its revolution in the same period as that luminary employs in its rotation. The solar atmosphere cannot, therefore, extend to the orbit of Mercury; and consequently it cannot produce the zodiacal light, which appears to extend even to the orbit of the earth.

ATMOSPHERIC *Tides*, are certain periodical changes in the atmosphere, similar, in some respects, to those of the ocean, and produced in a great measure by the same causes.

ATOM, a particle of matter indivisible, on account of solidity, hardness, and impenetrability, which preclude all division, and leave no vacancy for the admission of any foreign force to separate or disunite its parts.

ATOMICAL *Philosophy*, is the doctrine of Atoms; a system which accounts for the origin and formation of things from the hypothesis, that atoms are endowed with weight and motion.

ATTRACTION, in *Physics*, a general term used to denote the cause, power, or principle, real or imaginary, by which all bodies mutually tend towards each other, and cohere, till separated by some other power. The laws, phenomena, &c. of attraction, form the chief subject of Newtonian philosophy, these being found to obtain in almost all the wonderful operations of nature.

The principle of attraction, in the Newtonian sense of it, was first hinted at by Copernicus. Kepler calls gravity a corporeal and mutual affection between similar bodies, in order to their union. And he pronounced more positively, that no bodies whatever were absolutely light, but only relatively so; and, consequently, that all matter was subjected to the power and law of gravitation.

The first who, in this country, adopted the notion of attraction, was Dr. Gilbert, in his book De Magnete; and the next was the celebrated Lord Bacon. In France it was received by Fermat and Roberval; and in Italy, by Galileo and Borelli. But till Newton appeared, this principle was very imperfectly defined and applied.

Before Newton, no one had entertained such correct and clear notions of the doctrine of universal attraction as Dr. Hooke, who observes, that the hypothesis upon which he explains the system of the world is founded upon the three following principles: 1. That all the celestial bodies have not only an attraction or gravitation towards their proper centres, but that they mutually attract each other within their sphere of activity. 2. That all bodies which have a simple and direct motion, continue to move in a right line, if some force, which operates without ceasing, does not constrain them to describe a circle, an ellipse, or some other more complicated curve. 3. That attraction is so much the more powerful, as the attracting bodies are nearer to each other. But Hooke was not able to solve the general problem relative to the law of attraction, which would occasion a body to describe an ellipse round another quiescent body placed in one of its foci; this discovery being reserved for Newton.

Attraction may be considered as it regards celestial bodies, terrestrial bodies, and the minuter particles of bodies. The first case is usually denoted by the word attraction, or universal gravitation, the second by gravitation, and the third by the words affinity, chemical attraction, or molecular attraction. Many philosophers are now of opinion, that it is the same force contemplated under different aspects, yet constantly subject to the same law.

At a finite distance, all the bodies in nature are said to attract one another in the direct ratio of the masses, and the inverse ratio of the square of the distance.

According to a law of Kepler, deduced from observation, the radii vectores of planets and comets describe about the sun areas proportional to the times; but this law can only have place so long as the force which incessantly deflects each of these bodies from the right line is constantly directed towards a fixed point, which is the origin of the radii vectores. The tendency, therefore, of the planets and comets towards the sun, follows necessarily, from the proportionality of the areas described by the radii vectores to the times of description: this tendency is reciprocal. It is, in fact, a general law of nature, that action and re-action are equal and contrary: whence it results, that the planets and comets re-act upon the sun, and communicate to it a tendency towards each of them.

The satellites of Uranus tend towards Uranus, and Uranus towards his satellites: the satellites of Saturn tend towards Saturn, and Saturn towards them. The case is the same with regard to Jupiter and his satellites. The earth and moon tend likewise reciprocally the one towards the other. The proportionality of the areas described by the satellites to the times of description, concur with the equality of action and re-action, to render these assertions unequivocal.

All the satellites have a tendency towards the sun; for they are all animated by a regular motion about their respective planets, as if they had been immoveable; whence it results that the satellites are impelled with a motion common also to their planets; that is to say, that the same force by which the planets tend incessantly towards the sun, acts also upon the satellites, and that they are carried towards the sun with the same velocity as the planets. And since the satellites tend towards the sun, it follows that the sun tends towards them, because of the equality of action and re-action.

Observations have convinced us that Saturn deviates a little from his path when he is near Jupiter; whence it follows, that Saturn and Jupiter tend reciprocally the one towards the other.

It therefore appears, that all the heavenly bodies tend reciprocally towards one another: but this tendency, or rather the attractive force which occasions it, appertains not solely to their aggregate mass; all their moleculæ partake of it, or contribute to it. If the sun acted exclusively upon the centre of the earth, without attracting each of its particles, the undulations of the ocean would be incomparably greater, and very different from those which are daily presented to our view. The tendency of the earth towards the sun is, therefore, the result of the sum of the attractions exerted upon all the moleculæ, which consequently attract the sun in the ratio of their respective masses;

besides, every body upon the earth is attracted towards its centre proportionally to its mass. It re-acts therefore upon it, the attraction following the same ratio. If it were otherwise; if all the parts of the earth did not exert upon one another a reciprocal attraction, the centre of gravity of the earth would move by a constantly-accelerating motion, till at length it would be lost beyond the limits of the system.

The attraction is therefore universal, reciprocal, and proportional to the mass. It remains to demonstrate, that this force is inversely as the square of the distance.

Observations have shown, that the squares of the periodic times of the celestial bodies are proportional to the cubes of the mean distances. Farther, it is rigorously demonstrable, that when bodies circulate in such manner that the squares of the periodic times are proportional to the cubes of the distances, the central force which actuates them is in the inverse ratio of the square of the distance: therefore, supposing the planets to move in circular orbits (from which they, in fact, differ but little), they are solicited towards the sun by a force which varies inversely as the square of the distance. This supposition is not rigorous. But the constant relation of the squares of the periodic times, to the cubes of the distances, being independent of the eccentricity, would doubtless subsist in the case where the eccentricity vanishes, that is, if the planets moved in circular orbits. Indeed, the truth of the position may be readily established with regard to elliptical orbits: but we omit the demonstration, rather than protract this article to too great a length.

If the planets revolve about the sun in virtue of a central force, which is reciprocally as the square of the distance, it is natural to infer that the moon is retained in her orbit by a central force directed towards the earth, and which only differs from the gravity of terrestrial bodies, in the ratio of the diminution that is occasioned by the augmentation of the square of the moon's distance. Now, it may be shown that the revolution of the moon about the earth is a phenomenon of the same kind and to be accounted for in the same manner (that is, by considering the joint operation of the projectile and gravitating forces) as the curvilinear motion of a stone, bullet, or any other projectile near the surface of the earth. If we had engines of a sufficient force to project a body in a right line parallel to the horizon, with the velocity of 24326 Paris feet (nearly five English miles) in a second of time, that body, setting aside the resistance of the air, would revolve about the earth like a moon. For, 24326 is a mean proportional between 39231600, the diameter of the earth, and $15\frac{1}{12}$, the space described in the first second of time by a heavy body falling from quiescence towards the earth. And the periodical time of such a projectile would be nearly equal to 1 hour, 24 minutes, 27 seconds. If this body could be carried to the distance of the moon, and projected in the same direction as that in which the moon moves, with such a velocity as would carry it through 188489 Paris feet in a minute, it would revolve about the earth in the same orbit as is described by the moon. We know from experience, that the motion with which a body near the surface of the earth tends to its centre, is such as in a second of time makes it descend through $15\frac{1}{12}$ Paris feet. Supposing this motion to decrease inversely as the square of the distance; at the distance of the moon, which is equal to 60 semidiameters of the earth, it would be 60×60 times less than at the surface of the earth; and therefore at that distance would be sufficient to make a body descend through $25\frac{1}{12}$ Paris feet in a minute of time. This is, in fact, the space through which the moon at the distance of 60 semidiameters of the earth, descends from the tangent of its orbit, towards the centre of the earth in a minute of time. For that space is a third proportional to the diameter of the moon's orbit, and the arc described in the same time. And 235389600 (the diameter of the moon's orbit in Paris feet) is to 188489 (the arc described in a minute,) as 188489 is to 15$\frac{1}{12}$. Thus the motion agrees in quantity as well as in direction, with the legitimate inferences from the motions of projectiles near the earth. And these phenomena are so perfectly coincident and similar, that they must be referred to the same principles, namely, a projectile force and a gravitating force varying inversely as the squares of the distances.

ATTRACTION of *Mountains*.—According to the Newtonian theory of attraction, this principle pervades the minutest particles of matter, and the combined action of all the parts of the earth forms the attractions of the whole. For the same reason, therefore, that a heavy body tends downwards in a perpendicular to the earth's surface, it must be attracted towards the centre of a neighbouring mountain by a force greater or less, according to the quantity of matter contained in it; and the effect of this attraction, or the accelerative force produced by it, must depend on the distance of the mountain from the gravitating body, because this force increases as the squares of the distances decrease. Upon these principles, it is obvious that the plumb of a quadrant, or of any other astronomical instrument, must be deflected from its proper situation, by a small quantity, towards the mountain; and the apparent altitudes and zenith distances of the stars, taken with the instrument, be altered accordingly; viz. if the zenith distance of a star were observed at two stations, under the same meridian, one on the south side of a mountain, the other on the north; and the plumb-line of the instrument were attracted out of its vertical position by the mountain, the star must appear too much to the north, by the observation at the southern station, and too much to the south, by that at the northern station; and, consequently, the difference of the latitudes of the two stations resulting from the observations, would be greater than it really is. If, then, the true difference of their latitudes be determined by measuring the distance between the two stations on the ground, the excess of the difference, found by the observations of the star above that found by this measurement, must have been produced by the attraction of the mountain; and the half of it will be the effect of such attraction on the plumb-line at each observation, provided that the mountain attracts equally on both sides.

The first idea for determining the quantity of this attraction was suggested by Newton, in his treatise of the System of the World; but no farther notice was taken of it till M. Bougner and M. de la Condamine, in the year 1738, while they were employed in measuring three degrees of the meridian, near Quito in Peru, thought they perceived a deflection of their plumb-line from the effect of the attraction of Chimboraco, a mountain in that neighbourhood, which by a rough computation they supposed to be equal to about the 2000th part of the attraction of the whole earth.

Nothing was afterwards done, till Dr. Maskelyn made a proposal to the Royal Society for this purpose, in the year 1772; and, in 1774, he was deputed to make the trial, accompanied with proper assistants, and furnished with the most accurate instruments. For the scene of his observations, he made choice of the mountain Schehallien, in Scotland, the direction of which is nearly from east to west: its mean height above the surrounding valley about 2000 feet, and its highest part above the level of the sea 3550 feet. Two stations for observations were selected, one on the north and the other on the south side of the mountain. Every circumstance that could contribute to the accuracy of the experiment was regarded; and from the observation of ten stars near the zenith, it was found that the apparent difference of the altitudes of the two stations was 54′ 6″; and from a measurement by triangles, formed from two bases on different sides of the mountain.

These data seemed to offer the means of determining the mean density of the earth; but the calculation necessarily required great accuracy, as well as immense labour. The task, however, was undertaken by Dr. Hutton, who gave an account of it, and the result of his investigation, in the Phil. Trans. as also in his Tracts lately published, from which it appears, that the mean density of the earth is to that of common water in about the ratio of 5 to 1.

Sir Richard Phillips (Essays, p. 8. *et seq.*) treats the doctrine of attraction as a mere assumption; and he sums up the objections which may be urged against it in these words:

1. "That matter is not itself an agent, or source of power.

2. "That it has no consciousness of any distant matter to be moved or attracted.

3. "That it has no means of knowing the relative quantity, and of moving accordingly.

4. "That the direction of motion in bodies said to attract one another, is from their further, remote, or opposite sides; consequently, each is assumed to receive a mutual impulse from parts in which neither are situated.

5. "That, through a vacuum, no obligation of equal mechanical momenta, in a system of bodies exists, such as we discover to be the fact in nature.

6. "That the sun, planets, and satellites, do not fall together, as the system requires.

7. "That, for the first four reasons, bodies do not attract or repel one another in any other branch of philosophy, more than in physics."

Centre of **Attraction.** See **Centre.**

ATTRITION, the striking or rubbing of bodies against each other.

ATWOOD's *Machine*, a very ingenious apparatus invented by the late Mr. Atwood, of Trinity College, Cambridge, to illustrate the doctrine of accelerated motion.

AVOIRDUPOISE *Weight*, a weight used in England for weighing all the larger and coarser sorts of goods; and all metals, except gold and silver. Avoirdupoise weight is thus divided:

16 dr. or drams, 1 ounce . . oz.
16 oz. . . . . 1 pound . . lb.
28 lb. . . . . 1 quarter . . qr.
4 qrs. . . . 1 hun. weght cwt.
20 cwt. . . . 1 ton.

The avoirdupoise is less than the troy ounce, in the proportion of 700 to 768; but the avoirdupoise pound is greater than the troy pound, in the proportiou of 700 to 576; *viz.*

700*oz.* troy = 768*oz.* avoirdupoise.
700*lbs.* troy = 576*lbs.* avoirdupoise.

AURORA, the morning twilight, or that faint light which appears in the morning when the sun is within eighteen degrees of the horizon. It is produced by the refractive power of the atmosphere.

Aurora *Borealis*, or **Northern** *Light*, sometimes also called *streamers*, is an extraordinary meteor, or luminous appearance, showing itself in the night-time in the northern parts of the heavens, and most commonly in frosty weather.

It is usually of a reddish colour inclining to yellow, and sends out frequent corruscations of pale light, which seem to arise from the horizon in pyramidal undulating forms, and shoots with great velocity up to the zenith.

AUSTRAL, the same as *southern*.

AUTOMATON, a seemingly self-moving machine; or one so constructed, by means of weights, levers, pulleys, springs, &c. as to move for a considerable time as if it were endued with animal life. And according to this description, clocks, watches, and all machines of that kind, are automata.

AUTUMN, the third season of the year: this begins at the descending equinox, which, in the northern hemisphere, is when the sun enters the sign Libra, about the twenty-second of August, and ends about the same day in December.

AUTUMNAL, something relating to autumn.

Autumnal *Equinox*, the time when the sun enters the descending point of the ecliptic where it crosses the equinoctial.

Autumnal *Point*, the point of the ecliptic answering to the autumnal equinox.

Autumnal *Signs*, are the signs Libra, Scorpio, Sagittary, through which the sun passes during the autumn.

AXIOM, a self-evident truth, or a proposition, the truth of which is perceived at first sight. It is also an established principle in some science. Thus, it is an axiom in geometry that things which are equal to the same thing, are equal to each other; that if to equal things, equal things be added, the wholes will be equal, &c.

AXIS, in Astronomy, an imaginary right line, supposed to pass through the earth, sun, planets, satellites, &c. and about which they perform their respective diurnal rotations.

The earth and planets, in their progress through the annual orbit, move in such a manner that the axis of each always keeps parallel to itself, or points to the same parts of the heavens.

The axis of the earth is inclined to the ecliptic, in an angle of nearly 66½°, a position which is well adapted for promoting the fertility of the earth and rendering it habitable.

Axis of the *Horizon*, *Equator*, &c. is a right line drawn through the centre of the respective circle, perpendicular to its plane.

Axis, in *Geometry*, the straight line in a plane figure, about which it revolves, to produce or generate a solid.

Axis is yet more generally used for a right line conceived to be drawn from the vertex of a figure to the middle of the base. So the

Axis of a *Circle* or *Sphere*, is any line drawn through the centre, and terminated at the circumference, on both sides.

Axis of a *Cone*, is the line from the vertex to the centre of the base.

Axis of a *Cylinder*, is the line from the centre of the one end to that of the other.

Axis of a *Conic Section*. See Conic *Section*.

Transverse Axis, in the *ellipse* and *hyperbola*, is the diameter passing through the two foci, and the two principal vertices of the figure. In the hyperbola it is the shortest diameter, but in the ellipse it is the longest.

Conjugate Axis, or *Second* Axis, in the *ellipse* and *hyperbola*, is the diameter passing through the centre, and perpendicular to the transverse axis. It is the shortest of all the conjugate diameters.

Axis of a *Curve Line*, is still more generally used for that diameter which has its ordinates at right angles to it, when that position is possible.

Axis, in *Mechanics*, a certain line about which a body may turn. Axis are of various kinds; as

Axis of a *Balance*, the line upon which it moves or turns.

Axis of *Rotation*, the line about which a body really revolves, when it is put into motion. The impulse given to a homogeneous sphere, in a direction which does not pass through its centre, will cause it to revolve constantly round the diameter, which is perpendicular to a plane passing through its centre, and the line of direction of the impressed force. New forces acting on all its parts, and of which the result passes through its centre, will not change the parallelism of its axis of rotation. Thus it is that the axis of the earth remains always nearly parallel to itself in its revolution round the sun, without its being necessary to suppose, with Copernicus, an annual motion of the poles of the earth round those of the ecliptic.

If the body possess a certain figure, its axis of rotation may change every instant. The determination of these changes, whatever may be the forces acting on the bodies, is one of the most interesting problems of mechanics respecting hard bodies, on account of its connection with the precession of the equinoxes, and the libration of the moon. The solution of this problem has led to a curious and very useful result; namely, that in all bodies there exist three axes perpendicular to each other, round which the body may turn uniformly when not solicited by external forces. On this account these axes are properly called principal axes of rotation.

Axis of *Oscillation*, is a line parallel to the horizon, passing through the centre, about which a pendulum vibrates, and perpendicular to the plane in which it oscillates.

Axis in *Peritrochio*, one of the five mechanical powers, consisting of a peritrochium or wheel, and moveable together with it about its axis. The power is applied at the circumference of the wheel, and the weight is raised by a rope that is gathered up on the axis while the machine turns round. The power may be conceived as applied at the extremity of the arm of a lever, equal to the radius of the wheel; and the weight as applied at the extremity of a lever, equal to the radius of the axis; only those arms do not meet at one centre of motion, as in the lever; but in place of this centre we have an axis of motion, *viz.* the axis of the whole machine.

Axis, in *Optics*. The optic axis, or visual axis, is a ray passing through the centre of the eye, or falling perpendicularly on the eye.

Axis of a *Lens*, or *Glass*, is the axis of the solid of which the lens is a segment. Or the axis of a glass, is the line joining the two vertices or middle points of the two opposite surfaces of the glass.

Axis of a *Magnet*, is a line passing through the middle of a magnet lengthwise, in such a manner as that, however the magnet is divided, provided the division is made according to a plane in which such line is found, the magnet will be cut or separated into two loadstones: and the extremes of such lines are called the poles of the magnet.

AZIMUTH, in *Astronomy*, an arc of the horizon intercepted between the meridian of the place, and the verticle circle passing through the centre of an object, and is equal to an angle at the zenith, formed by the said meridian and verticle circle, which is measured by the forementioned arc.

The azimuth is reckoned eastward in the morning, and westward in the afternoon, and is usually estimated to the north or south, as it is nearer to one or other of those points:

To find the Azimuth, at the Time of the Equinox.

Say,
As radius,
Is to the tangent of the latitude;
So is the tangent of the altitude of the sun or star,
To the cosine of the azimuth from the south.

Magnetical Azimuth, an arc of the horizon contained between the azimuth circle of the celestial object and the magnetical meridian; or it is the apparent distance of the object from the north or south point of the compass. This is found by observing the object with an azimuth compass, when it is about 10° or 15° high, either in the forenoon or afternoon.

Azimuth *Compass*, is a compass used at sea, for finding the sun's magnetical azimuth. See Compass.

Azimuth *Circles*, called also *vertical circles*, are great circles of the sphere, intersecting each other in the zenith and nadir, and cutting the horizon at right angles.

Azimuth *Dial*, is a dial whose style, or gnomon, is at right angles to the plane of the horizon.

B.

BACK-*Staff*, an instrument formerly used for taking the sun's altitude at sea; being so called because the back of the observer is turned towards the sun when he makes the observation. This instrument is also called *Davis's Quadrant*.

It consists of two co-centric arches of box-wood, and three vanes, the arch of the longer radius being of 30°, and that of the shorter of 60, forming together 90°, or a quadrant.

BALANCE, in *Mechanics*, a peculiar application of the lever, in order to determine the difference or equality of weights in heavy bodies; and, consequently, their masses or quantity of matter. There are various kinds of balances; the principal of which, however, are the common balance, the bent lever balance, the Roman balance, and the Swedish or Danish balance. Balances also receive other denominations, according to the circumstances under which they are employed, or the principles on which they act, as assay balance hydrostatic balance, &c.

Assay BALANCE, a very delicate balance used for determining the exact weight of minute bodies, in the different processes of assaying, and of frequent use and application in chemical analysis: for a description of which, see Gregory's Mechanics, vol. ii. p. 94; Nicholson's Journal, 4to. vol. v. p. 303; see also the Annales de Chemie, xxxvi. 50.

Bent Lever BALANCE. This instrument operates by a fixed weight at the end of a bent lever, supported on its axis in a pillar, and having a scale suspended from the other extremity.

Common BALANCE, or *Scale-beam*. This instrument is too well known to need any particular description; it consists of a lever with equal arms, at the extremity of each of which is attached a scale and before loading it with any weights the whole ought to preserve a perfect equilibrium; and this equilibrium must arise from an exact distribution of the weight of each arm and scale of the balance, as well as from the equal length of the former; for on this depends the accuracy of its action.

Professor Playfair has the following remarks on the accuracy of the balance.

"1. It should rest in a horizontal position when loaded with equal weights. 2. It should have great sensibility. 3. It should have great stability; that is, when disturbed, it should quickly return to a state of rest. That the first requisite may be obtained, the beam must have equal arms, and the centre of suspension must be higher than the centre of gravity. Were these centres to coincide, the beam, when the weights were equal, would rest in any position, and the addition of the smallest weight would overset the balance, and place the beam in a vertical situation, from which it would have no tendency to return. The sensibility, in this case, would be the greatest possible; but the other two requisites of level and stability would be entirely lost. The case would be worse, if the centre of gravity were lower than the centre of suspension, as the balance, when deranged, would make a revolution of no less than a semi-circle. When the centre of suspension is higher than the centre of gravity, if the weights be equal, the beam will be horizontal; and if they be unequal, it will take an oblique position, and will raise the centre of gravity of the whole, making the momentum on the side of the lighter weight equal to that on the side of the heavier, so that an equilibrium will again take place.

"The second requisite is the sensibility of the balance, or the smallness of the weight, by which a given angle of inclination is produced. If a be the length of the arm of the balance, and b the distance between the centre of suspension and the centre of gravity, P the load in either scale, and W the weight of the beam, the sensibility of the balance is as $\frac{a}{b(2P+W}$;

it is therefore greater, the greater the length of the arm, the less the distance between the two centres, and the less the weight with which the balance is loaded.

"Lastly. The stability or the force with which the state of equilibrium is recovered, is proportional to $(2P+W)b$, the denominator of the preceding fraction.

"The diminution of b, therefore, while it increases the sensibility, lessens the stability of the balance. The lengthening of a will, however, increase the former of these quantities, without diminishing the latter. By means of these formulæ, one balance may be made having exactly the same sensibility and stability with another; it is only required that the ratio of the lengths of the arms should be the same with that which is compounded of the ratios of the distances of the centres of gravity and suspension, and of the weights of the beams."

Compound Balance, is a combination of several balances employed in weighing very heavy bodies.

Danish Balance, is a sort of steel-yard, in very general use in various parts of the continent of Europe; the principle of its action being as follows:

It consists of a bar of iron, or batten of wood, having a heavy lump or knob at one end, and a scale or hook at the other. The goods to be weighed are placed in the scale, or suspended from the hook, and the whole is then balanced on a piece of cord, by sliding the latter about to and fro till the equilibrium obtains; and the weight of the body is then indicated by the graduated divisions of the instrument, which are thus computed:

Put the whole length $=d$, the distance of the cord from the knob $=x$, the weight of the knob $=a$, and the weight in the scale or on the hook $=w$.

Then, in case of equilibrium,

$$x : d-x = a : w$$

by comp. $d : x = a+w : a$

therefore $x=\frac{ad}{a=w}$

and hence, by substituting $w=1, 2, 3$, &c. ℔. the distance x or AC, for each of those weights, will be determined. Suppose, for example, the whole length $=30$ inches, and the weight $A = 2$℔.; then, substituting these numbers for a and d, and the numbers 1, 2, 3, &c. for w, we shall have the following series:

$x = 20, 15, 12, 10$, &c.
$w = 1, 2, 3, 4$, &c.

We give this merely as the principle of computation; in real practice, of course, the weight of the bar, and the place of its centre of gravity, must enter into the calculation, which will render it a little more complicated. See Gregory's Mechanics, vol. ii.

Hydrostatic Balance, is an instrument contrived for determining accurately the specific gravity of bodies, both solid and fluid; of which there are various constructions. See Hydrostatic *Balance.*

Roman Balance, or *Steel-yard.* See Steel-*yard.*

Balance of a *Clock*, or *Watch*, is that part which by its motion regulates and determines the beat.

Balance, in *Astronomy*, is the same as Libra, which see.

BALL, in a popular sense, is any spherical body, whether natural or artificial.

BALLISTIC *Pendulum*, a machine invented by Robins, for ascertaining the velocity of military projectiles, and consequently the force of fired gunpowder. It consists of a large block of wood suspended vertically by a strong horizontal iron axis, to which it is connected by a firm iron stem. Now to determine the velocity with which a ball is projected, the pendulum is so situated that the ball impinges directly against it, and causes it to vibrate through a certain arc, which being accurately observed, the velocity of projection is computed as follows:

Let the weight of the pendulum $=w$
the weight of the ball $=p$

And let O, G, and R, be the centres of oscillation, gravity, and gyration, and P the point of impact; also let q be a weight, which, collected in the point P, a given force applied to P will generate the same angular velocity as if it were applied against the pendulum itself in the point P.

This equivalent weight $q = \frac{w \times SG \times SO}{SD^2} = \frac{w \times SR^2}{SD^2}$.

Again, let v represent the velocity communicated to the point P, and x the required velocity of the ball. Then the block of wood being considered as non-elastic, the laws of collision gives the following proportion; *viz.*

$$q + p : p = x : v, \text{ whence}$$

$$x = v \times \frac{q + p}{p}.$$

And here, since p and q are known, it only remains to assign the velocity v, which has been communicated to the point P, from having given the angle through which the pendulum is impelled by the stroke.

For this purpose, make $SO = s$, $SP = d$, then the velocity acquired by the centre of oscillation in a pendulum, which describing from rest any arc of a circle has arrived at its lowest point, is equal to that acquired by a heavy body, which has descended freely from rest by the acceleration of gravity, through a space equal to the versed sine of the arc described by the pendulum. In like manner, if any given velocity be communicated to the centre of oscillation of a pendulum when quiescent, it will rise through an arc whose versed sine is equal to the space through which a body falls freely from rest, in order to acquire that velocity. Let, then, b represent the versed sine of the angle described by the pendulum to radius $= 1$, then will the centre of oscillation O, describe an arc during its motion, the versed sine of which $= sb$; and consequently, if we represent $16\frac{1}{12}$ feet by g, the central velocity of $O = \sqrt{4gbs}$ feet per second, and the velocity of [illegible] $P = \sqrt{\frac{4lbd^2}{s}} = v$.

This, therefore, will give us the velocity of the ball the instant it struck the pendulum; that is, the required velocity

$$x = v \times \frac{p + q}{p} = \frac{p + q}{p} \times \sqrt{\frac{4lbd^2}{s}} \text{ feet per second.}$$

BAROMETER, an instrument for measuring the weight of the atmosphere and its variations, in order principally to determine the changes of the weather, the heights of mountains, and other eminences, &c. The barometer is founded upon the Torricellian experiment. Torricelli filled a glass tube, about three feet long, with quicksilver; and having sealed it hermetically at one end, he immersed the other in an open vessel of the same fluid. He found that the mercury descended in the tube, and finally settled at the height of about twenty-nine and a half Roman inches, and this, whether the tube was vertical or inclined, according to the known laws of hydrostatical pressure. This experiment was repeated and diversified in various ways, with tubes filled with other fluids, and the result being the same in all, except so far as relates to their specific gravities, the weight and pressure of the air were established beyond the possibility of doubt. The real cause of the suspension of the mercury in the tube, and of water in pumps, was thus admitted to be the atmospheric pressure, and repeated observations were made, connected with this subject; from which it was discovered that the column of mercury varied considerably in its height, at different times, and this variation was soon observed to be followed by changes of the weather. This led to farther and more accurate observations, and various alterations and improvements were suggested in the form and construction of the barometer, or *weather-glass*, as it is sometimes called, in consequence of its use in determining the changes in the weather.

Common **BAROMETER.** This is a glass tube, open at one end, and hermetically sealed at the other, having its diameter about one-third or one-fourth of an inch, and its length thirty-three or thirty-four inches. It is filled with purified mercury so justly as not to have any air over it, nor any bubbles adhering to the sides of the tube, which is best done by means of a small paper or glass funnel, with a capillary tube. The orifice of the tube, filled after this manner, so as to overflow, is then closely pressed by the finger, so as to exclude any air between it and the mercury; this done, invert the tube, and immerse the finger and end thus stopped, into a bason of like purified quicksilver, and in this position withdraw the finger, and the mercury will descend in the tube to some place between twenty-eight and thirty-one inches above the surface of the mercury in the bason, these being the limits between which it always stands near the surface of the earth or sea.

Instead, however, of the detached vessel, the modern barometer tubes are curved at the bottom, and terminate in a bulb, which ought to be as big as it can be conveniently made, in order that the variation in the altitude of the mercury in the tube may affect, as little as possible, the depth of that in the bulb.

The barometer tubes, under either of the above forms, are now to be enclosed in a wooden case or frame, to prevent their being broken, and the vessel or bulb, though open to the air, must be secured from dust; and thus far the construction will be completed.

Next measure from the surface of the mercury to 28 inches and 31 inches; dividing the spaces between them into inches and tenths, which are marked on a scale placed against the side of the tube; and these tenths are again subdivided into hundredth parts of an inch, by means of a sliding index carrying a *vernier* or *nonius*. In the common barometers, called weather-glasses, the lowest limit is marked *stormy*, and the highest point is marked on one side *very dry* for summer, and on the other, *very hard frost* for winter. To the next half-inch below the highest point are annexed *set fair*, on the one side, and *set frost* on the other. At the height of 30 inches, *fair* is marked on one side, and *frost* on the other. At the height of 29½ inches is marked *changeable*, both for summer and winter; and at 29 inches, *rain* on one side, and *snow* on the other. At 28½ inches, *much rain* on one side, and *much snow* on the other; the lowest division being marked *stormy*, as we have before observed.

The bore of the tube should be large (not less than one-fourth of an inch), in order to prevent the effects of the attraction of cohesion. If a cistern be used as a reservoir for the stagnant mercury, it should be at least ten times the diameter of the tube, that the addition or subtraction of the mercury contained between the greatest and least altitudes, may not sensibly affect its depth. In order more effectually to preserve the lower surface at the same height from the divisions on the scale affixed to the instrument, a floating gage has been applied, by means of which the same screw that renders the barometer portable, regulates the surface of the mercury in the cistern, so that it is always at the place from whence the divisions on the scale commence. The tube should be preserved free from dust till it is used; and for this purpose it may be hermetically sealed at both ends, and one end may be opened with a file, when it is to be filled. The mercury should be pure; and may be purged of its air by previously boiling it in a glazed earthen pipkin covered close; and when the tube has been uniformly heated, and rendered electrical by rubbing it, the hot mercury should be poured into it in a regular current, through a glass funnel, with a long capillary tube, so that the air may not have room to pass between the parts of the quicksilver.

Another circumstance, that requires attention in the construction and use of barometers, is the temperature of the air; for unless this remains the same, the dimensions of a given quantity of mercury will be variable; and the altitude of the mercury will be an uncertain measure of the weight of the atmosphere, because it is dilated by heat, and contracted by cold, when probably its weight and pressure are unchanged. M. de Luc attended particularly to this circumstance, and contrived to estimate the effects of heat on the quicksilver in the barometer, when it is used for accurate observation, by means of the thermometer; the scale of which is divided in such manner as to indicate, with little labour of calculation, the correction to be made on account of heat. The interval between the freezing and boiling points (answering to six lines of the barometer) is divided into 96 equal parts; each of which corresponds to the sixteenth of a line in the motion of the mercury in the barometer dilated by heat, which must be added to, or subtracted from the height of the mercury in the barometer, for every degree of the variation of the thermometer so graduated.

As soon as it was observed that the different heights of the mercury indicated by the barometer were, in some measure, connected with the state of the weather, and that it might be applied to the purpose of a weather-glass, many attempts were made to render the changes in it more sensible, whereby to measure the variations in the weight of the atmosphere the more accurately, which attempts have given rise to a great number of barometers of different structures, deviating from the simplicity of the common barometer, and frequently much less accurate.

Diagonal BAROMETER. This acts on the same principle as the common barometer above described, except, that the upper end of the tube is bent at about an angle of 45° from the vertical, whereby the scale of variations is increased in about the ratio of three to two, or more accurately, in the ratio of the diagonal of a square to the length of its side. If the upper end be bent in a greater angle, the scale of variation will be so much the more increased; but in practice it is not found convenient to have the angle much exceeding 45°.

Horizontal BAROMETER. This consists of a tube bent at right angles, having a pretty wide cylindrical part at the upper end of the vertical leg, which is hermetically sealed, the horizontal leg being open, where, however, the mercury cannot run out, being opposed by the pressure of the atmosphere, the variations of which are indicated by a scale attached to that branch of the instrument.

Marine BAROMETER. This instrument is intended to be used on shipboard, being contrived so as not to be affected or injured by the motion of the vessel. It consists of a double thermometer, or two tubes half filled with spirits of wine; the one sealed at both ends, with a quantity of air included; the other sealed at one end only. The former of these is affected only by the temperature of the air; but the other, both by the external temperature, and by the variable pressure of the atmosphere. Hence, considering the spirit thermometer as a standard, the excess of the rise or fall of the other, beyond the former, will shew the increase or decrease of the pressure of the atmosphere.

Pendant BAROMETER. This instrument is rather pretty than useful. It consists of a conical tube placed vertically; its upper and smallar extreme is hermetically sealed, and its larger and lower end open: it has no vessel or cistern, its conical figure supplying that defect.

Portable BAROMETER. This is so constructed that it may be carried from one place to another without damage or derangement. The end of the tube is tied up in a leathern bag, not quite full of mercury, which, being pressed by the air, forces itself into the tube, and keeps suspended at its proper height. This bag is usually enclosed in a box, through the bottom of which passes a screw, by means of which the mercury may be forced to the upper end of the tube, and prevented from breaking it by dashing against the top, an accident very likely to happen without this precaution. Mr. Patrick, it seems, first had a contrivance of this kind; but the portable barometer has received various improvements since, of which, perhaps, the most complete is that described by De Luc, in his Recherches. Improvements in this instrument have also been suggested by Sir G. Shuckburgh and General Roy, which were carried into execution by Ramsden, with farther improvements of his own. Sir H. Englefield, and Humboldt, the celebrated traveller, have also much simplified the structure of the portable barometer; and an improvement as important, perhaps, as any other was suggested by Professor Playfair; namely, making the tube, as high as the lowest range of the mercury, of iron, by which means the hydrostatical pressure, the most usual casuality in the use of this barometer, is, in a great measure, guarded against.

Reduced BAROMETER. This instrument is only three inches long, and shows the dilatations of the air in the receiver of an air-pump, for which purpose it is commonly employed.

Wheel BAROMETER. This instrument is meant to render the alterations in the air more sensible. Here the barometer tube has a large ball at top, and is bent up at the lower or open end, where an iron ball floats on the surface of the mercury in the tube, to which is connected another ball by a cord hanging freely over a pulley, the axis of which carries an index that traverses on a dial. The circumference of the dial is so fitted to that of the pulley, that the whole range of the mercury is equal to the former, and thus what is but one inch in the common barometer, extends to three or four inches in this. But then it must be observed, that the friction of the pulley and axis greatly obstructs the free motion of the quicksilver. This imperfection in wheel barometers has been in some measure obviated by introducing two pulleys, moving on friction wheels.

The phenomena of the barometer, considered as a weather-glass, have been very differently explained by various writers; and although we have reason to believe, that the barometer never fails to indicate a storm, or any very great change of weather, for some hours before it occurs; yet its variations afford no indications or prognostics that are absolutely certain, with respect to those less considerable changes, to which the weather is subject in our variable climate. With certain restrictions they afford some ground for probable conjecture; and these restrictions are to be determined merely by the sagacity of long-continued observation and experience. Strictly speaking, the height of the mercury in the barometer has no immediate and necessary connection either with rain or fair weather. That its variable height is the immediate consequence of the variable pressure of the atmosphere, is a fact that admits of no doubt; but the causes of this variable pressure have not yet been fully and satisfactorily ascertained; and how far the state of the weather, in all its minute and sudden changes, depends upon it, is a question that is still to be determined.

The principal observations that have been made on the variations of this instrument, are summed up by Mr. Kirwan in the following particulars:

I. The more considerable elevations and depressions of the mercury in the barometer happen at a very short interval of time, in places very remote from each other. But the coincidence is more remarkable when the places differ only in latitude.

II. The variations are far more frequent and extensive in the neighbourhood of the poles than in that of the equator.

III. The variations without the tropics are greater and more frequent in winter than in summer.

IV. They are considerably smaller in very elevated situations than at the level of the sea.

V. The mean height of the barometer at the level of the sea, in most parts of the globe hitherto examined, is about 30 inches.

Dr. Halley and Mr. Patrick have given rules for judging of the weather, which are as follow:

Dr. Halley's Rules.

1. In calm weather, when the air is inclined to rain, the mercury is commonly low.
2. In serene, good, and settled weather, the mercury is generally high.
3. Upon very great winds, though they be not accompanied with rain, the mercury sinks lowest of all, according to the point of the compass the wind blows from.
4. The greatest heights of the mercury are found upon easterly or north-easterly winds, other circumstances alike.
5. In calm, frosty weather, the mercury commonly stands high.
6. After very great storms of wind, when the mercury has been very low, it generally rises again very fast.
7. The more northerly places have greater alterations of the barometer than the more southerly, near the equator.
8. Within the tropics, and near them, there is little or no variation of the barometer, in all weathers.

Mr. Patrick's Rules.

These are esteemed the best of any general rules hitherto made:

1. The rising of the mercury presages, in general, fair weather; and its falling, foul weather, as rain, snow, high winds, and storms.
2. In very hot weather, the falling of the mercury indicates thunder.
3. In winter, the rising presages frost: and in frosty weather, if the mercury falls three or four divisions, there will certainly follow a thaw. But in a continued frost, if the mercury rises, it will certainly snow.
4. When foul weather happens soon after the falling of the mercury, expect but little of it; and, on the contrary, expect but little fair weather when it proves fair shortly after the mercury has risen.
5. In foul weather, when the mercury rises much and high, and so continues for two or three days before the foul weather is quite over, then expect a continuance of fair weather to follow.
6. In fair weather, when the mercury falls much and low, and thus continues for two or three days before the rain comes, then expect a great deal of wet, and, probably, high winds.
7. The unsettled motion of the mercury denotes uncertain and changeable weather.
8. You are not so strictly to observe the words engraven on the plates, as the mercury's rising and falling; though, in general, it will agree with them. For if it stands at much rain, and then rises up to changeable, it presages fair weather; though not to continue so long as if the mercury had risen higher. And so, on the contrary, if the mercury stood at fair, and falls to changeable, it presages foul weather; though not so much of it as if it had sunk lower.

M. Chiminello observed the barometer twenty-two times a day for three years, but he left a chasm in the night which he supplied by calculation. The principal positions which he thence deduced are, that the barometer falls towards noon, as well as towards night.

M. Hemmer, from a great number of accurate observations, deduced the three following general rules:

1st. When the sun passes the meridian, the barometer, if in the act of falling, continues to fall, and the falling is accelerated.

2d. When the sun passes the meridian, the barometer, if in the act of rising, falls, or becomes stationary, or rises more slowly.

3. When the sun passes the meridian, the barometer, which is stationary, falls, if it has not risen before or after being stationary; in which case it usually becomes stationary during the sun's passage.

BAROSCOPE, an instrument intended only to show that the air has weight; whereas the barometer measures that weight, and determines its true quality.

BARREL, an English vessel or

cask, which contains 36 gallons beer measure, and 32 gallons ale measure. The barrel of beer, vinegar, or liquor preparing for vinegar, ought to contain 34 gallons, according to the standard of the ale quart.

Barrel is also used as a measure in various commodities; thus,

Barrel of Essex butter is 106 lb.
Barrel of Suffolk butter 256 lb.
Barrel of herrings holds 32 gal. wine measure, and contains about · · · · · · · 1000 her.
Barrel of salmon holds · 42 gal.
Barrel of soap is · · · · 256 lb.

Barrel, in *Machinery*, is also applied to any thing hollow and cylindrical, as the barrel of a pump, of a gun, of a watch, &c.

BASE of a *Figure*, denotes the lowest part of its perimeter; in which sense the base stands opposed to the *vertex*, which denotes the highest part.

Base of a right-angled triangle, is properly the hypothenuse, though it is generally used to denote one of the sides about the right angle, the other side being called the perpendicular.

That side on which a solid body stands is called the base of the solid.

Base of a *Conic Section*, is a right line in the parabola and hyperbola, formed by the common intersection of the cutting plane and the base of the cone.

Base, in *Surveying*, is a line measured with the greatest possible exactness, on which a series of triangles are constructed, in order to determine the position of objects and places.

BATTERING *Ram*, or *Aries*, an ancient military engine, employed for destroying the walls of fortified places, of which there were three different sorts.

The first seems to have been nothing more than a great beam, having a heavy head of iron, which the soldiers bore in their arms, and with one end of it assailed the walls by main strength.

The second sort, as described by Josephus, consisted of a very heavy beam, headed with iron, and suspended in the middle from another strong beam supported on two posts.

The third sort was mounted on wheels. Desaguliers has demonstrated, that the momentum of a battering ram, 28 inches in diameter, 180 feet long, with a head of cast iron of 1½ ton, the whole ram weighing, with its iron hoops, 41,112 pounds, and moved by the united strength of 1000 men, would only be equal to that of a ball of 36 pounds weight shot point blank from a cannon. And Atwood, comparing the effect of the battering ram, having its metal extremity equal to a twenty-four pounder, with a cannon-ball of 24 pounds weight, observes, that in order to their producing the same effect in penetrating a wall or making a breach in it, the weight of the ram must exceed that of the cannon-ball in the proportion of the square of 1700, the velocity of the ball, to the square of the velocity with which the battering ram could be made to impinge against the wall, expressed in feet. Estimating this at 10 feet in a second, the proportion of the weights will be that of about 2,890,000 to 100, or 28,900 to 1.

The ram was frequently used in the fourteenth century. Sir Christopher Wren employed it in demolishing the walls of the old church of St. Paul.

BATTERY, in *Electricity*, is a combination of coated surfaces of glass, so connected together that they may be charged at once, and discharged by a common conductor. Mr. Gralath, a German electrician, first contrived to increase the shock by charging several phials at the same time. Dr. Franklin, having analysed the Leyden phial, and found that it lost at one surface what it received at the other, constructed a battery consisting of eleven panes of large sash glass, coated on each side, and connected in such a manner, that the whole might be charged together, and with the same labour as one single pane; and by bringing all the giving sides into contact with one wire, and all the receiving sides with another, he contrived to unite the force of all the plates, and to discharge them at once. Dr. Priestley describes a still more complete battery: this consists of 64 jars, each 10 inches long, and 2½ inches in diameter, all coated within an inch and a half of the top, forming in the whole about 32 square feet of coated surface.—But the largest and most powerful battery of all is that employed by Dr. Van Marum, to the electrical machine, constructed for Teyler's museum at Haarlem. This grand battery consists of a great number of jars coated as above, to the amount of about 130 square feet; and the effects of it, which are truly astonishing, are related by Dr. Van Marum in his description of this machine, and of the experiments made with it at Haarlem, 1785.

Galvanic **Battery**, or *Pile*, an apparatus employed for accumulating galvanism. It was invented by Volta, from whose labours the science of galvanism derived many advantages and much improvement. Mr. Henry, in his valuable Epitome of Chemistry, gives the following easy directions for the construction of this pile; "Procure, at a brazier's or coppersmith's, 30, 40, or 50 pieces of zinc or speltre, cast in sand, of the size of half-crowns or shillings, but rather thicker. A corresponding number of half-crowns or shillings will also be required, according to the sizes of the pieces of zinc that may be employed. Let an equal number of pieces of woollen cloth be cut, of a circular shape, to correspond with the pieces of zinc, and steep these in a strong solution of common salt in water. Then dispose the three substances alternately, in the following order: silver, zinc, moistened cloth; silver, zinc, &c. till a sufficient number of these triplicates, not less than 20 or 30, have been thus arranged, the silver terminating the pile at top. In order to facilitate the touching of the bottom piece of silver, it may be well to put under it a slip of tinfoil, or Dutch leaf, which may project a few inches. Next, let the hands be moistened with salt and water, and, on touching the piece of tinfoil with one hand, and the uppermost piece of silver with the other, a shock will pass through the arms, which will be strong in proportion to the number of pieces of zinc &c. employed." Of late, copper has been used instead of silver, on account of its being cheaper; and solutions of muriate of ammonia (sal ammoniac,) of nitrous acid, and of muriatic acid, have been substituted for the solution of common salt, with increased effect. Any two metallic substances, which are perfect conductors of electricity, will answer the purpose, on condition that the interposed fluid is capable of oxidizing at least one of them.

The galvanic trough consists of a box of baked wood, in which plates of copper and zinc, or of silver and zinc, soldered together at their edges, are cemented in such a manner as to leave a number of water-tight cells corresponding to the number of the series.

Piles and batteries of various forms and powers have been constructed by different experimentalists. It has been found that the power of chemical decomposition is in the ratio of the surfaces, and that of exciting the muscular energy in a subject recently dead in the ratio of the number of combinations. Very powerful batteries have been made with concentric cylinders of zinc and copper. See *Galvanism.*

BEARD of a *Comet*, the rays which it emits in the direction in which it moves, as distinguished from the tail, or the rays emitted or left behind it as it moves along.

BEARING, in *Geography* and *Navigation*, the position of one place with regard to another, as estimated by the points of the compass.

BELTS, zones or girdles, surrounding the planet Jupiter, more lucid than the other parts of his body, and terminated by parallel straight lines, being sometimes broader and sometimes narrower, varying both in magnitude and position.

BENDING, the reducing a body to a curved or crooked form. The bending of boards, planks, &c. is effected by means of heat, usually by boiling, by which their fibres are so relaxed that they may be bent into any figure at pleasure.

BEVEL *Angle*, is a term used by artificers to denote any angle except those of ninety or forty-five degrees. Bevel is also the name of an instrument used by workmen in setting off angles.

BILLION, in *Numeration*, a *million* of *millions*, or 10000·0000·0000; The French mathematicians understanding billion to mean thousands of millions, and the English millions of millions; thus, with the French, the place of billions is said to be the *tenth* from the right towards the left, whereas we make it in our arithmetics the *thirteenth*.

BIMEDIAL *Line*, in *Geometry*, is the sum of two medials. Thus, when two medial lines, commensurable only in power, and containing a rational rectangle, are compounded, the whole shall be irrational with respect to either of the two; and is called, by Euclid, a first *bimedial line*. But if two medial lines, commensurable only in power, and containing a medial rectangle, be compounded, the whole will be irrational; and is called a *second bimedial line*.

BINARY *Number*, is a number consisting of two units.

Binary *Arithmetic*, or **Binary** *Scale* of *Notation*. See *Arithmetic*.

BINOCULAR *Telescope*, is a telescope to which both eyes may be applied. It consists of two tubes, with two sets of glasses of the same power, and adjusted to the same axis.

BINOMIAL, in *Algebra*, a quantity consisting of two terms or names, and connected by the signs $+$ *plus*, $-$ *minus*, or $=$ *equal*; thus, $a + b$, $a - b$, $a = b$, are all binomials; the difference $a - b$, being also frequently called a *residual*; Thus we say, *Binomial Curve*, *Equation*, *Surd*, *Theorem*, &c.

Binomial *Curve*, is a curve whose ordinate is expressed by a binomial quantity; as, for instance, $y = x^m \sqrt{b + dx^n}$.

Binomial *Equation*, is any equation of two terms, but more commonly applied to the higher order of equations of the form $x^n = 1$, or $x^n - 1 = 0$; the solution of which forms one of the most interesting researches of the modern algebra. It is obvious, that if we have $x^n = 1$, where n is a prime number; and also $x^{nm} = 1$, that the roots of the former equation are likewise roots of the latter. For let r, s, t, &c. be the roots of the equation $x^n = 1$, so that $r^n = 1$, $s^n = 1$, &c.; then it is evident also that $r^{nm} = 1$, $s^{nm} = 1$, &c. and, consequently, the roots of the former are, at the same time, roots of the latter.

In every binomial equation, $x^n - 1 = 0$, n being a prime number, there are contained one real root, and $n - 1$ imaginary roots, which latter are all powers of each other, and all different from each other, the sum of which is equal to their continued product, being both equal to -1.

Every equation having as many roots as their units in the index of its highest power, it is evident that the whole number of roots is n, and of these only one can be real, viz. $x = 1$; consequently, all the other $n - 1$ roots are imaginary, and that they are all powers of each other may be shown as follows:

Let r represent any one of these imaginary roots, then since $r^n = 1$, so likewise will $r^{2n} = 1$, $r^{3n} = 1$, $r^{4n} = 1$, &c.; therefore, if r be one root, so likewise is every term in the series r, r^2, r^3, r^4, &c. r^{n-1} for each of these quantities raised to the nth power is equal to unity, which is the condition of the equation. Whence it follows, that any one of these roots may be considered as the leading one in the series, and all the others will still be powers of this, and of each other.

These roots are also all different. For let $r^{pa} = r^{qa}$, where p and q are each necessarily less than n, and since by the supposition p and q are not equal, let $p > q$, then dividing the equation $r^{pa} = r^{qa}$ by r^{qa}, we have $r^{(p-q)a} = 1$, so that $r^{(p-q)a}$ would likewise be a root of the same equation, but this is impossible; no two of the series of roots are equal to each other.

It follows also, from the known theory of equations, that the sum of all the roots is equal to the co-efficient of the second term, which in the present case is 0: and that the continued product is equal to the absolute term, which is here -1.

$$1 + r^{a} + r^{2a} + r^{3a}, \text{ \&c. } r^{(n-1)a} = 0;$$

or, . . . $$r^{a} + r^{2a} + r^{3a} +, \text{ \&c. } r^{(n-1)a} = -1;$$

Again, $1 \times r^{a} \times r^{2a} \times r^{3a} \times$, &c. $r^{(n-1)a} = -1$;

or, . . . $$r^{a} \times r^{2a} \times r^{3a} \times, \text{ \&c. } r^{(n-1)a} = -1;$$

as is evident.

The whole series of the roots of these equations, both real and imaginary, is expressible by means of certain general trigonometrical formulæ; viz.

All the roots of the equation $x^{n} - 1 = 0$ are contained in the general formula

$$x = \cos. \frac{2k\pi}{n} \pm \sqrt{\left(\cos.^{2} \frac{2k\pi}{n} - 1\right)}$$

k being equal to zero, or any integer number whatever, and π representing the semicircumference.

And in equations of the form $x^{n} + 1 = 0$, the general formula, in which the roots are contained, is

$$x = \cos. \frac{(2k+1)\pi}{n} \pm \sqrt{\left(\cos.^{2} \frac{(2k+1)\pi}{n} - 1\right)}$$

k and π being the same as above. In the first of these formulæ, if k be divisible by n, it gives the real root; as does the second, when $2k + 1$ is divisible by the same number.

Imaginary, or *Impossible* BINOMIAL, is one of which one of the branches is imaginary; as,

$$a \pm \sqrt{-b}, \text{ or } -a \pm \sqrt{-b}.$$

BINOMIAL *Surd*, is a binomial of which one or both of the branches are surd quantities; thus, $a + \sqrt{b}$, $\sqrt{a} + \sqrt{b}$, &c. are binomial surds.

BINOMIAL *Theorem*, a general algebraical expression or formula, by which any power or root of a quantity of two terms is expanded into a series.

This theorem, in its most simple form, is as follows; viz.

$$(a+b)^{n} = a^{n} + \frac{n}{1} a^{n-1}b + \frac{n(n-2)}{1.2} a^{n-2}b^{2} + \frac{n(n-1)(n-2)}{1.2.3} a^{n-3}b^{3},$$

&c. where the law of the series is immediately obvious. Thus, $(a+b)^{3} = a^{3} + 3a^{2}b + 3ab^{2} + b^{3}$ and $(a+b)^{7} = a^{7} + 7a^{6}b + 21a^{5}b^{2} + 35a^{4}b^{3}$, &c. which series, it is obvious, must always terminate when the index n is an integer number.

The signs of the several terms in the series are affected by the signs of the powers of b; that is, if b be negative, as in the quantity $a - b$, all those terms into which the odd powers of b enter, will be preceded by a negative sign; but when b is positive, then all the signs of the series will be positive. Thus,

$$(a \pm b)^{7} = a^{7} \pm \frac{7}{1} a^{6}b + \frac{7.6}{1.2} a^{5}b^{2} \pm \frac{7.6.5}{1.2.3} a^{4}b^{3} +, \text{ \&c.}$$

the signs being *plus* and *minus* alternately in the latter case.

If we raise $(1 + 1)$ to any power n, we shall have the co-efficients only; hence the sum of all the co-efficients of a binomial are equal to the same power of 2, as that to which the binomial is raised. Also, the sum of the positive co-efficients is equal to the sum of the negative ones, which therefore destroy each other.

If also we consider the latter part of the series, it will be found that the co-efficients from either extreme are the same, increasing from each end to the centre term, when the number of terms is odd, or to the two centre terms when the number is even.

The number of terms is 1 greater than the exponent of the power. It is odd for even powers and even for odd powers.

The index of the first, or leading quantity, is the same as that of the power, and in the succeeding terms it decreases always by 1; while that of the second part increases by 1, whereby the sum of the indices is always the same in each term.

As to the co-efficients, the first is always unity, and the second the same as the index of the power and for the rest multiply the co efficient of each preceding term by the index of the leading quantity in that term, and divide the product by the number of terms to that place, and the quotient will be the co-efficient of the following term.

And for the signs, they will be all *plus* when both terms of the root are plus, and alternately *plus* and *minus* when the second term is negative.

If we assume the index $\frac{m}{n}$, in which form it may relate to roots as well as powers.

This theorem may be expressed in its most simple form; viz. put $a = \text{P}$, and $b = \text{PQ}$, then we shall have

$$(\text{P} + \text{PQ})^{\frac{m}{n}} = \text{P}^{\frac{m}{n}} + \frac{m}{n}\text{AQ} + \frac{m \quad n}{2n}\text{BQ} + \frac{m - 2n}{3n}\text{CQ} + \text{\&c.}$$

where $\frac{m}{n}$ is the index, P the first term, Q the second term divided by the first, and A, B, C, D, &c. the several foregoing terms, with their proper signs.

An example will sufficiently illustrate this formula.

1. Let it be proposed to convert $\sqrt{(a^2 + b^2)}$, or $(a^2 + b^2)^{\frac{1}{2}}$, into an infinite series. Here $\text{P} = a^2$, $\text{Q} = \frac{b^2}{a^2}$, $\frac{m}{n} = \frac{1}{2}$; that is, $m = 1$, and $n = 2$; therefore,

$$\text{P}^{\frac{m}{n}} = (a^2)^{\frac{m}{n}} = (a^2)^{\frac{1}{2}} = a = \text{A}$$

$$\frac{m}{n}\text{AQ} = \frac{1}{2} \times a \times \frac{b^2}{a^2} = \frac{b^2}{2.a} = \text{B}$$

$$\frac{m - n}{2n}\text{BQ} = \frac{1 - 2}{4} \times \frac{b^2}{2.a} \times \frac{b^2}{a^2} = \frac{-1.b^4}{4.2.a^3} = \text{C}$$

$$\frac{m - 2n}{3n}\text{CQ} = \frac{1 - 4}{6} \times \frac{-1.b^4}{4.2.a^3} \times \frac{b^2}{a^2} = \frac{3.1.b^6}{6.4.2.a^5} = \text{D}$$

consequently,

$$(a^2 + b^2)^{\frac{1}{2}} = a + \frac{b^2}{2.a} - \frac{1.b^4}{4.2.a^3} + \frac{3.1.b^6}{0.4.2.a^5} \quad \frac{5.3.1b^9}{8.6.4.2a^7} +, \text{\&c.}$$

where the terms may be continued at pleasure, the law of series being obvious.

When the quantity to be expanded comes under the form of a fraction, the denominator must be placed under a negative index, and brought up into the numerator.

BIQUADRATE, or **BIQUADRATIC** *Power*, in *Algebra*, is the square squared, or fourth power of a quantity; thus, 16 is the biquadrate of 2.

BIQUADRATIC *Root*, is the fourth root of any proposed quantity; thus 2 is the biquadratic root of 16.

The biquadratic root of a number is found by extracting the square root of it, and then the square root of that root.

BIQUADRATIC *Equation*, is an equation in which the unknown quantity rises to the fourth power; as $x^4 + ax^3 + bx^2 + cx + d = 0$ in which a, b, c, and d, may be any numbers whatever, positive or negative, or any of them equal 0.

A biquadratic equation is the highest order of equation that admits of a general solution; all higher ones being soluble only in particular cases.

The following are two of the methods of solution.

1. *Ferrari's Method.*

Let there be proposed the following general equation, of the fourth degree; viz.

$$x^4 + ax^3 + bx^2 + cx + d = 0;$$

and let us suppose this equation to be the same as

$$(x^2 + \tfrac{1}{2}ax + p)^2 - (qx + r)^2 = 0,$$

where p, q, and r, are unknown, the values of which are to be so determined as to make the latter equation equal to that proposed. This is effected as follows:

$$(x^2 + \tfrac{1}{2}ax + p)^2 = \begin{cases} x^4 + ax^3 + \frac{1}{4}a^2x^2 + \\ \quad apx + p^2 + 2px^2 \end{cases}$$

$$-(qx + r)^2 = \; -q^2x^2 - 2qrx - r^2$$

by comparing which with the original, we shall have

$$\begin{aligned} \tfrac{1}{4}a^2 + 2p - q^2 &= b \\ ap - 2qr &= c \\ p^2 - r^2 &= d \end{aligned}$$

By means of these three equations, the values of p, q, and r, may be found in terms of the known quantities a, b, c, and d; for we have

$$8p^3-4bp^2+(2ac-8d)p-a^2d+4bd-c^2=0$$

which being a cubic equation, the value of p becomes determined, and may now be considered as known; and q and r are also determined from the equations

$$q=\sqrt{(\tfrac{1}{4}a^2+2p-b)}$$

$$r=\frac{ap-c}{2q}$$

Having thus found the values of p, q, and r, the four values of x, in the proposed equation, are also determined from the assumed equation

$(x^2+\frac{1}{2}ax+p)^2-(qx+r)^2=0$, or
$x^2+\frac{1}{2}ax+p=\pm(qx+r)$; whence
$x^2+(\frac{1}{2}a-q)\,x=r-p$, or
$x^2+(\frac{1}{2}a+q)\,x=-r-p$,

by taking the ambiguous sign by which $(qx+r)$ is effected both $+$ and $-$; whence the four required roots are

$$x=-\frac{\frac{1}{2}a-q}{2}\pm\sqrt{\left\{\left(\frac{\frac{1}{2}a-q}{2}\right)^2+r-p\right\}}$$

$$x=-\frac{\frac{1}{2}a+q}{2}\pm\sqrt{\left\{\left(\frac{\frac{1}{2}a+q}{2}\right)^2-r-p\right\}}$$

2. *Euler's Method.*

He assumes a general biquadratic under the form

$$x^4-ax^2-bx-c=0,$$

and supposes its root to be

$$x=\sqrt{p}+\sqrt{q}+\sqrt{r};$$

which, squared, becomes

$$x^2=p+q+r+2(\sqrt{pq}+\sqrt{pr}+\sqrt{qr});$$

or, making $p+q+r=f$, it becomes

$$x^2-f=2(\sqrt{pq}+\sqrt{pr}+\sqrt{qr};$$

squaring this, we have

$$x^4-2fx^2+f^2=\begin{cases}4(pq+pr+qr)\\+8(\sqrt{p^2qr}+\sqrt{q^2pr}\\\quad+\sqrt{r^2pq}).\end{cases}$$

Making $pq+pr+qr=g$, and put- he latter part under the form

$rq(\sqrt{p}+\sqrt{q}+\sqrt{r})$,

nting at the same time h, we obtain

$f^2=4g+8x\sqrt{h}$

$8\sqrt{h}.x+(f^2-4g)=0$;

$=a$, or $f=\frac{1}{2}a$

$$8\sqrt{h}=b,\ \text{or}\ h=\frac{b^2}{64}$$

$$f^2-4g=-c,\ \text{or}\ g=\tfrac{1}{16}a^2+\tfrac{1}{4}c;$$

and since
$$\begin{aligned}p+q+r&=f,\\ pq+pr+qr&=g,\\ pqr&=h,\end{aligned}$$

it is obvious that p, q, and r, are the three roots of the cubic equation

$$p^3-fp^2+gp-h=0;$$

whence the three quantities p, q, and r, become known, and consequently the roots of the proposed equation, $x^4-ax^2-bx-c=0$, being as follows; viz.

When b is positive,

1st. root $x=\sqrt{p}+\sqrt{q}+\sqrt{r}$
2d. . . $x=\sqrt{p}-\sqrt{q}-\sqrt{r}$
3d. . . $x=-\sqrt{p}+\sqrt{q}-\sqrt{r}$
4th. . . $x=-\sqrt{p}-\sqrt{q}+\sqrt{r}$

When b is negative,

1st. root $x=\sqrt{p}+\sqrt{q}-\sqrt{r}$
2d. . . $x=\sqrt{p}-\sqrt{q}+\sqrt{r}$
3d. . . $x=-\sqrt{p}+\sqrt{q}+\sqrt{r}$
4th. . . $x=-\sqrt{p}-\sqrt{q}-\sqrt{r}$

Let us illustrate this rule by an example.

Given $x^4-25x^2+60x-36=0$, to find the four values of x.

Here $a=25$, $b=-60$, $c=36$;

therefore $f=\frac{25}{2}$, $g=\frac{625}{16}+9$,

$$=\frac{769}{16};\quad h=\frac{225}{4};$$

consequently the cubic equation will be

$$p^3-\frac{25}{2}p^2+\frac{769}{16}p-\frac{225}{4}=0;$$

the three roots of which are

$$p=\frac{9}{4},\ q=4,\ r=\frac{25}{4};$$

and the square roots of these are

$$\sqrt{p}=\frac{3}{2},\ \sqrt{q}=2,\ \sqrt{r}=\frac{5}{2}$$

Hence, as the value b is negative, the four roots are the following:

1st. root $x=\frac{3}{2}+\frac{4}{2}-\frac{5}{2}=1$
2d. . . $x=\frac{3}{2}-\frac{4}{2}+\frac{5}{2}=2$
3d. . . $x=-\frac{3}{2}+\frac{4}{2}+\frac{5}{2}=3$
4th. . . $x=-\frac{3}{2}-\frac{4}{2}-\frac{5}{2}=-6$

For the construction of biquadratic equations, see Construction.

BISSEXTILE, or *Leap-Year*, in *Chronology*, a year consisting of 366 days, happening once every four years, by reason of the addition of a day in the month of February to recover the six hours which the earth occupies in its annual course, beyond the 365 days ordinarily allowed for it.

The day thus added is also called bissextile; Cæsar having appointed it to be introduced by reckoning the 24th of February twice; and as this day, in the old account, was the same as the sixth of the calends of March, which had been long celebrated among the Romans on account of the expulsion of Tarquin, it was called "*bis sextus calendas Martii*;" and from hence we have derived the name bissextile.

By the statute 21 Hen. III. to prevent misunderstandings, the intercalary day, and that next before it, are to be accounted as one day.

The astronomers concerned in reforming the calendar, by order of Pope Gregory XIII. in 1582, observing that the bissextile in four years added forty-four minutes more than the sun spent in returning to the same point of the zodiac, and computing that these supernumerary minutes in 133 years would form a day; to prevent any changes being thus insensibly introduced into the seasons, directed that in the course of 400 years there should be three bissextiles retrenched; so that every centisimal year, which according to the Julian account is bissextile or leap-year, is a common year in the Gregorian account, unless the number of centuries can be divided by four without a remainder. Thus, 1600 and 2000 are bissextile; but 1700, 1800, and 1900, are common. But, with the exceptions of the above even centuries, any year which exactly divides by four is leap-year; and when there is any remainder, it indicates the number of years since leap-year.

The bissextile, or number of years after it, is the remainder, upon dividing the date by 4. Thus, 1820 ÷ 4 leaves 0, therefore 1820 is bissextile; and 1823 ÷ 4 leaves 3, therefore 1823 is the third after bissextile.

BLACK, an epithet applied to any thing opaque and porous, which imbibes the greater part of the light that falls on it, reflects little or none, and therefore exhibits no colour.

Bodies of a black colour are found more inflammable, because the rays of light falling on them are not reflected outwards, but enter the body, and are often reflected and refracted within it, till they are stifled and lost. They are also found lighter, *cæteris paribus*, than white bodies, being more porous.

BLUE, one of the seven primitive colours of the rays of light, into which they are divided when refracted through a glass prism.

The blue colour of the sky is a remarkable phenomenon, which has been variously accounted for by different philosophers. Newton observes, that all the vapours when they begin to condense and coalesce into natural particles, become first of such a bigness as to reflect the azure rays, before they can constitute clouds of other colours. Bouguer ascribes this blueness of the sky to the constitution of the air itself, being of such a nature that the fainter coloured rays are incapable of making their way through any very considerable portion of it.

BODY, or Solid, in *Geometry*, has three dimensions, viz. length, breadth, and thickness.

BODY, in *Physics*, is a solid, extended, palpable substance; of itself merely passive, and indifferent either to motion or rest; but capable of any sort of motion, and all figures and forms.

Bodies are either *hard*, *soft*, or *elastic*.

A *hard Body* is that whose parts do not yield to any stroke or percussion, but which retains its figure unaltered.

A *soft Body* is that whose parts yield to the stroke or impression, without restoring themselves again.

An *elastic Body* is that whose parts yield to any stroke, but immediately restore themselves again, and the body retains the same figure as at first.

We know not, however, of any bodies that are perfectly hard, soft, or elastic; but all possess these properties in a greater or less degree.

Bodies are also either *solid* or *fluid.*

A *solid Body* is that in which the attractive power of the particles of which it is composed exceed their repulsive power, and, consequently, they are not readily moved one among another; and, therefore, the body will retain any figure that is given to it.

A *fluid Body* is that in which the attractive and repulsive powers of the particles are in exact equilibrio, and therefore yields to the slightest impression.

Fluid bodies are also distinguished into *nonelastic* and *elastic*, or *fluids* properly so called, and *æriform fluids* or *gases*.

Regular Bodies, or *Platonic* Bodies, are those which have all their sides, angles, and planes, similar and equal, of which there are only five, *viz.*

1. Tetraedron, contained under	4 equilateral triangles.
2. Hexaedron, . . .	6 squares.
3. Octaedron, . . .	8 triangles.
4. Dodecaedron, . .	12 pentagons.
5. Icosaedron, . . .	20 triangles.

BOW *Compass*, for drawing arches of very large circles; it consists of a beam of wood or brass, with three long screws that govern or bend a lath of wood of steel to any arch. The term is also sometimes used to denote very small compasses employed in describing archs, too small to be accurately drawn by the common compasses.

BRACHYSTOCHRONE, is the name which John Bernoulli gave to his celebrated problem of the "Curve of swiftest Descent," namely, to find the curve along which a body would descend from a given point A, to another given point B, both in the same vertical plane, in the shortest time possible.

At first view of this problem, it would be imagined that a right line, as it is the shortest path from one point to another, must likewise be the line of swiftest descent; but the attentive geometer will not hastily assert this, when he considers, that in a concave curve, described from one point to another, the moving body descends at first in a direction more approaching to a perpendicular, and consequently acquires a greater velocity than down an inclined plane; which greater velocity is to be set against the length of the path, which may cause the body to arrive at the point B sooner through the curve than down the plane.

A keen contest was for some time carried on among the continental mathematicians, respecting the solution of this problem, which was first accurately given by James Bernoulli. It is the same as the cycloid.

BRANCH of a *Curve*, in *Geometry*, is a term used to denote certain parts of a curve, which are infinitely extended without returning upon themselves; being called also infinite branches: such are the legs of the parabola and hyperbola.

The infinite branches of curves, are either of the parabolic or hyperbolic kind.

Parabolic Branches are those which may have for an asymptote a parabola of a superior or inferior order: thus, for example, the curve of which the equation is

$$y = \frac{x^2}{a} + \frac{b^2}{x}$$

will have an infinite parabolic branch, which will have the common parabola for its asymptote, of which the equation is $y = \frac{x^2}{x}$. For x being supposed infinite, the last term vanishes, and the equation becomes simply $y = \frac{x^2}{d}$, which is the equation of the common parabola.

Hyperbolic Branches, are those which have a right line, or an hyperbola of a superior or inferior degree for their asymptote. For example, the curve whose equation is $y = \frac{x^2}{a} + \frac{b^2}{x}$, of which we have been speaking above, is reduced when $x = 0$, to $y = \frac{b^2}{x}$, and its asymptote will be the infinite ordinate passing through its origin. It may also have for its asymptote the common hyperbola.

BRIDGE, a work of carpentry or masonry, built over a river or canal, for the convenience of passing from one side to the other.

BURNING *Glass*, a convex lens which transmits the rays of light, but in their passage refracts or inclines them towards a common point in the axis called the focus; and by thus combining together in a single point the power of all the rays transmitted through the glass, a very great degree of heat is accumulated in that point, which will fuse bodies that are infusible in the greatest culinary heat that can be produced.

BURNING *Mirrors*, or *Specula*, are concave reflecting surfaces, commonly of metal, which reflect the rays of light falling upon them, but at the same time incline them towards a determined point or focus, where their accumulated effect operates in the most powerful manner, burning and dissipating the hardest and most infusible bodies.

Convex lenses were very imperfectly understood by the ancients; but they seem to have had burning mirrors in greater perfection than the moderns, at least if we may credit the relations of several eminent historians, who assert that Archimedes, by means of such mirrors, burned and destroyed the Roman fleet, which, under Marcellus, was employed at the siege of Syracuse; and that Proclus in the same way destroyed the navy of Vitellius, at the siege of Byzantium.

"When the fleet of Marcellus was within bow-shot," says Tzetzes, chil. 2. hist. 35, "the old man (Archimedes) brought an hexagonal mirror which he had previously prepared, at a proper distance from which he also placed other smaller mirrors, of the same kind, that moved in all directions on hinges, which, when placed in the sun's rays, directed them upon the Roman fleet, whereby it was reduced to ashes."

Of the moderns, the most remarkable burning glasses are those of Magine, of 20 inches diameter; of Sepatala, of Milan, near 42 inches diameter, and which burnt at the distance of 15 feet; of Settala, of Vilette, of Tschirnhausen, of Buffon, of Trudaine, and of Parker.

Mr. Parker, of Fleet-street, London, succeeded in the construction of a lens of flint glass, 3 feet in diameter, which, when fixed in its frame, exposed a surface of 32 inches; the distance of the focus 6 feet 9 inches, and its diameter 1 inch. The rays from this large lens were received and transmitted through a smaller one of 13 inches diameter, its focul length 29 inches, and diameter of its focus $\frac{3}{8}$ inch; so that this second lens increased the power of the former, as 8^2 to 3^2, or rather more than 7 to 1.

From a variety of experiments made with these lenses, the following are selected to serve as a specimen of their powers:

Substances fused, with their Weight and Time of Fusion.	Wt. in Grs.	Time in Sec.
Gold, pure	20	4
Silver, ditto	20	3
Copper, ditto	33	20
Platina, ditto	10	3
Nickell	10	3
Bar iron, a cube	10	12
Cast iron, ditto	10	3
Steel, ditto	10	12
Scoria of wrought iron	12	2
A topaz, or chrysolite	3	45
An oriental emerald	2	42
Crystal pebble	7	6
Topaz	10	30
Flint oriental	10	30
Rough cornelian	10	75
Jasper	10	25
Onyx	10	20
Garnet	10	17
White rhomboidal spar	10	60

A diamond weighing 10 grains, exposed to this lens for 30 minutes, was reduced to 6 grains; but gold remained in its metallic state, without apparent diminution, notwithstanding an exposure at intervals of many hours.

BUSHEL, a measure for dry goods, which, by an act of parliament, is to contain 2150·42 cubic inches.

C.

CALCULATION, a reckoning, the result of arithmetical operation; in allusion to the *calculi*, or little stones, used by the antients as we now use figures.

CALCULUS, among mathematicians, denotes a certain way of performing mathematical operations, investigations, &c. Thus we say *antecedental* calculus, *differential* calculus, *fluxional* calculus, &c.

Calculus of *Derivations*. See *Derivations*.

Calculus *Differentialis*, *Exponentialis*, *Integralis*, of *Partial Differences*, of *Variations*, &c. See *Differential*, *Exponential*, and *Integral*.

Fluxional Calculus. See *Fluxions*.

Literal Calculus is the same as Algebra.

Numeral Calculus is the same as Arithmetic.

CALENDAR, Calendarium, or Kalendar, a distribution of time accommodated to the uses of life, or a table or almanac containing the order of the days, weeks, months, feasts, &c. happening throughout the year. The word is derived from *calendæ*, anciently written in large characters at the head of each month.

The days in the calendar were originally divided into octades, or periods of eight days; but afterwards, in imitation of the Jews, into periods of seven, agreeably to the Mosaic law.

The *Roman* Calendar was first formed by Romulus, for the use of his followers and people. The year was first supposed to consist of only 304 days, which was divided into 10 months, some of these months containing 20 days, others 35 days, and some more: it began with the first of March, and ended with December.

Numa reformed this calendar, and added the months of January and February, making it to commence on the first of January, and to consist of 355 days. But as this was evidently deficien, of the true year, he ordered an intercalation of 45 days to be made every four years in this manner, viz. every two years an additional month of 22 days between February and March; and at the end of each two years more, another month of 23 days; the month thus interposed being called Macedonius, or the intercalary February.

Julius Cæsar, with the aid of Sosigines, a celebrated astronomer of those times, farther reformed the Roman calendar, from whence arose the Julian calendar, and the Julian, or old style. Finding that the sun performed his annual course in 365¼ days nearly, he divided the year into 365 days, but every fourth year 366 days, adding a day that year before the 24th of February. This was farther reformed by order of Pope Gregory XIII.; whence arose the terms Gregorian calendar and style, or new style.

Gregorian Calendar, which, by means of epacts rightly disposed through the several months, determines the new and full moons, with the time of Easter, and the moveable feasts depending upon it.

Corrected Calendar, is that which, rejecting all the apparatus of golden numbers, epacts, and dominical letters, determines the equinox, and the paschal full moon, with the moveable feasts depending upon it, by computation from astronomical tables. This calendar was introduced in the year 1700.

CALENDS, or Kalends, in the Roman chronology, the first day of every month.

CALIBER, or Caliper, properly denotes the diameter of any round or cylindrical body.

Caliber, or Caliper *Compasses*, or simply *Calipers*, a sort of compass made with bowed legs, for the purpose of taking the diameter of any round body, or of a cylinder, whether external or internal.

CAMERA, in *Optics*, a name given to those machines or contrivances by which the images of objects are pourtrayed by reflection upon a plane. There are two kinds, the Camera Lucida, and the Camera Obscura.

Camera Lucida. The instrument now known by this name, and which is the invention of Dr. Wollaston, is one of the most ingenious applications of optical reflection. The principal part of it consists of a glass prism, whose section is similar to the trapezium A, B, C, E, (Plate II. fig. 1.) A is a right angle, C is obtuse, and B and E acute. The prism, which may be about ½ inch in the side, and ¾ long, is fixed on a stand, and has over the angle E a plate of metal, with a sight-hole, half covered by the angle of the prism. The ray of light P *a*, coming from the point P, will enter the face AB of the prism at right angles, and consequently undergo no refraction, but proceed in a straight line, and impinge on the face BC, at *a*. There it will, in consequence of the smallness of the angle of incidence, be wholly reflected, and again impinge upon the face CE at *b*, and there be again reflected. The two angles of reflection being = a right angle, it will pass at right angles through AE, to the eye above the sight at E, and thus the point P will, to that eye, appear to be at P in the line *x y*. The eye will, at the same time, see that line through the point of the sight which is uncovered by the prism. If now any object be substituted for P, and a sheet of paper for *xy*, the outlines of the object will be seen, and can be delineated on the paper. The Camera Lucida is exceedingly portable, and as the image is not reflected or refracted by a curve surface, the whole of it is equally clear. It is, however, difficult to procure glass for the prism completely free of internal refractions. A convex lens is sometimes placed over the sight, for the purpose of magnifying the image.

Camera Obscura. This instrument is formed by admitting the light into a dark box or chamber, through a convex lens, from a concave mirror, or even through a small hole, and the image when received without reflection is inverted. Let AB (Plate II. fig. 2) be an object opposite to C, a lens in one side of a dark box, the rays from it will fall inverted and produce on the opposite side the image *ba*. In order to procure a direct image, the rays are made to impinge upon a mirror, placed at an angle of 45°, and be reflected from that upon the receiving surface. In observatories the Camera Obscura is usually made with a concave mirror placed at 45°, and the image received upon a convex table, which shews all the parts equally vivid. Beautiful instances are to be seen at Greenwich, and at the Calton-hill, Edinburgh.

Cancer, *Tropic* of, a small circle of the sphere parallel to the equinoctial, and passing through the beginning of the sign Cancer.

CAPILLARY *Tubes*, pipes whose canals or bores are exceedingly narrow, being so called from their resemblance to a hair in size.

One of the most singular phenomenon of these tubes is, that if you take several of them of different sizes, open at both ends, and immerse them a little way into water, or any other fluid, it will immediately rise in the tubes to a considerable height above the surface of that into which they are immersed.

Another phenomenon of these tubes is, that such of them as will naturally discharge water only by drops, when electrified, yields it in a perpetual stream.

CARDINAL *Points*, in *Geography* and *Navigation*, the four principal points of the compass; viz. east, west, north, and south.

CARDIOIDE, the name of a curve so denominated by Castilliani, from its resemblance to a heart, καρδια; the construction of which is as follows:

Through one extremity of the diameter of the circle, draw a number of lines cutting the circle. Upon these set off without the circle parts equal to the diameter; then the curve passing through all the points is the Cardioide.

CARTESIAN *Philosophy*. The first principle of the Cartesian philosophy is this, "I think, therefore I am;" this is the foundation of Des Cartes's metaphysics: that on which his physics is built is, 'That nothing exists but substances.'" Substance he makes of two kinds; the one that thinks, the other is extended; so that actual thought and actual extension make the essence of substance. The essence of matter being thus fixed in extension, Des Cartes concludes that there is no vacuum, nor any possibility of it in nature, but that the universe is absolutely full: by this principle, mere space is quite excluded; for extension being implied in the idea of space, matter is so too.

Motion is defined to be the translation of a body from the neighbourhood of others that are in contact with it, and considered as at rest, to the neighbourhood of other bodies: by which the distinction is destroyed between motion that is absolute or real, and that which is relative or apparent. He maintains that the same quantity of motion is always preserved in the universe, because God must be supposed to act in the most constant and immutable manner; and hence also he deduces his three laws of motion.

He supposes that God created matter of an indefinite extension, which he separated into small square portions or masses, full of angles: that he impressed two motions on this matter; the one, by which each part revolved about its own centre; and another, by which an assemblage, or system of them, turned round a common centre. From whence arose as many different vortices, or eddies, as there were different masses of matter, thus moving about common centres.

The consequence of these motions in each vortex, according to Des Cartes, is as follows:—The parts of matter could not thus move and revolve amongst one another, without having their angles gradually broken; and this continual friction of parts and angles must produce three elements: the first of these an infinitely fine dust, formed of the angles broken off; the second, the spheres remaining after all the angular parts are thus removed; and those particles not yet rendered smooth and spherical, but still retaining some of their angles and hamous parts from the third element.

Now the first or subtilest element, according to the laws of motion, must occupy the centre of each system, or vortex, by reason of the smallness of its parts: and this is the matter which constitutes the sun, and the fixed stars above, and the fire below. The second element, made up of spheres, forms the atmosphere, and all the matter between the earth and the fixed stars, in such sort, that the largest spheres are always next the circumference of the vortex, and the smallest next its centre. The third element, formed of the irregular particles, is the matter that composes the earth, and all terrestrial bodies, together with comets, spots in the sun, &c.

He accounts for the gravity of terrestrial bodies from the centrifugal force of the ether revolving round the earth: and upon the same general principles he pretends to explain the phenomena of the magnet, and to account for all the other operations in nature.

CATACAUSTIC *Curves*, in the higher *Geometry* and in *Optics*, are the species of caustic curves formed by reflection, which are generated in the following manner: If there be an infinite number of rays proceeding from the radiating point, and reflected by any given curve, so that the angles of incidence be equal to the angles of reflection; then the curve to which the reflected rays are always tangents, is the catacaustic or caustic by reflection: a caustic curve is that formed by joining the points of concurrence of the several reflected rays.

If the reflected ray be produced, so that it becomes equal to the incident ray, the curve formed will be the evolute of the caustic, beginning at the point where the first ray enters; and the portion of the caustic intercepted, is the difference of the two incident rays added to the difference of the two reflected ones.

When the generating curve is a geometrical one, the caustic will be so too, and will always be rectifiable. The caustic of the circle is a cycloid, or epicycloid, formed by the revolution of a circle upon a circle.

If the inside of a smooth bason, containing in it any white liquor, as milk, be placed in the sun's rays, or in a strong candle-light, it will exhibit a very perfect catacaustic curve.

CATENARY, Catenaria, in the higher *Geometry*, a mechanical curve, into which a chain or rope forms itself, by its own weight, when hung freely between two points of suspension.

The equation of this curve is

$$y = a \times \text{hyp. log.}\ \frac{(z+x)^2}{z^2 - x^2}\text{, or}$$

$$y = a \times \text{hyp. log.}\ \frac{z+x}{z-x}$$

CATOPTRICS, that branch of *Optics* which illustrates the laws and properties of light, reflected from mirrors or specula. See the articles *Reflection*, *Mirror*, *Light*, and *Vision*.

Catoptric *Telescope*, the same as *Reflecting* Telescope.

Central, something relating to centre.

Central *Forces* are those forces which tends directly to or from a certain point or centre; being called, in the former case, *centripetal* force, and in the latter, *centrifugal*.

The doctrine of central forces depends on the first Newtonian law of motion; *viz.* "Every body perseveres in its state of rest, or uniform motion in a right line, until a change is effected in it by the agency of some external force."

M. de Moivre, in his "Miscel. Analyt." p. 231, as well as in Phil. Trans. has treated on this subject, and to him we owe many elegant theorems relating to the doctrine of central forces. Varignon, Maclaurin, Simpson, Euler, Emerson, and de l'Hopital, &c. have treated on this subject; to the latter of whom we owe the following general and comprehensive proposition; *viz.*

If a body of any determinate weight move uniformly round a centre, with any given velocity, its centrifugal force may be computed by this proportion:

As the radius of the circle it describes,
Is to double the height due to its velocity;
So is the weight of the body,
To its centrifugal force.

So that if b represent the weight of the body, and $2\,g = 32\frac{1}{6}$ the force of gravity, v its velocity, and r the radius of the curve; we shall have, from the laws of falling bodies,

$$4\,g^2 : v^2 = g : \frac{v^2}{4\,g} = \text{the height}$$

due to its velocity. Whence, by the above proportion,

$$r : \frac{v\ 2}{2\,g} = b : \frac{bv^2}{2gr} = f\text{, the centrifugal force.}$$

Consequently, when the centripetal and centrifugal forces are equal, the velocity of the body is equal to that which it would acquire in falling through half the radius.

2. The central force of a body, moving in a circle, is as the versed sine of an indefinitely small arc, or it is as the square of that arc directly, and as the diameter inversely.

3. If two bodies revolve uniformly in different circles, their central forces are in the duplicate ratio of their velocities directly, and the diameters or radii of the circles inversely;

that is, $F : f = \frac{V^2}{D} : \frac{v^2}{d} = \frac{V^2}{R} : \frac{v^2}{r}$

4. And hence, if the radii or diameters be reciprocally in the duplicate ratio of the velocities, the central forces will be reciprocally in the duplicate ratio of the radii, or directly as the fourth power of the velocities; that is, if $V^2 : v^2 = r : R$, then $F : f = r^2 : R^2 = V^4 : v^4$.

5. The central forces are as the diameters of the circles directly, and squares of the periodic times inversely. For if c be the circumference described in the time t, with the velocity v; then the space $c = tv$, or $v = \frac{c}{t}$; hence, using this value of v in the third rule, it becomes

$$F : f = \frac{C^2}{DT^2} : \frac{c^2}{dt^2} = \frac{D}{T^2} : \frac{d}{t^2} = \frac{R}{T^2} : \frac{r}{t^2}$$

; since the diameter is as the circumference.

6. If two bodies, revolving in different circles, be acted on by the same central force; the periodic times are in the subduplicate ratio of the diameters or radii of the circles; for when $F = f$, then $\frac{D}{T^2} = \frac{d}{t^2}$, and $D : d = T^2 : t^2$, or $T : t : \sqrt{D} : \sqrt{d} = R : \sqrt{r}$.

7. If the velocities be reciprocally as the distances from the centre, the central forces will be reciprocally as the cubes of the same distances, or directly as the cubes of the velocities. That is, if $V : v = r : R$, then is $F : f = {}^3R^3 = V^3 : v^3$.

8. If the velocities be reciprocally in the subduplicate ratio of the central distances, the squares of the times will be as the cubes of the distances; for if $V^2 : v^2 = r : R$, then is $T^2 : t^2 = R^3 : r^3$.

9. Wherefore, if the forces be reciprocally as the squares of the central distances, the squares of the periodic times will be as the cubes of the distances; or when $F : f = r^2 : R^2$, then is $T^2 : t^2 = R^3 : r^3$.

From the preceding theorems, we may deduce the velocity and periodic time of a body revolving in a circle, by means of its own gravity, at any given distance from the earth's centre. Let g be the space through which a heavy body falls, at the surface of the earth, in the first second of time, or $16\frac{1}{12}$ feet; then $2g$ will measure the force of gravity at the surface; and r being assumed for the earth's radius, the velocity of the body in a circle at its surface, in one second, will be $= 2600$ feet nearly; the radius of the earth being taken $= 21000000$ feet.

Again, putting $c = 3{\cdot}14159$, &c. we shall have

$$\sqrt{2gr} : 2cr = 1'' : c\sqrt{\frac{2r}{g}} = 5078$$

seconds nearly $= 1^h\ 24^m\ 38''$; the periodic time at the circumference.

Let now R represent the radius of another circle, described by a projectile, about the earth's centre; since the force of gravity varies inversely, as the square of the distance, we shall have

$$R^{\frac12} : r^{\frac12} = v : v\frac{r^{\frac12}}{R^{\frac12}} = v\sqrt{\frac{r}{R}}$$

the velocity in the circle whose radius is R.

$$\text{And } r^{\frac32} : R^{\frac32} = t : t\frac{R^{\frac32}}{r^{\frac32}} = \sqrt{\frac{R}{r^3}}$$

the periodic time in the same circle.

Or since we have found $v = 26000$, and $t = 5078$, these formulæ become.

$26000, \sqrt{\frac{r}{R}}$ for the required velocity.

$5078\sqrt{\frac{R^3}{r^3}}$ for the periodic time.

In the case of the moon $R = 60r$, we have therefore

$26000\sqrt{\frac{1}{60}} = 3357$ feet per second, the velocity.

$5078\sqrt{216000} = 27\frac{3}{10}$ days, nearly the periodic time.

And in the same way the velocities of the planets, and their several periodic times may be determined, their distances being given; or their periodic times being given, their distances may be found by the converse operation; the periodic time of the earth's revolution, and its distance from the sun, being supposed known.

It may be proper to observe, that though our first theorems related merely to circular motion, yet they are equally true for eliptic orbits.

CENTRE, in a general sense, denotes a point equally remote from the extremes of a line, surface, or solid.

Centre of *Attraction* of a *Body*, is that point into which, if all its matter was collected, its action upon any remote particle would still be the same.

The common centre of attraction of two or more bodies, is sometimes used to denote that point in which, if a particle of matter were placed, the action of each body upon it would be equal, and where it will therefore remain in equilibrio.

Let M and *m* represent the masses of the two bodies, and *d* their distance from each other. Put x for the distance of the point of equal attraction from M. and y the distance of the same from m; then by the laws of attraction

$$m : \mathrm{M} = y^2 : x^2, \text{ or}$$
$$\sqrt{m} : \sqrt{\mathrm{M}} = y : x$$
$$\sqrt{m} + \sqrt{\mathrm{M}} : \sqrt{m} = y + x\ (=d) : y$$
$$\sqrt{m} + \sqrt{\mathrm{M}} : \sqrt{\mathrm{M}} = y + x\ (= d : x$$
$$\text{Whence...}\ y = \frac{d\sqrt{m}}{\sqrt{m} + \sqrt{\mathrm{M}}}$$
$$\text{And.........}\ x = \frac{d\sqrt{\mathrm{M}}}{\sqrt{m} + \sqrt{\mathrm{M}}}$$

Centre of a *Circle*, is that point in a circle which is equally distant from every point of the circumference; and, if more than two equal lines can be drawn from any point within a circle to the circumference, that point will be the centre.

Centre of *Equilibrium*, is the same with respect to bodies immersed in a fluid as the centre of gravity is to bodies in free space.

Centre of *Friction*, is that point in the base of a body on which it revolves, into which, if the whole surface of the base and the mass of the body were collected, and made to revolve about the centre of the base of the given body, the angular velocity destroyed by its friction would be equal to the angular velocity destroyed in the given body by its friction in the same time.

Centre of *Gravity* of any body, or system of bodies, is that point upon which the body or system of bodies, acted upon only by the force of gravity, will balance itself in all positions; hence it follows, that if a line or plane passing through the centre of gravity be supported, the body or system will be also supported.

To find the Centre of Gravity of bodies, mechanically, it is only necessary to dispose the body successively in two positions of equilibrium, by the aid of two forces in vertical directions, applied in succession to two different points of the body, and the point of intersection of these two directions will show the centre required.

Prop. 1. To find the centre of gravity of two given bodies, geometrically.

Divide the line joining them in the inverse ratio of the bodies. Thus, if *a* be two pounds, *b* one pound, and they be three feet asunder, then will the centre of gravity be one foot from *a* and two from *b*.

Prop. 2. To find the centre of gravity of a triangle.

Bisect any two of its sides, draw lines to the opposite angles, and the intersection of these lines will be the centre of gravity; for, as the triangle balances itself upon each of the lines, it must also balance itself upon the point where they meet.

Prop. 3. To find the centre of gravity of any rectilinear figure.

Divide it into triangles, find their centres of gravity, join them, and the result will be a new figure, having fewer sides. Repeat this operation till the figure be reduced to one triangle or to two. If to one, find its centre of gravity: If to two find both, and divide their distance inversely as the triangles: The ultimate point in either case is the centre required.

General Laws of the Centre of Gravity.

Prop. 1. To find the centre of any number of bodies placed in a right line.

Multiply each body by its distance from some fixed point in the line, and divide the sum of the products by the sum of the bodies, the result is the distance of the centre from the point.

Prop. 2. If perpendiculars be drawn from any number of bodies to a given plane, the sum of the products of each body into its respective perpendicular distance from the plane, is equal to the product of the sum of all the bodies into the perpendicular distance of their common centre of gravity from the plane.

From this may be derived a general method for finding the centre of gravity of a system of bodies. For A, B, C, &c. (the bodies of the system,) may be considered as the elementary particles of a body, whose sum or mass is M = A + B + C +, &c.; and A × A*a*, B × B*b*, C × C*c*, &c., the several momenta of all these parts. Hence, in any body, find a general expression for the sum of the momenta, and divide it by the content of the body, so shall the quotient be the distance of the centre of gravity, from the vertex or any other fixed point, from which the momenta are estimated. But, in order to find the general expression for the sum of the momenta, the problem divides itself into several cases, according as it is required to find the centre of gravity of a *solid*, of a plane *surface*, of a *curve surface*, or of a *curve line* of any description.

Prop. 3. To find the centre of gravity of a body considered as an *area, solid, surface of a solid*, or *curve line.*

Let ALV (Plate II. fig. 3.) be any curve, RL the axis, in which the centre of gravity must lie; for as it bisects every ordinate IF in N, the parts on each side RL will always balance each other, and therefore the body will balance itself upon RL, and consequently the centre of gravity must be somewhere in that line.

Put LN $= x$, IN $= y$, IL $= z$, and draw PQ parallel to IF; then if we consider this body to be made up of an indefinite number of corpuscles, and multiply each into its distance from PQ, the sum of all the products divided by the sum of all the corpuscles, or by the whole body LG, will give us the distance of the centre of gravity from L; as is shown above in the preceding proposition.

Now, to get the sum of all the products, we must first have the fluxion of the sum, the fluent of which will be the sum itself.

Put $\dot{s}$ for the fluxion of the body at the distance LN, or x, then will $x\dot{s}$ be the fluxion of the sum of all the products; also, $\dot{s}$ the fluxion of the sum of all the corpuscles. Hence, taking G to represent the centre of gravity, and $p = 3{\cdot}14159$, &c., we have

$$\text{LG} = \frac{flu.\ x\dot{s}}{flu\ \dot{s}}$$

1. Now in the case of an *area*, $\dot{s} = 2y\dot{x}$,

$$\therefore \text{LG} = \frac{flu.\ 2yx\dot{x}}{flu.\ 2y\dot{x}} = \frac{flu.\ yx\dot{x}}{flu.\ y\dot{x}}$$

2. In the case of a simple *curve line*, $\dot{s} = 2\dot{z}$,

$$\therefore \text{LG} = \frac{flu.\ 2x\dot{z}}{flu.\ 2\dot{z}} = \frac{flu.\ x\dot{z}}{flu.\ \dot{z}}$$

3. For the *solid*, we have, $\dot{s} = py^2\dot{x}$,

$$\therefore \text{LG} = \frac{flu.\ py^2x\dot{x}}{flu.\ py^2\dot{x}} = \frac{flu.\ y^2x\dot{x}}{flu.\ y^2\dot{x}}$$

4. For the simple *surface of the solid*, $\dot{s} = py\dot{z}$,

$$\therefore \text{LG} = \frac{flu.\ pyx\dot{z}}{flu.\ py\dot{z}} = \frac{flu.\ yx\dot{z}}{flu.\ y\dot{z}}$$

The centre of gravity of the following bodies, putting generally a for the line joining the vertex and middle of the base, is

A plain triangle $= \frac{2}{3}a$ } from the
A right cone $= \frac{3}{4}a$ } vertex.

A circular sector, as *arch* : *chord* $= \frac{2}{3}$ radius, to the distance of centre of gravity from the centre.

The altitude of the segment of a sphere, or spheroid or conoid, being represented by x, and the whole axis by a, the distance of the centre of gravity in each of these bodies from the vertex will be

The sphere or spheroid $= \dfrac{4a - 3n}{6a - 4x}$

Semi-sphere or semi-spheroid $= \frac{5}{8}x$

Parabolic conoid $= \ldots\ldots \frac{2}{3}x$

Hyperbolic conoid $= \ldots \dfrac{4a + 3x}{6a + 3x}$

In mechanical questions, the centre of gravity of this being found, the whole body is considered as condensed into that single point, by which means the greatest possible simplicity is obtained.

Centre of *Gyration*, is that point into which, if the whole mass were collected, a given force applied at a given distance would produce the same angular velocity in the same time as if the bodies were disposed at their respective distances. This point differs from the centre of oscillation only in this, that in the latter case the motion is produced by the gravity of the body; but in the former, the body is put in motion by some other force acting at one place only.

To determine the Centre of Gyration.

Let A, B, C, &c. (Plate II. fig. 4,) be the bodies which together form a system; P the given force applied at D; R the centre of gyration.

Then the force which accelerates D, while these bodies are at their respective distances, is

$$\frac{P \times SD^2}{A \times SA^2 + B \times SB^2 + C \times SC^2 +}, \text{ \&c.}$$

Let the whole mass be collected in R; and the accelerating force upon D is

$$\frac{P \times SD^2}{(A + B + C +, \text{ \&c.}) \times SR^2}$$

But since P, and the angular velocity of D, are by the definition the same in both cases, the absolute velocity of D is the same, and, therefore, the accelerating force upon D must be the same; that is,

$$\frac{P \times SD^2}{A \times SA^2 + B \times SB^2 +, \text{ \&c.}} = \frac{P \times SD^2}{(A + B + C +, \text{ \&c.}) \times SR^2}$$

Whence SR =

$$\sqrt{\left(\frac{A \times SA^2 + B \times SB^2 +, \text{ \&c.}}{A + B + C +, \text{ \&c.};}\right)}$$

Consequently, if $\dot{s}$ be the fluxion of the body at the distance x from the axis,

$$SR = \sqrt{\frac{\mathit{flu.}\ x^2 \dot{s}}{S}}$$

This, in the case of a right line, becomes

$$\sqrt{\frac{\mathit{flu.}\ x^2 \dot{x}}{x}} = x = \sqrt{\tfrac{1}{3}}$$

In a circle, or cylinder, revolving about the axis, = radius × $\sqrt{\frac{1}{2}}$.

The periphery of a circle, about the diameter, = radius × $\sqrt{\frac{1}{2}}$.

A wheel with a very thin rim, revolving about its axle, = radius.

The plane of a circle, about the diameter, = $\frac{1}{2}$ radius.

The surface of a sphere, about the diameter, = radius × $\sqrt{\frac{2}{3}}$.

A globe, about the diameter, = radius × $\sqrt{\frac{2}{5}}$.

In a cone, about the axis = radius × $\sqrt{\frac{3}{10}}$.

The distance of the centre of gyration from the axis of motion, is a mean proportional between the distance of the centre of gravity and centre of oscillation from the same axis. Hence, when any two of these distances are known, the third may be readily determined.

Centre of *Magnitude*, is the point which is equally distant from the similar external parts of a body.

Centre of *Motion*, that point in a revolving body which remains at rest.

Centre of *Oscillation* is that point in the axis of suspension of a vibrating body, in which, if all the matter of the system were collected, any force applied there would generate the same angular velocity in a given time, as the same force at the centre of gravity, the parts of the system revolving in their respective places.

Let several bodies oscillate about a point of suspension, as if the mass of each were concentrated into points referred to the same plane perpendicular to the axis of motion. Then the gravity of each of them may be decomposed into two forces, of which the one passing through the centre of suspension is destroyed by its resistance; and so the other, perpendicular in direction to the former, is alone efficacious in moving the body or system. Now gravity tends to impress the same velocity upon the points in the vertical direction; which velocity we shall denote by g, and by m, n, p, the sines of the angles, which the supposed inflexible bars, joining the bodies with the centre of suspension, form with the perpendicular. Drawing lines parallel to this perpendicular, and each equal to g, they will represent the accelerating forces of the bodies, or the spaces which they would describe in the first unit of time, if they were left to themselves. But because of the obliquity of these forces upon the inflexible bars, if rectangles be constructed, the spaces run over will be only the sides of those rectangles which are at right angles to the bars; and as the angles have for their sines m, n, p, we shall have these respectively equal to gm, gn, and gp. Hence it follows, that the bodies taken separately, move with different velocities. But if we suppose them connected together, so that they all perform their vibrations in the same time, the velocity of some will be augmented while that of others will be diminished; and as the aggregate of the forces which solicit the system is always the same, it is necessary that the sum of the motions lost should be equal to that of the motions gained.

Let us represent, by A, B, C, the masses of the three bodies, by a, b, c, their distances from the point of suspension, and by α, β, γ, the initial velocities which they lose; the quantities of motion lost will be $A\alpha$, $B\beta$, $C\gamma$, which must be in equilibrio; therefore, the sum of their momentums taken with regard to the point of suspension is nothing; and as these respective distances from that point are a, b, c, we shall have

$$Aa\alpha + Bb\beta + Cc\gamma = 0.$$

Let f be the velocity which the point A subjected to the laws of the system would receive in the first unit of time; as all the points describe similar arcs, their initial velocities are proportional to the distances from the centre of suspension; therefore that of B will be $\frac{bf}{a}$ and that of C will be $\frac{cf}{a}$. Now the velocity lost by each body is equal to the velocity which it would have had, *minus* that which it really has; therefore $\alpha = gm - f$, $\beta = gn - \frac{bf}{a}$, $\gamma = gp - \frac{cf}{a}$; whence, by substituting these values in the preceding equation, we have

$$Aa\,(gm - f) + Bb\left(gn - \frac{bf}{a}\right) + Cc\left(gp - \frac{cf}{a}\right) = 0.$$

Multiplying by a to clear this equation of fractions, and finding the value of f, we have

$$f = \frac{g\,(A\,a^2m + B\,abn + C\,acp)}{A\,a^2 + B\,b^2 + C\,c^2}.$$

From A, B, C, let fall the perpendiculars upon the line passing vertically through the point of suspension, and from the centre of gravity of the system draw perpendicular to the same line. The sum of the momentums of the weights, referred to the point of suspension, is equal to the momentum of their resultant which passes through the centre of gravity.

Whence,

$$f = \frac{ga\,(A + B + C)\,hr}{Aa^2 + Bb^2 + Cc^2}.$$

To ascertain the actual position of the point whose invariable connection with the system does not change its velocity, let x be its distance from the centre of suspension, and S the sine of the angle which the inflexible rod that retains it to that point makes with the vertical: its accelerating force when it moves singly is gs; in the contrary case it is proportional to its distance from the point S, and of consequence is equal to $\frac{x}{a}\,f$ but these two forces, or the initial velocities they produce, must be equal; therefore $\frac{x}{a} = gs$, or, putting the preceding value of f for it, there arises

$$\frac{(A + B + C)\,ghrx}{Aa^2 + Bb^2 + Cc^2} = gs,$$

from which we find

$$x = \frac{s}{r}\cdot\frac{Aa^2 + Bb^2 + Cc^2}{(A + B + C)\,h}.$$

That the point sought may be the centre of oscillation, it is not merely necessary that these two velocities be equal in the first instant, they must continue so in every instant of the descent; therefore x remaining the same, this equation should hold whatever be the position of the point sought, and that of the centre of gravity, relatively to the vertical), that is to say, whatever be s and r, the ratio $\frac{s}{r}$ is therefore constant; and consequently we have at the same time $r = 0$, $s = 0$; which shows that the centre of oscillation, the centre of gravity, and the point of suspension are in one and the same right line. Hence it results that $s = r$, and that

$$x = \frac{A a^2 + B b^2 + C c^2}{(A + B + C) h}.$$

The same kind of reasoning applies exactly, however many the number of particles may be; therefore, to find the centre of oscillation of a system of particles or of bodies, we must multiply the weight of each of them by the square of its distance from the point of suspension, and divide the sum of these products by the weights multiplied by the distance of the centre of gravity from the centre of suspension: this quotient expresses the distance of the centre of oscillation from the point of suspension measured on the continuation of the line joining the centre of gravity and that point.

Call S the point of suspension, O the centre of oscillation, or SO the distance of the centre of oscillation from the point of suspension; also let s be the fluxion of the body at the distance x; then the above formula becomes

$$SO = \frac{flu.\ x^2 \dot{s}}{flu.\ x \dot{s}}$$

As an example, let it be proposed to find the centre of oscillation of a right line, or cylinder, suspended at one end.

In this case

$$SO = \frac{flu.\ x^2 \dot{x}}{flu.\ x \dot{x}} = \frac{\frac{1}{3} x^3}{\frac{1}{2} x} = \frac{2}{3} x$$

that is, the centre of oscillation is $\frac{2}{3}$ of the whole length from the point of suspension.

If the centre of oscillation be made the point of suspension, the point of suspension will become the centre of oscillation.

Centre of *Percussion*, in a moving body, is that point where the percussion or stroke is the greatest, in which the whole percutient force of the body is supposed to be collected.

When the percutient body revolves about a fixed point, the centre of percussion is the same with the centre of oscillation. For instance, when the body moves with a parallel motion, or all its parts with the same celerity, then the centre of percussion is the same as the centre of gravity.

Centre of *Position*, in *Mechanics*, denotes a point of any body, or system of bodies, so selected, that we may properly estimate the situation and motion of the body, or system, by those of this point.

Centre of *Pressure*, is that point against which a force being applied equal and contrary to the whole pressure, will sustain it, so as that the body pressed on will not incline to either side.

Centre of *spontaneous Rotation*, is that point which remains at rest the instant a body is struck, or about which the body begins to revolve.

CENTRIFUGAL *Force*, is that by which a body revolving about a centre or about another body, has a tendency to recede from it.

CENTRIPETAL *Force*, is that by which a body is perpetually urged towards a centre, and thereby made to revolve in a curve instead of a right line.

CENTRIPETATION, a term used by Sir Richard Phillips to indicate the tendency which bodies or planets, and parts of planets, have to fall or move towards their centres, which tendency he ascribes to the orbicular and rotatory motions of the entire masses. He uses the term *Centripetation*, as descriptive of the *local* effect, to avoid the ambiguity of the term *Attraction*, which is used to express a cause, and the term *universal Gravitation*, used to express a *universal* cause; which cause is local and particular in each planet, as resulting from its own motions. Centripetation, therefore, according to the new system, is a local effect, producing aggregation in the planetary masses, while the mutual action and re-action of distant planets, and of the sun on the planets, and the re-actions of the latter arise from their several and respective impulses on the medium of space within which they are situated. Of course, the *diverging* action and re-action, through a gazeous medium, is inversely as the squares of the distances, and directly on the masses; and hence the laws of the two systems accord with each other and with nature, though the explications are very different.

CENTRO *Baryco*, is the same as the Centre of Gravity.

CENTRO BARYC *Method*, in *Mechanics*, is a method of measuring or determining the quantity of any surface or solid, by considering it as generated by motion, and multiplying the generating line or surface into the path of its centre of gravity, viz.

Every figure, whether superficial or solid, generated by the motion of a line or surface, is equal to the product of the generating magnitude into the path of its centre of gravity.

CERES, the name given by Piazzi, of Palermo, to a planet which he discovered on the 1st of January, 1801.

Elements.

Inclination of orbit	0^{s}	10°	$30'$	$57''$
Node • • • • • •	2	21	0	44
Epoch of 1801 • •	2	16	28	0
Mean anomaly •	3	15	55	0
Aphelion • • • •	10	26	27	38
Eccentricity • •	0.0825017			
Equation • • • •		0°	$28'$	
Distance • • • •	2.7355			
Revolution • • •	1681^{d}	12^{h}	9^{m}	

CHAIN, an instrument used in surveying, of which there are different kinds; but that which is most commonly employed for this purpose, is the *Gunter chain*, so called from the name of its inventor.

This chain is 4 poles, or 66 feet long, and is divided into 100 parts or links, each link being 7.92 inches in length;

1 square chain = 10,000 links = 16 poles
10 square chains = 100,000 links = 160 poles = 1 acre.

Hence we have the following easy method of converting links or chains to acres.

From the number of links point off 5 figures to the right-hand for decimals, and those on the left will be *acres*.

CHALDRON, an English dry measure of capacity, mostly used in measuring coals. The chaldron contains 36 bushels, and it weighs about 28 cwt. By act of parliament, the Newcastle chaldron is 52½ cwt.

CHANCES, a branch of analysis, which treats of the probability of events taking place, by contemplating the different ways in which they may happen or fail.

The probability of an event is the ratio of the chance for its happening to all the chances, both for its happening and failing.

The expectation of an event, is the present value of any sum or thing which depends either on the happening or on the failing of such an event.

Events are independent, when the happening of any one of them neither increases or lessens the probability of the rest.

Prop. 1. If an event may take place in n different ways, and each of these be equally likely to happen, the probability that it will take place in a specified way is properly represented by $\frac{1}{n}$, certainty being represented by unity.

For the sum of all the probabilities is certainty, or unity, because the event must take place in some one of the ways, and the probabilities are equal, therefore each of them is $\frac{1}{n}$. And if the certainty be a, the value of the expectation will be $\frac{a}{n}$.

Prop. 2. If an event happen in a ways, and fail in b ways, all being equally probable, the chance of its happening is $\frac{a}{a+b}$, and of failing $\frac{b}{a+b}$.

Thus, the probability of casting an ace with a single die in one throw is $\frac{1}{6}$, of casting an ace or deuce is $\frac{2}{6}$, and so on.

Suppose it were required to determine the probability of drawing out of the 52 cards in a pack the four aces in four draws.

Here $m=4$ and $n=52$, whence the probability is $\frac{1\,.\,2\,.\,3\,.\,4}{52\,.\,51\,.\,50\,.\,49}$.

Prop. 3. If two events be independent of each other, and the probability that one will happen be $\frac{1}{m}$; and the probability that the other will happen be $\frac{1}{n}$, the probability that they will both happen is $\frac{1}{mn}$. For each of the ways in which the first can happen or fail may be combined with each of the n ways in which the other can happen or fail, and thus form mn combinations, and there is only one in which both can happen, therefore the probability is $\frac{1}{mn}$.

Hence: If there be any number of independent events, the probability that they will all happen is equal to the product of all the separate probabilities.

The same has also place with regard to the failure of an event.

Ex. 1. What is the chance of throwing sixes upon two dice?

Ans. Each is $\frac{1}{6}$, therefore both is $\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$.

2. What is the chance of throwing one six with two dice?

Ans. The chance that we shall not throw one is $\frac{5}{6}\times\frac{5}{6}=\frac{25}{36}$, and $1-\frac{25}{26}=\frac{11}{36}=\frac{1}{3}$ nearly.

CHANGES, in *Mathematics*, denote the various arrangements that may take place in the order or situation of a given number of things; and is distinguished from the more general term permutations, in this; that in the latter there may be any number of things, and any number taken at a time; while in the former, the whole number is always supposed to enter. The whole number of changes that a given number of things n admits of, is equal to the continued product, thus the number of changes of 6 things

$$=1.2.3.4.5.6=720.$$

CHARGE, in *Electricity*, the supposed accumulation of the electric matter on one surface of an electric, whilst an equal quantity passes off from the opposite surface.

CHARGE, in *Gunnery*, is the quantity of powder and ball, or shot, put into a piece of ordnance, in order to prepare it for execution.

Different charges of powder, with the same weight of ball, produce different velocities in the ball, which are in the subduplicate ratio of the weights of powder; and when the weight of powder is the same, and the ball varied, the velocity produced is in the reciprocal subduplicate ratio of the weight of the ball. Calling the length of the bore of the gun b, the length of the charge producing the greatest velocity ought to be $\frac{b}{2.718281848}$, or about $\frac{3}{8}$ths of the length of the bore.

CHART, or *Sea Chart*, a hydrographical or sea-map for the use of navigators; being a projection of some part of the sea in plano, shewing the sea-coasts, rocks, sands, bearings, &c.

Plain CHARTS have the meridian, as well as the parallels of latitudes, drawn parallel to each other, and the degrees of longitude and latitude every where equal to those at the equator.

Mercator's CHART, like the plain charts, has the meridians represented by parallel right lines, and the degrees on the parallels, or of longitude, every where equal to those at the equator, so that they are increased more and more, above their natural size, as they approach towards the pole; but then the degrees of the meridians, or of latitude, are increased in the same proportion at the same part; so that the same proportion is preserved between them as on the globe itself.

CHORD in *Geometry*, is the right line joining the extremities of any arc of a circle.

A line drawn from the centre to

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bisect a chord, is perpendicular to the chord; or if perpendicular to the chord, it bisects both the chord and the arch.

Chords equally distant from the centre are equal; or if they are equal, they are equally distant from the centre.

The chord is a mean proportional between the diameter and versed sine.

CHROMATICS, that part of optics which explains the several properties of the colours of light, and of natural bodies.

CHRONOLOGY, the art of measuring time, and distinguishing its several constituent parts, such as centuries, ages, years, months, weeks, &c. by appropriate marks and characters.

CHRONOMETER, a kind of clock, so contrived as to measure very small portions of time with great accuracy.

CIRCLE, in *Geometry*, a plane figure bounded by a curve-line, every where equally distant from a point within it, called the centre. The periphery or circumference, is sometimes called the circle, though that name denotes the space contained within the circumference, and not the circumference itself.

To find the Area, or Circumference, of a Circle, the Diameter being given.

1. Multiply the diameter by 3·14159, and the product will be the circumference.

2. Multiply the square of the diameter by ·7854, and the product will be the area.

Or general, if we put diameter = D, circumference = C, area = A, and 3·14159 = P, we have the following relations between those four quantities; *viz.*

$$1.\ .\ .\ D=\frac{C}{P}=\frac{4A}{C}=2\sqrt{\frac{A}{P}}$$

$$2.\ .\ .\ C=PD=\frac{4A}{D}=2\sqrt{PA}$$

$$3.\ .\ .\ A=\frac{PD^2}{4}=\frac{C^2}{4P}=\frac{DC}{4}$$

$$4.\ .\ .\ P=\frac{C}{D}=\frac{4A}{D^2}=\frac{C^2}{4A}$$

To find the centre of a given circle draw any chord, bisect it, and through the point of section draw a line at right angles. This line is a diameter, which bisected, gives the centre.

To describe a circle through any three given points not in the same straight line.

Join the points by two straight lines, bisect those lines at right angles, and the intersection of the bisecting lines is the centre.

To divide a given circle into any number of co-centric parts, equal to each other.

Divide the radius into as many equal parts as are required; and from the parts of division, erect perpendiculars upon the radius; describe a semicircle meeting the perpendiculars; and through the points of contact draw the circles.

To divide a circle into any number of parts, equal both in area and periphery.

Divide the diameter into the number of parts, and describe a semicircle upon the alternate sides of each division, so as to touch the point of contact, and also the extremities of the diameter.

Quadrature and Rectification of the Circle.

The simplest and most ancient approximation is as follows:

As 7 to 22, so is the diameter to the circumference.

Other approximations, nearer than the above, but in larger numbers, are as follows:

$$\left.\begin{array}{r}\text{as }\ 106:333\\ 113:355\\ 1702:5347\\ 1815:5702\end{array}\right\}\text{so is diam. to the cir.}$$

of which each is more accurate than the preceding.

Vieta shows, that if the diameter of the circle be 1000, the circumference will be

greater than 3141·5926,535
but less than 3141·5926,537

a greater and greater degree of approximation.

Van Ceulen carried the approximation to thirty-six places of figures; Sharp, to seventy-two places of figures; Machin, to 100 places; and Lagny, to 128; that is, if the diameter of a circle be one, the circumference will be

31415,92653,58979,32384,62643,38327, 95028,84197,16939,93751,05820,97494, 45923,07816,40628,62089,98628,03482, 53421,17067,98214,80865,13272,30664, 70938,446 + or 447 —

Lord Brounker found the ratio of the square of the diameter, to the area of a circle in a continued fraction, to be as

$$1 \text{ to } 1 + \frac{1}{2 + \frac{9}{2 + \frac{25}{2 + \frac{49}{2} \;\&c.}}}$$

If the diameter of a circle be 1, and c be taken to represent the circumference; then

$$c = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} \;\&c.\right)$$

$$c = \sqrt{8}\left(1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \frac{\ }{11} \;\&c.\right)$$

$$c = \sqrt{12}\left(1 - \frac{1}{3 \cdot 3} + \frac{1}{5 \cdot 3^2} - \frac{1}{7 \cdot 3^3} + \frac{1}{9 \cdot 3^4} \;\&c.\right)$$

$$c = 8\left(\frac{1}{1 \cdot 1 \cdot 3} + \frac{1}{1 \cdot 3 \cdot 5} - \frac{1}{3 \cdot 5 \cdot 7} + \frac{1}{5 \cdot 7 \cdot 9} - \&c.\right)$$

CIRCLES of the *higher Orders* are curves, the properties of which are expressed by the following equations:

$$x^m : y^m :=: y : \overline{a - x}, \text{ or } y^{m+1} = x^m\,\overline{a - x}$$

$$x^m : y^m :=: y^n : \overline{a - x}|^n. \text{ or } y^{m+n} = x^m\,\overline{a - x}|^n$$

where a is the axis, x the absciss, and y the ordinate. Curves defined by this equation will be ovals, when m is an odd number. But when m and n are each equal to one, the equation becomes that of the common circle.

CIRCLE of *Curvature* in *Geometry*, that circle, the curvature of which is equal to that of any curve at a certain point. See RADIUS of *Curvature*.

CIRCLES of the *Sphere*, either great or small.

A *great* CIRCLE of the *Sphere*, is that which divides it into two equal parts or hemispheres, having the same centre and diameter with it; as the horizon, meridian, &c.

A small of the *Sphere*, divides the sphere into two unequal parts, having neither the same centre nor diameter with the sphere; its diameter being only some chord of the sphere less than its axis. Such as the parallels of latitude, &c.

CIRCLES of *Altitude*. Parallels to the horizon.

CIRCLES of *Declination*, are great circles intersecting each other in the poles.

Diurnal CIRCLES, are parallels to the equinoctial, supposed to be described by the stars, and other points of the heavens, in their apparent diurnal rotation about the earth.

It may here be observed, that most circles of the sphere are transferred from the heavens to the earth; and have thus a place in geography, as well as in astronomy; all the points of each circle being conceived as let fall perpendicularly on the surface of the terrestrial globe, and hence tracing our circles perfectly similar to them.

CIRCLE of *Illumination*, a circle passing through the centre of the earth or moon, perpendicular to a line drawn from the sun to the respective body.

CIRCLES of *Celestial Latitude*, are great circles perpendicular to the plane of the ecliptic, passing through its poles, and through every star and planet.

CIRCLE of *perpetual Apparition*, one of the less circles, parallel to the equator; described by any point of the sphere touching the northern or southern point of the horizon; and carried about with the diurnal motion.

CIRCLE of *perpetual Occultation*, is another circle at a like distance from the equator; and contains all those stars which never appear at the place to which it refers.

Polar CIRCLES, are at a distance from the poles equal to the greatest declination of the ecliptic.

CIRCULAR, any thing relating to the circle.

Circular *Arc,* any part of the circumference.

To find the Lengths of Circular Arcs.

Let r represent the radius, d the diameter, c the circumference of the circle, s the sine of the arc, and v the versed sine of the half arc, and m its measure, in degrees, &c.; then

1. The arc $= rm \times \cdot 0174533$.

2. The arc $= 2\sqrt{dv}\left\{1 + \frac{v}{2.3d} + \frac{3v^2}{2.4.5d^2} + \frac{3.5x^3}{2.4.6.7d^3}\right.$ &c.
or
$2d\sqrt{q} + \frac{q}{2.3}A + \frac{3^2q}{4.5}B + \frac{5^2q}{6.7}C$, &c.

where $q = \frac{v}{d}$; and A, B, C, &c. are the 1st, 2d, 3d, &c. terms.

3. The arc $= 2s \times \left\{1 + \frac{s^2}{3.3r^2} + \frac{3s^4}{5.2.4r^4} + \frac{3.5s^6}{7.2.4.6r^6}\ \text{\&c.}\right\}$ or
$2s + \frac{q}{2.3}A + \frac{3^2q}{4.5}B + \frac{5^2q}{6.7}C + \frac{7^2q}{8.9}D$, &c.

where $q = \frac{s^2}{r^2}$; and A, B, C, D, &c. the preceding terms.

To which may be added the following approximations:

4. The arc $= 2d\sqrt{\frac{3v}{3d-v}}$ nearly.

5. . . . $= \frac{2}{9} \times \left\{5d\sqrt{\frac{5v}{5d-3v}} + 4\sqrt{dv}\right\}$ nearly.

6. . . . $= \frac{8c' - C'}{3}$ nearly.

where C' is the chord of the whole arc, and c' the chord of half the arc.

Circular *Instruments,* or *Reflecting,* or *Multiplying Circles,* are instruments which possess all the advantages of accuracy attending large instruments, in diminishing the errors of division and eccentricity at pleasure by means of reflection, though the instruments themselves are small and portable. Circular instruments may be considered as improvements of Hadley's octant, and the marine sexant, which are for the same purpose; viz. for observing the altitudes, distances, &c. of the heavenly bodies, extremely useful for navigators in finding the moon's distance, and other nautical purposes. The best instruments of this kind, are those of Mayer, Borda, and Rios.

Circular *Parts* (Napier's), are the legs, the complement of the hypothenuse, and the complements of the two oblique angles of a right angled spherical triangle.

Napier's general rule is this: the rectangle, under the radius and the sine of the middle part, is equal to the rectangle under the tangents of the adjacent parts, and to the rectangle under the cosines of the opposite parts. The right angle or quadrantal side being neglected, the two sides and the complements of the other three natural parts are called the circular parts; as they follow each other, as it were, in a circular order. Of these, any one being fixed upon as the middle part, those next it are the adjacent, and those farthest from it the opposite parts. This rule contains all the particular rules for the solution of right-angled spherical triangles.

Circular *Sailing,* the method of navigating a ship upon a great circle of the globe.

Circular *Sectors,* are the areas bounded by any arc of a circle and two radii; the measure of which is found as follows:

Let l represent the length of the arc of the sector, and m its measure, in degrees, minutes, &c.; then

1. Area of sector $= \frac{1}{2} rl$

2. . . . . $= \cdot 7854\, d^2 \times \frac{m}{360}$

Circular *Segment* is the space bounded by any arc and its chord; the area of which may be found by the following formula:

If A' represent the area of the circular sector, and C' the chord of the arc, then

1. Area of segment $= A' - \frac{1}{2} C (r - v)$

2. Area $= 2v \sqrt{dv} \times \left\{ \frac{2}{3} - \frac{v}{5d} - \frac{v^2}{28d^2} - \frac{v^3}{72d^3} - \text{\&c.} \right\}$ or $2\sqrt{dv} \times \left\{ \frac{2}{3} - \frac{3v}{5.2d} A - \frac{5v}{7.4d} B - \frac{7.3v}{9.6d} C - \frac{9.5v}{11.8d} D \right\}$

A, B, C, &c. being the preceding terms.

3. Area $= \frac{4}{3} v \sqrt{vV} + \frac{v}{5V} A - \frac{v}{7V} B + \frac{3v}{9V} C - \frac{5v}{11V} D$, &c. where $V = (d - v)$.

4. Area $= 2rc'' - \frac{1}{2.3} q^2 A - \frac{1.3}{4.5} q^2 B - \frac{3.5}{6.7} q^2 C$, &c. where c'' represents the cosine of half the arc, and $q = \frac{c''}{r}$. A, B, C, &c. being the preceding terms.

To which, may be added the following approximations; viz.

5. Area $= \frac{4}{5} \left\{ \sqrt{(dv - v^2)} + \frac{2}{3} \sqrt{dv} \right\}$ nearly.

6. Area $= \frac{4}{3} v (dv - \frac{3}{5} v^2)$ nearly.

The area of circular zones is found by finding the difference of the two segments. And the area of circular rings, by finding the difference of the areas of the two circles. Or by making D and *d* the diameters.

CIRCULATING *Decimals*, consist of a repetition of a number of digits, as ·646464, &c. 4127127127, &c.; in fact, every decimal that is not finite, is a circulating decimal, or is such, that if continued far enough the same figures will again recur.

When the figures recur from the beginning the decimal is called *pure;* and when there are some figures before the circulation, it is called *mixed.*

The denominator of every pure circulating decimal consists of as many 9's as there are figures in the circle.

Let .315 be a circulating decimal, the denominator is 999.

$.315 \times 1000 = 315.\ 315, 315$, &c.
$.315 \times \quad 1 = \quad .\ 315, 315$, &c.
Hence
$.315 \times \ 999 = 315.$ wherefore
$.315 = \frac{315}{999}$. Q. E. D.

The denominator of a mixed circulating decimal, after the terminate figures are subtracted from the whole, consists of as many 9's as there are circulating figures, with as many 0's as there are terminate ones.

The denominator of .63'315, is $99900.63 = \frac{63}{100}$ and $.00315' = \frac{00315}{99900}$, reducing to a common denominator
$.00315 \times 100 = \ .315$ and
$.63 \times 99900 \ = 62\,927 \ (= 63000 - 63)$

Hence $.63,315 = \frac{.63'252'}{99900}$ Q, E, D.

Hence, to reduce a circulating decimal to a fraction, subtract the terminate figures from the whole; and place for denominator of the remainder as many 9's as there are circulating figures, with as many 9's as there are terminate ones.

CIRCUMFERENCE, in a general sense, denotes the line, or lines, bounding any figure.

CIRCUMFERENTOR, an instrument used by surveyors in taking angles; it consists of a brass circle and index, in one piece, commonly about seven inches in diameter, and index about fourteen inches long, and one and a half inches broad. On the circle is a card or compass, divided into 360 degrees; the meridian line of which answers to the middle of the breadth of the index. There is also soldered on the circumference a brass ring, on which screws another ring with a flat glass in it, so as to form a kind of box for the needle, suspended on the needle in the centre of the circle. There are also two sights to screw on, and slide up and down the index, as also a ball and socket screwed on the under side of the circle, to receive the leg of the three-legged staff.

CIRCUMSCRIBED *Figure*, is that which is drawn about another figure so as to touch all its angles, or to touch it by every side.

CISSOID, in the higher *Geometry*, is a curve line of the second order, invented by Diocles, a Greek geometrician, for the purpose of finding two continued mean proportionals between two other given lines. The generation of this curve is as follows; at the extremity of the diameter of a circle draw a tangent, and from the opposite extremity of the diameter draw any number of lines meeting the tangent. Set off upon these from the tangent, the same distances that are within the circle; and the curve drawn through these points is the Cissoid.

The cissoid has the following properties: 1. The curve has two infinite legs, meeting in a cusp, and tending continually towards the indefinite line, which is their common asymptote. 2. The curve bisects each semi-circle. 3. Letting fall perpendiculars from any corresponding points, the portions at each extremity of the diameter, and also of the line meeting the tangent, are equal to each other. 4. If the diameter be $= a$, the abscissa $= x$, and the ordinate $= y$; then is $x : a - x = : y^2 : x^2$, or $x^3 = y^2 . (a - x)$, which is the equation of the curve. 5. The whole infinitely long cissoidal space, contained between the infinite asymptote, and the curves of the cissoid, is equal to triple the generating circle. 6. All cissoids are similar figures.

CLEPSYDRA, an instrument or machine serving to measure time by the fall of a certain quantity of water. The use of clepsydræ is very ancient; they were invented in Egypt, under the Ptolemies; as were also sun-dials. Their use was chiefly in the winter, as the sun-dials served in the summer: but they had two great defects; the one, that the water ran out with a greater or less facility, as the air was more or less dense; the other, that the water ran more readily at the beginning than towards the conclusion. Suppose a cylindrical vessel, whose charge of water flows out in twelve hours, were required to be divided into two parts, to be evacuated each hour. 1. As the part of time 1 is to the whole time 12, so is the same time 12 to a fourth proportional 144. 2. Divide the altitude of the vessel into 144 equal parts: here the last will fall to the last hour; the three next above to the last part but one; the five next to the tenth hour; lastly, the twenty-three last to the first hour. For since the times increase in the series of the natural numbers 1, 2, 3, 4, 5, &c. and the altitudes, if the numeration be in a retrograde order from the twelfth hour, increase in the series of the unequal numbers 1, 3, 5, 7, 9, &c. the altitudes computed from the twelfth hour will be as the squares of the times, 1, 4, 9, 16, 25, &c. Therefore the squares of the whole time, 144, comprehend all the parts of the altitude of the vessel to be evacuated. But a third proportional to 1 and 12 is the square of 12, and consequently it is the number of equal parts into which the altitude is to be divided, to be distributed according to the series of the unequal numbers, through the equal interval of hours. There were many kinds of clepsydræ among the ancients; but they all had this in common, that the water ran generally through a narrow passage, from one vessel to another, and in the lower was a piece of cork, or light wood, which as the vessel filled, rose up by degrees, and showed the hour.

CLIMATE, or **Clime**, in the ancient *Geography*, a part of the surface of the earth, or zone, bounded by two lesser circles parallel to the equator; and of such a breadth, as that the longest day in the parallel nearer the pole exceeds the longest day in that next the equator, by some certain space, as half an hour, or an hour.

Vulgarly, the term climate is bestowed on any country or region differing from another, either in respect of the seasons, the quality of the soil, or even the manners of the inhabitants, without any regard to the length of the longest day

CLOCK, a well-known instrument for measuring time; it is regulated by means of a *Pendulum*, the laws of which will be found under that article.

CLOUD, a visible aggregate of minute drops of water suspended in the atmosphere.

It is concluded, from numerous observations, that the particles of which a cloud consists, are always more or less electrified; and this fluid has hence been considered as the cause of the formation of all clouds whatever, whether of thunder, hail, rain, or snow.

The hypotheses which assumes the existence of vesicular vapour, and makes the particles of clouds to be hollow spheres, which unite and descend in rain when ruptured, however sanctioned by the authority of several eminent philosophers, does not seem necessary to the science of meteorology in its present state; it being evident that the buoyancy of the particles is not more perfect than it ought to be, if we regard them as mere drops of water. In fact, they always descend, and the water is elevated again only by being converted into invisible vapour.

CO-EFFICIENTS, in *Algebra*, are numbers or letters prefixed to other letters, or unknown quantities, into which they are supposed to be multiplied; and with such letters, or the quantities, making a product, or co-efficient product.

When a quantity stands alone, without being preceded by any number or letter, it is always supposed to have unity for its co-efficients.

In equations the absolute term is sometimes classed under the general term co-efficient, in which it is supposed to be prefixed to x^0 or y^0, &c. all such quantities being equal to unity: thus, in the formula x^3+ax^2+cx+d, the co-efficients are 1, a, c, and d; the first 1 being understood, and the last being supposed to precede $x^0=1$.

In equations we have the following remarkable property of the co-efficients; viz.

1. The co-efficient of the second is equal to the sum of the roots of the equation with their signs changed. 2. The co-efficient of the third term is equal to the sum of all the product, taken two and two. The co-efficient of the fourth term is equal to the sum of all the product, taken three and three together, with their signs changed, and so on; and, finally, the absolute term is equal to the product of all the roots, with their signs changed, if the number of terms be even; but without being changed, if the number of terms be odd; this term being here supposed to stand on the left-hand side of the equation.

Thus, for example, in the cubic equation, $x^3+ax^2+bx+c=0$; supposing the roots to be p, q, and r, we shall have

$$\begin{aligned} a &= -(p+q+r) \\ b &= pq+pr+qr \\ c &= -pqr \end{aligned}$$

and the same for every order of equations.

The sum of all the co-efficients of the binomial $(a+x^n)=2^n$, and of $(a-x)^n=0^n=0$; that is, the sum of the positive co-efficients is equal to the sum of the negative ones; and consequently their sum is equal to zero. See *Binomial Theorem*.

COFFER *Dam*, a term applied by engineers to denote the enclosures formed for laying the foundation of piers and other works in water, to exclude the surrounding fluid, and thus forming a protection both to the work and workmen.

COHESION, that species of attraction which, uniting particle to particle, retains together the component parts of the same mass; being thus distinguished from *adhesion*, or that species of attraction which takes place between the surfaces of similar or dissimilar bodies.

The following table shows the weights necessary to tear asunder rods of different substances, whose bases were each a square inch, the weights being applied in the direction of their length.

METALS.	*lbs. Avoirdupoise.*
Steel	135000
Iron bar	74000
Cast iron	50100
Copper, cast	28600
Silver, ditto	41500
Gold, ditto	22000
Tin, ditto	4000

Bismuth	2900
Zinc	2600
Antimony	1000
Lead, cast	860
WOODS.	*lbs. Avoirdupoise.*
Beech, Oak	17300
Alder	13900
Elm	13200
Mulberry	12500
Willow	12500
Ash	12000
Plum	11800
Elder	10000
Fir	8330
Walnut	8130
Pitch, Pine	7650
Cypress	6000
Poplar	5500
Cedar	4880

Other experiments have been made to ascertain the strength of cohesion in bodies, when placed horizontally, and loaded with weights in different parts. The weights, and their distances from the point of support, are shown in the following table:

Woods	*Dist. in Inch.*	*Wt. in oz.*
Pine	9½	36½
Fir	9	40
Beech	7	56½
Elm	9	44
Oak	8½	48
Elder	9¼	48

In the above table the rods were rectangular parallelopipedons, and the side of their square section .26 of an inch.

Coulomb found the lateral cohesion of brick and stone only $\frac{1}{44}$ more than the direct cohesion, which, for stone, was 215lb. for a square inch; for good brick, from 280 to 300.

Count Rumford found the cohesive strength of a cylinder of iron, an inch in diameter, 63466, or 63173lb.; the mean 63320; which is only $\frac{1}{20}$ more than Emerson's result.

Sickingen makes the comparative cohesive strength of gold, 150955; of silver, 190771; of platina, 262361; of copper, 304696; of soft iron, 362927; of hard iron, 559880. Guyton makes platina a little stronger.

In Buffon's experiments, *b*, *d*, and *l*, being the breadth, depth, and length of a beam of oak, in inches; the weight which broke it in pounds was $= bd^2\left(\frac{54.25}{l} - 10.\right)$

For farther information on this subject, the reader may consult Ritter on Cohesion, Gilbert's Journal, iv. 1; Benzenberg on Cohesion, Gilbert, xvi. 76; Fontana on Solidity and Fluidity, Soc. Ital. i. 89; and Dr. T. Young on the Cohesion of Fluids, in the Phil. Trans. for 1805, or in the second volume of his "Natural Philosophy."

COLD, in common language, denotes the sensation which is felt, or the effect which is produced, by the abstraction of heat. Thus the climate of Great Britain is a cold climate, in comparison with that of the West India islands; and a hot climate in comparison with that of Siberia. If a man warms one of his hands near a fire, whilst he cools his other hand by means of ice; and if afterwards he plunges both his hands in a bason of water of the common temperature of the atmosphere, that water will feel cold to the hand that has been heated, and hot to the other hand. From this, it appears that cold is not any thing real, but merely a privation of heat; so that instead of saying that a body has been cooled to a certain degree, it may with equal truth and propriety be said that the body has been deprived of heat to that certain degree.

Notwithstanding the simplicity of this theory, and the conviction which seems to accompany it, philosophers have often entertained doubts concerning it; and they have endeavoured to inquire into the real state of the matter, by devising experiments capable of demonstrating whether the cause of heat was any thing real, and that of cold only a privation or diminution of the former; or, *vice versa*, whether the cause of cold was any thing real, and that of heat a diminution; or, lastly, whether the production of heat, and the production of cold, were not owing to two distinct principles or elements. On the supposition that the cause of one of those effects only is real, it is much more natural to suppose that the cause of heat is the real principle or element, since its ef fects, viz. enlargement of the bulk of bodies, the separation of their parts, &c. are such as must be produced by the introduction of something real; and the abstraction of this principle may naturally produce the effects of cold, such as contraction of the bulk of bodies, agglutination, &c.; whereas it would be unnatural to suppose that a body contracts its bulk, as its parts come into closer contact, because something else has been introduced amongst them. With respect to the last supposition, viz. whether the effects of heat and those of cold be not owing to two distinct principles, a few arguments, and the equivocal result of a few experiments, have, at times, been adduced in support of it. But the general and prevailing opinion among philosophers is, that a single element, called caloric, produces heat, or the effects of expanding bodies separating their parts, &c.; and that cold is only a relative expression; that is, meaning only the decrement of heat; so that real or absolute cold consists only in the total abstraction of caloric; and, that such a point, viz. the zero of heat may be determined, has been shown by the experiments, the discoveries, and the calculations of some late eminent philosophers, viz. Irvine, Black, Crawford, and others.

COLLIMATION, *Line* of, in a telescope, is a line passing through the intersection of those wires that are fixed in the focus, and the centre of the object glass.

COLLISION, in *Mechanics* and *Physics*, is the meeting and mutual striking of two or more bodies, one of which, at least, is in motion. The most simple of the problems relating to collision, is that of a body proceeding to strike against another at rest, or moving before it with less velocity, or approaching towards it. Des Cartes supposing that the same quantity of absolute motion always exists in the world, concluded that the sum of the motions after the impact was equal to the sum of the motions before it. But the proposition is true only in the first and second of these cases; it is false when the two bodies meet each other; for, in that case, the sum of the motions after the impact is equal to the difference of their motions before it, not to their sum.

If the impact of two perfectly hard bodies be direct, they will, after impact, either remain at rest, or move on uniformly together with different velocities, according to the circumstances under which they met.

Let B and b represent two perfectly hard bodies, and let the velocity of B be represented by V, and that of b by v, which may be taken either positive or negative, according as b moves in the same direction as B, or contrary to that direction, and it will be zero when b is at rest. This notation being understood, all the circumstance of the motion of the two bodies, after collision, will be expressed by the formula, velocity =

$$\frac{BV + bv}{B + b},$$

which being accommodated to the three circumstances under which v may enter, becomes

$$\text{velocity} = \frac{BV + bv}{B + b}$$

when both bodies moved in the same direction,

$$\text{velocity} = \frac{BV - bv}{B + b}$$

when the bodies moved in contrary directions,

$$\text{velocity} = \frac{BV}{B + b}$$

when the body b was at rest.

These formula arise from the supposition of the bodies being perfectly hard, and consequently that the two after impact move on uniformly together as one mass. In cases of perfectly elastic bodies, other formulæ have place which express the motion of each body separately, as in the following proposition.

If the impact of two perfectly elastic bodies be direct, their relative velocities will be the same both before and after impact, or they will recede from each other with the same velocity with which they met; that is, they will be equally distant, in equal times, both before and after their collision, although the absolute velocity of each may be changed. The circumstances attending this change of motion in the two bodies, using the above notation, are expressed in the two following formulæ:

$\frac{2bv+(B-b)V}{B+b}$ the velocity of B

$\frac{2BV+(B-b)v}{B+b}$ the velocity of b

which needs no modification, when the motion of b is in the same direction with that of B.

In the other case of b's motion, the general formulæ become

$\frac{-2bv+(B-b)V}{B+b}$ the velocity of B

$\frac{2BV-(B-b)v}{B+b}$ the velocity of b

when b moves in a contrary direction to that of B, which arise from taking v negative. And

$\frac{(B-b)V}{B+b}$ the velocity of B

$\frac{2BV}{B+b}$ the velocity of b

when b was at rest before impact, that is, when $v=o$.

If a perfectly hard body impinge obliquely upon a perfectly hard and immoveable plane, it will, after collision, move along the plane.

And its velocity before impact
Is to its velocity after impact
As radius
Is to the cosine of the angle of impact. But if the body be elastic it will rebound from the plane with the same velocity, and at the same angle with which it met it.

COLOUR, in natural philosophy, that property of bodies which affects the sight only; or that property possessed by the elementary rays of light, separated by any means whatever, of exciting in us different sensations according to their different refrangibility. Thus colour may be considered in two respects, as it regards bodies in general, and as it is produced by solar light.

Sir Isaac Newton's theory of light and colours is deduced from clear and decisive experiments.

1st. That lights which differ in colour, differ also in degrees of refrangibility.

2d. That the light of the sun, notwithstanding its uniform appearance, consists of rays differently refrangible.

3d. That those rays which are more refrangible than others, are also more reflexible.

4th. That as the rays of light differ in degrees of refrangibility and reflexibility, so they also differ in their disposition to exhibit this or that particular colour; and that colours are not qualifications of light derived from refractions or reflections of natural bodies, as was generally believed, but original and connate properties, which are different in different rays, some rays being disposed to exhibit a red colour and no other, and some a green and no other, and so of the rest of the prismatic colours.

5th. That the light of the sun consists of violet-making, indigo-making, blue-making, green-making, yellow-making, orange-making, and red-making rays; and all of these are different in their degrees of refrangibility and reflexibility; for the rays which produce red colours are the least refrangible, and those that make the violet the most; and the rest are more or less refrangible as they approach either of these extremes, in the order already mentioned: that is, orange is least refrangible next to red, yellow next to orange, and so on; so that to the same degree of refrangibility there ever belongs the same colour, and to the same colour the same degree of refrangibility.

6th. Every homogeneal ray, considered apart, is refracted according to one and the same rule, so that its sine of incidence is to its sine of refraction in a given ratio; that is, every different coloured ray has a different ratio belonging to it.

7th. The species of colour, and degree of refrangibility and reflexibility, proper to any particular sort of rays, is not mutable by reflection or refraction from natural bodies, nor by any other cause that has been yet observed. When any one kind of rays has been separated from those of other kinds, it has obstinately retained its colours, notwithstanding all endeavours to bring about a change.

8th. Yet seeming transmutations of colours may be made, where there is any mixture of divers sorts of rays; for, in such mixtures, the component colours appear not, but, by their mutually alloying each other, constitute an intermediate colour.

9th. There are, therefore, two sorts of colour, the one original and simple, the other compounded of these; and all the colours in the universe are either the colours of homogeneal, simple light, or compounded of these mixed together in certain proportions. The colours of simple light are, as we observed before, violet, indigo, blue, green, yellow, orange, and red, together with an indefinite variety of intermediate gradations. The colours of compounded light are differently compounded of these simple rays, mixed in various proportions: thus a mixture of yellow-making and blue-making rays exhibits a green colour, and a mixture of red and yellow makes an orange; and in any colour the same in specie with the primary ones may be produced by the composition of the two colours next adjacent in the series of colours generated by the prism, whereof the one is next more refrangible, and the other next less refrangible. But this is not the case with those which are situated at too great a distance; orange and indigo do not produce the intermediate green, nor scarlet and green the intermediate yellow.

10th. The most surprising and wonderful composition of light is that of whiteness; there is no one sort of rays which can alone exhibit that colour: it is ever compounded, and to its composition all the aforesaid primary colours are requisite.

11th. As whiteness is produced by a copious reflection of rays of all sorts of colours, when there is a due proportion in the mixture; so, on the contrary, blackness is produced by a suffocation and absorption of the incident light, which being stopped and suppressed in the black body, is not reflected outward, but reflected and refracted within the body till it be stifled and lost.

COLURES, in *Astronomy* and *Geography*, two great circles supposed to intersect each other, at right angles, in the pole, and to pass through the solstitial and equinoctial points of the ecliptic; that which passes through the former point being called *solstitial colure*, and the other the *equinoctial colure*.

COMBINATIONS, in *Mathematics*, denote the different collections that may be formed out of any given number of things, taken a certain number at a time, without regard to the order in which they may be arranged; and are thus distinguished from *permutations*, or *changes*, which have reference to the order in which the several quantities may be disposed.

In order to find the number of combinations that a given number of things will admit of, let us take the continued product of the factors, $(x + a)$ $(x + b)$ $(x + c)$ $(x + d,)$ &c. *viz.*

$$\begin{array}{ll}
(1) & x + a \\
 & \underline{x + b} \\
 & x^2 + \left.\begin{array}{l} a \\ b \end{array}\right\} x + ab \\
(2) & \underline{x + c\qquad\qquad\qquad\qquad} \\
 & x^3 + \left.\begin{array}{l} a \\ b \\ c \end{array}\right\} x^2 + \left.\begin{array}{l} ab \\ ac \\ bc \end{array}\right\} x + abc \\
(3) & \underline{x + d\qquad\qquad\qquad\qquad\qquad\qquad\qquad} \\
 & x^4 + \left.\begin{array}{l} a \\ b \\ c \\ d \end{array}\right\} x^3 + \left.\begin{array}{l} ab \\ ac \\ ad \\ bc \\ bd \\ cd \end{array}\right\} x^2 + \left.\begin{array}{l} abc \\ abd \\ acd \\ bcd \end{array}\right\} x + abcd
\end{array}$$

Now it is obvious, that in each of these formulæ the coefficient of the *second* term is equal to the sum of all the quantities $a, b, c,$ &c. that enter into the expression; the *third* is the sum of all the possible combinations of every two of them; the *fourth* of every three; the *fifth* of every four, and so on; that is, the number of combinations that may be formed out of any number of things (n,) taken a certain number (m,) at a time, will be expressed by the coefficient of the $m+1$ term of the above expanded function, carried to n factors; or making a, b, c, d, &c. each equal to 1, the same will be expressed by the $m+1$ coefficient of the expanded binomial ($x+1^n$;) which from the known law of the binomial theorem is equal to

$$\frac{n(n-1)(n-2)(n-3)\cdot\cdot(n-\overline{m-1})}{1.2.3.4\cdot\cdot m}$$

which is, therefore, a general formulæ for expressing the number of combinations that may be formed out of n things taken any number m at a time.

Suppose, for example, it were required to find how many combinations may be formed out of 13 cards, all different from each other, taken 4 at a time?

Here we have $\frac{13.12.11.10}{1.2.3.4}=$ 715 combinations.

The above formulæ leads us also to the method of finding the whole possible number of combinations that may be formed out of a given number of things n, by taking 1, 2, 3, 4, &c. at a time, to n, at a time; for this it is obvious will be equal to the sum of all the coefficients of the expanded binomial ($x+1^n$,) wanting the first term. But the sum of all these coefficients is equal to 2^n, therefore the number of possible combinations of n things is equal to 2^n-1. The single quantities a, b, c, &c. being classed under the general term combination, for the sake of analogy, if these be excluded the formulæ will be $2^n-(n+1.)$

Thus the number of all the combinations that may be formed out of the five letters $a, b, c, d, e = 2^5-1=31$, or excluding the single terms, the number of combinations is $2^5-6=26$.

To determine the number of combinations that may be formed out of a given number of things, in which there are m things of one sort, n of another sort, p of another sort, &c., by taking 1 at a time, 2 at a time, 3 at a time, &c. to any given number of things at a time.

Rule. Place in one horizontal row $m+1$ units, annexing ciphers on the right hand, till the whole number of units and ciphers exceeds the greatest number of things to be taken at a time by unity

Under each of these terms write the sum of $n+1$, left-hand terms, including that as one of them under which the number is placed; and under each of these, the $p+1$ left-hand terms of the last line; then the $q+1$ terms of this, and so on through all the number of different things, and the last line will be the answer.

The theory of combinations is of the greatest use in the doctrine of chances and probabilities, for the probability of an event happening or failing depends generally upon the number of combinations that may be formed, or that may take place amongst the circumstances on which the event ultimately depends.

COMBUSTIBLE *Bodies*, are those bodies which when once set on fire will continue to burn without the farther accession of fuel.

COMET, in *Astronomy*, a heavenly body, appearing at uncertain periods, and which, during the time of its appearance, has a motion in some respects similar to those of the planets. The orbits of the comets however differ from those of the planets, in their being more eccentric, and being inclined to the plane of the ecliptics in angles of various magnitudes, the plane of some of them being nearly coincident, and others nearly perpendicular to the plane of the ecliptic. The motions of comets are also some of them direct, and others retrograde, whereas those of the planets are all direct.

Comets are popularly divided into three distinct classes, *viz. bearded*, *tailed*, and *hairy comets;* though this distinction relates rather to the circumstance under which they are seen, than to any difference of the bodies themselves.

Thus, when the comet is eastward of the sun, and moves from it, it is said to be *bearded;* when the comet is westward of the sun, and sets after it, it is said to be *tailed;* and when the sun and the comet are in opposition, the train is hid behind the body of the comet, excepting a little which appears round it in the form of a border of hair, hence it is called *hairy*, and hence the name of comet is derived.

The estimates that have been given of the magnitudes of comets, are not sufficiently accurate to be depended upon; for it does not appear that they distinguish between the nucleus and the surrounding atmosphere. Some comets, from the apparent magnitude and distance compared, have been judged to be much larger than the moon, and even equal to some of the primary planets. The diameter of that of 1744, when at the distance of the sun from us, measured about 1′, and therefore its diameter must be about three times the diameter of the earth: at another time the diameter of its nucleus was nearly equal to that of Jupiter. Hence it has been conjectured, that some of the solar eclipses recorded in history, that cannot be verified by calculation from tables of the sun and moon, have been occasioned by the interposition of comets between the sun and the earth.

The principal phenomena which have been observed with respect to the tails of comets are: 1. Their tails appear the largest and brightest immediately after the passage through their perihelion, or nearest approach to the sun. 2. The tail of a comet always declines from a just opposition to the sun, towards those parts which the body or nucleus pass over, in its progress through its orbit. 3. This declination is the smallest when the head or nucleus approaches nearest the sun, and is still less near the neucleus of the comet, than towards the extremity of the tail. 4. The tails are somewhat brighter and more distinctly defined in their convex than in their concave part. 5. They are also broader at the upper extremity than near the centre of the comet. 6. Their tails are always transparent, and the smallest stars are seen through them without any sensible diminution of their light, and without those effects of refraction which might be expected from viewing them through a visible medium, which circumstance seems to indicate that the tails are composed of extremely rare and attenuated matter; but with regard to their formation we may be said to be totally ignorant, notwithstanding the numerous hypotheses that have been advanced on this subject. The tail of the comet of 1811, when at its maximum, subtended an angle of at least 16 degrees, and was computed to be at least 23 million miles in length.

The comet of 1759 is known to perform its revolution in 76 years, nearly, whence it appears that its mean distance is about 18 times that of the earth, or a little less than the mean distance of Uranus; but, in consequence of the great eccentricity of its orbit, its aphelion point, or greatest distance from the sun, is nearly double that of the above planet. The perihelion distance of this comet is about 6 of the mean distance of the earth, which being taken from 36, the mean transverse axis of its orbit, leaves 35·4 for its aphelion distance, which is nearly double the greatest distance of Uranus, and about four times that of Saturn.

It is extremely difficult to determine, from computation, the elliptic orbit of a comet, to any degree of accuracy: for when the orbit is very eccentric, a very small error in the observation will change the computed orbit into a parabola, or hyperbola. Now, from the thickness and inequality of the atmosphere with which the comet is surrounded, it is impossible to determine, with any precision, when either the limb or centre of the comet pass the wire at the time of observation. And this uncertainty in the observations will subject the computed orbit to a great error. The only safe way to get the period of comets, is to compare the elements of all those which have been computed, and where you find they agree very well, you may conclude that they are elements of the same comet, it being so extremely improbable that the orbits of two different comets should have the same inclination, the same perihelion distance, and the places of the perihelion and node the same. Thus, knowing the periodic time, we get the major axis of the ellipse; and the perihelion distance being known, the minor axis will be known. When the elements of the orbits agree, the comets may be the same, although the periodic time should vary a little; as that may arise from the attraction of the bodies in our system, and which may also alter all the other elements in a small degree.

COMETARIUM, a machine for conveying an idea of the revolution of a comet about the sun. It is contrived in such a manner, as by elliptical wheels to show the unequal motion of a comet in every part of its orbit.

COMMENSURABLE, among geometricians, an appellation given to such quantities as are measured by one and the same common measure.

Commensurable *Numbers*, whether integers, surds, or fractions, are such as can be measured or divided by some other number without any remainder: such are 12, and 18, as being measured by 6 and 3: also $2\sqrt{2}$, and $3\sqrt{2}$, being measured by $\sqrt{2}$.

Commensurable in *Power*, is said of right lines, when their squares are measured by one and the same space or superficies.

Commensurable *Surds*, those that being reduced to their least terms, become true figurative quantities of their kind; and are therefore as a rational quantity to a rational one.

COMMON *Measure* or *Divisor*, in *Arithmetic*, is that number which will divide two other numbers without leaving a remainder; and the greatest of such divisors is called the *greatest common measure*, or *greatest common divisor*.

COMMUNICATION of *Motion*, that act of a moving body by which it transfers its motion to another body.

COMMUTATION, in *Astronomy*. *Angle* of commutation is the distance between the sun's true place seen from the earth, and the place of a planet reduced to the ecliptic; and is, therefore, found by subtracting the same longitude from the heliocentric longitude of the planet.

COMPASS, or *Mariner's Compass*, an instrument used at sea by mariners, to direct and ascertain the course of their ship. The invention of this instrument is commonly ascribed to Flavio Gioia, or Flavio, of Malphi, about the year 1302.

The common construction of the mariner's compass is extremely simple. It consists of a circular brass box, which contains a paper card, on which is drawn the 32 points of the compass; and this card is fixed on a magnetic needle, which always turns to the north, except a small deviation which is variable at different places, and at the same place at different times.

Azimuth **Compass.** This differs from the common sea compass only in this, that the circumference of the card or box is divided into degrees; also to the box is fitted an index with two sights, which are upright pieces of brass placed diametrically opposite to each other, having a slit down the middle of them, through which the sun or star is to be viewed at the time of observation. The use of this instrument is to take the bearing of any celestial object, when it is in or above the horizon, in order to find from the magnetical azimuth, or amplitude, the variation of the needle.

These are the thirty-two principal points of divisions drawn on the compass card; and are otherwise called *Rhumbs;* each of which has a particular denomination expressed by means of the initials of the four first points, North, East, South, West.

Each point contains 11° 15′, and is divided into ¼ points, containing 2° 48′ 5″.

Compasses, or *Pair of Com-*

passes, a mathematical instrument for describing circles, measuring and dividing lines, or figures, &c.

The *common* Compasses consists of two sharp-pointed branches or legs, joined together at the top.

Triangular Compasses; the construction of which is like that of the common compasses, with the addition of a third leg or point, which has a motion every way. Their use is to take three points at once, and so to form triangles, and lay down three positions of a map to be copied at the same time.

Beam Compasses consist of a long straight beam or bar, carrying two brass cursors; one of these being fixed at one end, the other sliding along the beam, with a screw to fasten it on occasionally.

Elliptic Compasses, commonly called a trammel, consists of a cross with two dovetail grooves, at right angles, and a ruler with two dovetail knobs and a tracing point. The two knobs are adjusted to the local distance of the ellipse, and the distance between the remotest one and the tracing point is made equal to the semi-transverse (the distance of the nearer knob from the same point being equal to the semiconjugate). Then the cross being laid with its centre over that of the ellipse, and the knobs let into the grooves, the turning round of the ruler will trace the ellipse.

German Compasses, have their legs a little bent outwards towards the top; so that when shut the points only meet.

Hair Compasses have an adjusting screw attached to one of the legs, by means of which measures may be taken to a very great degree of accuracy.

Proportional Compasses, are those in which the joint lies, not at the end of the legs, but between the points terminating each leg. These are either simple or compound. In the former sort the centre or place of the joint is fixed; so that one pair of them serves only for one proportion. In the compound ones the joint may be set at any distance, and consequently any proportion whatever easily obtained.

Spring Compasses, or *Dividers*, are made of hardened steel, with an arched head, which by its springs opens the legs; the opening being directed by a circular screw fastened to one of the legs, let through the other, and worked with a nut.

Geometry of the Compasses, a species of geometry invented by M. Mascheroni, of Milan, by which all the elementary problems of plane geometry are performed by the compasses only, without the use of the ruler; it is, however, more ingenious than profound, and may be considered rather as a subject of curiosity than of real utility.

COMPLEMENT of an *Arch* or *Angle*, is what it wants of 90 degrees.

Arithmetical Complement, of a logarithm, is what the logarithm wants of 10·00000, &c.; and the easiest way to find it is, beginning at the left hand, to subtract every figure from 9, and the last from 10.

Complement, in *Astronomy*, denotes the distance of a star from the zenith.

Complement of *Life*, a term much used in the doctrine of life-annuities, by De Moivre; to denote the number of years which a given life wants of 86, this being the age which he considered as the utmost probable extent of life.

Complements of a *Parallelogram*, are the two smaller parallelograms made by drawing two right lines through a point in the diagonal, and parallel to the sides of the parallelogram. In every parallelogram these complements are equal to each other.

COMPOSITE *Number*, is that which is produced by the multiplication of two or more numbers or factors, and is thus distinguished from a prime number, which cannot be so produced.

COMPOSITION, is a species of reasoning by which we proceed from things that are known and given, step by step, till we arrive at others, which were before unknown.

Composition of *Forces*, in *Mechanics*, is the method of finding the quantity and direction of a single force, equivalent to two or more forces of which the quantity and direction are given. It is thus distinguished from *Resolution of Forces*, which is the method of resolving a given force into two or more forces, the combined effect of which shall be equivalent to the single given force.

Composition of *Proportion*, is when, of four proportionals, the sum of the 1st and 2d is to the 2d, as the sum of the 3d and 4th is to the 4th.

Composition of *Ratios*, in *Arithmetic* and *Algebra*, is performed by multiplying the quantities or exponents of two or more ratios together, which product is then said to be compounded of all the other ratios whose exponents were multiplied.

COMPOUND *Interest*, is that which arises by the continual addition of the interest to the principal as it becomes due.

Compound *Motion*, is that which arises from the effect of several conspiring forces, which may render it either rectilinear or curvilinear, according to the nature of the forces and the circumstances under which they act.

Compound *Quantities*, in *Algebra*, are those connected together by the signs + and —; they are distinguished into *binomials*, *trinomials*, &c. according to the number of terms of which they are composed.

Compound *Ratio*, is that which arises from the composition of ratios.

COMPRESSIBILITY, that quality of a body by which it yields to the pressure of another body or force, so as to be brought into a narrower compass.

The following table shows the quantity of compression of these fluids and mercury, when the thermometer was at 50° and barometer 29½ inches.

	Compress. of Mill. pts.	*Sp grav.*
Spirits of wine	66	846
Oil of olives	48	918
Rain water	46	1000
Sea water	40	1028
Mercury	3	13595

CONCAVE, an expression used to denote the curvilinear vacuity of hollow bodies.

Concave *Lenses*, or *Mirrors*, have either one or both sides concave. See Lens and Mirror.

CONCAVITY, from *concave*, the hollow or vacuity of bodies.

CONCAVO-*Concave Lens*, is that which is concave on both sides.

Concavo-*Convex Lens*, is that which is concave on one side, and convex on the other.

CONCENTRIC, having a common centre, as concentric *circles*, *ellipses*, &c.

CONCHOID, the name of a curve invented by Nicomedes; and hence commonly called the *Conchoid of Nicomedes*, which was much used by the ancients in the construction of solid problems.

It is thus constructed. From a point draw any number of lines, each cutting another line, then from this line set off equal distances, either toward the point, or in the opposite direction; when the lines and the curve drawn through these points is the conchoid.

There are two kinds of Conchoid the first, or external one, on the side opposite to the point; and the second, or internal one, on the side next the point. Of the second there are two forms, 1st. pointed, when the curve passes between the line and the point; and 2d looped, when it passes beyond the point.

If a be the portion cut off from the lines, b the perpendicular distance of the point from the line cut by the others, ϕ the angle which any of the lines cut makes with the perpendicular, and Z the total length of the line so cut; then $Z = \frac{b}{\cos. \phi} \pm a$. In which + answers for the first conchoid, and — for the second.

The swell of architectural columns is usually a first conchoid.

Newton approves of the use of this curve for the trisection of angles, finding two mean proportionals, and in the construction of problems, for which purposes it was employed by the ancients.

CONDENSATION, the act

whereby a body is rendered more dense, compact, and heavy.

CONDENSER, a pneumatic engine, or syringe, whereby an uncommon quantity of air may be condensed into a given space.

CONDUCTOR, in *Electricity*, a term used to denote those substances which are capable of receiving and transmitting electricity, in opposition to electrics, in which the matter or virtue of electricity may be excited and accumulated, or retained. The former are also called non-electrics, and the latter non-conductors. And all bodies are ranked under one or other of these two classes, though none of them are perfect electrics, nor perfect conductors, so as wholly to retain, or freely and without resistance to transmit electricity.

To the class of conductors belong all metals and semi-metals, ores, and all fluids, (except air and oils), the effluvia of flaming bodies, ice (unless very hard frozen), and snow, most saline and stony substances, charcoals, of which the best are those that have been exposed to the greatest heat; smoke, and the vapour of hot water.

It is commonly supposed, that the electric fluid passes through the substance, and not merely over the surfaces or metallic conductors; because, if a wire of any kind of metal be covered with some electric substance, as resin, sealing-wax, &c. and a jar be discharged through it, the charge will be conducted as well as without the electric coating.

It has also been alleged, that electricity will pervade a vacuum, and be transmitted through it almost as freely as through the substance of the best conductor; but Mr. Walsh found that the electric spark or shock would no more pass through a perfect vacuum than through a stick of solid glass; in other instances, however, when the vacuum has been made with all possible care the experiment has not succeeded.

CONDUCTOR *Prime*, is an insulated conductor, so connected with the electrical machine, as to receive the electricity immediately from the excited electric.

CONDUCTORS of *Lightning*, are pointed metallic rods fixed to the upper parts of buildings, to secure them from strokes of lightning.

CONE, is a solid body, having a circular base, and its other extremity terminated in a single point or vertex.

Cones are either *right*, or *oblique*.

A *Right* CONE, is that in which the right line joining the vertex and centre of the base, is perpendicular to the plane of the base. It may be conceived to be generated by revolution of the right-angled triangle, about its perpendicular. And thus, Euclid defines a cone to be a solid figure, whose base is a circle, and is produced by the entire revolution of the plane of a right-angled triangle about its perpendicular, being called the axis of the cone.

Right cones are called acute, obtuse, right-angled, or equilateral, according to the species of the vertical angle.

An *Oblique* CONE, is that in which the line joining the vertex and centre of the base is not perpendicular, but oblique, to the plane of the base.

Every cone, whether right or oblique, is equal to one-third of a cylinder of equal base and altitude. And therefore the solidity of a cone is found by multiplying the area of its base by one-third of its perpendicular altitude.

The curve surface of a right cone is equal to a circular sector, having its radius equal to the slant height of the cone, and its arc equal to the whole circumference of the cone's base. And therefore this surface is equal to half the product of the slant side into the circumference of the base.

The surface of an oblique cone is not quadrable; indeed, no rule has yet been found that will even lead to a practical approximation of its area, notwithstanding the attempts of several ingenious and able mathematicians.

The solidity of a cone with an elliptic base, forming part of a right cone, is equal to the product of its surface by a third of one of the perpendiculars, drawn from the point in which the axis of the right cone intersects the ellipse; and it is also equal to one-third of the height of the cone multiplied by the area of the elliptic base. Consequently, the above perpendicular is to the height of the cone, as the elliptic base is to the curve surface. Its curve surface is equal to half the sum of the circumferences multiplied by the slant side; and its solidity to the sum of the squares and product of the diameters multiplied by .7854, and by $\frac{1}{3}$ the perpendicular height.

Frustrum of a Cone, is that which is formed by cutting off the upper part of a cone, by a plane parallel to its base. Put the greater diameter $= D$, the less diameter $= d$, the height $= h$, and $7854 = p$; then

$$\text{Surface} = 2p\,(D + d$$

$$\text{Solidity} = \tfrac{1}{3}ph\,(D^2 + Dd + d^2)$$

$$\text{Solidity} = \tfrac{1}{3}ph + \frac{D^3 - d^3}{D - d}$$

CONGELATION, the transition of a liquid into a solid state, in consequence of an abstraction of heat: thus, metals, oil, water, &c. are said to congeal when they pass from a fluid into a solid state. With regard to fluids, congelation and freezing mean the same thing. Water congeals at 32°; and there are few liquids that will not congeal, if the temperature be brought sufficiently low. Every particular kind of substance requires a different degree of temperature for its congelation.

CONGRUOUS *Quantities*, are those which are of the same kind, and therefore admit of comparison; and quantities which cannot be so compared, are incongruous quantities. All abstract numbers are congruous; but concrete numbers are not congruous, unless the quantities they represent be so. Thus, three and four, as abstract numbers, are congruous; but if they denote three pounds and four miles, they are incongruous.

CONICAL, any thing of a conical form, or relating to the cone.

Conical *Ellipse*, *Hyperbola*, *Parabola*, denote those figures, under their most simple form, as cut from the cone, to distinguish them from the same figures of a higher order. See the respective articles.

Conical *Ungula*, or Conic *Ungula*, is a solid formed by a plane passing through the side and base of a cone.

Conic *Sections*, as the name implies, are such curve-lines and plane-figures as are produced by the intersection of a plane with a cone.

From the different positions of the cutting plane, there arise five different sections; *viz.* a *triangle*, *circle*, *ellipse*, *parabola*, and *hyperbola*. But only the three latter are particularly denominated *conic sections*.

1. If the cutting plane pass through both base and vertex, the section is *a triangle*.

2. If it pass through neither, it is a *circle* when parallel to the base, and an *ellipse* when not; the obliquity determining the eccentricity of the ellipse.

3. If it pass through the base and slant side, it is a *parabola* when parallel to the opposite side; and an hyperbola when it makes an angle with that side, and would meet it beyond the vertex of the cone.

4. The *vertices* of any section are the points where the cutting plane meets the opposite sides of the cone. Hence the ellipse and hyperbola have each two vertices, but the parabola only one, unless we consider the other as at an infinite distance.

5. The *axis*, or *transverse diameter*, of a conic section, is the line joining its vertices; therefore the axis of an ellipse is within the figure, of the hyperbola without it, and in the parabola it is infinite in length.

6. The *centre* of a conic section, is in that point which bisects the axis. Hence the centre of an ellipse is within the figure, of the hyperbola without the figure, and in the parabola it is at an infinite distance from the vertex.

The definition of the other lines, in and about the conic sections, will be found under their respective heads.

The conic sections are of themselves a system of regular curves

Conic Sections.

Fig. 1.

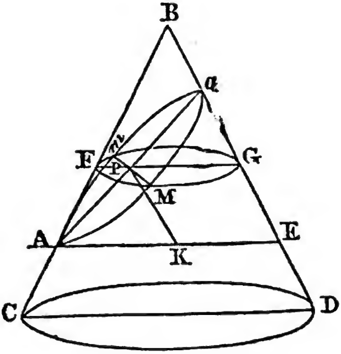


Fig. 2.

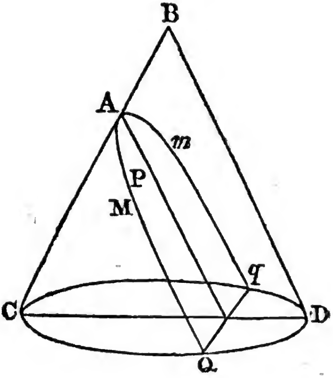
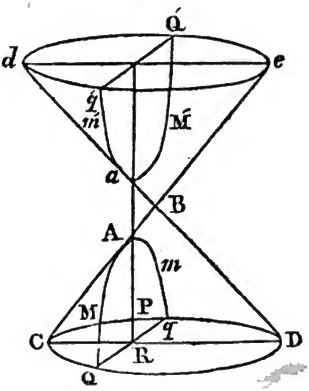


Fig. 3. See Fig. 1.

Fig. 4.



allied to each other, the doctrine of which is of the greatest use in physical astronomy, as well as in the physico-mathematical sciences, and has been much cultivated by both ancient and modern mathematicians.

There are three methods of investigating and demonstrating the various properties of the conic sections. The first is to consider them as they are really, cut from the cone itself, which is the way adopted by all the ancients. In the second method, the properties are deduced from arbitrary descriptions of the curves in plano; the properties belonging to these curves being shown to apply to those actually cut from the cone. In the third method, the chief properties are inferred from the different modifications of the general algebraic equation of lines of the second order, and the established analogies between the properties of equations and those of curves.

Origin and General Equation of the Conic Sections.

Let a right cone B C D (pl. *conic sections*, fig. 1,) be cut by any plane A M P; required the equation of the curve M A *m*, which results from the intersection.

If through the summit B we cause to pass a plane B C D, perpendicular to the base of the cone, and to the cutting plane A M P, the intersection of these two planes will be a right line A *a*; and if we cut the cone parallel to its base by a plane F M G, we shall obtain a circle whose plane will be perpendicular to the triangle B C D, and whose intersection with the plane A M P will be a right line P M, perpendicular to the two right lines A *a*, F G. The line P M will be an ordinate common to both the circle and the section M A *m*.

This premised, call A P $= x$, P M $= y$, A B $= c$, the angle A B *a* $=$ B, the angle B A *a* $=$ A; the well known property of the circle gives $yy = \mathrm{FP} \times \mathrm{PG}$. To find analytical expressions for the lines F P and P G, draw A E parallel to C D, and P K parallel to B D, both in the plane B C D; this gives A B : sin A E B : : A E : sin B. But the

$$\angle \mathrm{AEB} = \frac{180^\circ - \mathrm{B}}{2} = 90 - \tfrac{1}{2}\mathrm{B}:$$

therefore $\sin \mathrm{AEB} = \sin (90 - \frac{1}{2}\mathrm{B}) = \cos \frac{1}{2}\mathrm{B}$, and consequently from the above proportion $\mathrm{AE} = \dfrac{c \times \sin \mathrm{B}}{\cos.\ \frac{1}{2}\mathrm{B}}$.

Again, the triangle A P K gives sin A K P : sin A P K, or sin A E B : sin A *a* E, or rather $\cos \frac{1}{2}\mathrm{B} : \sin (\mathrm{A} + \mathrm{B}) : : x : \mathrm{AK}$ (because the external angle A *a* E = the two internal angles A and B). Hence $\mathrm{AK} = \dfrac{x \sin (\mathrm{A} + \mathrm{B})}{\cos \frac{1}{2}\mathrm{B}}$; and therefore K E or $\mathrm{PG} = \dfrac{c \sin \mathrm{B} - x \sin (\mathrm{A} + \mathrm{B})}{\cos \frac{1}{2}\mathrm{B}}$

To find an expression for the part F P; in the triangle A P F we have sin A F P, or sin B F G, or sin B G F, $\cos \frac{1}{2}\mathrm{B} : x : : \sin \mathrm{A} : \mathrm{FP} = \dfrac{x \sin \mathrm{A}}{\cos \frac{1}{2}\mathrm{B}}$; therefore, by substituting these values of F P, P G, we have $yy = \dfrac{\sin \mathrm{A}}{\cos^2 \frac{1}{2}\mathrm{B}} \left\{ cx \sin \mathrm{B} - xx \sin (\mathrm{A} + \mathrm{B}) \right\}$ for the equation required.

Now there can occur but three cases. *First*. That $\mathrm{A} + \mathrm{B} = 180^\circ$, in which case the cutting plane is parallel to the side B D (pl. *conic sections*, fig. 2). This conic section is called a *parabola*, and its equation is $yy = \dfrac{\sin \mathrm{A} \times \sin \mathrm{B}}{\cos^2 \frac{1}{2}\mathrm{B}} cx = \dfrac{\sin^2 \mathrm{B}}{\cos^2 \frac{1}{2}\mathrm{B}} cx = 4cx \sin^2 \frac{1}{2}\mathrm{B}$, by (12 fig.) (because $\sin 18^\circ 0 = o$, and $\sin \mathrm{A} = \sin \mathrm{B}$), or $y = \pm 2 \sin \frac{1}{2}\mathrm{B} \sqrt{cx}$. Consequently, the parabola is a curve formed by two equal and similar branches, infinitely extended, and constantly receding from each other.

Second. If A + B is less than 180°, it is easy to perceive that the plane A M P, if continued, must meet the other side B D (pl. *conic sections*, fig. 3). The conic section which results is called an *ellipse*, and is a curve formed by two equal, similar, and finite branches A M *a*, A *m a*. Its equation is $yy = \dfrac{\sin \mathrm{A}}{\cos^2 \frac{1}{2}\mathrm{B}} \left\{ cx \sin \mathrm{B} - xx (\sin \mathrm{A} + \mathrm{B}) \right\}$

Third. If A + B is more than 180°, the section is called an *hyperbola*, and its equation is

$$y\,y = \frac{\sin A}{\cos^2 \frac{1}{2} B} \left\{ c\,x \sin B + x\,x \sin (A + B - 180^\circ) \right\}$$ (pl. *conic sec.* fig. 4.)

If we conceive a cone B *c d* equal, and opposite at the vertex to the right cone B C D, it is evident that the cutting plane A M P, if prolonged, will meet it, and that from their intersection will result a curve M′ *a m′*; equal, similar, and opposite to the lower curve M A *m*; or rather these two curves which are called *opposite hyperbolas*, will form but one same curve represented generally by the same equation. (Pl. *conic sec.* fig. 4.)

If these sections had been made in an oblique cone, such as would have been the cone B C D (fig. 1) if the angle C had not been equal to the angle D, we should have had for the general equation

$$y^2 = \frac{\sin A}{\sin C \sin D} \left\{ c\,x \sin B - x^2 \sin (A + B) \right\}$$

This equation, as well as the foregoing one, belongs to an ellipse, a parabola, or an hyperbola, according as the sum of the angles A and B is less, equal to, or greater than 180°.

In the first case it expresses a circle, whenever the angle A is equal to either of the angles C or D; for then we have one or other of the following equations

$$y^2 = \frac{c\,x \sin B}{\sin C} - x^2,$$

$$\text{or } y^2 = \frac{c\,x \sin B}{\sin D} - x^2$$

which are evidently equations of the circle. This circumstance shews that in an oblique cone we may form circular sections in two different ways; one by cutting the cone parallel to its base, the other by cutting it by a plane which makes, with one of the sides of the triangle through the axis, an angle equal to that which the other side of the same triangle forms with its base. This is called a *subcontrary section.*

In the third case, in which the equation expresses a hyperbola, we may suppose $c = o$, and then we have

$$y^2 = \frac{\sin A \sin (A + B - 180^\circ)}{\sin C \sin D} x^2$$

Suppose the co-efficient of x^2 equal to a constant quantity, as $\frac{a^2}{b^2}$, we then find $y = \frac{a\,x}{b}$, which is an equation to a right line; hence we may conclude, which indeed is sufficiently evident, that the hyperbola degenerates into a triangle, when $c = o$, that is, when the cutting plane passes through the vertex of the cone.

CONJUNCTION, in *Astronomy,* the meeting of two or more stars or planets in the same degree of the zodiac. It is *true* when they have the same latitude and longitude, and *apparent* when they have the same latitude only. It is *heliocentric* when it would appear to an observer in the sun, and *geocentric* when it would appear to an observer on the earth. *Superior* when the object is beyond the sun, and *inferior* when between the sun and earth. The planets which are more remote from the sun than the earth, can have no inferior conjunctions, and hence they are called superior planets.

CONOID, a solid figure generated by the revolution of any conic section about its axis.

CONSEQUENTIA, a Latin term, commonly employed by astronomers to denote the real or apparent motion of a planet or comet, when it is moving from west to east, or according to the order of the signs; and is thus opposed to *antecedentia,* which denotes a contrary motion.

CONSTANT *Quantities,* in *Algebra,* are those whose values are known, or which remain constantly the same. These are commonly denoted by the leading letters of the alphabet, *a, b, c,* &c. to distinguish them from the variable and unknown quantities, which are represented by the final letters, *z, y, x,* &c.

CONSTELLATION, in *Astronomy,* an assemblage or system of several stars, expressed and represented under the name and figure of some animal or other emblem.

TABLE

OF THE ANCIENT AND MODERN CONSTELLATIONS.

	Names.	Number of the Stars.				Principal Stars.	
		Ptolemy.	Tycho.	Hevelius.	British.		Mag.
Zodiacal Constellations.	Aries	18	21	27	66		
	Taurus	44	43	51	141	Aldebaran	[illegible]
	Gemini	25	25	38	85	Castor & Pollux	1, 2
	Cancer	13	15	29	83		
	Leo	35	30	49	95	Regulus	1
	Virgo	32	33	50	110	Spica Virginis	1
	Libra	17	10	20	51	Zubenich Meli	2
	Scorpio	24	10	20	44	Antares	1
	Sagittarius	31	14	22	69		
	Capricornus	28	28	29	51		
	Aquarius	45	41	47	108	Scheat	3
	Pisces	38	36	30	113		
Northern Constellations.	Ursa minor	8	7	12	24	Pole-star	2
	Ursa major	35	29	73	87	Dubhe	1
	Perseus	29	29	46	59	Algenib	2
	Auriga	14	9	40	66	Capalla	1
	Bootes	23	18	52	54	Arcturus	1
	Draco	31	32	40	80	Rastaber	3
	Cepheus	13	4	51	35	Alderamin	3
	**Canis venatici*, i. e. *Asterion et Chara*	—	—	23	25		
	Cor. Caroli	—	—	—	3		
	Triangulum	4	4	12	16		
	Triangulum minus	—	—	—	10		
	Musca	—	—	—	6		
	Lynx	—	—	19	44		
	Leo minor	—	—	—	53		
	Coma Berenices	—	14	21	43		
	Camelopardalus	—	—	32	58		
	Mons Menelaus	—	—	—	11		
	Corona Borealis	8	8	8	21		
	Serpens	13	13	22	64		
	Scutum Sobieski	—	—	7	8		
	Hercules cum Ramo et Cerbero	29	28	45	113	Ras Algiatha	3
	Serpentarius sive Ophiuchus	29	15	40	74	Ras Alhagus	3
	Taurus Poniatowski	—	—	—	7		
	Lyra	10	11	17	22	Vega	1
	Vulpeculus et Anser	—	—	27	37		
	Sagitta	5	5	5	18		
	Aquila cum *Antinous*	— —	12 3	23 19	71	Altair	[illegible]
	Delphinus	10	10	14	18		
	Cygnus	19	18	47	81	Deneb Adige	1
	Cassiopea	13	26	37	55		
	Equulus	4	4	6	10		
	Lacerta	—	—	—	16		
	Pegasus	20	19	38	89	Markab	2
	Andromeda	23	23	47	66	Almaac	2

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* Those in *Italics* are modern Constellations.

TABLE

OF THE ANCIENT AND MODERN CONSTELLATIONS.

	Names.	Number of the Stars.				Principal Stars.	
		Ptolemy.	Tycho.	Hevelius.	British.		Mag.
Southern Constellations.	*Phœnix*	—	—	—	13		
	Officina Sculptoria .	—	—	—	12		
	Eridanus . . .	34	10	27	84	Achernar .	1
	Hydrus . . .	—	—	—	10		
	Cetus	22	21	45	97	Menkar . .	2
	Fornax Chemica .	—	—	—	14		
	Horologium . .	—	—	—	12		
	Reticulus Rhomboidalis . . .	—	—	—	10		
	Xiphias Dorado .	—	—	—	7		
	Celapraxitellis . .	—	—	—	16		
	Lepus . . .	12	13	16	19		
	Columba Noachi .	—	—	—	10		
	Orion	38	42	62	78	Betelgeuse .	1
	Argo Navis . .	45	3	4	64	Canopus . .	1
	Canis major . .	29	13	21	31	Sirius . .	1
	Equuleus Pictorious .	—	—	—	8		
	Monoceros . .	—	—	19	31		
	Canis minor . .	2	2	13	14	Procyon .	1
	Chamælon . . .	—	—	—	10		
	Pyxis Nautica . .	—	—	—	4		
	Piscis Volans . .	—	—	—	8		
	Hydra . . .	27	19	31	60	Cor Hydræ .	1
	Sextans . . .	—	—	11	41		
	Robur Carolinum .	—	—	—	12		
	Machina Pneumatica.	—	—	—	3		
	Crater . . .	7	3	10	31	Alkes . .	3
	Corvus . . .	7	4	—	9	Algorab . .	3
	Crosiers el Cruzero .	—	—	—	6		
	Apis Musca . .	—	—	—	4		
	Apus, or Avis Indica.	—	—	—	11		
	Circinus . . .	—	—	—	4		
	Centaurus . . .	37	—	—	35		
	Lupus	19	—	—	24		
	Quadra Euclidis .	—	—	—	12		
	Triangulum Australe	—	—	—	5		
	Ara	7	—	—	9		
	Telescopium . .	—	—	—	9		
	Corona Australis .	13	—	—	12		
	Pavo	—	—	—	14		
	Indus . . .	—	—	—	12		
	Microscopium . .	—	—	—	10		
	Octans Hadleianus .	—	—	—	43		
	Grus	—	—	—	14		
	Toucan . . .	—	—	—	9		
	Piscis Australis . .	18	—	—	24	Fomelhaut.	1

	Const.	Magnitudes						Total.
		1st.	2d.	3d.	4th.	5th.	6th.	
Zodiac British Catalogue	12	5,	16,	44,	120,	183,	646	1014
Northern	34	6,	24,	95,	200,	291,	635	1251
Southern	45	9,	36,	84,	190,	221,	323	863
		20,	76,	223,	510,	695,	1604	3128

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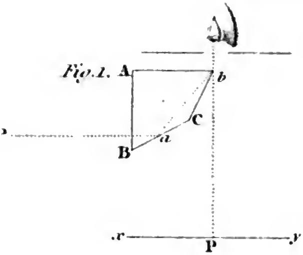
Fig. 1.



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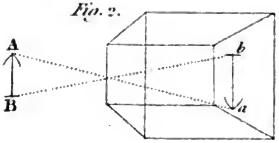

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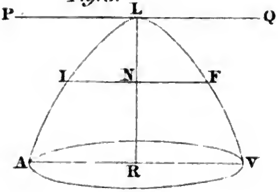

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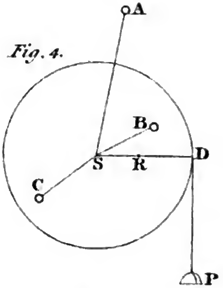

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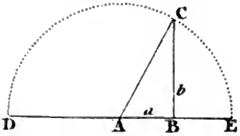

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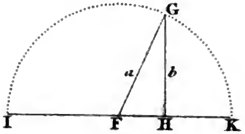

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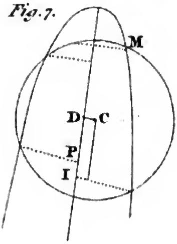

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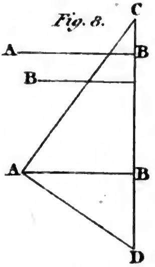

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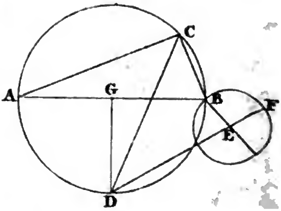

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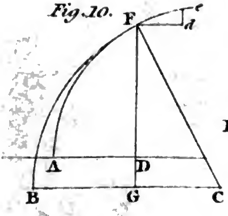

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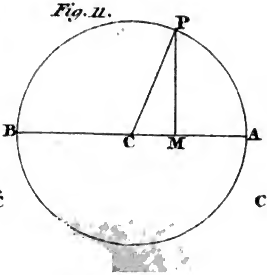

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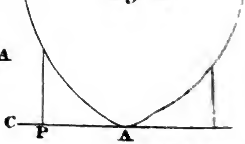

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CONSTRUCTION of *Equations*, in *Algebra*, is the method of finding the roots of equations by means of geometrical figures, which is effected by lines or curves, according to the order or degree of the equation.

To construct a Simple Equation.—This is done by resolving the equation into a proportion, thus: the general form for simple equations being $ax=b$, or $ax=1.b$, it is only necessary to find a fourth proportional to the three quantities, a, 1, and b; which fourth proportional will give the value of x.

To construct a Quadratic Equation.—If it be a simple quadratic, it will reduce to this form $x^2=ab$; whence $a:x::x:b$, or $x=\sqrt{ab}$, a mean proportional between a and b.

Adfected quadratics will be of three forms, according to the signs of the terms, the general equation $x^2 \pm 2ax = \pm b$, not admitting of a construction that will apply to all the varieties.

1. If the quadratic be $x^2+2ax=b^2$; then form the right-angled triangle whose base AB (plate II. fig. 5,) is a, and perpendicular BC is b; with the centre A and radius AC, describe the semi-circle DCE; then DB and BE will be the two roots of the given quadratic; x being $=\sqrt{(b^2+a^2)} \pm a$.

2. If the quadratic be $x^2-2ax=b^2$, the construction will be the same as that of the preceding one, $x=a\pm\sqrt{(b^2+a^2)}$ having one positive and one negative value.

3. But if the form be $x^2-2ax=-b^2$, or $2ax-x^2=b^2$; construct a right-angled triangle whose hypothenuse (plate II. fig. 6,) FG is a, and perpendicular GH is b; with the radius FG and centre F, describe a semicircle IGK; then shall IH and HK be the roots of the given equation, x being $=a\pm(a^2-b^2)$. In this form, if a be greater than b, the equation will have two positive roots; but if a be less than b, the solution is manifestly impossible.

To construct Cubic and Biquadratic Equations.—The roots of a biquadratic equation may be determined by the intersection of two conic sections; for the equation by which the ordinates, from the four points in which these conic sections may cut one another, can be determined, will arise to four dimensions, and the conic sections may be so assumed as to make this equation coincide with any proposed biquadratic; so that the ordinates from these four intersections will represent the four roots of the proposed biquadrate.

Let M M′ M″ M‴ (plate II. fig. 7,) be a parabola whose axis is AP, and M M′ M″ M‴ a circle whose centre is C and radius C M, cutting the parabola in the points M M′ M″ M‴; and from these points draw the ordinates to the axis M P, M′ P′, M″ P″, M‴ P‴, and from C draw C D perpendicular to the axis, and C N parallel to it meeting P M in N.

Let $AD=a$, $DC=b$, $CM=n$, the parameter of the parabola $=p$, $AP=x$, $PM=y$; then $px=y^2$.

Also $CM^2=CN^2+NM^2$, or $n^2=(x-a)^2+(y-b)^2$;

or $x^2-2ax+a^2+y^2-2by+b^2=n^2$,

and substituting for x its value $\frac{y^2}{p}$, and arranging the terms according to the powers of y, this equation becomes

$y^4-(2pa-p^2)y^2-2bp^2y+(a^2+b^2-n^2)p=0$, which is a biquadratic equation, whose roots are PM, P′ M′, P″ M″, and P‴ M‴.

If one of the intersections of the conic section fall upon the axis, then one of the ordinates vanishes, and the equation by which these ordinates are determined will then be only of three dimensions, or a cubic, to which any proposed cubic equation may be accommodated, so that the three remaining ordinates will be the three roots of the proposed cubic.

Construction *of Geometrical Problems.*

Prob. 1. Giving the base of a right-angled triangle, and the difference between the hypothenuse and perpendicular, to construct the triangle.

Const. Let A B (plate II. fig. 8,) be the given base, and B D the difference of the other two sides.

From the extremity B, of the given base A B, demit the perpendicular B D, equal to the given difference, and produce it indefinitely towards C. Join A D, and from the point A draw A C, making the angle D A C = A D C, and meeting B D produced in C; so shall A B C be the triangle required.

Demons. Since the angle DAC = ADC, the side AC = CD (Euc. vi. 1), therefore the difference between AC and CB is equal to BD, but BD is equal to the given difference by construction, and AB is equal to the given base; also the angle B is a right angle: consequently, ABC is the right-angled triangle required.

If the difference BD is equal to, or exceeds the base AB, the problem is impossible; for in either of those cases the line AC would not meet the continuation of BD, which is necessary for the construction. This limitation arises from a known geometrical problem, though it is not found in Euclid's "Elements;" viz. "The difference of any two sides of a triangle is less than the third side."

PROB. 4. Giving the hypothenuse of a right-angled triangle, and the side of its inscribed square, to construct the triangle.

Construc. On the given hypothenuse AB (Plate II. fig. 9) describe the circle ACD, and from the centre G demit the perpendicular GD; join BD, and perpendicular to it, at the point B, draw BE = to half the diagonal of the given square; join DE, and produce it to F, making EF = BE. Then from D as a centre, and with DF as a radius, describe the arc FC cutting the circle in C; join AC, BC, and ABC will be the triangle required.

CONTACT, *Angle* of, is that made by a curve line and a tangent to it, at the point of contact, is less than any right-lined angle whatever; though it does not therefore follow that it is of no magnitude or quantity.

CONTINUAL *Proportionals* are a series of quantities in which the ratio between each two adjacent terms is equal.

CONTINUATION, in the *Fluxional Analysis*, or *the finding of Fluents by Continuation*, is a method of finding one fluent from another.

Few general rules can be given in these operations, but they principally consist in assuming some quantity y, in the form of a rectangle, or product of two factors, which are such, that the one of them multiplied into the fluxion of the other, may be of the form of the fluxion proposed; then taking the fluxion of the proposed rectangle, there will be deduced a value of the original fluxion, in terms that will frequently admit of finite fluents.

This will be better illustrated by an example.

1. Let it be proposed to find the fluent of $\frac{x^2 \dot{x}}{\sqrt{(x^2 + a^2}}$.

Here it is evident that by assuming $y = x \sqrt{(x^2 + a^2)}$, we shall have one part of the fluxion of this product; that is, $x \times F \sqrt{(x^2 + a^2)}$ of the same form as the proposed fluxion.

Now putting the assumed rectangle into fluxions, we have

$$\dot{y} = \dot{x} \sqrt{(x^2 + a^2)} + \frac{x^2 \dot{x}}{\sqrt{(x^2 + a^2)}}$$

or $$\dot{y} = \frac{x^2 + a^2}{\sqrt{(x^2 + a^2)}} \dot{x} + \frac{x^2 \dot{x}}{\sqrt{(x^2 + a^2)}}$$

or $$y = \frac{2 x^2 \dot{x}}{\sqrt{(x^2 + a^2)}} + \frac{a^2 \dot{x}}{\sqrt{(x^2 + a^2)}}$$

Whence

$$\frac{x^2 \dot{x}}{\sqrt{(x^2 + a^2}} = \tfrac{1}{2}\dot{y} - \tfrac{1}{2}a^2 \times \frac{\dot{x}}{\sqrt{(x^2 + a^2)}}$$

now the fluent of $\frac{\dot{x}}{\sqrt{x^2 + a^2}}$ = hyp. log. $x + \sqrt{(x^2 + a^2)}$

therefore flu. $\frac{x^2 \dot{x}}{\sqrt{(x^2 + a^2)}} = \tfrac{1}{2} x \sqrt{(x^2 + a^2)} - \tfrac{1}{2} a^2 \left\{ \text{hyp. log. } x + \sqrt{(x^2 + a^2)} \right\}$

CONTINUED *Fractions*, are certain species of fractions extremely useful in various arithmetical and indeterminate problems, in the reduction of ratios, the extraction of roots, &c.

Every fraction of the form

$$\frac{a}{b \pm \frac{c}{d \pm \frac{e}{f \pm \&c.}}}$$

is called a *continued fraction*, and the number of its terms will be finite or infinite, according as the quantity which it expresses, or from which it is deduced, is rational or irrational.

The above is the most general form that can be given to continued fractions, but there are few cases in which it is necessary to have any others than those whose numerators are unity, and signs +, and therefore it will be sufficient to consider the latter form only.

The series of fractions formed of the first term, the first two terms, the first three terms, &c. of any continued fraction, are called *converging fractions.*

To reduce any proposed rational Fraction to a continued Fraction.

Divide the denominator by the numerator, then the divisor by the remainder, and so on; and the successive quotients will be the several denominators of the continued fraction, 1 being the numerator.

If the numerator of the proposed fraction be greater than its denominator, the continued fraction will be preceded by an integer.

Exam. Reduce $\frac{1171}{9743}$ to a continued fraction.

1171)9743(8
375)1171(3
46)375(8
7)46(6
4)7(1
3)4(1
1)3(3

therefore the proposed fraction,

$$\frac{171}{9743}=\cfrac{1}{8+\cfrac{1}{3+\cfrac{1}{8+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3}}}}}}}$$

And in the same way any other rational fraction may be reduced to a continued fraction. It is not, however, in this form that they are applied to any useful arithmetical purpose. In order to their being thus employed they must be reduced to a series of converging fractions, as in the following proposition.

To reduce any given continued Fraction to a series of converging Fractions.

Take the first numerator and denominator for the first fraction. For the second fraction, multiply the terms of the first by the second denominator, adding 1 to the product of the denominator. For every succeeding one multiply the terms of the last by the corresponding denominator, and add to the products the terms of the last except one.

From this it follows that the difference of any two contiguous fractions is 1 divided by the product of their denominators, and that the difference between the product of the numerator of each, by the other's denominator, is also 1.

To extract the Square Root by continued Fractions.

Since the quantity that we have to represent by a continued fraction is irrational, the fraction which expresses it must consist of an indefinite number of terms; but still it has this property, that after a certain period, the denominators of the several fractions recur again in the same order as at first; and consequently, after having arrived at that period, the series may be carried on to any length at pleasure.

Let it be required to extract the square root of 19 by continued fractions; we first find the greatest integer contained in $\sqrt{19}$, which is 4; therefore,

$\sqrt{19}=4+\frac{\sqrt{19}-4}{1}$, or multiplying this last fraction by $\frac{\sqrt{19}+4}{\sqrt{19}+4}$, we obtain $\sqrt{19}=4+\frac{19-16}{\sqrt{19}+4}=4+\frac{3}{\sqrt{19}+4}=4+\cfrac{1}{\cfrac{\sqrt{19}+4}{3}}$. We next find the greatest integer in $\frac{\sqrt{19}+4}{3}$, which is 2, whence $\frac{\sqrt{19}+4}{3}=2+\frac{\sqrt{19}-2}{3}$; and proceeding as before. $\frac{\sqrt{19}+4}{3}=2+\frac{\sqrt{19}-2}{3}=2+\frac{19-4}{3(\sqrt{19}-2)}=2+\frac{5}{\sqrt{19}+2}$, &c. and in the same way we may proceed with any other integer not a square.

But this may be more readily effected by means of the following formula; where N is the given number, a the greatest integer contained in $\sqrt{N}$, and u, u', u'', &c. the greatest integers contained in the respective fractions to which they correspond; viz.

$$\frac{\sqrt{N}+o}{1} = a + \&c. \qquad 1.\,a - o = m;\ \frac{N-m^2}{1} = n$$

$$\frac{\sqrt{N}+m}{n} = u + \&c. \qquad n.\,u - m = m';\ \frac{N-m'^2}{n} = n'$$

$$\frac{\sqrt{N}+m'}{n'} = u' + \&c. \qquad n'\,u' - m' = m'';\ \frac{N-m''^2}{n'} = n''$$

$$\frac{\sqrt{N}+m''}{n''} = u''' + \&c. \qquad n''u'' - m'' = m''';\ \frac{N-m'''^2}{n''} = n'''$$

&c. &c. &c.

$N = 19$, and $a = 4$, the calculation will stand thus:

$$\frac{\sqrt{19}+0}{1} = 4 + \&c. \qquad 1\,.\,4 - 0 = 4;\ \frac{19-4^2}{1} = 3$$

$$\frac{\sqrt{19}+4}{3} = 2 + \&c. \qquad 3\,.\,2 - 4 = 2;\ \frac{19-2^2}{3} = 5$$

$$\frac{\sqrt{19}+2}{5} = 1 + \&c. \qquad 5\,.\,1 - 2 = 3;\ \frac{19-3^2}{5} = 2$$

$$\frac{\sqrt{19}+3}{2} = 3 + \&c. \qquad 2\,.\,3 - 3 = 3;\ \frac{19-3^2}{2} = 5$$

&c. &c. &c.

By this latter formula the square root of any irrational square may be easily extracted, particularly as the computation on the right hand is very readily supplied without setting down the steps of the operation, as is done in the above example, which is merely for the purpose of explanation. It is not, however, for the sake of the arithmetical extraction of the square root, that this method has been devised, but in consequence of its application to indeterminate equations of the second degree, which admit of no other general method of solution.

Continued *Proportion*, is that which the consequent of the first ratio is the same as the antecedent of the second. Thus 2 : 6 = 6 : 18 is a continued proportion.

CONTINUITY, *Law of*, is that by which variable quantities passing from one magnitude to another, pass through all the intermediate magnitudes, without ever passing over any of them abruptly. Many philosophers and metaphysicians have asserted the probable conformity of natural operations to this law; but father Boscovich proves that the law is universal.

CONTRACTION, in *Physics*, the diminishing the extent or dimensions of a body, or the causing its parts to approach nearer to each other, in which sense it stands opposed to dilatation or expansion.

Water and all aqueous fluids are gradually contracted by a diminution of temperature, until they arrive at a certain point, which is about 8° above the freezing point; but below that point they begin to expand, and continue to do so according as the temperature is lowered; and similar effects have been observed with regard to some metals. In speaking of contraction, a remarkable phenomenon, of considerable importance in manufactures, obtrudes itself on our notice; which is, the hardness that certain bodies acquire in consequence of a sudden contraction, and this is particularly the case with glass and some of the metals. Thus, glass vessels, suddenly cooled after having been formed, are so very brittle that they hardly bear to be touched with any hard body. The cause of this effect is thus explained: When glass in fusion is very suddenly cooled, its external parts become solid first, and determine the magnitude of the whole piece, while it still remains fluid within. The internal part, as it cools, is disposed to contract still further, but its contraction is prevented by the resistance of the external parts, which form an arch or vault round it, so that the whole is left in a state of constraint; and as soon as the equilibrium is disturbed in any one part, the whole aggregate is destroyed.

CONVERGENT, or *Converging*, the tendency of different things, variously disposed, to one common point. It is also sometimes used to denote an approximation towards the real value of a thing.

CONVERGING *Fractions*. See def. 2, CONTINUED *Fractions*.

CONVERGING *Lines*, those which tend to a common point.

CONVERGING *Rays*, those which tend to a common focus.

CONVERGING *Series*, those series whose terms continually diminish.

CONVERSE, in *Mathematics*, commonly signifies the same thing as *reverse*. Thus, one proposition is called the converse of another, when, after a conclusion is drawn from something supposed in the converse proposition, that conclusion is supposed; and then, that which in the other was supposed, is now drawn as a conclusion from it.

Converse propositions are not necessarily true, but require a demonstration; and Euclid always demonstrates such as he has occasion for. An instance will shew this. If two right-lined figures are so exactly of a size and form (both respecting their sides and angles) that being laid one on the other, their boundary lines do exactly coincide and agree, then no one doubts that these figures are equal. Now try the converse. If two right-lined figures are equal, then if they be laid the one on the other, their boundary lines exactly coincide and agree. It is manifest that this proposition, though the converse of the former, is by no means true. A triangle and a square may have equal areas; but it is impossible the sides of the former can all coincide with those of the latter

CONVERSION of *Proportion*, is when of four proportionals it is inferred that the 1st is to its excess above the 2d, as the 3d to its excess above the 4th.

CONVEX, round or curved, or protuberant outwards, as the out side of a globular body.

CONVEX *Lens Mirror*. See LENS *Mirror*, &c.

CONVEXITY, the exterior or outward surface of a convex or round body.

CONVEXO-*Concave Lens*, is one that is convex on one side, and concave on the other.

CONVEXO-*Convex Lens*, is one that is convex on both sides.

CO-ORDINATES, in the theory of *Curves*, signify any absciss, and its corresponding ordinate. See ABSCISS, ORDINATE, and CURVE.

COPERNICAN *System*, is that system of the world in which the sun is supposed at rest; and the earth and the several planets to revolve about him as a centre, while the moon and the other satellites revolve about their respective primaries in the same manner.

The heavens and stars are here supposed at rest; and that diurnal motion that they appear to have from east to west, is imputed to the earth's motion from W. to E.

COROLLARY, a consequent truth, which follows immediately from some preceding truth or demonstration.

CORPUSCLE, the diminutive of *corpus*, is used to denote the minute particles that constitute natural bodies.

CORPUSCULAR *Philosophy*, that scheme or system of physics, in which the phenomena of bodies are accounted for, from the motion, rest, position, &c. of the corpuscles or atoms of which the bodies consist.

CORRECTION of a *Fluent*, in the *Inverse Method* of *Fluxions*, is the determination of the constant or invariable quantity which belongs to the fluent of a given fluxion; and which does not naturally arise from that fluxion, but depends wholly on the nature of the problem whence the fluxion was deduced.

That such correction is frequently necessary will be obvious, if we consider, that the fluxion of x is $\dot{x}$, and the fluxion of $x \pm c$ is also $\dot{x}$, whatever be the value of the constant part c, and with whatever sign it is connected with the variable x. And hence, conversely, the fluent of x may be either x, or $x \pm c$; and this, as well as the precise value of the quantity c, must be determined from the nature of the problem, which may be done by the following rule.

First, take the fluent according to the proper rules for that purpose, and then observe whether this fluent becomes equal to zero, or to some constant quantity, when the nature of the problem requires that it should; if it do, it is the complete fluent, and no correction is necessary; but if not, it wants a correction, and this correction is the difference between the two general sides when reduced to that particular state. Hence, connect the constant, but indeterminate quantity c, with one side of the fluxional equation, found as above; then in this equation substitute for the variable quantities, such values as they are known to have at any particular state, place, or time, and then from that particular state of the equation find the value of c, which will be the correction required.

Suppose that a body is projected with a velocity of a feet per second from a given place, to determine its velocity, after it has passed over a certain space x, it having been acted upon by some law which rendered it subject to the fluxional calculus; and suppose the resulting fluxional equation to be

$$\dot{v} = x\dot{x} + 3\,ax^2\dot{x}$$

the fluent of this is

$$v = \tfrac{1}{2}x^2 + ax^3 + c$$

c being the correction.

Now, from the nature of the equation, when $x = o$, or before the body had passed over any space, the velocity was a; therefore, when $x = o$, $v = a$; and writing these values we have

$$a = o + c. \text{ or } c = a,$$

therefore the correct fluent is

$$v = \tfrac{1}{2}x^2 + a\,x^3 + a$$

Whereas, if the body had in the first instance been only solicited by gravity, then when $x = o$, v would also have been equal to o; and we should have found $c = o$, or the general fluent would have been the correct fluent required.

COSECANT, *Cosine*, *Cotangent*, *Coversed Sine*, are the secant, sine, tangent, and versed sine of the complement of the arch or angle. *Co* being a contraction of the word complement.

COTESIAN *Theorem*, in the *higher Geometry*, an appellation distinguishing an elegant property of the circle discovered by Mr. Cotes, and of great use in the integration of differentials by rational fractions. The theorem is this:

If the factors of the binomial $a^n + x^n$ be required, the index n being an integer number. With radius $= a$ describe a circle, and divide its circumference into as many equal parts as there are units in $2n$, then in the radius, produced if necessary, take from the centre a distance $= x$, and from the extremity of this distance draw lines, and to all the points of division in the circumference, these lines taken alternately shall be the factors sought.

COUNTERPOISE, any weight which, placed in opposition to another weight, produces an equilibrium.

COURSE, in *Navigation*, the point of the compass, or horizon, which the ship steers on, or the angle which the rhumb line on which it sails makes with the meridian; and is sometimes reckoned in degrees, and sometimes in points and quarter points of the compass.

CRANE, in Mechanics, a machine for the raising of weights, the power of which depends upon a double cone, two barrels of different diameters, a combination of pullies or of wheel work, but more commonly of the last.

CREPUSCULUM, in *Astronomy*, twilight; the time from the first dawn or appearance of the morning to the rising sun; and again, between the setting of the sun and the last remains of day. The crepusculum is usually computed to begin and end when the sun is about 18 degrees below the horizon; for then the stars of the sixth magnitude disappear in the morning, and appear in the evening. It is of longer duration in the solstices than in the equinoxes, and longer in an oblique than in a right sphere. The crepuscula are occasioned by the sun's rays refracted in our atmosphere, and reflected from the particles thereof to the eye. See TWILIGHT.

CROSS, an instrument used in surveying for the purpose of raising perpendiculars. It consists merely of two pair of sights set at right angles to each other, mounted on a staff, of a convenient height for use.

CUBES, or *Cube Numbers*, in *Arithmetic*, and the *Theory of Numbers*, are those whose cube root is a complete integer.

All cube numbers are of one of the forms $4n$, or $4n + 1$, that is, all cube numbers are either divisible by 4, or when divided by 4 leave 1 for the remainder.

All cube numbers are of one of the forms $9n$, or $9n \pm 1$; that is, they are either divisible by 9, or when divided by 9, they leave for a remainder ± 1.

Cube numbers divided by 6 leave the same remainder as their root, when divided by 6. And consequently the difference between any integral cube and its root is divisible by 6.

Neither the sum nor difference of two cubes can be a cube, that is, the equation $x^3 \pm y^3 = z^3$ is impossible.

The sum of a number of consecutive cubes beginning with unity in a square whose root is equal to the sum of the roots of all the cubes.

The third differences of consecutive cubes are equal to each other, being each equal to 6.

To find the Cube Root of a given Number.

1. Separate the given number into periods of three figures each, by putting a point over the place of units, another over the place of thousands, and so on over every third figure, to the left-hand in integer, and to the right in decimals; and then find the nearest cube root of the first period, and set it in the quotient.

2. Subtract the cube of the figure of the root, thus found, from the first period, to the left-hand, and annex the following period to the remainder for a dividend.

3. Divide this dividend by 3 times the square of the figure of the root above determined, and the first figure of the quotient will be the second figure of the root.

4. Subtract the cube of these two figures of the root from the first two periods on the left, and to the remainder annex the following period, for a new dividend, which divide as before; and so on till the whole is finished. And finally point off as many figures for integers as there are periods of integers in the proposed number.

Ex. Required the cube root of 41278·242816

41278·242816(34·56 root
$3^3 = 27$
$3^2 \times 3 = 27$) 14278(4 2d figure of the root,
41278 1st two periods
$34^3 =$ 39304
$34^2 \times 3 = 3468$) 1974·242(5, 3d figure.
41278·242 1st three periods
$345^3 =$ 41063·625
$345^2 \times 3 = 357075$) 214·617816(6 4th figure

and thus the operation may be carried on till the root be obtained to any degree of accuracy required. This method, however, is extremely laborious, and is seldom or never employed; as other methods have been found in which the approximation is carried on much more rapidly.

2. *Method.* Write the given number under the form $a^3 \pm b$, then by the binomial theorem,

$$\sqrt{}(a^3 \pm b^1) = a \pm \frac{b}{3\,a^2} - \frac{2\,b^2}{3.6\,a^5} \pm \frac{2\cdot 5\cdot b^3}{3\cdot 6\cdot 9\;a^8} - \frac{2.5.8.\,b^4}{3.6.9.12\,a^{11}} \pm \&c.;$$

in which formula substituting the values of a and b, the root may as above be found to any degree of accuracy required. It should be observed, however, that this formula can only be advantageously employed when the given number is only a little more or less than an exact cube.

Ex. Extract the cube root of 1001.

Here $1001 = 10^3 + 1$, therefore $a = 10$ and $b = 1$; consequently

$$\sqrt[3]{1001} = 10 + \frac{1}{2\cdot 10^2} - \frac{1}{9\;10^5} + \frac{5}{81\cdot 10^8} - \&c.$$

$\sqrt[3]{1001} = 10 + .003333 - 000001111 + \&c. = 10.0033322.$

4th Method, by *logarithms*. Divide the logarithm of the given number by 3, and the quotient will be the logarithm of the root; the natural number of which will be the root itself.

Thus to find the cube root of 7867.

$$\begin{array}{ll} \text{log. } 7867 = & 3)\underline{3{\cdot}897404} \\ \text{root } 19715 = & \underline{\underline{1{\cdot}299135}} \end{array}$$

CUBIC *Equation*, in *Algebra*, is that in which the highest power of the unknown quantity rises to the third degree, as $x^3 + ax^2 + bx + c = o$.

Having first reduced the equation to the form $x^3 + ax = b$ Make $x = p + q$, then

$$\begin{cases} x^3 = p^3 + 3\,pq\,(p + q) + q^3 \\ ax^3 = \qquad a\,(p + q) \end{cases}$$

Whence by addition, since $p + q = x$, we have

$x^3 + ax = p^3 + (3pq + a)\;x\;\; q^3 = b.$

If, therefore, we assume $a = -3pq$, we have $p^3 + q^3 = c$, that is, we obtain the two following equations:

$$\begin{cases} p^3 + q^3 = b \\ 3pq = -a. \end{cases}$$

The second gives $p = \frac{-a}{3q}$, which cubed, and substituted, in the first gives,

$$\frac{-a^3}{27\;q^3} + q^3\;b;$$

or, by reduction,

$$q^6 - bq^3 = \frac{a^3}{27}.$$

Whence,

$q^2, = \frac{1}{2}\,b \pm \sqrt{\left(\frac{1}{4}b^2 + \frac{1}{27}\,a^3\right)}$

and, $q = \sqrt[3]{\ }\left\{\frac{1}{2}\,b \pm \sqrt{\left(\frac{1}{4}\,b^2 + \frac{1}{27}a^3\right)}\right\}$

in the same way,

$p = \sqrt[3]{\ }\left\{\frac{1}{2}b \pm \sqrt{\left(\frac{1}{4}b^2 + \frac{1}{27}\,a^3.\right)}\right\}$

Whence x in the proposed equation, becomes

$r = \sqrt[3]{\ }\left(\frac{1}{2}b \times \sqrt{\frac{1}{4}b^2 + \frac{1}{27}\,a^3}\right) \times \sqrt[3]{\ }\left(\frac{1}{2}b - \sqrt{\frac{1}{4}b^2 + \frac{1}{27}\,a^3}\right)$

Which formula is commonly called *Carden's Rule for Cubic Equations*.

In the above formula $\frac{1}{4}\,b^2$ is always positive, being the square of $\frac{1}{2}\,b$, but $\frac{1}{27}\,a^3$ will be negative, or positive, according as a, in the proposed equation, is negative or positive, being the cube of $\frac{1}{3}\,a$; in the latter case, each branch of the root has a real value, which may be ascertained by the proper rules; and in the former case, the same will have place, if $\frac{1}{27}\,a^3 < \frac{1}{4}b^2$; but if a be negative, and $\frac{1}{27}\,a^3 > \frac{1}{4}\,b^2$, then each branch taken separately becomes imaginary, and the root cannot be found by this rule; notwithstanding that the two together are equal to a real quantity. This is what is termed the *Irreducible Case* in *Cubic Equations*, the solution of which, by means of formula, has bid defiance to the attempts of many of the ablest mathematicians of modern times.

CULMINATION, in *Astronomy*, the passage of any heavenly body over the meridian, or its greatest altitude during its diurnal revolution.

CURVATURE of a *Line*, is its bending or flexure; by which it becomes a curve of any particular form and properties. Thus the nature of the curvature of the circle is such, that every point in the periphery is equally distant from a point within called the centre; and so the curvature of the same circle is everywhere the same, but the curvature of all other curves is continually varying.

The curvature of a circle is so much the more, as its radius is less, being always reciprocally as the radius and the curvature of other curves, is measured by the reciprocal of the radius of a circle, having the same degree of curvature as any curve has, at some certain point.

Every curve is bent from its tangent by its curvature, the measure of which is the same as that of the angle of contact formed by the curve and tangent. Now the same tangent is common to an infinite number of circles, or other curves, all touching it, and each other in the same point of contact. So that any curve may be touched by an infinite number of different circles at the same point: and some of these circles fall wholly within it, being more curved, or having a greater curvature than that curve; while others fall without it near the point of contact, or between the curve and tangent at that point, and so, being less deflected from the tangent than the curve is, they have a less degree of curvature there. Consequently, there is one, of this infinite number of circles, which neither falls below it nor above it, but, being equally deflected from the tangent, coincides with it most intimately of all the circles; and the radius of this circle is called the radius of curvature of the curve; also the circle itself is called the circle of curvature, or the osculatory circle of that curve, because it touches it so closely that no other circle can be drawn between it and the curve.

To determine the Radius and Circle of Curvature.

Let AE*e* (Plate II. fig. 10.) be any curve, concave towards its axis AD; draw an ordinate DE to the point E where the curve is required to be found; and suppose EC perpendicular to the curve, and equal to the radius of the circle BE*e* or curvature sought; lastly, draw E*d* parallel to AD, and *d e* parallel and indefinitely near to DE; thereby making E*d* the fluxion or increment of the absciss AD, also *de* the fluxion of the ordinate DE, and E*e* that of the curve AE. Now put $x =$ AD, $y =$ DE, $z =$ AE, and $r =$ CE the radius of curvature; then is E*d* $= \dot{x}$, *d e* $= \dot{y}$, and E*e* $= \dot{z}$.

Now, by
sim. tri. the 3 lines · E*d*, *de*, E*e*,
or · $\dot{x}$, $\dot{y}$, $\dot{z}$,
are respectively
as the three · · · · GE, GC, CE;
therefore · · · · · · $GC.\dot{x} = GE\dot{x}$;
and the flux. of this equa. is
$GC.\ddot{x} + \dot{GC}.\dot{x} = GE.\ddot{y} + \dot{GE}y$,
or because $\dot{GC} = -\dot{BG}$, it is
$GC.\dot{x} - \dot{BG}.\dot{x} = GE.\ddot{y} + \dot{GE}.\dot{y}$.

But, since the two curves AE and BE have the same curvature at the point E, their abscisses and ordinates have the same fluxions at that point, that is E*d* or $\dot{x}$ is the fluxion both of AD and BG, and *d e* or $\dot{y}$ is the fluxion both of DE and GE. In the above equation therefore substitute $\dot{x}$ for $\dot{BG}$, and $\dot{y}$ for $\dot{GE}$, and it becomes

$$GC.\ddot{x} - \dot{x}\dot{x} = GE.\ddot{y} + \dot{y}\dot{y},$$

or $$GC\,\ddot{x} - GE.\ddot{y} = \dot{x}^2 + \dot{y}^2 \text{ or } \dot{z}^2.$$

Now multiply the three terms of this equation respectively by these three quantities,

$\frac{\dot{y}}{GC}, \frac{\dot{x}}{GE}, \frac{\dot{z}}{CE}$, which are equal, and it becomes

$\dot{y}\ddot{x} - \dot{x}\ddot{y} = \frac{\dot{z}^3}{CE}$, or $\frac{\dot{z}^3}{r}$; and

hence $r = \frac{\dot{z}^3}{\dot{y}\ddot{x} - \dot{x}\ddot{y}}$, which is the general value of the radius of curvature for all curves whatever, in terms of the fluxions of the absciss and ordinate.

CURVE in *Geometry*, is a line, the several parts of which lie in different directions, and are successively posited towards different points in space.

A plane CURVE, is that of which the several points in it lie in the same plane; when this is not the case, the curve is said to be one of double curvature.

CURVE *Lines* are distinguished into algebraical or geometrical, and transcendental or mechanical.

Algebraical or *Geometrical* CURVES, are those in which the relation of the abscisses to the ordinates can be expressed by a common algebraical expression; and *Transcendental* or *Mechanical* CURVES, are such as cannot.

Lines are distinguished into ders, according to the dimensions of the equation by which they are expressed; so that a line represented by the general equation

$$0 = a + bx + cy$$

is a line of the first order; but this equation comprises the straight line only, which is the most simple of all lines.

Lines of the second order, or

curves of the first order, are comprised in the general equation

$0 = a + bx + cy + dx^2 + exy + fy^2$; that is to say, we may class among lines of the second order all the curve lines which this equation expresses, x and y denoting the rectangular co-ordinates; which order of lines are more commonly called conic sections, because they all result from sections of the cone, and consist of four species only, *viz.* the circle, ellipse, parabola, and hyperbola. The particular equations of these, when the abcisses begin at the vertex, and the ordinates perpendicular to the respective abscisses, are as follows, *viz.*

Circle • • $y^2 = dx - x^2$, d being the diameter.

Ellipse • $y^2 = \frac{c^2}{t^2}(tx - x^2)$ where t and c are the two axis of the ellipse.

Parabola • • $y^2 = px$, where p is the parameter.

Hyperbola $y^2 = \frac{c^2}{t^2}(tx + x^2,)$ t and c representing the two axis of the hyperbola.

Lines of the third order, or curves of the second order, are comprised in the general equation

$0 = a + bx + cy + dx^2 + exy + fy^2 + gx^3 + hx^2y + kxy^2 + ly^3$.

Lines of the fourth order, or curves of the third order, are comprised in the general equation

$0 = a + bx + cy + dx^2 + exy + fy^2 + gx^3 + hx^2y + kxy^2 + ly^3 + mx^4 + nx^3y + px^2y^2 + qxy^3 + ry^4$; taking always x and y for rectangular co-ordinates.

To find the Equation of a Curve, its description or characteristic properties being given.

For the Circle.—Here the characteristic property is, that all lines drawn from the centre to the circumference are equal to each other.

Let therefore C P (Plate II. fig. 11.) $= r$, $MP = y$, $MA = x$, then $CM = r - x$, and $CP^2 - CM^2 = PM^2$ (Euclid, xlvii. 1;) that is,

$r^2 - (r - x)^2 = y^2$, or $y^2 = 2rx - x^2$, which is the equation to the circle.

Having given the Equation of a Curve, to describe it, and trace its principal properties.

Let the proposed equation be $ay = x^2$.

Assume the point A in the right line CAB, (Plate II. fig. 12.;) and set off $AP = x$, $AP' = -x$; now in both cases y will be positive, and y increases as x increases; the curve, therefore, has two infinite branches on the same side of the line CB. This curve is the *common parabola*.

CURVE of *Double Curvature*, is used to denote the curve-line, all the parts of which are not situated in the same plane.

CYCLE, a certain period or series of numbers, circulating perpetually.

The Cycle of the Sun, or *Solar Cycle*, is a revolution of 28 years, in which time the days of the months return again to the same days of the week.

The Cycle of the Moon, commonly called the *Golden Number*, is a revolution of 19 years; in which time the conjunctions, oppositions, and other aspects of the moon, are within an hour and a half of being the same as they were on the same days of the month 19 years before.

The year of our Saviour's birth was the ninth year of the solar cycle, and the first year of the lunar cycle.

Hence, *to find the Year of the Solar Cycle*, add 9 to any given year, and divide by 28, the quotient is the number of cycles, and the remainder is the cycle for the given year: if nothing remains, the cycle is 28.

To find the Lunar Cycle.—Add one to the given year, and divide the sum by 19; the quotient is the number of cycles elapsed, and the remainder is the cycle for the given year: if nothing remains, the cycle is 19.

The Cycle of Easter, is a revolution of 532 years; found by multiplying the solar cycle 28 by the lunar cycle 19. If the new moons did not anticipate upon this cycle, Easter-day would always be the Sunday next after the full moon which follows the twenty-first of March; but, on account of the above anticipation, the ecclesiastic

Easter has several times been a week different from the true Easter. The earliest Easter possible, is the 22d of March; the latest, the 25th of April. Within these limits are thirty-five days, and the number belonging to each of them is called the number of direction; because thereby the time of Easter is found for any given year.

CYCLOID, is a mechanical or transcendental curve, possessing several very curious properties. It is generated by making a circle revolve along a line, which a point in a radius of the circle traces the curve. The line is called the *base*, the diameter of the circle the *axis*, and the circle the *generating circle*. If the point be without the circle, the cycloid is called *curtate;* and if within, it is called *prolate*. If the point were in the centre of the circle, the cycloid would become a line.

If the circle revolve about another curve, the path described is called an epicycloid. In Mechanics, the teeth of a wheel should be formed into the epicycloid made by the revolution of the pinion.

Properties of the Cycloid.—Draw any right ordinate FGH, (Plate III. fig. 1.) above, join CG, and from H draw HI parallel to CG, meeting the axis DC produced in I. Then,

1. IH is a tangent to the cycloid at the point H.

2. The circular arc CG is equal to the right line HG.

3. The semi-circumference CGD = semi-base DB.

4. The cycloidal arc CH = double the chord CG.

5. The semi-cycloidal arc CB = double the diameter CD.

6. The area of the cycloid ACBA = triple the area of the circle CGD.

7. The three spaces ACD, the circle CGD, and CBD, are equal to each other.

8. The upper segment of a cycloid cut off by a line parallel to the base, at $\frac{1}{4}$ of the axis from the vertex, is equal to the regular hexagon inscribed in the generating circle.

9. The solid generated by the revolution of the cycloid about its base AB, is to its circumscribing cylinder as 5 to 8.

10. The solid generated about the tangent parallel to the base, is to its circumscribing cylinder as 7 to 8.

11. And the solid generated about the tangent parallel to the axis, is to its circumscribing cylinder as 6 to 8.

12. The centre of gravity of the whole cycloid, is $\frac{5}{8}$ths of the axis from the vertex.

13. The evolute of a cycloid is another equal cycloid. Hence, if two equal semi-cycloids OP, OQ, (Plate III. fig. 2.) be joined at O, so that OM = MK, the diameter of the generating circle, and the string of a pendulum be suspended at O, having its length = OK = OP; then by plying the string round the curve OP, to which it is equal, and the ball of the pendulum be let go, it will describe and vibrate in the other cycloid PKQ. And all its vibrations, whether in great arcs or in small ones, will be performed in the same time. And the time of one vibration is to the time of falling perpendicularly through the axis MK, as the circumference of a circle to its diameter, or as 3·1416 : 1.

14. The cycloid is the curve of swiftest descent; that is, a body will fall from Q to K, through the arc QK, in less time than by any other rout.

CYLINDER, is a solid having two equal circular ends parallel to each other; and every plane section parallel to the ends is also a circle, and equal to them.

Cylinders are either *right* or *oblique*.

A *Right* CYLINDER, is that whose side is perpendicular to the plane of its base.

An *Oblique* CYLINDER, is that whose side is oblique to the plane of its base.

To find the surface of a cylinder: Multiply the circumference of the base by the height; add the surfaces of the two bases. To find the solidity: Multiply the area of the base by the height.

D.

DATA, in *Mathematics*, denote certain quantities which are given or known, and by means of which other quantities, which are unknown, are to be determined.

DATUM, the singular of data.

DAY, in *Astronomy*, is that portion of time which elapses between two successive transits of the sun over the same meridian; and the hours are counted from one period to another, from one to twenty-four.

Mean Solar DAY. The solar day is the time between one transit of the sun and another; but this period is not of equal duration at all seasons of the year. See EQUATION of *Time*.

Sidereal DAY, is that period in which the earth makes one complete revolution on its axis, and is measured by the interval between two successive transits of any fixed star over the same meridian. The sidereal day is about four minutes less than the mean solar day, the difference arising from the motion of the earth in its orbit. Hence, the number of sidereal day's exceed the number of mean solar days; the year consisting of 366 of the former, and only 365 of the latter. The sidereal day is the most uniform of all astronomical periods, neither observation nor theory having yet detected in it the least variation.

DECAGON, a plane geometrical figure of ten equal sides and angles.

If the radius of a circle, or the side of the inscribed hexagon, be divided in extreme and mean proportion, the greater segment will be the side of a decagon inscribed in the same circle. And, therefore, as the side of the decagon is to the radius, so is the radius to the sum of the two. Whence, if the radius of the circle be r, the side of the inscribed decagon will be $\frac{\sqrt{5}-1}{2} \times r$, and if its side be s its area $= s^2 \times 7{\cdot}694209$.

DECIMAL *Arithmetic*, in a general sense, denotes the common arithmetic, in which we count by periods of tens; and is otherwise, and more properly called *Denary Arithmetic*.

DECIMAL *Fraction*, is a fraction having always some power of 10 for its denominator; which consists of either 10, 100, 1000, &c. denoting the number of equal parts into which the integer or whole is supposed to be divided; as $\frac{2}{10}$, $\frac{2}{100}$, $\frac{2}{1000}$, &c. But, for the sake of brevity, the numerator only is expressed, like a whole number with a point on the left of it; as ·2, ·02, ·002, &c. and which must always consist of as many figures as there are ciphers in the denominator; the places between the significant figures and the point being supplied with ciphers. For the several operations of MULTIPLICATION, DIVISION, &c. of *Decimals*, see the several articles.

DECLINATION, in *Astronomy*, the distance of the sun, a star, planet, or other point of the sphere, from the equator, either northward or southward; and is the same with latitude in geography. Declination is either real or apparent, according as the real or apparent place of the object is considered.

The declination of any heavenly body, as of a star, may be easily found by the following rule: take the meridian altitude of the star, at any place where the latitude is known; the complement of this is the zenith distance, and is called north or south, as the star is north or south at the time of observation. Then, 1. When the latitude of the place and zenith distance of the star are of different kinds, namely, one north, and the other south, their difference will be the declination; and it is of the same kind with the latitude, when that is the greatest of the two, otherwise it is of the contrary kind. 2. If the latitude and the zenith distance are of the same kind, *i. e.* both north or both south, their sum is the declination; and it is of the same kind with the latitude.

Circles of DECLINATION, are great circles of the sphere passing through the poles, on which the declination is measured.

Parallels of DECLINATION, are small circles of the sphere parallel to the equator.

Parallax, or *Refraction* of DECLINATION, is such an arch of a meridian as is equal to the change produced in the declination by parallax or refraction respectively.

DECREMENTS, are the small parts by which a variable quantity decreases

DEFINITION, an enumeration or specification of the simple ideas of which a compound idea consists, in order to ascertain its nature and character.

Definitions are of two kinds; the one nominal, or of the name; the other real, or of the thing.

Nominal definition is that which explains the sense or signification appropriated to a word, so as to leave it out of doubt what the subject is that is intended or denoted by the name.

DEFLECTION, the turning any thing aside from its former course by some adventitious or external cause.

DEFLECTION of the *Rays* of *Light*, is the same property which Sir Isaac Newton calls inflection. It is called by others diffraction.

DEFLECTIVE *Forces*, are those forces which act upon a moving body in a direction different from that of its actual course, in consequence of which the body is deflected, or turned, or drawn aside, from the direction in which it is moving.

DEGREE, in *Algebra*, a term applied to equations, to distinguish the highest power of the unknown quantity.

DEGREE, in *Geometry* or *Trigonometry*, is the 360th part of the circumference of any circle. Every circle being considered as divided into 360 parts, called degrees; which are marked by a small ° near the top of the figure; thus 45° is 45 degrees.

The degree is subdivided into 60 smaller parts, called minutes, meaning first minutes; the minute into 60 others, called seconds; the second into 60 thirds, &c. Thus 45° 12′ 20″, are 45 degrees, 12 minutes, 20 seconds.

The magnitude or quantity of angles is accounted in degrees; for because of the uniform curvature of a circle in all its parts, equal angles at the centre are subtended by equal arcs, and by similar arcs in peripheries of different diameters; and an angle is said to be of so many degrees as are contained in the arc of any circle comprehended between the legs of the angle, and having the angular point for its centre. Thus we say, an angle of 90°, or of 45° 24′. It is also usual to say, such a star is elevated so many degrees above the horizon, or declines so many degrees from the equator; or such a town is situate so many degrees of latitude or longitude. A sign of the ecliptic or zodiac contains 30 degrees.

DEGREE of *Latitude*, is the space or distance on the meridian through which an observer must move, to vary his latitude by one degree, or to increase or diminish the distance of a star from the zenith by one degree; and which, on the supposition of the perfect sphericity of the earth, is the 360th part of the meridian.

The quantity of a degree of a meridian, or other great circle, on the surface of the earth, is variously determined by different observers; and the methods made use of are also various.

In the infancy of this part of science, the increase of the degrees of latitude toward the pole was considered as proof that the earth was *elongated;* but this was soon corrected, by considering that the plumb-line is a perpendicular to the tangent, and not a continuation of the diameter.

TABLE

Of the different lengths of a degree, as measured in various parts of the earth, the time of its measurement, the latitude of its middle point, &c.

Date.	Latitude.			Extent in Eng.miles and dec.	Measures.	Countries.
1525	49°	20¼′	N″	68·763	M. Fernel	France
1620	52	4	N.	66· 91	Snellius	Holland
1635	53	15	N.	69·545	Norwood	England
1644				75 066	Riccioli	Italy
1669	49	22	N.	68·945	Picard	France
1718				69·119	Cassini	
1737	66	20	N.	69·403	Maupertius, &c. . .	Lapland
	49	22	N.	69·121	Cassini and La Caile	France
1740	45	00	N.	69·092		
1744	0	0		68·751	Juan and Ullca . .	Peru
				68·732	Bouguer	
				68·713	Condamine	
1752	33	18½	S.	69·076	La Caile.	C. of Good Hope
1755	43	0	N.	68·998	Boscovitch	Italy
1764	44	44	N.	69·061	Beccaria	
1766	47	40	N.	69·142	Leisganig	Germany
1768	39	·12	N.	68·893	Mason and Dixon .	America
1802	51	29 54½	N.	69·146	Lt. Colonel Mudge	England
1803	66	20½	N.	69·292	Swanberg, &c. . .	Lapland
	12	32	N.	68·743	Lambton	Misore
1808	44	52½	N.	68 769	Biot, Arago, &c. . .	France

Ellipticities of the earth, expressed in parts of its equatorial diameter.

Authors.	Ellipticities.	Principles.
Huyghens . . .	$\frac{1}{579}$	Theory of gravity
Newton	$\frac{1}{230}$	
Maupertius, &c.	$\frac{1}{314}$	Mensuration of arcs
	$\frac{1}{129}$	
	$\frac{1}{213}$	
Swanberg . . .	$\frac{1}{323{\cdot}063}$	
Clairault . . .	$\frac{1}{11{\cdot}33}$	Rotatory motion
	$\frac{1}{337}$	Vibrations of the pendulum
Treisnsoker . .	$\frac{1}{329}$	Occultations of the fixed stars
Laplace	$\frac{1}{334}$	Precession and nutation
	$\frac{1}{305}$	Theory of the moon.

Degree of *Longitude*, is the space between two meridians that make an angle of 1° with each other at the poles; the quantity or length of which is variable, according to the latitude, being every where as the cosine of the latitude; viz. as the cosine of one latitude is to the cosine of another, so is the length of a degree in the former latitude to that in the latter, on the supposition that the earth is spherical. But taking the earth as a spheroid, the degree of longitude may be found in any given latitude L, by saying,

1. As the equatorial diameter to the polar, so is tang. (90° — L,) to tang. of an angle A; then, 2. As radius to sine of A, so is the length of a degree of the equator to the length of a degree on the parallel of the given latitude.

DENOMINATOR of a *Fraction*, is that number written below the line expressing the number of parts into which the unit is supposed to be divided; thus in the fractions $\frac{5}{12}$, $\frac{a}{b}$; 12 and b are the denominators.

DENSITY, strictly speaking, denotes vicinity or closeness of particles; but in mechanical science it is used as a term of comparison, expressing the proportion of the number of equal moleculæ, or the quantity of matter in one body to the number of equal moleculæ in the same bulk of another body. It is, therefore, directly as the quantity of matter, and inversely as the magnitude of the body.

Since it may be shewn experimentally that the quantities of matter, or the masses in different bodies, are proportional to their weight; of consequence, the density of any body is directly as its weight, and inversely as its magnitude; or the inverse ratio of the magnitudes of two bodies, having experimentally equal weights (in the same place), constitutes the ratio of their densities.

No body is absolutely or perfectly dense; that is, no space is perfectly full of matter, so as to have no vacuity or interstices.

DENSITY of the *Earth*. The determination of the density of the earth, as compared with that of water, or any other known body, is a subject which has excited considerable interest amongst modern mathematicians, and nothing can at first sight seem more beyond the reach of human science than the due solution of this problem; yet this has been determined, and on such principles, that if it be not correctly true, it is probably an extremely near approximation.

The first idea of determining the density of the earth was suggested by M. Bouguer, in consequence of the attraction of Chimboraco, which affected his plumb-line while engaged with Condamine in measuring a degree of the meridian, near Quito, in Peru. This led to the experiments on the mountain Schehallien, in Scotland, which were carried on under the direction of Dr. Maskelyne, and afterwards submitted to calculation by Dr. Hutton, who determined the density of the earth to be to that of water as 4½ to 1. But in consequence of the specific gravity of the mountain being assumed rather less than it ought to have been, the above result is less than the true density, as has since been shewn both by Dr. Hutton and Professor Playfair, the former of whom makes it, in his corrected paper, as 99 to 20, or nearly as 5 to 1.

The same problem has been attempted on similar principles, but totally in a different manner, by the late Mr. Cavendish, who found the density of the earth to be to that of water as 5·48 to 1. Taking a mean of all these, we have the density of the earth to that of water as 5·24 to 1.

DENSITY of the *Planets*. Since the density of a body is directly as its mass, and reciprocally as its magnitude; any two of these being given, the third may be determined. Now the magnitude of the several planets, as also of the sun, being supposed known from observation, if we can determine their masses, their densities will thus also become known.

The power of attraction with which any central body acts upon another body revolving about it, is directly as the mass of the central body, and reciprocally as the square of the distance of the revolving body; and this power may be measured by the deflection of the revolving body in a given time from the direction of its tangent. Now, if we consider the earth as the central, and the moon as the revolving body, the deflection of the latter in one second is known to be $\frac{16}{3600}$ of a foot, that is, it deviates so much from the direction of its tangent in one second of time, as may be readily ascertained by computation, being equal to the versed sine of the arc described in one second. If the distance between the earth and moon had been double what it is, this deflection would have been only $\frac{1}{4}$ of the above; if the distance were only half what it is, this deflection would be four times as much, and so on, because the power of the central body is reciprocally as the square of its distance. Again, if the distance of the bodies remain the same, and the mass of the central body be doubled, the deflection will be doubled; if tripled the reflection will be tripled, and so on; because the power of the central body is directly as its mass.

Now Jupiter's first satellite revolves about that planet at the same distance as the moon does about the earth, but its deflection in one second is 256 times that of the moon; whence, the distances being equal, it follows, that the mass of Jupiter is 256 times greater than that of the earth.

The density of the sun, and the several planets, as deducted from Laplace's "System du Monde," latest edition, are as follows; viz.

Sun	0·25226
Mercury . .	2·58330
Venus . . .	1·02400
Earth . . .	1·00000
Mars	0 65630
Jupiter . . .	0·20093
Saturn . . .	0·10349
Uranus . . .	0·21085

DEPARTURE, in *Navigation*, is the easting or westing of a ship in respect of the meridian it departed or sailed from; or it is the difference of longitude in miles either east or west, between the present meridian the ship is under, and that where the last reckoning or observation was made.

DEPRESSION of *Equations*, in *Algebra*, is the reduction of an equation to one of lower dimensions.

If an equation have two equal roots it may be depressed one degree; if three equal roots it may be depressed two degrees, and so on: and in reciprocal equations of even dimensions may be depressed to half the original degree; and if they are of odd dimensions, to half the original degree minus 1.

Generally if the original equation $x^n - p\,x^{n-1} + qx^{n-2} - \&c. = o$, have m, equal roots, the equation $nx^{n-1} - (n-1)\,px^{n-2} + (n-2)\,qx^{n-3} - \&c. = o$, has $m - 1$ of those roots, as may be readily shown. Hence, when there are m equal roots, the two equations have a common measure of this form $(x-a)^{n-1}$ (a being one of the roots) which may be obtained in the usual way, and m roots of the original equation may thus be known. Divide this equation by $(x-a)^m$, and the resulting equation of $n - m$ dimensions contains the other roots.

DEPRESSION of the *Pole*, is its approximation towards the horizon as we proceed towards the equator.

DEPRESSION of the *Sun*, or of a *Star*, is its distance below the horizon, measured by an arc of a vertical circle.

DEPRESSION of the *Visible Horizon*, denotes its sinking or dipping below the true horizontal plane; whether caused by some variation of the atmosphere, or by the different height of the observer's eye above the surface of the sea.

The depression is easily calculated; for, calling the radius of the earth AC, and the radius + the height CB, we have

CA : CB = rad. : sec. C : or
CB : CA = rad. : cosin. C. In which C is the angle of depression.

DERIVATIONS, the *Calculus* of, a general method of considering quantities, deriving themselves one from another.

It is a fact which has been repeatedly demonstrated, and now generally admitted, that in the developement of $(a + x^m,)$ whatever operation is indicated by m, whether involution, evolution, division, &c., the terms of the series involve the successive integral powers of x, and that the second term of the series is always of the form $ma^{m-1}x$, so that the coefficient of the term involving x is always found by multiplying the index *m* of the first term, into that term when its index has been diminished by unity.

Let then D be assumed to denote that operation; then, $D\,a^m$ represents ma^{m-1}, $D\,a^{m-1}=(m-1)\,a^{m-2}$; $D\,a^{-m}=-ma^{-m-1}$, $D.1m=m\,1^{m-1}$, or *m*, $D.1\tfrac{1}{2}=\tfrac{1}{2}.1^{-\frac{1}{2}}$, so also $DD\,a^m=m\,D\,a^{m-1}=m\,(m-1)\,a^{m-2}$ $DDD\,a^m=$

$$\left\{\begin{array}{l} m\,DD\,a^{m-1}=m\,(m-1)\,D\,a^{m-2}= \\ m.(m-1)\,(m-2)\,a^{m-3}. \end{array}\right.$$

Whence it readily follows, that DDD....D, supposing there to be *n* D's, will represent *n* such operations as those above, and expressing these *n* D's by the abridged expression D^n, we shall have

$$\begin{array}{l} D^n a^m = m\,(m-1)\,(m-2)\,(m-3,) \\ \quad \&c.\ (m-n+1)\,a^{m-n} \\ D^3\,a^3\ = 3\,.\,2\,.\,1\,a^{\circ}, \\ D^3\,a^4\ = 4\,.\,3\,.\,2\,a \\ D^3\,1^m\ = m\,(m-1)\,(m-2)\,1^{m-3} \\ D^2\,a^{-m} = -m-(m+1)\,a^{-m-2} \\ \quad \&c. \qquad\qquad\qquad \&c. \end{array}$$

Whence it follows that the binomial $(a+x^m)$ according to this notation becomes

$$(a+x)^m=a^m+D\,a^m x+\frac{D^2\,a^m\,x^2}{1\,.\,2}+\frac{D^3\,a^m\,x^3}{1\,.\,2\,.\,3}$$

Or if $\underset{c}{D}^n$ be understood to represent $\dfrac{D^n}{1.2.3\,..\,n}$ that is $\underset{c}{D}^2=\dfrac{D^2}{1\,.\,2}$, $\underset{c}{D}^3=\dfrac{D^3}{1.2.3}$, &c.

then the above is more simply expressed $(a+x)^m=a^m+D\,a^m\,x+\underset{c}{D}^2\,a^m\,x^2+\underset{c}{D}^3\,a^m\,x^3+$ &c.

DESCENSION, in *Astronomy,* is either *right* or *oblique*.

Right DESCENSION, is the arch of the equator which descends with the sign or star below the horizon of a direct sphere.

Oblique DESCENSION, is the arch of the equator which descends with the sign or star below the horizon of an oblique sphere. See ASCENSION.

DESCENSIONAL *Difference,* is the difference between the right and oblique descension of the same star, or point of the heavens.

DESCENT of *Bodies,* in *Mechanics,* is their motion or tendency towards the centre of the earth, either in a direct or oblique direction.

A body dropt from the top of a tower, instead of falling to the westward of the tower, ought to fall a little to the eastward of it, in consequence of the velocity of rotation being greater at the top than at the foot of the tower. La Place gives the following theorem for the computation of this deflection: Let $h=$ the height the body falls, $g=$ double the space a body will descend in the first second from the action of gravity, $n=$ the angle of the earth's rotation in the same time, at the rate of 360° ÷ 0·99727 in a day, and θ the colatitude of the place, also D the deviation towards the east, then

$$D=\tfrac{2}{3}\,n\,h\,\sin.\,\theta\,\sqrt{\frac{2\,h}{g}}$$

DESCRIPTIVE *Geometry,* a species of geometry almost entirely new, and which we owe in great measure to M. Monge.

When any surface whatever penetrates another, there most frequently results from their intersection curves of double curvature, the determination of which is necessary in many arts, as in groined vault-work, cutting arch-stones, wood-cutting, for ornamental work, &c. the form of which is frequently very singular and complicated: it is in the solution of problems appertaining to these subjects that descriptive geometry is especially useful.

We can only mention one or two of the problems and theorems. Thus, among the problems: 1st, Two right lines being given in space, and which are neither parallel nor in the same plane, to find in both of them the points of their least distance, and the position of the line joining these points. 2d, Three spheres being given in space, to determine the position of the plane which touches them. There are also curious problems relative to lines of double curvature, and to surfaces resulting from the application of a right line that leans continually upon two or three others given in position in space. Among the theorems, the following may be mentioned: if a plane surface given in space is projected upon three planes, the one horizontal and the two others vertical and perpendicular to each other, the square of that surface will be equal to the sum of the squares of the three surfaces of projection. This theorem is as interesting in the geometry of solids, as Euc. 1. xlviii. is in plane geometry.

DETERMINATE *Problem*, and *Analysis*, in *Geometry*, is that which admits but of a certain and determinate number of solutions, being thus opposed to indeterminate problems, the number of whose solution is unlimited.

DEVELOPEMENT, is a term used to denote the transformation of any function into the form of a series: thus the developement $(a + b)^m$, is the expansion of it into the form $a^m + ma^{m-}b + \frac{m.(m-1)}{1.2} a^{m-2}b$ + &c.

DIACAUSTIC *Curve*, or *Caustic by Refraction*, is a species of caustic curves, formed by a curve touching all the rays from a point that are refracted by another curve in such a manner as that the sines of the angles of incidence and refraction are always to each other in a given ratio.

DIAGONAL, in *Geometry*, a right line drawn across a quadrilateral or other figure, whether plane or solid, from the vertex or summit of one angle to that of another.

It is demonstrable, 1. That every diagonal divides a parallelogram into two equal parts. 2. That two diagonals drawn in any parallelogram bisect each other. 3. A line passing through the middle point of the diagonal of a parallelogram, divides the figure into two equal parts. 4. The diagonal of a square is incommensurable with one of its sides. 5. The sum of the squares of the two diagonals of every parallelogram is equal to the sum of the squares of the four sides. 6. In any trapezium, the sum of the squares of the four sides is equal to the sum of the squares of the two diagonals, together with four times the square of the distance between the middle points of the diagonals. 7. In any trapezium, the sum of the squares of the two diagonals is double the sum of the squares of two lines bisecting the two pairs of opposite sides. 8. In any quadrilateral inscribed in a circle, the rectangle of the two diagonals is equal to the sum of the two rectangles under the two pairs of opposite sides. 9. In every parallelopiped, the sum of the squares of the four diagonals of the solid is equal to the sum of the squares of its twelve edges. 10. In every hexaëdron, regular or not, the sum of the squares of the twelve edges, plus the sum of the squares of the twelve diagonals of the faces, is equal to three times the sum of the squares of the four diagonals which cross the solid, plus four times the sum of the squares of the six right lines which join, two by two, the middle points of those four latter diagonals. 11. In every polygon, and in every polyëdron, the sum of the squares of the lines which join, two by two, the middle points both of sides and diagonals, is the quarter of the sum of the squares of those sides and diagonals; multiplied by the number of summits of the polygon or polyëdron, diminished by two units. 12. A farther generalization of the latter property leads to the most celebrated property of the centre of gravity.

DIAGRAM, a scheme for the explanation or demonstration of any figure or of its properties.

DIAL, or *Sun-Dial*, an instrument serving to measure time, by means of the shadow of the sun.

Universal DIAL, there are several kinds of dials called universal, because they serve for all latitudes. One of a very ingenious construction has lately been invented by Mr. G. Wright, of London. The hour circle, or arch and latitude arch, are the portions of two meridian circles; one fixed, and the other moveable. The hour or dial plate at top is fixed to an arch, and has an index that moves with the hour circle; therefore, the construction of this dial is perfectly similar to the construction of the meridians and hour-circles upon a common globe. The peculiar problems to be performed by this instrument are, 1. To find the latitude of any place. 2. The latitude of the place being known, to find the time by the sun and stars. 3. To find the sun or stars' altitude and azimuth.

But the dial being properly adjusted, a great variety of astronomical and trigonometrical problems may be wrought upon it.

DIALLING, the art of drawing dials on the surface of any given body, whether plane or curved.

Dialling is wholly founded on the diurnal motion of the earth.

The edge of the plane by which the time of the day is found is called the stile of the dial, which must be parallel to the earth's axis; and the line on which this plane is erected is called the substile. The angle included between the substile and stile is called the elevation or height of the stile.

If the stile be at right angles to the substile, the distances of the hour lines will be equal, each making an angle of 15°; and if the stile be parallel to the substile, the distances will be the tangents of 15°, 30°, 45°, the distance of the style and substile being radius. If the position of the stile and substile be oblique, the measures may be determined by cutting the lines drawn to the points of the tangents at the same obliquity, or they may be found by calculation. Thus for a *Horizontal* dial, that is, a dial having its substile parallel to the horizon;

As Radius
Is to the sine of the latitude of the place,
So is tangent of sun's distance from the meridian
To the tangent of the angle from the meridian on the dial.

The preceding construction and computation, for the case of an horizontal dial may be said to include the whole theory of dialling, for there is no plane, however obliquely situated with respect to any given place, but what is parallel to the horizon of some other place; and therefore, if we find that other place by a problem on the terrestrial globe, or by a trigonometrical calculation, and construct a horizontal dial for it, that dial, applied to the plane where it is to serve, will be a true dial for that place.—Thus, an erect direct south dial in 51½° N. lat. would be a horizontal dial on the same meridian, 90° southward, which falls in with 38½° S. lat.: but if the upright plane declines from facing the south at the given place, it would be a horizontal plane 90° from that place; but for a different longitude, which would alter the reckoning of the hours accordingly.

DIAMETER, in *Geometry*, the line which passing through the centre of a circle, or other curvilinear figure, divides it or its respective ordinates into equal parts.

Conjugate DIAMETER of a *Conic Section*. See *Conjugate*.

Transverse DIAMETRE of a *Conic Section*, is a line drawn through the foci to the curve.

DIAMETER of the *Planets*, &c. in *Astronomy*, is either real or apparent.

The real diameters are the absolute measure of them in miles, &c. and their apparent diameters are the angles under which they appear to spectators on the earth.

The general position of the style being thus constant, the whole art of dialling is reduced to one problem, viz. finding the points at which rays drawn from the style, and, at 15° apart, would cut the substyle; and this may, in a great variety of cases, be done by means of a globe: for if the pole of the globe be so elevated as to form with the horizon the angle which the style makes with the substyle, then the hour circles on the globe will cut the horizon in the angles required; and to whatever angle the pole is raised, the six o'clock lines will cut the east and west points.

TABLE

OF THE MEAN AND APPARENT DIAMETERS OF THE PLANETS.

	Apparent mean diam as seen from the earth.	Apparent diam. as seen from the sun.	Mean diam. in English miles.	Apparent diam. of the sun seen from the planets.	Proportional diam.
Sun . . .	32′ 2″	. . .	882260	. .	110·000
Mercury	11″·8	16″	3124	80′	·4
Venus .	57″·9	30″	7702	46′	·9
Earth . .	. . .	17″ 4	7916	32′	1·0
Mars . .	8″·94	10″	4398	21′	·5
Jupiter .	39″	37″	91522	6′·1	11 6
Saturn .	18″	16″	76068	3′·4	9·8
Uranus .	3″·54	4″	35112	1′·6	4·25
Moon . .	31′ 26″·5	4″·6	2160	34	·27

DIFFERENCE of *Longitude* of two places, is an arc of the equator contained between the meridians of those places.

DIFFERENTIAL, in the higher *Geometry*, is an infinitely small quantity, or a part of quantity so small as to be less than any assignable one, and is thus denominated, because it is frequently considered as the difference of two quantities, and as such is the foundation of the differential calculus.

DIFFERENTIAL *Calculus*, is a method of differencing quantities, or of finding an infinitely small quantity, which being taken infinite times shall be equal to a given quantity; or it is the arithmetic of the infinitely small differences of variable quantities; and thus differs in its metaphysics, though not in its principles or method of application, from the fluxional calculus; those quantities which in the former are considered as infinitely small differences, being in the latter supposed to be the infinitely small or momentary increments of variable or flowing quantities. There is this difference in the notation, a fluxion is denoted by a dot (·) over the quantity, and a differential by *d* placed as a coefficient. Thus,

Fluxional $\dot{x}, \ddot{x}, \dddot{x}, \dot{x}^n, \dot{x}^2, \dot{x}^3$, &c.
Differential $dx, d^2x, d^3x, d^nx, dx^2, dx^3$, &c.

Fluxional $\frac{\dot{u}}{\dot{x}}, \frac{\ddot{u}}{\dot{x}^2}, \frac{\dddot{u}}{\dot{x}^3}, \frac{\ddot{u}}{\dot{x}\,\dot{y}}, \frac{\dddot{u}}{\dot{x}^2\dot{y}}, \left(\frac{\dot{y}}{\dot{z}}\right)$

Differential $\frac{du}{dx}, \frac{d^2u}{dx^2}, \frac{d^3u}{dx^3}, \frac{d^2u}{dx.dy}, \frac{d^3u}{dx^2dy}, d^3\left(\frac{dy}{dz}\right)$

The above notation being attended to, the principles of the differential calculus will be understood from the following illustration.

Let $u=fx$ be any function of x, then if for x we substitute $x+h$, the developement of this will be of this form:

$$f(x+h)=fx+f'xh+\frac{f''xh^2}{1.2}+\text{\&c.}$$

the coefficients $f'x, \frac{f''x}{1.2}$ being derived from the primitive function fx, and independent of h.

Hence $f(x+h)-fx=f'xh+\frac{f''xh^2}{1.2}+$ &c. which quantity represents the difference between fx, and what fx becomes when x

is increased by h, or when it becomes $x+h$.

Let the first term of this difference be called the differential, and be denoted by the expression $d.fx$; then we have $d.fx = f'xh$;

therefore $f'x = \frac{d.fx}{h}$.

Hence to find $f'x$, divide the difference betwen two successive values of fx by the increment of x; but since $f'x$ is independent of h, h must disappear by this division, and may be represented by any symbol at pleasure.

Therefore, for the sake of uniformity, let h be represented by dx; then $f'x = \frac{dfx}{dx}$. Hence, in order to find dfx, or the differential of fx, write in the function fx, $x + dx$ instead of x, then develope $f(x+dx)$ as far as the terms effected with the first power of dx, and subtract fx from it.

Hence

$$\begin{aligned} f\ (x+dx) - f\ x &= f'\ xdx \\ f'\ (x+dx) - f'\ x &= f''\ xdx \\ f''\ (x+dx) - f''\ x &= f'''\ xdx \\ \text{\&c.} \qquad & \qquad \text{\&c.} \end{aligned}$$

Also $f'xdx = d.fx.f''xdx = d.f'x$, &c.

therefore $f''x = \frac{d.f'x}{dx} = d\left(\frac{dfx}{dx}\right)\frac{1}{dx}$.

But since dx is invariable,

$$f'(x+dx)\,dx - f'xdx = f''xdx^2$$

hence $d.f'xdx = f''xdx^2$

but $df'xdx = d\,dfx = d^2fx$, d^2, representing the second differential of x according to the preceding notation.

Whence it appears, since

$$f''x = d\left(\frac{dfx}{dx}\right)\frac{1}{dx} \text{ also } fx = \frac{dfx}{dx^2};$$

$$\text{that } d\left(\frac{dfx}{dx}\right)\frac{1}{dx} = \frac{d^2fx}{dx^2}$$

and thus the derived functions $f'x$, $f''x$, $f'''x$, &c. may be represented by the quantities $\frac{dfx}{dx}$, $\frac{d^2fx}{dx^2}$, $\frac{d^3fx}{dx^3}$, &c. so that the developement of $f(x+dx)$ takes the following form; viz. $f(x+dx) = fx + \frac{dfx}{dx}h + \frac{d^2fx}{1.2.dx^2} + \frac{d^3fx}{1.2.3.dx^3}$, &c.

DIGIT, in *Arithmetic*, signifies any one of the ten numerals, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. The word comes from *digitus*, a finger; thus indicating the humble means originally employed in computations.

DIGIT is also a measure equal to three-fourths of an inch.

DIGIT, in *Astronomy*, is the measure by which we estimate the quantity of an eclipse. The diameter of the sun or moon's disc is conceived to be divided into 12 equal parts, called digits.

DIMENSION, in *Geometry*, is either length, breadth, or thickness. Thus a line has only one dimension, *length*; a surface two, *length* and *breadth*; and a body or solid, *length*, *breadth*, and *thickness*.

DIMENSION of an *Equation*, or any other quantity in *Algebra*, is used with regard to the highest power that enters into its composition.

DIOPHANTINE *Analysis* or *Problems*, in *Algebra*, are certain questions relating to square, cube, &c. numbers, and rational right-angled triangles; the properties of which were first discussed by Diophantus.

For instance, to divide a given square number into two other square numbers.

Let a^2 represent the given square, and x^2 and y^2 the required squares; then we have only to satisfy the equation

$$\begin{cases} a^2 = x^2 + y^2, \text{ or} \\ a^2 - y^2 = x^2 \end{cases}$$

In order to which, assume

$$a + y = \frac{px}{q}$$

$$a - y = \frac{qx}{p}$$

From which we readily deduce

$$2a = \frac{px}{q} + \frac{qx}{p} = \frac{(p^2+q^2)x}{pq}$$

$$2y = \frac{px}{q} - \frac{qx}{p} = \frac{(p^2-q^2)x}{pq}$$

Whence $x = \frac{2pqa}{p^2+q^2}$

$$y = \left(\frac{p^2-q^2}{2pq}\right) \times \left(\frac{2pqa}{p^2+q^2}\right) = \frac{(p^2-q^2)a}{p^2+q^2}$$

Where the indeterminates p and q may be assumed at pleasure.

Cor. If a be the sum of two squares, p and q may be so assumed that $p^2+q^2=a$, or any factor of a; in which case the above expressions will be integral; and as many different integer values may be found for x and y as there are different ways of resolving a into the sum of two squares, or any of its factors.

Exam. 1. Resolve 65^2 into two other squares.

Here $x=\frac{2pq \times 65}{p^2+q^2}$, and $y=\frac{(p^2-q^2)65}{p^2+q^2}$

And since $65=8^2+1^2=7^2+4^2$; we may take

$$\left\{\begin{array}{l} p=8, \text{ and } q=1; \text{ which gives} \\ \quad x=16, \text{ and } y=63; \\ p=7, \text{ and } q=4; \text{ which gives} \\ \quad x=56, \text{ and } y=33; \end{array}\right.$$

Also since $13=3^2+2^2$ is a factor of 65, we may take

$p=3$, and $q=2$; which gives $x=60$, and $y=25$;

And again, $5=2^2+1^2$ is a factor of 65, therefore we may take

$p=2$, and $q=1$; which gives $x=52$, and $y=39$;

So that $65^2=16^2+63^2=56^2+33^2=60^2+25^2=52^2+39^2$.

Which are the only integral solutions that the equation admits of; but fractional answers may be obtained, *ad libitum*.

DIOPTRICS, or ANACLASTICS, the doctrine of refracted vision, which investigates and explains the effects of light refracted by passing through different mediums.

DIPPING *Needle*, or INCLINATORY *Needle*, a magnetical needle, so hung as that, instead of playing horizontally, and pointing north and south, one end dips or inclines to the horizon, and the other points to a certain degree or elevation above it.

DIRECTION, in *Astronomy*, the motion and other phenomena of a planet when direct.

Line of DIRECTION, in *Gunnery*, is the direct line in which the piece is pointed; and, in *Mechanics*, the same term implies the line in which a body moves, or in which a force is applied. When two conspiring forces act on a body at the same time, the angle included between the lines of their direction is called the *angle of direction*.

Quantity of DIRECTION, in *Mechanics*, is a term sometimes used to denote what is more commonly called *Momentum*.

DIRECTLY, is used in nearly the same sense as direct; thus we say, quantities are directly proportional, which is only another way of stating them to be in direct proportion; and in *Mechanics*, one body is said to impinge directly upon another, when the former strikes the latter perpendicular to its surface.

DIRECTRIX, in the *Conic Sections*, is a certain right line perpendicular to the axis of the curve, and frequently referred to in treating of the properties of those figures, from the description of them in plano.

DIRECTRIX, or DIRIGENT, is also that line or plane along which another line or plane is supposed to move, in the generation of a surface or solid.

DISCHARGE of *Fluids*.

1. If a fluid runs through a pipe or tube of an uniform shape, equal quantities of it will pass through every parallel section of the tube.

2. If a fluid runs through a tube or pipe, kept constantly full by means of a proper supply, but which is not of uniform shape; then the velocity of the fluids, in different sections, will be inversely as the areas of the sections.

3. If a fluid, flowing through a small orifice in the bottom of a vessel, be kept constantly full by means of an uniform supply at top, the velocity of the effluent fluid will be equal to that acquired by a heavy body in falling freely through the height of the surface of the fluid above the orifice.

4. If a pipe be inserted horizontally in a vessel, the plate of fluid will be discharged with the same velocity as before (if its centre of pressure be of the same depth), whatever be the thickness of the plate; this velocity not depending upon a continual acceleration through the length of the tube, otherwise the effluent fluid could not attain its full velocity, until a column had been discharged whose

base is equal to the orifice, and height equal to the length of the tube; whereas we find by experience, that this full velocity can be attained by the thinnest plate which can be let escape from the aperture.

5. The velocities and quantities discharged at different depths, are as the square roots of the depths.

6. The quantity run out in any time is equal to a cylinder, or prism, whose base is the area of the orifice, and its altitudes the space described in that time by the velocity acquired by falling through the height of the fluid.

So that if h denote the height of the fluid, a the area of the aperture, $g = 32\frac{1}{6}$th feet, or 386 inches, and t the time of efflux, we shall have, for the quantity discharged, $Q = at\sqrt{2gh}$.

Or when a and h are expressed in feet, $Q = 8{\cdot}0208\, at\sqrt{h}$ feet.

When a and h are expressed in inches, $Q = 27{\cdot}7387\, at\sqrt{h}$ inch.

If the orifice is a circle whose diameter is d, then $0{\cdot}785398\, d^2$ must be substituted for a;

And when d and h are expressed in feet, $Q = 6{\cdot}29952 d\sqrt{h}$ feet.

When d and h are expressed in inches, $Q = 21{\cdot}78592\, d\, 1^2\sqrt{h}$ inches. And from either of these it will be easy to find either a, t, or h, when the other three quantities are given.

7. The force with which the effluent water impinges against any quiescent body, is proportional to the altitude of the fluid above the orifice. For the force is as the velocity multiplied by the quantity of matter; but the quantity discharged, in a given time, is as the velocity; therefore the force is as the square of the velocity; that is, as the height of the fluid.

8. The water spouts out with the same velocity whether it be downwards or upwards, or sideways; because the pressure of fluids is the same, in all directions, at the same depth.

9. When a vessel is left to discharge itself gradually, through an orifice in its bottom, if the area of the section parallel to the bottom be every where the same, the velocity of the surface of the fluid, and consequently the velocity of the efflux, will be uniformly retarded.

The quantities of water in a prismatic vessel, discharged through an aperture in the bottom, decrease in equal times; as the series of odd numbers, 1, 3, 5, 7, 9, &c. taken in an inverted order.

The quantity of water contained in an upright prismatic vessel, is half that which would be discharged in the time of the entire gradual evacuation of the vessel, if the water be kept always at the same altitude.

10. If upon the altitude of the fluid in a vessel, as a diameter, we describe a semicircle, the horizontal space described by the fluid spouting from a vertical orifice, at any point in the diameter, will be as the ordinate of the circle drawn from that point; the horizontal space being measured on the plane of the bottom of the vessel.

When the aperture is vertical, and indefinitely small (as supposed here), the fluid will spout out horizontally with the velocity due to the altitude of the fluid above the orifice (by prop. 3); and this velocity, combined with the perpendicular velocity arising from the action of gravity, will cause every particle, and consequently the whole jet, to describe the curve of a parabola. Now the velocity with which the fluid is expelled from any hole, is such, as if uniformly preserved would carry a particle through a space equal to twice the height of the fluid above the hole, in the time of the fall through the height above that plane; but after quitting the orifice, it describes the parabolic curve, and arrives at the horizontal plane in the same time as a body would fall freely through the height above that plane; so that to find the distance from the perpendicular at which it will meet the horizontal plane, since the times are as the roots of the spaces, we have,

As the square root of height of fluid,
To square root of height above the horizon;

So is twice the height of the fluid,
To the distance which it will spout.

If the apertures be made at equal distances from the top and bottom of the vessel, (kept full of fluid), the horizontal distances to which the water will spout from these apertures will be equal; and when the orifice is made at the point bisecting the altitude of the fluid in the vessel, the fluid will spout to the greatest distance on the horizontal plane.

From experiment we may derive the following conclusions; viz.

1. The quantities of fluid discharged, in equal times, from different-sized apertures, the altitude of the fluids being the same, are to each other nearly as the areas of the apertures.

2. The quantities of water discharged, in equal times, by the same aperture, with different altitudes of water in the reservoir, are nearly as the square roots of the corresponding altitudes of the water in the reservoir above the centre of the aperture.

3. That, in general, the quantities of water discharged, in the same time, by different apertures, and under unequal altitudes of the water in the reservoir, are to each other in a compound ratio of the areas of the apertures, and the square roots of the altitudes.

4. That on account of the friction, the smallest apertures discharge proportionally less water than those that are larger and of a similar figure, the water in the respective reservoirs being at the same height.

5. That of several apertures whose areas are equal, that which has the smallest circumference will discharge more water than the other, the water in the reservoirs being at the same altitude, and this because there is less friction. Hence, circular apertures are most advantageous, as they have less rubbing surface under the same areas.

To this we can only add, that if instead of the orifice being pierced in a plate of tin or other thin plate, a cylindrical pipe or tube be inserted in the vessel whose length is from two to four times the diameter of the orifice, then a greater quantity of water will be discharged through it than through the simple aperture, in an equal portion of time, all other circumstances remaining the same; the quantity of the fluid discharged, in the two cases, being to each other as 133 to 100 nearly.

DISCHARGER, or **Discharging** *Rod*, in *Electricity*, is a rod used for the purpose of discharging a jar or battery of its contents, without injury or pain to the operator.

DISCOUNT, or **Rebate**, is an allowance made on a bill or any other debt not yet become due, in consideration of present payment.

The rule for finding discount is this:

As the amount of £1. for the given rate and time
Is to the given sum or debt,
So is the interest of £1. for the given rate and time
To the discount of the debt.

So that if p be the principal or debt, r the rate of interest, and t the time;

then as $1+rt : p :=: rt : \frac{prt}{1+rt}$, which is the true discount. Hence also $p-\frac{prt}{1+rt}=\frac{p}{1+rt}$ is the present worth, or sum to be received.

Or, making a to represent the present worth, we have

$a=\frac{p}{1+rt}=$ the present value
$p=a(1+rt)=$ the sum due
$t=\frac{p-a}{ar}=$ the number of years
$r=\frac{p-a}{at}=$ the rate of interest.

Bankers and others who discount bills of exchange, usually charge the interest.

DISCRETE, or **Disjunct** *Proportion*, is that in which the ratio between two or more pairs of numbers is the same, and yet the proportion is not continued, so as that the ratio may be the same between the consequent of one pair and the antecedent of the next pair.

Thus $3:6:=:5:10$ is a disjunct

proportion; but 3 : 6 : = : 12 : 24 is a continued proportion.

DISTANCE, in *Astronomy*, as of the sun, planets, &c. is either real or proportional; it is also further distinguished into mean, perihelion, and aphelion distances.

Aphelion DISTANCE of the *Planets*, is when they are at their greatest distance from the sun.

Perihelion DISTANCE of the *Planets*, is when they are at their least distance from the sun.

Mean DISTANCE of the *Planets*, is a mean between their aphelion and perihelion distances.

Proportional DISTANCES of the *Planets*, are the distances of the several planets from the sun, compared with the distance of any one of them considered as unity.

Real DISTANCES, are the absolute distances of those bodies, as compared with any terrestrial measure, as miles, leagues, &c.

The proportional distances of the planets from the sun, any one of them (as for example that of the earth) being considered as unity, are readily determined from Kepler's third law; viz. that the squares of the periodic times of revolving bodies about the same central body, are as the cubes of their respective distances: and hence also the real distance of any one of the planets being known, the absolute distance of all the others may be determined. Now the real distance of the earth from the sun has been determined, by means of the transit of Venus, to be about 93,000,000 miles. And hence we have the following tablet of the real and proportional distances of the several planets.

	Proportional mean Dist.	*Real mean Dist.*
Mercury	·3870981	36 million miles
Venus	·7233323	68
Earth	1·0000000	93
Mars	1·5236935	142
Vesta	2·2373000	221
Juno	2·6671630	248
Ceres	2·7674060	257
Pallas	2·7675920	257
Jupiter	5·2027911	485
Saturn	9·5387705	890
Uranus	19·1833050	1800

For the distance of the Moon, and the other secondaries from their several primaries, see *Satellites*.

DISTANCE of the *fixed Stars* from the *Earth* or *Sun*, has never yet been determined; we only know it is so great, that the whole diameter of the earth's orbit, which is near two hundred million miles, is but as a point compared with their distance, and therefore forms no sensible measure whereby it may be estimated.

Accessible DISTANCES, are such as may be measured by the application of any lineal measure.

Inaccessible DISTANCES, are those which cannot be measured by the application of any lineal measure, but by means of angles and trigonometrical rules and formulæ.

The distance of objects may also be ascertained by means of sound; for as this has been found by experiment to travel at the rate of about 1142 feet per second, if the time which elapses between the firing of a gun and the report of the same be duly observed, the distance in feet will be found by multiplying the number of seconds by 1142; and in this way we may estimate the distance of a thunder cloud, by the number of seconds being observed that elapses between the flash of lightning, and the clap of thunder by which it is succeeded.

Apparent DISTANCE, in *Optics*, is that distance at which we judge an object to be placed, when seen afar off, and which is usually very different from the true distance; because we are apt to think that all very remote objects whose parts cannot well be distinguished, and which have no other object in view near them, are at the same distance from us, though perhaps one of them is thousands of miles nearer than the other; as is the case with the sun, moon, and planets.

The most universal, and frequently the most sure means of judging of the distance of objects is, the angle made by the optic axis. To convince ourselves of the usefulness of this method of judging, suspend a ring in a thread, so that its side may be towards us, and the hole in it to the right and left hand; and taking a small rod, crooked at the end, retire from the ring two or three paces; and having with one hand covered one of our eyes, to endeavour with the other to pass the crooked end of the rod through the ring. This appears very easy; and yet upon trial, perhaps once in 100 times we shall not succeed, especially if we move the rod a little quickly.

Distance, in *Navigation*, is the number of miles or leagues that a ship has sailed from one point to another.

DIVING *Bell*, an apparatus used for the purpose of diving. It is most commonly made in the form of a truncated cone, the smallest end being closed and the larger one open. It is weighted with lead, and so suspended that it may be sunk full of air, with its open base downwards, and as near as may be parallel to the horizon, so as to close with the surface of the water.

DIVISIBILITY, that quality of a body by which it admits of separation into parts. The divisibility of quantity is a subject which has given rise to various arguments amongst philosophers, some contending that this separation may be carried on *ad infinitum*, while others maintain that it cannot be extended beyond certain limits. To the metaphysical divisibility there unquestionably is no end, but in the real division there is always a limit, though sometimes the number of parts is astonishing.

Again, a whole ounce of silver may be gilt with eight grains of gold, which may be afterwards drawn into a wire 13000 feet long.

In odoriferous bodies we can still perceive a greater subtilty of parts, and even such as are actually separated from one another; several bodies scarce lose any sensible part of their weight in a long time, and yet continually fill a very large space with odoriferous particles.

The particles of light, if light consists of particles, furnish another surprising instance of the minuteness of some parts of matter. A lighted candle placed on a plane will be visible two miles, and consequently fill a sphere, whose diameter is four miles, with luminous particles, before it has lost any sensible part of its weight. And as the force of any body is directly in proportion to its quantity of matter multiplied by its velocity, and since the velocity of the particles of light is demonstrated to be at least a million of times greater than the velocity of a cannon-ball, it is plain that if a million of these particles were round, and as big as a small grain of sand, we durst no more open our eyes to the light, than to expose to sand shot point blank from a cannon.

By help of microscopes, such objects as would otherwise escape our sight appear very large: there are some small animals scarcely visible with the best microscopes; and yet these have all the parts necessary for life, as blood and other liquors.

DIVISION, is one of the principal rules in *Arithmetic* and *Algebra*; it consists in finding how often a less number or quantity is contained in a greater. The number to be divided is called the *dividend*, the number by which the division is made is the *divisor*; the number of times that this is contained in the former, is called the *quotient*, and if any thing remains, after the operation is finished, it is called the *remainder*. Division is either simple or compound.

Simple **Division**, is when both the divisor and dividend are integral numbers.

Rule. Find how many times the divisor is contained in as many of the left-hand figures of the dividend as are just necessary, and place that number on the right. Multiply the divisor by this number, and place the product under the figure of the dividend above mentioned. Subtract this product from that part of the dividend under which it stands, and bring down the next figure of the dividend, or more if necessary, to the right of the remainder. Divide this number, so increased, as before, and so on till the whole is finished.

Note 1. When it is necessary to bring down more than one figure to the remainder, a cipher must be placed in the quotient for every additional figure thus brought down.

Note 2. If the divisor do not exceed 12, the quotient may be written down as it arises, immediately under the dividend.

Proof of DIVISION. Multiply the divisor and quotient together, and add to this product the remainder, which ought to be equal to the dividend if the work be right.

Sometimes, for the sake of abridging the operation, the successive products are omitted, and the subtraction is made figure for figure as the work is carried on.

Compound DIVISION, is when the dividend is a compound quantity.

Rule. Divide the highest denomination of the dividend by the divisor, as in the former rule. Reduce the remainder, if any, to the next inferior denomination, and divide as before; reduce this remainder again, and divide as before, and so on till the whole is finished.

Note 1. If the divisor exceed 12, and be a composite number, divide by its factors successively, instead of the whole number at once.

Note 2. Both divisor and dividend may be compound quantities; but then they must be of the same kind, and the quotient either an abstract number or a quantity different from the others: thus, it may be asked, how many parcels, each containing 1 quarter 7lbs. may be made out of 5 tons; or what quantity of goods, at 5*s*. 9½*d*. per pound, may be purchased for 100*l*. In such cases, the divisor and dividend must be reduced to the same denomination; and if the quotient be such as to admit of lower denominations, then the remainder must be multiplied by the number of these, in the denomination of which the price or value is given. As this case of division is not found in any of the common books on Arithmetic, two examples are subjoined

Ex. 1. How much sugar at 2*l*. 7*s*. 6*d*. per cwt. may be bought, for 30*l*. 6*s*. 9*d*.?

```
    2  7  6)30  8  9(12 cwt.
            28 10  0
            --------
             1 18  9
multiply by        4
            --------
    2  7  6) 7 15  0(3 quarters
             7  2  6
            --------
             0 12  6
multiply by       28
            --------
    2  7  6)17 10  0(7 pounds
            16 12  6
            --------
             0 17  6
multiply by       16
            --------
    2  7  6)14  0  0(5 ounces
            11 16  6
            --------
             2  3  6
multiply by       16
            --------
    2  7  6)34 16  0(14 drams
            33  5  0
            --------
```

remainder $= \frac{1\ \ 11\ \ 0}{2\ \ 7\ \ 6} = \frac{62}{95}$

So that the quantity is
12 cwt. 3 qrs. 7 lbs. 5 oz. $14\frac{62}{95}$ drams.

Ex. 2. How much land, at 2*l*. 18*s*. 9*d*. per acre, may be rented for 375*l*.?

In this example we shall show how the operation may be simplified, by multiplying both divisor and dividend, by a number which will remove the shillings and pence from the divisor.

```
            2 18  9) 375
multiply by       4)   4
            ------------
           11 15  0)1500
multiply by       4)   4
            ------------
                47)6000(127 acres
                   564
                   ---
                   360
                   329
                   ---
                    31
                     4
                   ---
                47)124(2 roods
```

```
47) 124(2 roods
     94
    ---
     30
      40
    ----
47) 1200(25 poles
     1175
      ---
       25
       30¼
   -------
  47)756¼(16 yards
     752
     ----
       4¼
       9
     ----
   47)40½(0 feet
      141
     ----
 47) 5832(124 inches
     564
      ---
      192
      188
     ----
        4
```

The quantity is 127 acres and ¾ nearly.

Division of *Fractions*, is performed by the following rule. Reduce mixed numbers to improper fractions, then invert the terms of the divisor, and multiply the numerators and the denominators together as in multiplication; observing that such factors, as are common in the numerators and denominators, may be cancelled.

Division of *Decimals*, is performed the same as in the simple rule of division, observing only to point off in the quotient as many decimal places as those in the dividend exceed those in the divisor; and, if there be not so many, the defect must be supplied by prefixing ciphers.

Another way to know the place for the decimal point is this; the first figure of the quotient must be made to occupy the same place either in integers or decimals, as does that figure of the dividend that stands over the unit's place of the first product.

Division of *Circulating Decimals*, is performed either by performing the multiplications and subtractions as directed for those numbers, or by converting the repetends into their equivalent fractions, and then proceeding as in division of fractions.

Division, in *Algebra*, is the method of finding the quotient arising from the division of one indeterminate quantity by another, which may be considered under two cases.

Case 1. When the divisor and dividend are both simple quantities.

Rule. Divide the co-efficients, as in arithmetic, and to the quotient annex the result arising from the division of the indeterminate quantities.

Note When the divisor and dividend have like signs, the sign of the quotient is plus $+$; and when they are unlike, the sign of the quotient is minus $-$, as in multiplication.

Case 2. When the dividend is a compound quantity, and the divisor either simple or compound.

Rule. Set the divisor on the left of the dividend, and proceed in the operation the same as in division of numbers, observing still the same rule as above with regard to the signs.

Note.—When there is a remainder after the division, it must be written over the divisor, and annexed as a fraction to the quotient.

Division of *Algebraic Fractions*, is performed the same as in the case of simple fractions: viz. reducing all mixed expressions to improper fractions, then invert the terms of the divisor, and multiply the numerators and denominators as in the rule above quoted, observing to cancel all factors that are common to the numerators and denominators.

Division of *Surds*, is the method of ascertaining the quotient arising from the division of one irrational quantity by another.

Rule.—Reduce the given surds to their simplest form, and the radical parts thus arising to like radicals; then divide the co-efficient of the dividend by the co-efficient of the divisor for the new co-efficient, and one surd part by the other for the required surd, which being annexed with its proper radical sign to the co-efficient before found, will be the answer required.

Note.—If the radical signs be not the same, but the quantities under them be equal, the division will be effected by subtracting the index representing the radical of the divisor from the index of that representing the radical of the dividend.

Note.—When the proposed divisor is a binomial surd, consisting of the sum or difference of two square roots, it may be rendered rational by multiplying both numerator and denominator, or, which is the same, both divisor and dividend, by the same two quantities, but connected with a contrary sign to that by which they are connected in the denominator; that is by + when that is —; and by — when that is +; because the product of the sum of two quantities multiplied by their difference, is equal to the difference of their squares.

Division of *Ratios*, or *Divided Ratio*, is when of four proportional quantities, the differences of the antecedents and consequents are compared either with the antecedents or consequents; thus if

$$a : b = c : d$$

$$\text{then } \left\{\begin{array}{l} a-b : a = c-d : c \\ a-b : b = c-d : d \end{array}\right.$$

DIVISOR, is that number or quantity, which exactly divides another number or quantity, without leaving a remainder.

Common Divisor, in *Arithmetic*, is that number which will exactly divide two or more given numbers; and the greatest of all such divisors is called the greatest common divisor, or the greatest *common measure*.

If the last n digits of any number be divisible by 2^n, the whole number is divisible by 2^n. If the sum of the digits of any number be divisible by 3 or by 9, the whole number is divisible by 3 or 9; and if also the last digit be even, the whole number is divisible by 18. If a number terminate with 5, it is divisible by 5; and if it terminate in 0, it is divisible by either 10 or 5. If the sums of the alternate digits be equal, or if one sum exceed the other by 11, or by any multiple of 11, the whole number is divisible by 11.

When the greatest common divisor of two numbers is required we must divide the greater number by the less, then divide the divisor by the remainder; and thus continue always dividing the last divisor by the last remainder till nothing remains, and the last divisor will be that required.

If the greatest common divisor of three or more numbers be required; and the common divisor of two of them first, then of this common divisor and another of the given numbers, and so on to the last; so shall the last divisor be the greatest common divisor required.

To find the greatest common divisor of two given algebraical expressions.

Arrange the terms according to the dimensions of one of the letters, and then divide one of the proposed quantity by the other; then the last divisor by the last remainder, and so on, the same as in finding the greatest common divisors of two numbers, and the last divisor will be that required.

Note.—All the letters or figures, which are common to each term of the divisor, may be thrown out of them before they are used in the operation.

Divisors, *Theory* of, relates to the investigation of certain properties of divisors, with regard to their forms, powers, sums, &c.; which forms one of the most important and interesting researches in the theory of numbers.

Let N represent any number, and a, b, c, &c. its prime factors; then may N be expressed by $N = a^m b^n c^p$, &c.; thus, $1728 = 2^6 3^3$; $8640 = 2^6 . 3^3 . 5$, &c.

Now any number N, being reduced to the above form, or $N = a^m b^n e^p$, &c., then will the number of all the divisors of N be expressed by the formula.

$$(m+1)(n+1)(p+1), \text{ \&c.}$$

To find a number that shall have any given number of divisors.

Let w represent the required number of divisors; resolve w into any factors, as $w = x \times y \times z$, &c.

Take $m=x-1$, $n=y-1$, $p=z-1$, &c. so shall $a^m b^n c^p$, &c. be the number sought, a, b, and c being taken any prime numbers whatever, providing they be not equal to each other.

3. If $N=a^m b^n c^p$, &c. as before, then will the sum of all the divisors of N be expressed by the formula

$$\frac{a^{m+1}-1}{a-1} \times \frac{b^{n+1}-1}{b-1} \times \frac{c^{p+1}-1}{c-1}$$

4. If t and u be any two numbers prime to each other, then every divisor of the formula t^2+u^2, is itself also of the same form; that is, a number that is the sum of two squares, prime to each other, can only have for divisors numbers that are also the sums of two squares.

5. Every divisor of the formula t^2+2u^2, t and u being prime to each other, is of the same form. Or, the divisors of the sum of a square and double a square, are also the sum of a square and double a square.

6. Every divisor of the formula t^2-2u^2, is also of the same form. Or the divisors of the difference between a square and double a square, is also the difference of a square and double a square.

7. Every odd divisor of the formula t^2+3u^2, is also of the same form.

8. Every odd divisor of the formula t^2-5u^2 is itself also of the same form.

DIVISORS, *Method* of, a method discovering the roots of an equation, the principles of which may be briefly stated as follows: Since the absolute term of every equation is equal to the product of all the roots; and farther, since an equation having integral co-efficients, must have its roots either integral, irrational, or impossible; or, which is the same, since such an equation cannot have a fractional root, it follows that if it have any rational root it must be found amongst the factors of this absolute quantity, having either a positive or negative sign prefixed. In order, therefore, to find a root, we have only to try for the unknown quantity, the different factors of this number, and if by such trial none of them obtain, then we may conclude with certainty that the equation has no rational root.

DODECAGON, a regular polygon of 12 equal sides and angles.

To inscribe one in a circle. Apply the radius of the circle six times round the circumference, which will divide it into six equal parts; then bisect each of those parts, which will divide the circumference into 12 parts for the dodecagon required.

To find the area. If the side of a dodecagon be 1, its area is equal to $3 \times \tan. 75^\circ = 3(2+\surd 3) = 11{\cdot}1961524$ nearly: and as the areas of polygons are to each other as the square of their like sides, we have as

$1^2 : 11{\cdot}1961524 = s^2 : s^2 \times 11{\cdot}1961524 =$ the area of a dodecagon whose side is s.

DODECAHEDRON, in *Geometry*, one of the regular Platonic bodies, comprehended under 12 equal sides or faces, each of which is a regular pentagon.

Or, a dodecahedron may be conceived to consist of twelve equal pentagonal pyramids, whose vertices or tops all meet in one common point, which will be the centre of the sphere circumscribing the dodecahedron.

To find the surface and solidity of a dodecahedron, the side of one of its equal faces being given.

Let s represent the given side, then will surface

$$=15\surd(1+\tfrac{2}{5}\surd 5)s^2 = 20{\cdot}6457788s^2$$

$$\text{Solidity} = 5\surd\left(\frac{47+21\surd 5}{40}\right)s^3 =$$

$7{\cdot}66311896s^3$.

DOME, in *Architecture*, is a roof, or vault, rising from a circular, elliptic, or polygonal base, or plan; with a convexity outwards, or a concavity inwards, in such a manner, that all the horizontal sections made by planes will be similar figures round a vertical axis.

Domes are denominated by the figures of the basis on which they are erected; and are therefore called *polygonal*, *circular*, or *elliptic* domes. *Circular* domes are of several kinds, as *spherical*, *sphe-*

roidal, or *ellipsoidal*, *hyperboloidal*, *paraboloidal*, &c. Domes that rise higher than the radius of the base are called *surmounted domes;* and those which rise less than this dimension, are termed *diminished* or *surbased* domes. Domes that rise from circular basis are called also *cupolas*.

DOMINICAL *Letter*, in *Chronology*, properly called Sunday letter, one of the seven letters of the alphabet A B C D E F G, used in almanacs, ephemerises, &c. to denote the Sundays throughout the year.

In our almanacs, the first seven letters of the alphabet are commonly placed to show on what days of the week the days of the months fall throughout the year. And because one of those seven letters must necessarily stand against Sunday, it is printed in a capital form, and called the dominical letter; the other six being inserted in different characters, to denote the other six days of the week.

The dominical letter may be found universally, for any year of any century, thus:

Divide the centuries by 4, and take twice what remains from 6; then add the remainder to the odd years above the even centuries, and their 4th. Divide their sum by 7, and the remainder taken from 7 will leave the number answering to the letter required.

DOUBLE *Ounce*, a coinage of Naples, value £1 1*s*. 3*d*. sterling.

Double *Point*. (See Punctum *Duplex*.)

DRACHM, or Dram, a small weight, the 8th part of an ounce in apothecaries weight, and the 16th part of an ounce avoirdupoise.

DRACO, the *Dragon*, a northern constellation. (See *Constellation*.)

DRAGON. (See *Draco*.)

Dragon's *Head* and *Tail*, are the nodes of the planets, but more particularly of the moon. One of these points is to the northward, the moon beginning then to have north latitude, and the other southward, where she commences south latitude; the former point being represented by the knot ☊ for the head, and the other by the same reversed, or ☋ for the tail.

DUCTILITY, the extensibility and cohesion of particles which enables a metal to be drawn out into wire without breaking.

The great ductility of some bodies, especially gold, is very surprising: the gold-beaters and wire-drawers furnish us with abundant proofs of this property; they every day reduce gold into lamellæ inconceivably thin, yet without the least aperture or pore discoverable, even by the microscope: a single grain of gold may be stretched under the hammer, into a leaf that will cover a house, and yet the leaf remain so compact as not to transmit the rays of light, nor even admit spirit of wine to transude. Dr. Halley took the following method to compute the ductility of gold: he learned from the wire-drawers, that an ounce of gold is sufficient to gild, that is, to cover or coat a silver cylinder of forty-eight ounces weight, which cylinder may be drawn out into a wire so very fine, that two yards shall weigh only one grain; and consequently ninety-eight yards of the same wire, only forty-nine grains; so that a single grain of gold here gilds ninety-eight yards; and, of course, the ten-thousandth part of a grain is here above one-third of an inch long. And since the third part of an inch is yet capable of being divided into ten less parts visible to the naked eye, it is evident that the hundred-thousandth part of a grain of gold may be seen without the assistance of a microscope. Proceeding in his calculation, he found, at length, that a cube of gold, whose side is the hundredth part of an inch, contains 2,433,000,000 visible parts; and yet, though the gold with which such wire is coated is stretched to such a degree, so intimately do its parts cohere, that there is not any appearance of the colour of the silver underneath.

Ductility of *Glass*. Our ordinary spinners do not form their threads of silk, flax, or the like, with half the ease and expedition the glass-spinners do threads of this brittle matter. We have some of them used in plumes for children's heads, and divers other works, much finer than any hair, and which bend and wave like hair with every wind.

DUODECIMALS, or *Cross Multiplication*, is a rule used by workmen and artificers in computing the contents of their work; dimensions are usually taken in feet, inches, and parts; but of the last, all those less than $\frac{1}{4}$ of inches are frequently omitted as of little or no consequence, and the same is always done in casting up the contents, where they are of still less importance.

Rule. Multiply each term in the multiplicand, beginning at the lowest, by the feet in the multiplier, and set the result of each directly under its corresponding term, observing to carry 1, for every 12, from the parts to the inches, and from the inches to the feet.

In like manner multiply all the multiplicand, by the inches and parts of the multiplier, and set the result of each one place removed to the right-hand of those in the multiplicand, and the sum of these successive products will be the answer.

Or, instead of multiplying by the inches and parts, such parts of the multiplicand may be taken as these are of a foot.

The feet in the answer are square feet, but the numbers standing in the place of inches are not square inches, as one might at first infer, but 12th parts of square feet, each part being equal to 12 square inches. In like manner the numbers standing in the third place, or place of parts, are so many 12th parts of the preceding denomination, these therefore are square inches; and in the same manner, if the operation be carried farther, every successive place will be a 12th part of that preceding it.

DUODECIMO is used to denote the size of a book when the sheets are folded into twelve leaves.

DUODECUPLE, consisting of twelves.

DUODENARY *Arithmetic*, is that in which the local value of the figures increases, in a twelvefold proportion from right to left, instead of the tenfold proportion, in the common or denary arithmetic.

In the duodenary scale of notation, there must be introduced two new characters for expressing 10 and 11, and these may be represented by φ, and ϰ, that is 10 = φ; 11 = ϰ; so that the digits of this system becomes,

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, φ, ϰ,

with which characters any number whatever may be expressed according to the duodenary system, the same as in common arithmetic, by the nine simple digits.

To perform Duodecimal operations by means of the Duodenary scale.

Rule. Transform the feet, if above 12, into the duodenary scale, and set the inches and parts as decimals; then multiply as in common arithmetic, except carrying for every 12 instead of for every 10, as in common operations; and finally transform the integral part of the product into the denary scale.

DUPLICATE *Ratio*, is the square of a ratio, or the ratio of the squares of two quantities; thus $a^2 : b^2$ is in the duplicate ratio of $a : b$.

DUPLICATION, the doubling of a quantity, or multiplying by 2.

Duplication of a *Cube*, is the finding of the side of a cube which shall be double in solidity to a given cube. This cannot be effected geometrically, as it requires the solution of a cubic equation, or requires the finding of two mean proportionals, viz. between the side of the given cube and the double of the same, the first of which two mean proportionals is the side of the double cube.

DYNAMICS, is the science of moving powers, or of the action of forces on solid bodies, when the result of that action is motion. Mechanics, in its most extensive meaning, is the science which treats of quantity, of extension, and of motion. Now that branch of it which considers the state of solids at rest, such as their equilibrium, their weight, pressure, &c. is called *Statics;* and that which treats of their motion, *Dynamics.* So when fluids, instead of solids, are the subjects of investigation, that branch which treats of their equilibrium, weight, pressure, &c. is called *Hydrostatics;* and that which treats of their motion, *Hydrodynamics.*

E.

EARTH, in *Astronomy,* is a primary planet belonging to the solar system, the third in order from the sun, performing its revolution about that body, between the two planets Venus and Mars.

Various opinions were formerly held as to the figure of the earth; some of the ancients maintaining that it was cylindrical, others that it was an extended plane; while others, as far back as Pythagoras, gave to it that globular form which the voyages and discoveries of the moderns have since rendered evident to demonstration.

In the year 1671, Richer was sent to Cayenne; and among other objects of his voyage, he was charged by the academy to observe the length of the pendulum vibrating seconds. He returned in the year 1672, and mentioned that the same measure which had been marked at Cayenne on a rod of iron, according to the length which had been found necessary to make the pendulum vibrate seconds, being brought back and compared with that marked at Paris, the difference was found to be a line and a quarter, that at Cayenne being the shortest. Also the clock which Richer took to Cayenne having been adjusted to beat seconds at Paris, retarded two minutes a day at Cayenne; so that no doubt remained of the diminution of the force of gravity at the equator. This was the first direct proof of the diurnal motion of the earth. Huygens was then led to suspect that the same cause might produce a protuberance of the equatorial parts of the earth, and a corresponding depression of the poles.

He was the first person who attempted to determine the figure of the earth by direct calculation; but he assumed an hypothesis which has been since found to be inadmissible: this hypothesis supposes that the whole of the attractive force resides in the centre of the earth, and that its power varies inversely as the square of the distance. Upon this supposition, Huygens computes the ellipticity of the earth to be $\frac{1}{578}$, the centrifugal force at the equator being $\frac{1}{289}$ of the force of gravity. Newton, upon the supposition of the earth being an homogeneous fluid, estimates the ellipticity at $\frac{1}{230}$. But Clairaut was the first mathematician who gave a general solution of this problem, adapted to the hypothesis of a variable density. The result which he obtained from his investigation was as curious as unexpected; it appeared that if the density of the strata, of which the earth is composed, increases towards the centre, the ellipticity will be less than in the hypothesis of Newton, and greater than on that of Huygens; and, moreover, that the fraction expressing the ellipticity being added to the fraction expressing the augmentation of gravity at the poles, will together always make a constant quantity, which is equal to $\frac{5}{2}$ of the fraction which expresses the proportion which the centrifugal force at the equator bears to that of gravity.

The same problem has since been investigated by several eminent mathematicians; but considerable difference is found to exist in the results of the different calculations; and Playfair, in vol. v. of the Edinburgh Transactions, enters into the inquiry why, considering the care with which observations have been made, they do not agree more nearly in the results? Among the causes of this inconsistency, besides unavoidable mistakes and the imperfections of instruments, the professor mentions local irregularities in the direction of gravity, particularly those occasioned by the attraction of mountains. "There are," says he, "no doubt, situations in which the measurement of a small arch might, from this cause, give the radius of curvature of the meridian *infinite*, or even *negative*." Another kind of local irregularity is that arising from the unequal density of the materials under the surface of the earth, and not far from it: errors thus produced might amount to 10″ or 12″. And this cause of error is the more formidable, not only because it may go to a great extent, but because there is not any visible mark by which its existence can be distinguished. He also justly observes, that in order to avoid any material error in determining the figure of the earth, the arches measured should be large, consisting each of several degrees, as an error would then be rendered inconsiderable, by being spread over a greater interval.

Motion of the Earth.—There are three principal motions of the earth. 1. A motion of rotation on its own axis; 2. A motion in an orbit round the sun; 3. A motion of its axis about the poles of the ecliptic.

The rotation of the earth on its axis, called its diurnal motion, is the most uniform we are acquainted with. It is performed in 23^h 56′4″1, of mean solar time, or one sidereal day.

The earth, or more strictly speaking, the common centre of gravity of the earth and moon, describe an orbit round the sun, which orbit is of an elliptic form of small eccentricity; the sun being placed in one of its foci. If we suppose the plane of this orbit extended to the fixed stars, it will trace in the heavens a circle called the ecliptic. The plane of the earth's equator, which remains very nearly parallel to itself during the whole of this revolution, is inclined to the ecliptic at an angle of 23° 28′. The points in the earth's orbit, which are intersected by this plane, are called the equinoctial points.

The motion of the earth in its orbit is very far from uniform; but it is so far regular, that, with the exception of some small inequalities caused by the action of the moon and planets, the radius vector, or line joining the centres of the earth and sun, describes equal areas or sectors of the ellipse, in equal times.

The third motion of the earth, is that which produces the precession of the equinoxes. The motion of rotation having produced a protuberant form in the equatorial regions of the earth, the continual action of the sun and moon on this surrounding mass or annulus, produces a rotary motion in the axis of the earth round the axis of the ecliptic; the inclination of these axis remaining the same. This revolution is accomplished in 25,063 years.

The principal elements of the earth, as connected with the sciences of astronomy and geography, according to the determination of Laplace, are as follows:

Equatorial diamet.	7924	Eng. miles
Polar diameter	7908	———
Mean diameter	7916	———
Mean circumfer.	24869	———
Mean length of a degree	69.08	———
Surface . .	196862256	sq. mls.
Solidity .	259726936416	cubic miles.

Density of the earth is 3 9326 times greater than the density of the sun, and about five times that of common water.

Mass of the earth is $\frac{1}{337086}$ of the sun.

The weight of a body at the equator, is to the weight of the same body at the poles, as 1 : 1·00569.

The length of a second's pendulum at the equator is 39·027 inches, and at the poles 39·197 inches.

The centrifugal force at the equator is about $\frac{1}{28}$ of gravity.

If the rotatory motion of the earth was seventeen times greater than it is, the centrifugal force would be equal to that of gravity; and therefore bodies at the equator would then have no weight.

The mean distance of the earth from the sun is 23,578 times its own semi-diameter, or about 93,321,724 English miles.

Its aphelion distance is 94,889,528 miles.

Its perihelion distance is 91,753,920 miles.

We are, therefore, more than three million miles nearer the sun at the winter, than at the summer solstice.

The eccentricity of its orbit is ·0168, the semi-axis major being considered as unity.

The earth performs one complete sidereal revolution in $365^d\ 6^h\ 9'\ 11''\cdot5$; but the tropical year is only $365^d\ 5^h\ 48'\ 51''\cdot6$; being less than in the time of Hipparchus by $11''\cdot2$.

The mean velocity of the earth, in its orbit is $59'\ 10''\cdot7$ each day. When in its perihelion this velocity is $1°\ 1'\ 9''\cdot9$ per day; and in its aphelion $57'\ 10''\cdot7$.

The mean longitude of the earth, at the commencement of the present century, was $3^s\ 10°\ 9'\ 13''$.

The mean longitude of its perihelion, at the same period, was $3^s\ 9°\ 30'\ 5''$. But the line of the apsides has a direct sidereal motion of $19'\ 40''\cdot8$ in a century; which being referred to the ecliptic will give it a motion, according to the order of the signs, of $1'\ 1''\cdot9$ in a year, or $1°\ 43'\ 10''\cdot8$ in a century.

A complete revolution about the line of the apsides is called the anomalistic year; and is performed in $365^d\ 6^h\ 14'\ 2''$. The perihelion coincided with the vernal equinox about the year 4089 before the Christian æra; it coincided with the summer solstice about the year 1250 after Christ; and will coincide with the autumnal equinox about the year 6483.

A complete tropical revolution of the apsides is performed in 20,931 years.

The axis of the earth is inclined to the plane of the ecliptic in an angle of $23°\ 27'\ 57''$; which angle is observed to decrease at the rate of $52''\cdot1$ in a century; but this variation of the angle is confined within certain limits, and cannot exceed $2°\ 42'$.

The nutation of the axis is $19''\cdot3$.

The annual intersection of the equator with the ecliptic is not always in the same point, but is retrograde, or contrary to the order of the signs. Consequently, the equinoctial points appear to move forward on the ecliptic: and whence this phenomenon is called the *precession of the equinoxes*. The quantity of this annual change is $50''\cdot1$; or $1°\ 23'\ 30''$ in a century. A complete revolution is performed in 25,868 years.

The sidereal day, or the time employed by the earth in revolving on its axis, is always the same. If the mean astronomical or civil day be taken equal to 24 hours, the duration of the sidereal day will be $23^h\ 56'\ 4''\cdot1$.

The astronomical or civil day is constantly changing. This variation arises from two causes: 1. The unequal motion of the earth in its orbit; 2. The obliquity of that orbit to the plane of the equator. The mean and apparent solar days are never equal, except when the sun's daily motion in right ascension is $59'\ 8''$. This is the case about April 16, June 16, September 1, and December 25: on these days the difference vanishes, or nearly so. It is at its greatest about November 1, when it is $16'\ 16''$.

The astronomical year is divided into four parts, determined by the two equinoxes and the two solstices. The interval between the vernal and autumnal equinoxes is (on account of the eccentricity of the earth's orbit, and its unequal velocity therein) near eight days longer than the interval between the autumnal and vernal equinoxes. These intervals are, at present, nearly as follow:

	d h m	d. h. m.
From the spring equinox to the summer solstice · · · · · · · ·	92 21 45	= 185 35 20
From the summer solstice to the autumnal equinox · · · · · · ·	93 13 35	
From the autumnal equinox to the winter solstice · · · · · · · · ·	89 16 47	= 178 18 29
From the winter solstice to the spring equinox · · · · · · · ·	89 1 42	
	Difference · · ·	7 16 51

ECCENTRICITY in the *Orbit* of a *Planet*, is the distance between the centre and focus of the ellipse in which it revolves.

The eccentricity of the orbit is computed from the greatest equation of the centre by the following proportion:

As 57° 17′ 44″·8 (the arc = rad.)
Is to half the greatest equation,
So is radius = 1
To the eccentricity.

Another formula for computing the eccentricity is given by Lambert in the Ephem. of Berlin.

Eccentricities of the Planets, at the commencement of the present Century; the greater Semi-axis being expressed by unity.

Mercury	·20551494
Venus	·00685298
Earth	·01685318
Mars	·09313400
Jupiter	·04817840
Saturn	·05616830
Uranus	·04667030

Secular Variations of Eccentricities.

Mercury	·000003867
Venus	·000062711
Earth	·000041632
Mars	·000090176
Jupiter	·000159350
Saturn	·000312402
Uranus	·000025072

Eccentricity of the new Planets.

Vesta	·09322000
Juno	·25494400
Ceres	·07834860
Pallas	·24538400

The eccentricity of a planet being added to the mean distance, gives the greatest distance; or, taken from the mean distance, leaves the least distance from the sun.

ECHO, a reflected sound, from a solid body, returned or repeated to the ear.

The formation of echoes has generally been ascribed to the reflection of sound, similar to that experienced by light when it falls on a polished body.

Sound, it is known, is propagated in every direction by the vibration of the particles of the air; but if any column of air rests against some obstacle that prevents the direct movement of the elastic globules, which serve as the vehicle of sound, it must rebound in a contrary direction, and striking the ear, if it meets with one in the line of repercussion, convey to it a repetition of the same sound, provided the original sound does not affect that organ at the same instant. But we are taught by experience that the ear does not distinguish the succession of two sounds, unless there be between them the interval of at least one-twelfth of a second; for during the most rapid movement of instrumental music, each measure of which cannot be estimated at less than a second, twelve notes are the utmost that can be comprehended in a measure, to render the succession of the sounds distinguishable; consequently, the obstacle, which reflects the sound, must be at such a distance, that the reverberated sound shall not succeed the direct sound till after one-twelfth of a second; and as sound moves at the rate of about 1142 feet in a second, and consequently about 95 feet in the twelfth of a second, it thence follows that, to render the reverberated sound distinguishable from the direct sound, the obstacles must be at a distance of not less than about 48 feet.

There are single and compound echoes. In the former, only one repetition of the sound is heard; in the latter, there are 2, 3, 4, 5, &c. repetitions. We are even told of echoes that can repeat the same word 40 or 50 times.

Echo, in *Architecture*, a name given to such kinds of vaults and arches as are erected for the purpose of producing artificial echoes. These are generally chosen of a parabolic or elliptic form. Of this kind is the whispering gallery in St. Paul's Cathedral, and in some other large buildings.

ECLIPSE, in *Astronomy*, a privation of light of one of the luminaries by the interposition of some opaque body, either between it and the observer, or between it and the sun. To the first class belong solar eclipses, and occultations of the fixed stars by the moon or planets, and to the second lunar eclipses, and of the other satellites, particularly those of Jupiter.

Eclipses are divided with respect to the objects eclipsed into *solar* and *lunar* eclipses, and those of *Jupiter's satellites.* And with respect to circumstances, into *total, partial, annular,* and *central* eclipses.

Every planet and satellite in the solar system, is illuminated by the sun; and casts a shadow towards that point of the heavens which is opposite to the illuminating body; which shadow is nothing more than a privation of light in the space hid from the sun by the opaque body that intercepts his rays. When the sun's light is so intercepted by the moon, that to any place of the earth it appears partly or wholly covered, he is said to undergo an eclipse; though, properly speaking, it is only an eclipse of that part of the earth where the moon's shadow or penumbra falls. When the earth comes between the sun and moon, the moon falls into the earth's shadow; and having no light of her own, she suffers a real eclipse from the interception of the sun's rays. When the sun is eclipsed to us, the inhabitants of the moon on the side next the earth, see her shadow like a dark spot travelling over the earth, about twice as fast as its equatorial parts move, and in the same direction. When the moon is eclipsed, the sun appears eclipsed to her, total to all those parts on which the earth's shadow falls, and of as long continuance as they are in the shadow.

Lunar **Eclipses** happen only at the time of full moon; because it is only then that the earth is between the sun and moon: nor do they happen every full moon, because of the obliquity of the moon's path with respect to the earth's, but only in such full moons as happen either at the intersection of those two paths, called the moon's nodes, or very near them; *viz.* when the moon's latitude, or distance between the centres of the earth and moon, is less than the sum of the apparent semi-diameters of the moon and the earth's shadow.

The principal circumstances in lunar eclipses are as follow: 1. All lunar eclipses are universal, or visible in all parts of the earth which have the moon above their horizon, and are everywhere of the same magnitude and duration. 2. In all lunar eclipses, the eastern side (or the left-hand side, as we look towards her from the north) is that which first immerges into the shadow, and emerges again; for the proper motion of the moon being swifter than that of the earth's shadow, the moon approaches it from the west, overtakes and passes through it with the moon's east side foremost, leaving the shadow behind, or to the westward. 3. Although total eclipses of the longest duration happen in the node, yet there may be total eclipses within a small distance of the nodes, namely, within that distance where the moon's latitude is equal to the apparent semi-diameter of the earth's shadow, minus the semi-diameter of the moon's disc, but in these situations the duration of total darkness will be short; whereas in central eclipses it will continue nearly two hours. 4. If the earth had no atmosphere, the moon, when she was totally eclipsed, would be invisible; but as the earth has an atmosphere, some of the light from the sun will be refracted thereby, and transmitted to the moon, on which account she will be visible at that time, and appear of a dull red colour. Lastly, she grows sensibly paler and dimmer, before entering into the real shadow; owing to a penumbra which surrounds that shadow to a certain distance.

Solar **Eclipses**, happen only when the moon is in conjunction with the sun, that is at the new moon, and also in the nodes, or near them, the limit being about 17° on each side a node; such eclipses only happening when the latitude of the moon, viewed from the earth, is less than the sum of the apparent semi-diameters of the sun and moon. In the nodes, when the moon has no visible latitude, the occultation is total; with some continuance when the disc of the moon in perigee appears greater than that of the sun in apogee, and its shadow is extended beyond the surface of the earth; and without continuance, when the point of the moon's shadow barely covers the earth. Lastly, out of the nodes, but within the limits, the eclipses are partial.

An eclipse of the sun does not appear the same in all parts of the earth where it is seen, but is in some total or annular, while in others it is partial. A solar eclipse does not happen at the same time in all places where it is seen, but appears earlier in the western parts than in those to the eastward, because the motion of the moon beyond the sun, and consequently of her shadow, is from west to east. An eclipse of the sun begins on the western side, and ends on the eastern. No eclipse happens to all the places where the sun is visible, for the penumbra at no time covers an entire hemisphere of the earth. The portion of the cusp of the horns of the unobscured part of the sun's disc, may be easily found in the middle of the eclipse, for the line which joins them is parallel to the moon's apparent way. The middle of a solar eclipse will not be at the same time in all places on the same meridian; for the parallax of longitude will be different in different longitudes. The excess of the apparent semidiameter of the moon above that of the sun in a total eclipse is so small, that total darkness seldom constitutes more than four minutes in the latitude of London. In most solar eclipses the moon's disc is covered with a faint light, which is attribnted to the reflection of the light from the illuminated part of the earth. In total eclipses of the sun the darkness is so great as to render the stars and planets perfectly visible; these however are very rare occurrences in any particular place.

The *Number* of *Eclipses* of both luminaries cannot be fewer than two, nor more than seven, in one year; the most usual number is four, and it is rare to have more than six. The reason is obvious; for the sun passes by both the nodes of the moon's orbit but once in a year, unless he pass by one of them in the beginning of it, in which case he will pass by the same again a little before the end; because the nodes retrograde about $19\frac{1}{3}^\circ$ every revolution, and therefore the sun will come to either of them about 173 days after the other, for he will have to move $180^\circ - \frac{19\frac{1}{3}^\circ}{2}$, which will occupy nearly 173 days. And if either node be within 17° of the sun at the time of new moon, the sun will be eclipsed; and at the subsequent opposition, the moon will be eclipsed in the other node, and come round to the next conjunction before the former node be 17° beyond the sun, and eclipse him again. When three eclipses happen about either node, the like number commonly happens about the opposite one; as the sun comes to it in 173 days afterwards, and 6 lunations contain only 4 days more. Thus there may be two eclipses of the sun. and one of the moon, about each of the nodes. Speaking generally, there will be more solar than lunar eclipses, as they will nearly bear the proportion of their limits, *viz.* about 4 to 3. But more lunar than solar eclipses are seen at any given place; because a lunar eclipse is visible on a whole terrestrial hemisphere at once, whereas a solar eclipse is visible only in a small portion of it.

The moon's nodes move backwards $19\frac{1}{3}^\circ$ every year, therefore they would shift through all the points of the ecliptic in eighteen years and 225 days; and this would be the regular period of the return of eclipses, if any complete number of lunations were performed in it, without a fraction; which is however not the case. But in 223 mean lunations, after the sun, moon, and nodes, have been once in a line of conjunction, they return so nearly to the same state again, that the same node which was in conjunction with the sun and moon at the beginning of these lunations will be within 28′ 12″ of the line of conjunction, when the last of these lunations is completed; and in this period there will be a regular return of eclipses, till it be repeated about forty times, or in about 720 years, when the line of the nodes will be 28′ × 40 from the conjunction, and will, consequently, be beyond the ecliptic limits.

Eclipses are of considerable use in astronomy. Eclipses of the moon determine the spherical figure of the earth; they also show that the sun is larger than the earth, and the earth than the moon. Eclipses also, that are similar in all circumstances, and that happen at considerable intervals of time, serve to ascertain the period of the moon's motion.

ECLIPTIC, in *Astronomy*, a great circle of the sphere, supposed to be drawn through the middle of the zodiac. It is the apparent path of the earth, as viewed from the sun, and thence called the heliocentric circle of the earth. It obtained the name ecliptic, because all eclipses of the sun and moon happen when the moon is in or near a node or point where her path intersects this circle.

Upon the ecliptic are marked and counted the twelve celestial signs, Aries, Taurus, &c. and upon it the longitudes of planets and stars are reckoned. It is situated obliquely with respect to the equator, and cuts it in two points, *viz.* the beginning of Aries and Libra, which are directly opposite to each other; and, accordingly, we find the sun twice every year in the equator; and all the rest of the year, either on the north or south side thereof. The ecliptic is bisected by the horizon; consequently, the arch of the ecliptic, intercepted between the horizon and the meridian, is a quadrant. And again, the solstitial points of the ecliptic, those most remote from the equator, are a quadrant distant from the equinoctial points.

Obliquity of the *Ecliptic*, is the angle which it makes with the equator at their intersection. This obliquity may be found in the following manner: about the time of the summer solstice observe the sun's meridian altitude with the utmost care, for several days successively: from the greatest altitude observed, subtract the height of the equator; and the remainder is the greatest declination, which is the solstitial point. Or, it may be found by observing the meridian altitude of the sun's centre on the days of the summer and winter solstice; the difference of those altitudes will be the distance of the tropics; and half that distance will be the obliquity of the ecliptic.

This obliquity is now about 23° 27′ 57″; but it is variable. By a comparison with the most early and accurate observations, it appears that the diminution of the obliquity is on the average about the rate of 50¼″ in a century, or half a second in a year. It is at present 52″ 1′.

This change in the obliquity of the ecliptic arises wholly from the actions of the planets, particularly Venus and Jupiter, upon the earth. After a very long period, the change of obliquity will vary from a diminution to an increase, and never exceed 2°. Laplace makes the limit of variation 2° 42′.

Poles of the ECLIPTIC, are the two opposite points of the sphere, which are every way equally distant from the ecliptic, or 90°.

ELASTIC, having the power of returning to the form from which it has been distorted.

The principal phenomena observable in elastic bodies are thus enumerated: 1. A *perfectly* elastic body, has a tendency to restore itself with the same force with which it is pressed or bent. 2. An elastic body exerts its force equally on all sides, but the effect is found principally where there is the least resistance. 3. Elastic bodies, in whatever manner they are struck or bent, are reflected, and rebound with the same force in an opposite direction; thus an elastic body will rebound from the plane on which it impinges, with the same velocity with which it meets it, and so as to make the angle of reflection equal to the angle of incidence. 4. A perfectly fluid body (being defined, that which cannot be compressed) cannot be elastic. 5. A perfectly solid body cannot be elastic. 6. The elasticity of most bodies, particularly of long and slender forms, may be easily shown, and even in very hard and compact bodies their elasticity may be manifested in various ways; thus, for instance, let a marble slab, or a flat and smooth iron, be covered with black lead, or with printing ink; then drop an ivory ball upon it from different heights, and the degree of compression on it will be indicated by the magnitude of the spot which will be found upon the ivory ball.

Elastic *Curve*, is the name that James Bernouilli gave to the curve which is formed by an elastic blade, fixed horizontally by one of its extremities in a vertical plane, and loaded at the other extremity with a weight, which bends the blade into a curve. It is not the same as the catenary, though frequently confounded with that curve. If we call x the distance of any point from the line of direction of the weight, we shall have for the radius of the evolute

$$\left(\frac{\dot{x}^2 + \dot{y}^2}{\dot{x}\,\ddot{y}}\right)^{\frac{3}{2}} = \frac{1}{x}$$

whence regarding $\dot{x}$ as constant

$$\frac{x^2}{2} = -\frac{\dot{y}}{\sqrt{(\dot{y}^2 + \dot{x}^2)}}, \text{ and } \frac{x^2\ \dot{x}}{\sqrt{(4 - x^3)}}$$
$$= \dot{y}$$

which is the equation of the elastic curve.

ELASTIC *Fluids*, are those which are possessed of an elastic property, as air, steam, &c.

Elastic *Gum*, the same as caoutchouc, or Indian rubber.

ELASTICITY, is that property of bodies by which they restore themselves to their original figure after compression.

ELECTRIC, a body capable by friction, or otherwise, of exhibiting the phenomena of electricity; and which body is at the same time impervious to electricity; and it is to be observed that those bodies which are less pervious are more capable of being excited; and those which are more pervious are less capable of excitation. The former of these classes are called *electrics*, or *non-conductors*, and the latter *conductors* or *non-electrics*. Of the former are *glass*, *precious stones*, *amber*, *sulphur*, *resins*, *wax*, *silk*, &c.: and of the latter, all *metals*, *semi-metals*, *metallic ores*, *charcoal*, *water*, &c.

Electric *Fluid*, a fine, rare fluid, which is supposed to issue from and surround electrical bodies.

ELECTRICAL, belonging to Electricity.

Electrical *Apparatus*, the different machines and substances employed for exciting and explaining the phenomena of electricity.

Electrical *Battery*, consists of a number of coated jars, placed near each other in a convenient manner. These being charged, or electrified, and connected with each other, are then suddenly exploded or discharged, with a prodigious effect. (Plate III. Fig. 3.)

Electrical *Jar*, one of the jars of the battery, otherwise called the Leyden phial.

Electrical *Kite*, was contrived by Dr. Franklin, to verify his hypothesis of the identity of electricity and lightning. It consisted of a large, thin, silk handkerchief, extended and fastened at the four corners to two slight strips of cedar, and accommodated with a tail, loop, and string, so as to rise in the air like those of paper. To the top of the upright stick of the cross a sharp-pointed wire was fixed, rising a foot or more above the wood; and to the end of the twine, next the hand, was attached a silk ribband. From a key suspended at the union of the twine and silk, when the kite is raised during a thunder-storm, a phial may be charged, and electric fire collected, as is usually done by means of a rubbed glass tube, or globe.

Electrical *Machine*, the principal part of the electric apparatus, so constructed as to be capable of exciting electricity, and exhibiting its effects in a very sensible manner.

A great variety of forms have been given to electrical machines, either for the sake of convenience, or in order to render their effects more powerful; we shall only explain one of the most simple forms, as is represented at fig. 4, plate 3.

The cylinder F G H I, is supported by two strong perpendicular pieces D E. The axis of one

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ASTOR, LENOX AND
TILDEN FOUNDATIONS.

Fig. 1.

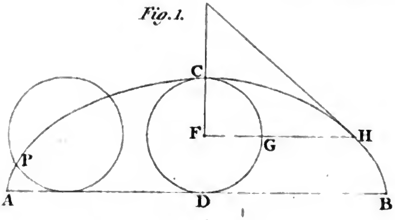


Fig. 2.

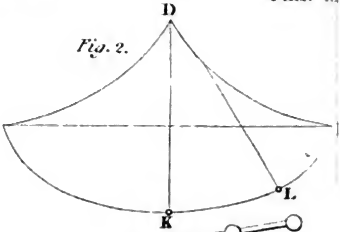


Fig. 3.

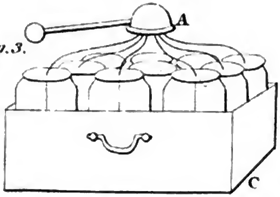


Fig. 4.

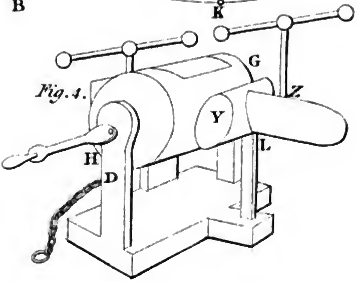


Fig. 5.

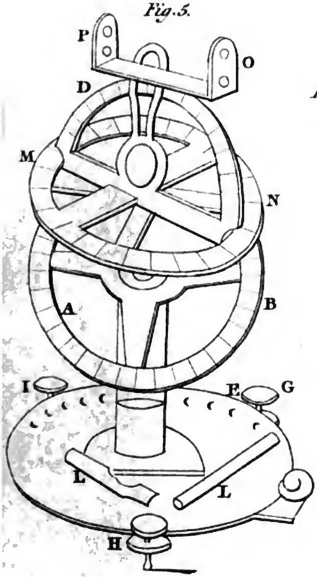


Fig. 6.

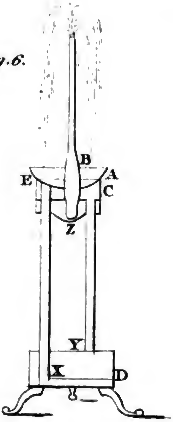


Fig. 7.

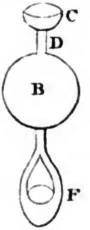
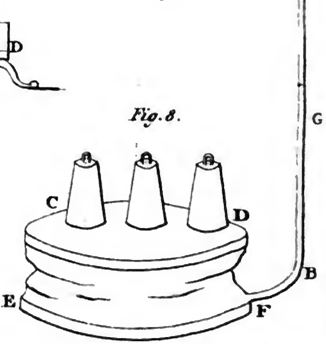


Fig. 8.



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cap of the cylinder moves in a small hole, at the upper parts of one of the supports, and the opposite axis passes through the upper part of the other support. To this end the handle or winch is fitted, and the cushion is supported and insulated by a glass pillar; the lower part of which is fitted into a wooden socket, to which a regulating screw is adapted, to increase or diminish the pressure of the cushion against the cylinder. A piece of silk comes from the under edge of the cushion, and lies on the cylinder, passing between it and the cushion, and proceeding till it nearly meets the collecting points of the conductor. The more strongly this conductor is made to adhere to the cylinder, the stronger is the degree of excitation. Before the cylinder, or opposite to the cushion, is a metallic tube Y Z, supported by a glass pillar L M. This is sometimes called the prime conductor, often only the conductor; and for the more conveniently trying experiments on the two powers, and exhibiting the different states of the cushion and conductor, there are two wires to be fixed occasionally, the one to the conductor, the other to the cushion; on the upper part of these are balls furnished with sliding wires, that they may be set apart from each other at different distances.

Having examined the different parts of the machine, and put them in order, the glass cylinder, and the pillars which support the cushion and conductor, should be well wiped with a dry silk handkerchief, to free them from the moisture which glass attracts from the air, being particularly attentive to leave no moisture on the ends of the cylinder, as any damp on these parts carries off the electric fluid, and weakens the force of the machine: in very damp weather it will be proper to dry the whole machine, by placing it at some little distance from the fire. Care should be taken that no dust, loose threads, or filaments, adhere to the cylinder, its frame, the conductors, or their insulating pillars; because these will gradually dissipate the electric fluid, and prevent the machine from acting powerfully. When satisfied of this, rub the glass cylinder first with a clean, coarse, dry warm cloth, or a piece of wash leather, and then with a piece of dry, warm, soft silk; do the same to all the glass insulating pillars of the machine and apparatus; these pillars must be rubbed more lightly than the cylinder, because, being varnished, they may be damaged by too much friction.

It may be proper in some cases to place a hot iron on the foot of the conductor, in order to evaporate the moisture which would otherwise injure the experiments.

It may also be observed that, 1. To excite the machine, it is requisite to clean the cylinder, and wipe the silk. 2. Grease the cylinder, by turning it against a greasy leather, till it is uniformly obscured. The tallow of a candle will answer this purpose. 3. Turn the cylinder till the silk flap has wiped off so much of the grease as to render it semi-transparent. 4. Spread some amalgam on a piece of leather, and apply this against the turning cylinder.

The best kind of amalgam is made of zinc and mercury. If a little of the latter be added to melted zinc it renders it easily pulverable, and more mercury must be added to the powder if very soft amalgam is required.

ELECTRICITY, the name of an unknown natural power which produces a great variety of peculiar and surprising phenomena. The electric property of amber was known to Thales, who lived about 600 years before our æra, though Theophrastus, who flourished about 300 years after Thales, is the first author who makes any distinct remark on this subject. Some other of the ancients also speak slightly on this head, but still they confined its action to the two substances amber and jet, and knew nothing farther of its effect except its power of attracting light substances after being excited by friction.

ELECTROMETER, an instrument contrived for measuring the quantity and determining the quality of electricity. The most common electrometer is composed of a pithball supported on a wire, and traversing a graduated arch.

ELECTROPHORUS is a machine consisting of two plates, one of which is a resinous electric, and the other metallic; and when the former is once excited by a peculiar application of the latter, the machine will furnish electricity for a considerable time, from which circumstance it derives its name.

ELIMINATION, in *Analysis*, that operation by means of which all the unknown quantities except one are exterminated out of an equation, whence the value of that one becomes determined, and hence by substitution the value of all the others.

This is a subject which in all its generality involves a variety of cases, and numerous and tedious investigations, which of course cannot be treated of in this place: but we shall give one or two of the most simple cases.

Let there be proposed the two following equations of the first degree:

$$ax + by + c = o$$
$$dx + ey + f = o$$

In order to eliminate one of the unknown quantities, for example y, the 1st equation is multiplied by e, and the 2d by b, so that the coefficient of y in both may be equal, then we have

$$eax + bey + ec = o$$
$$bdx + bey + bf = o$$

Subtracting the second of these equations from the first, we have

$$(ea - bd)\,x + ec - bf = o$$

whence again, $x = \dfrac{bf - ec}{ea - bd}$

Having thus obtained the value of x, that of y may be found either by substituting this value of x in either of the original equations, or by multiplying the first of them by d, and the second by e, and then subtracting as above, in either way we find

$$y = \frac{cd - af}{ae - bd}$$

which solutions are general for all equations of two unknown qualities of the first degree, and may therefore be considered as formulæ of solution in such cases.

By a similar proceeding, if there were given the three following equations of the first degree, *viz.*

$$ax + by + cz + d = o$$
$$ex + fy + gz + h = o$$
$$ix + ky + lz + m = o$$

we should find

$$x = \frac{bhl - chk + dgk - gbm}{\begin{array}{r} cek - gdk + afl - bel \\ + cfm - dfl \\ \hline + big - ifc \end{array}}$$

$$y = \frac{gam - cem + chi - ahl}{\begin{array}{r} cek - gdk + afl - bel \\ + del - gid \\ \hline + big - ifc \end{array}}$$

$$z = \frac{ahk - dek + dfi - afm}{\begin{array}{r} cek - gdk + afl - bel \\ + bem - bih \\ \hline + big - ifc \end{array}}$$

And by means of those general formulæ, we have the complete solution of all equations of the first degree, containing three unknown quantities.

Another method is the following;

Let u, x, y, z, &c. represent the unknown quantities, and let the number of them, as also the number of the equations, be n.

Let a, b, c, d, &c. be the respective co-efficients in the first equation.

a', b', c', d', those of the second.

a'', b'', c'', d'', those of the third, and so on.

And let us conceive the known term in each equation to be effected by some unknown quantity, which may be represented by t.

Form the product $uxyzt$, of all the unknowns, written in any order at pleasure; only when this order is once determined on in any case, the same must be observed throughout.

Change now successively each unknown for its co-efficient in the first equation, and observe to change the sign of each even term, and this result is called the *first line*.

Change in this *first line* each unknown for its co-efficient in the second equation, observing, as above, to change the sign of each even term, and this result is called the *second line*.

Change again in this ***second line***, each unknown for its co-efficient in the third equation, observing still the same change of sign, and call this the *third line.*

Continue thus the operation to the last equation inclusively, and the last line that you obtain will give you the value of each of the unknown quantities, as follows, *viz.*

Each unknown quantity will be expressed by a fraction, of which the numerator is the co-efficient of the same unknown letter in the last or *n*th line, and the general denominator will be the co-efficient corresponding with the unknown quantity *t*, which was introduced in the beginning of the operation.

This will be understood from the following example: Let

$$ax + by + c = o$$
$$a'x + b'y + c' = o$$

Introducing *t* these equations become,

$$ax + by + ct = o$$
$$a'x + b'y + c't = o$$

and form the product *xyt*.

Now change *x* into *a*, *y* into *b*, *t* into *c*, and change the sign of those terms which stand in an even place; thus we have

$$ayt - bxt + cxy$$

Now again change *x* into *a'*, *y* into *b'*, *t* into *c'*, changing signs as before, and we have

$$ab't - ac'y - a'bt + bc'x + a'cy - b'cx \text{ or}$$
$$(ab' - ba')t - (ac' - a'c)y + (bc' - b'c)x$$

$$\text{whence } x = \frac{bc' - b'c}{ab' - ba'}$$

$$\text{and} \quad y = \frac{a'c - ac'}{ab' - ba'}$$

ELLIPSE, one of the conic sections, formed by the intersection of a plane and cone when the plane makes a less angle with the base than that formed by the base and the side of the cone.

The word is derived from ἐλλειπης, *defective*, and is thus denominated, because the square of the ordinate in this figure is always less than the rectangle of the parameters and abscisses.

There are three ways in which we may define an ellipse; *viz.* 1. As being produced by the intersection of a plane and cone, as we find it treated of by Apollonius and all the ancients. 2. According to its description in plano, as it is treated of by several of the moderns. And 3. As being generated by the motion of a variable line or ordinate, along another line or directrix.

Properties of the Ellipse.—1. The squares of the ordinates of the axis, are to each other as the rectangle of the abscisses.

Or if one of the ordinates be taken at the centre.

As the square of the transverse axis

Is to the square of the conjugate,

So is the rectangle of the abscisses

To the square of their ordinate.

From which property is readily drawn the equation of the ellipse; for, make the transverse $= t$, the conjugate $= c$, the absciss $= x$, and the ordinate $= y$,

$$\text{then} \quad t^2 : c^2 : x(t - x) : y^2$$

$$\text{whence} \quad y^2 = \frac{c^2}{t^2}(tx - x^2) \text{ or}$$

$$y = \frac{c}{t}\sqrt{(tx - x^2)}$$

2. The sum of the two lines drawn from the two foci to any point in the curve is always equal to the transverse axis.

From this property is derived the common method of describing the curve mechanically by points, or by a thread, thus:

In the transverse axis take any point whatever. Then will the segments of the transverse describe, from the foci as centers, two arcs intersecting each other in a point in the curve. In like manner determine other points. Then with a steady hand a curve-line may be drawn through all the points of intersection, which will be the ellipse required.

Or, otherwise, take a thread of the length of the transverse axis, and fix its two ends in the foci. Then carrying a pen or pencil round by the thread, keeping it always stretched, and its point will trace out the ellipse, as is evident from the property above stated.

3. If a tangent be drawn to any point in an ellipse, and two lines drawn from the two foci to the point of contact, these two lines

will form equal angles with the tangent.

It is this property that gives the name foci to these two points, for as opticians find that the angle of incidence is equal to the angle of reflection; it follows from the above property, that rays of light issuing from the one focus, and meeting the curve in every point, will be reflected back into the other focus, and hence these points are denominated *foci*, or *burning points*.

4. If there be any number of ellipses described on the same transverse axis, and any ordinate be drawn so as to meet all those ellipses, the tangents to the several ellipses at those points will all terminate in one common point in the prolongment of the transverse axis.

And as this is necessarily true when the ellipse becomes a circle, we have hence an easy method of drawing a tangent to any point in an ellipse, namely, describe a semi-circle upon the transverse, produce the ordinate till it meet this circle, from the point of contact draw a radius, and a perpendicular to it at the point of contact, will cut the transverse produced in the subtangent.

5. The ordinate in the circle is to the ordinate of the ellipse, as the transverse axis of the ellipse is to its conjugate axis. And if a circle were described on the conjugate axis, and an ordinate drawn as before, then the ordinate of the circle would be to the corresponding ordinate in the ellipse, as the conjugate axis of the ellipse is to its transverse. And hence it follows, that the area of any ellipse is a mean proportional between the area of the circles described on its two axes.

6. Every parallelogram circumscribed about an ellipse, at the extremities of any pair of its conjugate diameters, is equal to the rectangle of its two axes.

7. The sum of the squares of any pair of conjugate diameters is always equal to the sum of the squares of the two axes of the ellipse.

Area of an Ellipse.—Multiply the two axes together, and that product again by ·7854, which will be the area required.

Area of an Elliptic Segment.—Find the area of the corresponding circular segment, described on the same axis to which the cutting line or base of the segment is perpendicular.

Then as this axis is to the other axis, so is the circular segment to that of the ellipse.

ELLIPTIC *Arc*, is any part of the periphery of an ellipse, the length of which is found as follows:

Let t represent the semi-transverse, and c the semi-conjugate axis of any ellipse, z the distance of the ordinate from the centre, then the arc bounded by the ordinate and parallel semi-axis will be,

1. Ellip. arc

$$= z\left\{1 + \frac{c^2}{6t^4}z^2 + \frac{4t^2c^2 - c^4}{40t^8}z^4 + \frac{8t^4c^2 - 4t^2c^4 + c^6}{112t^{12}}z^6 + \&c.\right\}$$

$$\text{make } \frac{t^2 - c^2}{t^2} = q\text{; then}$$

2. $\text{Arc} = z\sqrt{\frac{t^2 - \frac{1}{3}qz^2}{s^2 - \frac{1}{3}\ z^2}}$

3. $\text{Arc} = \frac{1}{5}\left\{q\sqrt{\frac{t^2 - \frac{1}{3}qz^2}{t^2 - \frac{1}{3}\ z^2}} - \left(1 + \frac{c^2z^2}{6t^4}\right)4z\right\}$

4. $\text{Arc} = \frac{15pC + (19C - 21p)y}{15pC + (qC - 21p)y} \times x$ nearly.

C being the whole axis, where the arc begins, and p, x, and y the corresponding parameter, absciss, and ordinate.

ELLIPTIC *Spindle*, is the solid generated by the revolution of any segment of an ellipse about its chord; the solidity of which may be found by the following formulæ: Put the perpendicular axe of the ellipse $= a$, the parallel axe $= b$; length of the spindle $= l$; distance of the centre of spindle and ellipse $= c$; and area of generating segment $=$ A; then,

1. $\text{Solidity} = 1{\cdot}57078 \times \left\{\frac{a^2 l^3}{3b^2} - 4\,c\,A\right\}$

$$2 \cdot \cdot \cdot \cdot \cdot = \tfrac{2}{3} l \times {\cdot}7854 \left\{ D^2 - 4e\left(\frac{3A}{l} - D\right) \right\}$$

where D is the greatest diameter of the spindle.

ELLIPTICITY of the *Terrestrial Spheroid*, is the difference between the major and the minor semiaxes; it is generally expressed in terms of the former, that is, of the radius of the equator.

ELONGATION, in *Astronomy*, the angle under which a planet would appear from the sun, when reduced to the ecliptic; or it is the angle formed by two lines drawn from the earth to the sun and planet, when reduced as above.

The greatest elongation is the greatest distance to which the planets recede from the sun; which, however, only relates to the inferior planets Mercury and Venus. The greatest elongation of Venus, is from 44° 57′ to 47° 48′, and that of Mercury from 17° 36′ to 28° 20′.

EPACTS, in *Chronology*, the excesses of the solar month above the lunar synodical month, and of the solar year above the lunar year of twelve synodical months.

Epacts, *Annual*, are the excesses of the solar year above lunar.

As the new moons are the same, that is, as they fall on the same day every 19 years, so the difference between the lunar and solar years is the same every 19 years. And because the said difference is always to be added to the lunar year in order to adjust or make it equal to the solar year; therefore, the said difference respectively belonging to each year of the moon's cycle is called the epact of the said year, that is, the number to be added to the same year, to make it equal to the solar year.

Rule for finding the Gregorian Epact.—Subtract 1 from the golden number, multiply the remainder by 11, and reject the 30's. This rule will serve till the year 1900; but after that year the Gregorian Epact will be found by this rule: divide the centuries of the given year by 4, multiply the remainder by 17; then to this product add 43 times the quotient, and also the number 86, and divide the whole sum by 25, reserving the quotient: next multiply the golden number by 11, and from the product subtract the reserved quotient, so shall the remainder, after rejecting all the 30's contained in it, be the epact sought.

EPHEMERIDES, in *Astronomy*, tables calculated by astronomers, showing the present state of the heavens for every day at noon; that is, the places wherein all the planets are found at that time. It is from these tables that the eclipses, conjunctions, and aspects of the planets are determined; horoscopes, or celestial schemes constructed, &c.

EPICYCLOID, a curve generated by a point in one circle, which revolves about another circle, either on the convacity or convexity of its circumference.

Epicycloids, are distinguished into *exterior* and *interior*.

Epicycloids, *Exterior*, are those which are formed by the revolution of the generating circle, about the convex circumference of the quiescent circle.

Epicycloids, *Interior*, are those that are formed when the generating circle revolves on concave circumference.

To these curves belong several curious properties, of which we shall only mention a few of the most remarkable.

1. If the generating and equiescent circle have to each other any commensurable ratio, then is the epicycloid both rectifiable and quadrable, although the area of the common cycloid, which is so much more simple in appearance, can never be completely obtained.

2. If the generating and quiescent circle are incommensurable with each other, then the area of the epicycloid cannot be found, but it is still in this case also rectifiable.

3. If in the interior epicycloid the diameter of the generant is equal to the radius of the quiescent circle, the curve becomes a right line, equal and coincident with the diameter of the latter.

To find the Length of any exterior Epicycloid.

As the semi-diameter of the quiescent circle,
Is to the sum of the diameters of the two circles;
So is double the versed sine of the arc of the generant, which has passed over any part of the quescent circle,
To the length of epicycloidal arc generated by the point which touched the quiescent circle at the beginning of motion.

When the whole arc is required, the versed sine becomes the diameter of the generant.

The length of any arc of an interior cycloid is found by the same proportion, only using the difference of the diameters, in the second term of the proportion, instead of the sum.

To find the Areas of Epicycloids.

As the radius of the quiescent circle,
Is to three times that radius, *plus* twice the radius of the generant;
So is the circular segment
To the epicycloidal sector;
Or, so is the whole area of the generant,
To the whole area of the epicycloid,

Which proportion holds good both for the exterior and interior epicycloids.

Dr. Halley gives us a general proposition for the measurement of all cycloids and epicycloids: thus, the area of a cycloid or epicycloid, either primary, or contracted, or prolate, is to the area of its generating circle; and also the areas of the parts generated in those curves, to the areas of analogous segments of the circle; as the sum of double the velocity of the centre and velocity of the circular motion, to the velocity of the circular motion.

Epicycloids, *Spherical*, are formed by a point of the revolving circle when its plane makes an invariable angle with the plane of the circle on which it revolves.

Epicycloids, *Parabolic, Elliptic, &c.* If a parabola be made to revolve upon another equal to it, its focus will describe a right line perpendicular to the axis of the quiescent parabola; the vertex of the revolving parabola will also describe the cissoid of Diocles, and any other point of it will describe some of Newton's defective hyperbolas, having a double point in the like point of the quiescent parabola. In like manner, if an ellipse revolve upon another ellipse equal and similar to it, its focus will describe a circle, whose centre is in the other focus, and consequently the radius is equal to the axis of the ellipse; and any other point in the plane of the ellipse will describe a line of the fourth order. The same also may be said of an hyperbola revolving upon another equal and similar to it; for one of the foci will describe a circle, having its centre in the other focus, and the radius will be the principal axis of the hyberbola; and any other point of the hyperbola will describe a line of the fourth order.

EPOCH, or **Epocha**, a term or fixed point of time, from whence the succeeding years are numbered or reckoned. The word is from εποχη, from επεχειν, *to sustain, stop;* because epochs define or limit a certain space or time.

Different nations make use of different epochs. The Christians chiefly use the epoch of the nativity or incarnation of Jesus Christ; the Mahometans, that of the Hegira; the Jews, that of the creation of the world, or of the deluge; the ancient Greeks used that of the Olympiads; the Romans, that of the building of their city; the ancient Persians and Assyrians, that of Nabonnassar, &c.

EQUABLY *accelerated* or *retarded Motion*, &c. is when the motion or change is increased or decreased by equal quantities or degrees, in equal times.

EQUAL, a term of relation between two or more things of the same magnitude, quantity, or quality.

It is an axiom in geometry, that two things which are equal to the same thing are also equal to each other. And again, if to or from equals you add or subtract equals, the sum or remainder will be equal.

Equal *Altitudes*, in *Practical Astronomy*, one of the most practicable and certain methods of determining the time, and thus ascertaining the error of a clock or chronometer, is by observing equal altitudes of the sun or of a fixed star. For this purpose, all that is necessary is to observe the instant the sun or star is at any altitude towards the east, before the meridian passage; and the instant must likewise be marked when the same object attains exactly the same altitude towards the west, after the meridian passage; the mean between the above quantities will be the instant marked by the clock at the moment the sun or star was on the meridian. The preceding operation, however, supposes that the declination of the object has not varied during the elapsed interval, but this with the sun seldom happens. The observation must, therefore, be corrected by a table, or by a direct calculation.

Equal *Angles*, are those whose containing lines are inclined alike to each other, or which are measured by similar arcs of circles.

Equal *Arithmetical Ratios*, are those wherein the difference of the two less terms is equal to the difference of the two greater.

Equal *Curvatures*, are such as have the same or equal radii of curvature.

Equal *Figures*, are those whose areas are equal, whether the figures be similar or not.

Equal *Geometrical Ratios*, are those whose least terms are similar aliquot or aliquant parts of the greater.

Equal *Solids* are those whose capacities are equal.

EQUALITY, in *Algebra*, is a comparison of two quantities which are in effect equal, though differently expressed or represented.

Equality is usually denoted by two parallel lines, as =; thus $4 + 4 = 8$; that is, 4 added to 4 is equal to 8.

Equality, *Double*, in the *Diophantine Analysis*, is when we have two formulæ containing the same unknown quantity or quantities, each of which is to be made equal to a perfect power.

Equality, *Triple*, is when there are three formulæ of the same kind as above to be made perfect powers.

Equality, *Ratio*, is the ratio of two equal quantities.

EQUATION, in *Algebra*, is any expression in which two quantities differently represented are put equal to each other, by the sign = placed between them.

A *Literal* **Equation**, is that in which all the quantities, both known and unknown, are expressed by letters.

A *Numeral* **Equation**, is that in which the co-efficients of the unknown quantity and absolute terms are given numbers.

A *Simple* **Equation**, is that in which the unknown quantity enters only in the first degree; or, which contains only one power of the unknown quantity.

An *Adfected* **Equation** is that which contains two or more powers of the unknown quantity It is called a *Quadratic*, a *Cubic*, a *Biquadratic*, &c., according to the highest power of the unknown quantity.

Binomial **Equations**, are such as have only two terms.

Determinate **Equations**, are those equations in which only one unknown quantity enters; or if more than one enter, there are always given as many independent equations as there are unknown quantities.

Indeterminate **Equations**, are those equations in which there are more unknown quantities than there are independent equations.

Reciprocal **Equations**, are those in which the co-efficients of each pair of terms, equally distant from the extremes, are equal to each other.

Exponential **Equations**, are those in which the exponent or index of the power is unknown.

Roots of an **Equation**, are those numbers or quantities which, when substituted for the unknown quantity, will make the whole expression become equal to 0.

The roots of an equation are *Positive*, when they have a positive sign prefixed or understood; *Negative*, when they have a negative sign prefixed to them; *Real*, when they are expressed by any real or possible quantity; and *Imaginary*, when any imaginary quantity enters, as $a \pm \sqrt{-b}$; which is imaginary, or impossible, because a negative quantity has no square root.

Every equation has as many roots, real and imaginary, as there are units in the exponent of the highest power of the unknown quantity.

If one of the imaginary roots of an equation be $a + \sqrt{-b}$, another of its roots will be $a - \sqrt{-b}$. Hence it follows, that the number of imaginary roots in any equation is always even; or, which is the same, they always enter in pairs; therefore in an equation of odd dimensions, that is, when the highest power of the unknown quantity is an odd number, there being an odd number of roots, one of them must necessarily be real; whereas in an equation of even dimensions, all its roots may be imaginary.

The co-efficient of the second term is equal to the sum of all the roots, having their signs changed; the co-efficient of the third term is equal to the sum of the product of every two roots; the co-efficient of the fourth term is equal to the product of every three, with their signs changed; and so on.

Reduction of EQUATIONS, is of two kinds; *viz.* first, the reduction of them from a higher to a lower dimension; and second, the reduction of them to some particular form, to prepare them for solution. The former of these cases is more commonly called the *Depression* of *Equations;* and the latter usually consists in exterminating the second term of the equation, this being the most eligible form for solution.

Solution of EQUATIONS, is the method of finding their roots; which, however, can only be done in a direct manner, for the first four degrees, *viz.* in *Simple, Quadratic, Cubic,* and *Biquadratic Equations;* and the several methods of procedure, in each of these, is given under the respective articles. Equations that exceed the fourth degree cannot be solved by any direct rule except in a few partial cases, in which there are certain relations either between the roots or co-efficients. We have, therefore, no means of obtaining the roots in those cases, but by approximation.

EQUATIONS in the *Differential* and *Integral Calculus,* are of different denominations; as *Differential Equations, Equation* of *Finite Differences, Equations* of *Partial Differences,* &c.

Differential EQUATION, is that which contains in it certain differential quantities; and it is said to be of the first, second, third, &c. order, according as it involves the first, second, third, &c. differential.

EQUATION of *Finite Differences,* in the *Theory* of *Finite Differences* or *Increments,* is that into which the finite differences of the variable quantities of any function enter.

EQUATION of *Partial Differences,* is that which has place between the differential of any function and the differentials of the variables on which it depends, combined with the variables them selves, and with or without constant quantities.

EQUATION of a *Curve,* in *Analysis,* is an equation showing the nature of a curve, by expressing the relation between any absciss and its corresponding ordinate, or the relation of their fluxions, &c.

EQUATION of *Payments,* in *Arithmetic,* is the finding the time to pay at once several debts due at different times, and bearing no interest till after the time of payment, so that no loss shall be sustained by either party.

The rule commonly given for this purpose is as follows: Multiply each sum by the time at which it is due; then divide the sum of the products by the sum of the payments, and the quotient will be the time required.

The true equated time for two

payments is expressed by the following formula:

Let p and p' be sums due, at the end of the time n, n', respectively; let also r represent the rate of interest, and x the required time of payment of the whole.

$$\text{Make } \frac{pr(n+n')+p'+p}{pr}=a$$

$$\text{and } \frac{prnn'+p'n'+pn}{pr}=b$$

$$\text{then } \frac{a\pm\sqrt{(a^2-4b)}}{2}=x,$$ the true equated time.

This formula is founded on the principle of allowing only simple interest. If compound interest be allowed it is more simple, being as follows:

Let a represent the sum of the several debts, and b the sum of their present values, computed as in the case of compound interest; also r the interest on 1*l.* for a year, and x the equated time required; then

$$x=\frac{\log.\,a-\log.\,b}{\log.\,(1+r)}$$

Equation, in *Astronomy*, is a term used to express the correction or quantity to be added to, or subtracted from, the mean position of a heavenly body, to obtain the true position; it also, in a more general sense, implies the correction arising from any erroneous supposition whatever.

Equation *to Corresponding Altitudes*, is a correction which must be applied to the apparent time of noon (found by means of the time elapsed between the instants when the sun had equal altitudes, both before and after noon,) in order to ascertain the true time.

Equation of the *Centre*, is the difference between the true and mean place of a planet, or the angles made by the true and mean place; or, which amounts to the same, between the mean and equated anomaly.

The greatest equation of the sun's centre may be obtained by finding the sun's longitude, at the times when he is near his mean distances, for then the difference will give the true motion for that interval of time; next find the sun's mean motion for the same interval of time; then half the difference between the true and mean motions will show the greatest equation of the centre.

When the mean anomaly and eccentricity of an orbit are given, the equation of the centre may be readily obtained by the following rule, as radius to the cosine of the given anomaly, so is five-fourths of the eccentricity of the orbit to a fourth number; which number add to half the greater axis, if the anomaly be less than 90° or more than 270°, otherwise subtract from the same. Then say, as the sum or remainder is to double the eccentricity, so is the logarithmic sine of the given anomaly to the sine of a first arch, from three times which sine subtract double radius, the remainder will be the sine of a second arch, whose one-third part, taken from the former, leaves the equation sought.

Equation of *Time*, in *Astronomy*, denotes the difference between mean and apparent time, or the reduction of the apparent unequal time, or motion of the sun or a planet, to equable and mean time, or motion.

The equation of time is calculated by tracing out the effects of three combined causes; the obliquity of the ecliptic, the sun's unequal apparent motion therein, and the precession of the equinoctial points. In consequence of the first of these, in the first and third quadrants of the ecliptic from Aries; that is, between Aries and Cancer, and between Libra and Capricorn, the right ascension being less than the mean longitude, the point of right ascension is to the west, and therefore the apparent noon precedes the mean noon; but in the second and fourth quadrants, namely, between Cancer and Libra, or Capricorn and Aries, the right ascension being greater than the longitude, or the mean motion taken in the equator, the mean noon is westward, and therefore precedes the apparent noon.

The mean and apparent solar days are never equal, except when the sun's daily motion in right ascension is 59′ 8″; this is nearly the case about April 15, June 15, September 1, and December 24: on these days the equation is nothing, or nearly so; it is at the greatest about November 1, when it is 16′ 14″.

EQUATOR, in *Astronomy*, is a great circle of the sphere, equally distant from the two poles. It is called the equator, because when the sun is in this circle the days and nights are equal in all parts of the world. Whence also it is called *Equinoctial;* and when drawn on maps and planispheres, it is called the *Equinoctial Line*, or simply the *Line*.

EQUATORIAL, *Universal*, or *Portable Observatory*, is an instrument intended to answer a number of useful purposes in practical astronomy, independent of any particular observatory. It may be employed in any steady room or place, and it performs most of the useful problems in the science. (See Plate III. fig. 5.)

The principal parts of this instrument are, 1. The azimuth or horizontal circle EF, which represents the horizon of the place, and moves on a long axis called the vertical axis. 2. The equatorial, or hour-circle, MN representing the equator, placed at right angles to the polar axis, or the axis of the earth upon which it moves. 3. The semi-circle of declination D, on which the telescope is placed, and moving on the axis of declination, or the axis of motion of the line of collimation.

The peculiar uses of this equatorial are, 1. To find the meridian by one observation only. For this purpose, elevate the equatorial circle to the co-latitude of the place, and set the declination-semicircle to the sun's declination for the day and hour of the day required; then move the azimuth and hour-circles both at the same time, either in the same or contrary direction, till you bring the centre of the cross hairs in the telescope exactly to cover the centre of the sun; when that is done, the index of the hour-circle will give the apparent or solar time at the instant of observation; and thus the time is gained, though the sun be at a distance from the meridian; then turn the hour-circle till the index points precisely at twelve o'clock, and lower the telescope to the horizon, in order to observe some point there in the centre of the glass, and that point is the meridian mark found by one observation only; the best time for this operation is three hours before or three hours after twelve at noon.

2. To point the telescope on a star, though not on the meridian, in full day-light. Having elevated the equatorial circle to the co-latitude of the place, and set the declination-semicircle to the star's declination, move the index of the hour-circle till it shall point to the precise time at which the star is then distant from the meridian, found in tables of the right ascension of the stars, and the star will then appear in the glass. Besides these uses peculiar to this instrument, it is also applicable to all the purposes to which the principal astronomical instruments, *viz.* a transit, a quadrant, and an equal altitude instrument, are applied.

EQUIANGULAR, figures whose angles are all equal; such are the square, and all regular figures. All equilateral triangles are also equiangular.

An equilateral figure, inscribed in a circle, is always equiangular; but an equiangular figure, inscribed in a circle, is not always equilateral, except when it has an odd number of sides. If the number of the sides be even, then they may be either all equal, or else half of them will always be equal to each other, and the other half to each other; the equals being placed alternately.

EQUIANGULAR, is also applied to any two figures of the same kind, when each angle of the one is equal to a corresponding angle in the other, whether each figure, separately considered, be an equiangular figure or not.

EQUIDISTANT *Ordinates, Method of*, is an approximation towards the area of a figure bounded by a right line and curve. Dr. Hutton, in his "Mensuration," gives the following formula for finding the area of figures by this method. Having measured any odd number of equidistant ordinates, put the sum of the first and last = A, the sum of the second, fourth, sixth, &c. = B, the sum of all the others = C, and the common distance of the ordinates = D; then

$$\frac{A + 4B + 2C}{3} \times D = \text{area nearly.}$$

EQUILIBRIUM *(to poise,) Equipoised,* in *Mechanics,* means an equality of forces acting in opposite directions, whereby the body acted upon remains at rest, or in *equilibrio;* in which state the least additional force being applied, on either side, motion will ensue.

A body in motion is also said to be in equilibrio when the power producing the motion, and the force whereby it is resisted, are so adjusted that the motion may be uniform.

EQUIMULTIPLES, the products arising from the multiplication of any two or more primitive quantities, by the same number or quantity. Equimultiples of any quantities have the same ratio as the quantities themselves.

EQUINOCTIAL, in *Astronomy,* a great circle of the sphere, under which the equator moves in its diurnal motion.

EQUINOCTIAL *Points,* are the two points wherein the equator and ecliptic intersect each other: the one, being in the first point of Aries, is called the vernal point; and the other, in the first point of Libra, the autumnal point.

ERATOSTHENES *Sieve.* The name given to a contrivance of Eratosthenes for finding prime numbers. The method was to write all the numbers from one to any extent, and then to begin at the fourth and mark out every second figure to the end, then beginning at the sixth mark out every third figure throughout, then at the tenth and mark out every fifth, and so on with all the prime numbers under the square root of the whole number proposed; by which means all composite numbers were excluded, and consequently those which remained were primes.

EVAPORATION, in *Natural Philosophy,* is the conversion of water into vapour, which, in consequence of its becoming lighter than the atmosphere, is carried considerably above the earth's surface; and afterwards by a partial condensation forms clouds, and finally descends in rain.

As a very considerable proportion of the earth's surface is covered with water, and as this water is constantly evaporating and mixing with the atmosphere in the state of vapour, a precise determination of the rate of evaporation must be of very great importance in meteorology. Accordingly, many experiments have been made to determine the point by different philosophers. From these we learn that, 1. The evaporation is confined entirely to the surface of the water: hence it is in all cases proportional to the surface of the water exposed to the atmosphere. Much more vapour of course rises in maritime countries, or those interspersed with lakes, than in inland countries. 2. Much more vapour rises during hot weather than during cold: hence the quantity evaporated depends in some measure upon temperature. The precise law has been discovered by Mr. Dalton, who says, in general, the quantity evaporated from a given surface of water per minute, at any temperature, is to the quantity evaporated from the same surface at 212°, as the force of vapour at the first temperature is to the force of vapour at 212°. 3. The quantity of vapour which rises from water, even when the temperature is the same, varies according to circumstances. It is least of all in calm weather, greater when a breeze blows, and greatest of all with a strong wind.

EVECTION, the most considerable of the lunar irregularities, caused by the action of the sun upon the moon; its general and constant effect is to diminish the equation of the centre in syzigies, and to increase it in the quadrature, to take place in the eccentricity of the lunar orbit, and at the same time a motion in the apogee.

EVOLUTE, a curve formed by the end of a thread unwound from another curve, the radius or curvature of which is constantly encreasing. The evolute of the cycloid is another equal cycloid, which property was first discovered by Huygens, who by this means contrived to make a pendulum vibrate in an arc of cycloid, by placing it between two cycloidal cheeks, and thus rendered the vibrations isochronous.

The evolute of a spiral, or indeed of any other curve, may be described by finding the radii of curvature at several points in the involute; for then we shall have as many points in the evolute, through which if a curve line be drawn, it will be the evolute sought.

EVOLUTION, in *Arithmetic* and *Algebra*, is the extraction of roots, being thus opposed to involution, which is the raising of powers.

EXCHANGE, in *Arithmetic*, is the reduction of different coins or any denominations of money, whether there be real coins answering to them or not, from one to another: or the method of finding how many of one species, or denomination, are equal in value to a given number of another; in order to which it is necessary to know the value of the coins and moneys of account of different countries, and their proportion to each other according to the settled rate of exchange. The several operations in this case are only different applications of the rule of three, which are explained in Bonnycastle's, Hutton's, and most books of arithmetic.

Arbitration, or *Comparison* of EXCHANGE, determines the method of remitting to, or drawing upon, foreign places, in such a manner as shall be most advantageous to the merchant.

Arbitration, is either simple or compound.

Simple Arbitration, respects three places only. Here, by comparing the par of arbitration between a first and second place, and between the first and a third, the rate between the second and third is discovered; from whence a person can judge how to remit or draw to the most advantage, and to determine what that advantage is.

Compound Arbitration, respects the cases in which the exchanges among three, four, or more places are concerned. A person who knows at what rate he can draw or remit directly, and also has advice of the course of exchange in foreign parts, may trace out a path for circulating his money, through more or fewer of such places, and also in such order, as to make a benefit of his skill and credit; and in this lies the great art of such negociations.

EXHAUSTIONS, in *Geometry*. Method of exhaustions is a way of proving the equality of two magnitudes, by a *reductio ad absurdum*, showing, that if one be supposed either greater or less than the other, there will arise a contradiction.

EXPANSION, in *Physics*, is the enlargement or increase in the bulk of bodies, in consequence of a change in their temperature. This is one of the most general effects of caloric being common to all bodies whatever, whether solid or fluid. The expansion of solid bodies is shown by the PYROMETER, and the expansion of fluids by the THERMOMETER.

The expansion of fluids varies very considerably; but, in general, the denser the fluid the less the expansion: for instance, water expands more than mercury, and spirits of wine more than water; and commonly the greater the degree of heat the greater is the expansion; but this is not universal, for there are cases in which expansion is produced, not by an increase, but by a diminution of temperature.

Water furnishes us with the most remarkable instance of this kind. Its maximum of density corresponds with 42°.5 of Fahrenheit's thermometer, as has been lately ascertained by Mr. Dalton. If it is cooled down below 42°.5, it undergoes an expansion for every degree of temperature which it loses; and at 32° the expansion amounts, according to Mr. Dalton, to $\frac{1}{160}$ of the whole expansion which water undergoes when heated from 42°.5 to 212°.

The prodigious force with which water expands in the act of freezing has been long known to philosophers. Glass bottles filled with water are commonly broken in pieces when the water freezes. The Florentine academicians burst a brass globe whose cavity was an inch in diameter, by filling it with water and freezing it. The force necessary for this effect was calculated by Muschenbroek at 27.720 lbs.

EXPECTATION, in the *Doctrine* of *Chances*, is the value of any prospect of prize or property depending upon the happening of some uncertain event, the value of which, in all cases, is equal to the whole sum multiplied by the probability that the event on which it depends may happen.

EXPECTATION, in the *Doctrine* of *Life Annuities*, denotes that particular number of years which a life of a given age has an equal chance of enjoying, or the time which a person of a given age may justly expect to live.

TABLE

Of the Expectations of Life in London.

Age.	Expectation.	Age.	Expectation.	Age.	Expectation.	Age.	Expectation.
1	27·0	21	28·3	41	19·2	61	12·0
2	32·0	22	27·7	42	18·8	62	11·6
3	34·0	23	27·2	43	18·5	63	11·2
4	35·6	24	26·6	44	18·1	64	10·8
5	36·0	25	26·1	45	17·8	65	10·5
6	36·0	26	25·6	46	17·4	66	10·1
7	35·8	27	25·1	47	17·0	67	9·8
8	35·6	28	24·6	48	16·7	68	9·4
9	35·2	29	24·1	49	16·3	69	9·1
10	34·8	30	23·6	50	16·0	70	8·8
11	34·3	31	23·1	51	15·6	71	8·4
12	33·7	32	22·7	52	15·2	72	8·1
13	33·1	33	22·3	53	14·9	73	7·8
14	32·5	34	21·9	54	14·5	74	7·5
15	31·9	35	21.5	55	14·2	75	7·2
16	31·3	36	21·1	56	13·8	76	6·8
17	30·7	37	20·7	57	13·4	77	6·4
18	30·1	38	20·3	58	13·1	78	6·0
19	29·5	39	19·9	59	12·7	79	5·5
20	28·9	40	19·6	60	12·4	80	5·0

From this table the expectation of life, at any age, is found on inspection; thus, a person of 20 years of age has an expectation of 28.9 years, and in the same manner may be found the expectation at any other age.

EXPERIMENTAL *Philosophy*, that system of philosophy which is founded upon the results of various experiments; which thus furnish certain data, that are assumed as the unalterable laws of nature, and on which finally rest every branch of modern philosophical investigation.

EXPONENT, in *Arithmetic* and *Algebra*, is a term used to denote the number or letter which expresses the degree of the power, or root, of a quantity; thus 2, 3, n, &c. are the exponents of 2d, 3d, nth, &c. powers; and $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{n}$, &c. the exponent of the 2d, 3d, nth, &c. root.

EXPONENTIAL *Calculus*, is the method of finding the fluctions and fluents of exponential quantities.

EXPONENTIAL *Curve*, is a curve which is defined by means of an exponential equation.

EXPONENTIAL *Equation*, is any equation into which an exponential quantity enters, as $a^x=b, x^x=a$, &c.

The readiest method of solving exponential equation is by means of a table of logarithms; thus:

1. Let there be proposed the equation $a^x=b$.

Taking the logarithms on both sides we have

$$x \log. a = \log. b, \text{ therefore } x = \frac{\log. b}{\log. a}$$

2. Let $a^{b^x}=c$, to find the value of x.

Assume $b^x=y$, then $a^y=c$, and $y \log. a = \log. c$; whence $y=$

$$\frac{\log. c}{\log. a}=d$$

Now, therefore, $b^x=d$, or $x=$

$$\frac{\log. d}{\log. b}$$

Exponential *Quantities*, are divided into different orders;

thus, x^x is called the first order,

x^{x^x} . . . . the second order;

and so on of others.

EXTENSION, in *Physics*, is one of the general and essential properties of matter, the extension of a body being the quantity of space which it occupies, the extremities of which limit, or circumscribe, the body. It is otherwise called the magnitude or size of the body. The word extension, however, is commonly used to denote the surface of a body only without regard to its thickness.

EXTERIOR *Angle*, that which is formed by producing the sides of a figure.

The sum of all the exterior angles of any right-lined plane figure are equal to four right angles.

EXTERMINATION, is a term frequently employed by algebraists, to denote any operation which has for its object the taking away any term or quantity, out of an equation or other algebraical expression; and is nearly the same as elimination.

EXTERNAL *Angles*, are those formed without a figure, by producing its sides.

The external angle of a triangle is equal to the sum of the two internal angles; and in any figure whatever the sum of all the external angles is equal to four right angles.

EXTRACTION of *Roots* is the finding the roots of given numbers, or algebraical quantities or equations.

Extraction of the *Roots* of *Numbers*, is to find a number which, multiplied by itself a certain number of times, will produce the given number.

Let N be the given power or number whose root is to be found, n the index of that power, r the assumed root (which is to be taken as near the true root as possible), and A its power.

Then as $(n+1)\ A+(n-1)\ N : (n+1)\ N+(n-1)\ A = r :$ the true root nearly.

Or as $(n+1)\ A+(n-1)\ N : 2\ (A \backsim N) = r :$ the correction, must be added to, or subtracted from, the preceding assumed root, according as it is less or greater than the root required.

Then call this new root r, and use it again as a new assumed root, with which proceed as before, and so on till you have obtained the correct result as far as the nature of the case requires.

Eye *Glass*, in a telescope or other optical instrument, is the glass next the eye.

F.

FACTORS, or *Multipliers*, in *Arithmetic* and *Algebra*, are those numbers or quantities, from the multiplication of which other numbers or quantities are produced, thus:

7 and 5 are the factors of 35;

3, 7, and 11, are the factors of 231.

Also $x+y$ and $x-y$, are the factors x^2-y^2, and x^2+xy+y^2, and $x-y$, are the factors of x^3-y^3, &c.

The resolution of algebraical formulæ into their factors is very frequently of considerable utility, both in the Diophantine and Fluxional Analysis.

To give an idea of the advantage of this resolution, we will assume one of the simplest cases.

Let it be proposed to find such values of x and y, such that their difference x^2-y^2, may be an integral square.

First, the factors of x^2-y^2, are $x+y$, and $x-y$; that is, $(x+y)$, $(x-y)=x^2-y^2$. And since this product is to be a square, we shall evidently have it so if we make each of its factors a square, or the same multiple of a square.

Assume therefore

$$\begin{cases} x+y=mr^2 \\ x-y=ms^2 \end{cases}$$

then the product $x^2-y^2=m^2r^2s^2$, which is evidently a square. But by addition and subtraction these equations give us

$$x=\frac{m(r^2+s^2}{2}$$

$$y=\frac{m(r^2-s^2)}{2}$$

where m, r, and s, may be assumed at pleasure, if we take $m=2$, then we have

$x=r^2+s^2$, and $y=r^2-s^2$;

which expressions will obviously give integral values of x and y, if r and s be taken integral numbers.

FELLOWSHIP, the name of a rule in *Arithmetic*, useful in balancing accounts between traders, merchants, &c.; as also in the division of common land, prize-money, and other cases of a similar kind.

Fellowship is of two kinds, single and double; or fellowship without time, and fellowship with time.

Simple FELLOWSHIP, is when all the moneys have been employed for the same time, and therefore the shares are directly as the stock of each partner. The rule in this case is as follows:

As the whole stock: the whole gain or loss = each man's particular stock: his particular share of the gain or loss.

Exam. A bankrupt is indebted to A 1,000*l.*, to B 2,000*l.*, to C 3,000*l.*; whereas his whole effects sold but for 1,200*l.*: required each man's share.

Here the whole debt is 6,000*l.*; therefore

$$\begin{matrix}\text{as } 6000 \\ 1200\end{matrix} :=\begin{cases} 1000:\pounds 200. & \text{A's share} \\ 2000:\pounds 400. & \text{B's share} \\ 3000:\pounds 600. & \text{C's share}\end{cases}$$

Compound FELLOWSHIP, is when equal or different stocks are employed for different periods of time. The rule in this case is as follows:

Multiply each person's stock by the time it has been engaged; then say,

As the sum of the products: the whole gain or loss = each particular product: the corresponding share of the gain or loss.

Exam. A had in trade 50*l.* for 4 months, and B 60*l.* for 5 months; with which they gained 24*l.*; required each person's particular share.

$50\times4=200$

$60\times5=300$

$$500:24=\begin{cases} 200:9l.\ 12s. \text{ A's gain} \\ 300:14l.\ 8s. \text{ B's gain}\end{cases}$$

FIELD *Book*, in *Surveying*, a book used for setting down angles, distances, and remarks, as they arise in the field practice, and from which the map is constructed and the contents computed. See SURVEYING.

FIGURATE *Numbers*, are all those that fall under the general form.

$$\frac{n(n+1)(n+2)(n+3)\cdot(n+m)}{1\cdot2\cdot3\cdot4\cdot(m+1)}$$

and they are said to be 1st, 2d, 3d, &c. order, according as $m=1, 2, 3$, &c. thus:

Nat. series, 1, 2, 3, 4, &c. n

1st order, 1, 3, 6, 10, &c.

$$\frac{n(n+1)}{1\cdot2}$$

2d order, 1, 4, 10, 20, &c.

$$\frac{n(n+1)(n+1)}{1\cdot2\cdot3}$$

3d order, 1, 5, 15, 35, &c.

$$\frac{n(n+1)(n+2)(n+3)}{1\cdot2\cdot3\cdot4}$$

&c. &c.

The most remarkable property of these numbers is, that if the nth term of any order be added to the $n+1$ term of the preceding order, the sum will be the $n+1$

term of the same order as the former.

Figure of a *Body*, in *Geometry* and *Mensuration*, denotes generally its form or shape.

Figure of the *Sines*, *Cosines*, *Versed Sines*, *Tangents*, or *Secants*, &c. are figures made by conceiving the circumference of a circle extend out in a right line, upon every point of which are erected perpendicular ordinates equal to the sines, cosines, &c. of the corresponding arcs; and then drawing the curve line through the extremity of all these ordinates; which is then the figure of the sines, cosines, &c.

FIRMNESS, denotes the consistence of a body, or that state wherein its sensible parts cohere, or are united together, so that a motion of one part induces a motion of the rest. In which sense firmness stands opposed to fluidity.

FIXED *Bodies*, generally denote those bodies which neither fire nor any corrosive has such an effect on, as to reduce or resolve them into their component elements, or absolutely to destroy them. Of fixed bodies, the principal are gold, platina, silver, precious stones, particularly the diamond, salts, &c.

Fixed *Ecliptic*, a certain imaginary plane, which never changes its position in the heavens from the action of any of the parts of the solar system on each other; but, like a centre of inertia, remains immoveably fixed. The existence of such a plane is demonstrated by Laplace, who has shown the method of determining it from the situations, velocities, masses, &c. of the planets and other bodies.

Fixed *Signs* of the *Zodiac*, an arbitrary denomination which some astronomers have given to the signs Taurus, Leo, Scorpio, and Aquarius. The particular corresponding season being supposed most fixed when the sun is in those signs.

Fixed *Stars*, are those which constantly maintain the same relative position with regard to each other, in contradistinction to the planets and comets which are constantly changing their relative positions. See Star and Constellation.

FLOATING *Bodies*, are those which swim on the surface of a fluid, the stability, equilibrium, and other circumstances of which form an interesting subject of mechanical and hydrostatical investigation, particularly as applied to the construction and management of ships and other vessels.

The equilibrium of floating bodies is of two kinds, *viz.* stable or absolute, and unstable or tottering.

In the one case, if the equilibrium be ever so little deranged, the bodies which compose the system only oscillate about their primitive position, and the equilibrium is then said to be firm, or *stable*. And this stability is *absolute* if it takes place, whatever be the nature of the oscillations; but it is *relative* if it only takes place in oscillations of a certain description.

In the other state of equilibrium, if the system be ever so little deranged, all bodies deviate more and more, and the system, instead of any tendency to establish itself in its primitive position, is overset and assumes a new position, entirely different from the former; and this is called a *tottering*, or *unstable*, equilibrium.

The stability of a floating body is the greater as its centre of gravity is lower than that of the displaced fluid, or as the distance between these centres is increased; it is for this reason that ballast is put in the lower part of vessels to prevent them from being overset. The nature of the equilibrium, as to stability, depends on the position of a certain point, called the *centre of pressure*.

When the centre of pressure is above the centre of gravity, the equilibrium is stable; on the contrary, when the meta centre is lower than the centre of gravity, the equilibrium is tottering; when the meta centre coincides with the centre of gravity, the body will remain at rest in any position it is placed in, without any tendency to oscillation.

Laplace gives the following rule

for determining whether the force which solicits the system, tends to restore it to the same state again when deranged from its primitive position, which is as follows:

"If through the centre of gravity of the section of the surface of the water on which a body floats, we conceive a horizontal axis to pass, such that the sum of the products of every element of the section, multiplied by the square of its distance from this axis, be less than any other horizontal axis drawn through the same centre, the equilibrium will be stable in every direction; when this sum surpasses the product of the volume of the displaced fluid, by the height of the centre of gravity of the body above the centre of gravity in this volume."

This rule is principally useful in the construction of vessels which require sufficient stability to enable them to resist the effects of storms, which tend to submerge them.

In a ship, the axis drawn from the stern to the prow, is that relative to which the sum above-mentioned is a minimum; it is easy, therefore, to ascertain and measure its stability by the preceding rule.

In order that the floating body may remain in equilibrium, it is also necessary that its centre of gravity be in the same vertical line with the centre of gravity of the displaced fluid, otherwise the weight of the solid will not be completely counteracted by the pressure of the displaced fluid.

When the lower surface of a floating body is spherical or cylindrical, the centre of pressure must coincide with the centre of the figure, since the height of this point, as well as the form of the portion of the fluid displaced, must remain invariable in all circumstances.

FLUENT, or *Flowing Quantity*, in the *Fluxional Analysis*, is the variable quantity which is considered as increasing or decreasing; or the fluent of a given fluxion is that quantity whose fluxion being taken according to the rules previously laid down, will be the same with the given fluxion.

Contemporary Fluents, are such as flow together, or for the same time. And the same is to be understood of contemporary fluxions. When contemporary fluents are always equal, or in any constant ratio, then also are their fluxions respectively either equal, or in that same constant ratio.

FLUID, or *Fluid Body*, is that whose parts yield to the smallest force impressed upon them, and by yielding are easily moved amongst each other; in which sense it stands opposed to a solid, whose parts do not yield, but constantly maintain the same relative situation.

The fluidity of bodies is accounted for by supposing them to be made up of a number of infinitely small particles, possessing, with regard to each other, an attractive and repulsive power, which being equal, places the whole system in equilibrium, whereby it obeys any external force impressed upon it.

This hypothesis evidently places the several particles of a fluid at small distances from each other, and consequently supposes them to be porous, or to possess certain vacuities, the existence of which may be demonstrated from various experiments. Thus, water will dissolve a certain quantity of salt; after which it will absorb a little sugar, and after that a little alum; and all this without increasing its bulk, which evidently shows that the particles of these solids are so far separated as to become smaller than the vacuities, or interstices, between the particles of the fluid.

Fluids are divided into *Elastic* and *Non-elastic*.

Elastic Fluids, are those which may be compressed into a smaller compass, but which on removing the pressure resume again their former dimensions; as air, and the various gases.

Non-elastic Fluids, are those which occupy the same bulk under all pressures, or if they be at all compressible, it is in a very trifling degree; such as water and other liquids.

FLUIDITY, that state or affection of bodies which renders them fluid.

FLUXION, in the *Newtonian Analysis*, denotes the velocity with which a flowing quantity increases by its generative motion, by which it stands contradistinguished from a fluent, or flowing quantity, which is constantly and indefinitely increasing, after the manner that a surface is described by the motion of a line, or a solid by the motion of a surface. Or, a fluxion may be otherwise defined, as the magnitude by which any flowing quantity would be uniformly increased in a given portion of time, with the generating celerity at any proposed position or instant, supposing it thence to continue invariable.

Fluxional Analysis, is the analysis of fluxions and flowing quantities, distinguishable from the differential calculus both by its metaphysics and notation, but in all other respects the two methods are identical.

It will be sufficient to observe, that all finite magnitudes are here conceived to be resolved into infinitely small ones, supposed to be generated by motion, as a line by the motion of a point, a superfices by a line, and a solid by a superfices; of which they are the elements, moments, or differences.

The art of finding these infinitely small quantities, or the velocities by which they are generated, and of discovering other infinite quantities by their means, makes what is called the *Direct Method of Fluxions*. And the method of finding the fluents or flowing quantities, these fluxions being given, constitutes the *Inverse Method*.

What renders the knowledge of infinitely small quantities of such great use and extent is that they have relations to each other, which the finite magnitudes, whereof they are the infinitesimals, have not.

Thus, for example, in a curve of any kind whatever, the infinitely small differences of the ordinate and absciss have the ratio to each other, not of the ordinate and absciss, but of the ordinate and subtangent; and, of consequence, the absciss and ordinate being known, will give the subtangent; or, which amounts to the same, the tangent itself.

The method of notation in fluxions is as follows:

The variable, or flowing quantity, to be uniformly augmented, as suppose the absciss of a curve is denoted by the final letters v, x, y, z; and their fluxions by the same letters with dots placed over them thus, $\dot{v}$, $\dot{x}$, $\dot{y}$, $\dot{z}$. The initial letters a, b, c, d, &c. are used to express invariable quantities.

Again, if the fluxions themselves are also variable quantities, and are continually increasing or decreasing, the velocities with which they increase or decrease, as the fluxions of the former fluxions or second fluxions, are denoted by two dots over them, thus, $\ddot{y}$, $\ddot{x}$, $\ddot{z}$.

After the same manner one may consider the increase and diminution of these, as their fluxions also, and thus proceed to the third, fourth, &c. fluxions, which will be denoted thus, $\dddot{y}$ $\dddot{x}$ $\dddot{z}$: $\ddddot{y}$ $\ddddot{x}$ $\ddddot{z}$, &c.

Lastly, if the flowing quantity be a surd, as $\sqrt{x-y}$, its fluxion is $(\sqrt{x-y})^{\cdot}$; if a fraction $\dfrac{xx}{d-y}$ it is denoted by $\left(\dfrac{xx}{d-y}\right)^{\cdot}$

Sometimes, however, the fluxions of compound quantities are expressed by placing the letter F, or f before them, thus, instead of $\left(\sqrt{(x-y)}\right)^{\cdot}$, is written F. $\sqrt{(x-y)}$ or $f.\ \sqrt{(x-y)}$. We shall, however, in the following article denote the fluxion by F. and the fluent by $f.$, so that,

$$\begin{array}{rl} \text{F.} & \sqrt{(x-y)} \\ & \dfrac{x+ax^2}{b+x} \end{array}$$

denote the fluxions of

$$\begin{array}{l} \sqrt{(x-y)} \\ \dfrac{x+ax^2}{b+x} \end{array}$$

$$f.\ \dot{x}\sqrt{(x+ax^2)}$$

$$f.\ \frac{xb}{x+ax^2}$$

denote the fluents of $\dot{x}\sqrt{(x+ax^2)}$ $\frac{b\dot{x}}{x+ax^2}$

Direct Method. It follows from the preceding definitions and notation that the fluxion of x being $\dot{x}$; of y, $\dot{y}$; of z $\dot{z}$, &c. that the fluxion of $(x+y+z+$, &c.$) = \dot{x}+\dot{y}+\dot{z}$, &c. Also the fluxion of x, being $\dot{x}$, the fluxion of ax will be $a\dot{x}$; of $ax \pm by \pm cz$, will be $a\dot{x} \pm b\dot{y} \pm c\dot{z}$, and so on of other similar expressions.

But when we have to find the fluxion of a rectangle of two variable quantities, as xy, the exact determination of it is by no means so obvious. The usual method of explaining it is this:

Let x and y be represented by two lines at right angles to each other, and complete the rectangle; it will express the product xy. Imagine each of the lines now to move over an infinitely small space, namely, x over $\dot{x}$ and y over $\dot{y}$, then a gnomen will be added to the rectangle, consisting of $x\dot{y}+y\dot{x}+\dot{x}\dot{y}$. But since both $\dot{x}$ and $\dot{y}$ are by hypothesis infinitely small, the last term $(\dot{x}\dot{y})$ may be omitted without error. In like manner it might be shown that the fluction of the product of three quantities, rejecting the terms which contain only fluctions is the sum of the products of the fluction of each quantity by all the other quantities.

Thus the difference between the integral and fluctionary calculus is purely metaphysical, or rather ideal, the powers and products of the differential being left out in the one and those of the fluction in the other. Thus the sum of the sides of the rectangle are $x+\dot{x}$ and $y+\dot{y}$, according to the one species of notation, and $x+d$ and $y+d$ in the other. Wherefore, the complete rectangle in the first case is $xy+x\dot{y}+y\dot{x}+\dot{x}\dot{y}$; and the whole product in the other is $xy+xd+yd+dd$, xy is the same in each, being constant; and $\dot{x}\dot{y}$ is equal to dd, because both are infinitely small. Hence, omitting the equal quantities, the fluction of the rectangle $x\dot{y}+y\dot{x}$, is equal to the differential, or increment $dx+dy$.

Hence it follows that the fluxion of a rectangle of two variable quantities, is equal to the sum of the fluction of each, multiplied into the other quantity; that is, flux. $xy = \dot{x}y+\dot{y}x$. Thus

$\text{F}.\,xyz = \dot{x}yz + \dot{y}xz + \dot{z}xy$

$\text{F}.\,xyzw = \dot{x}yzw + \dot{y}xzw + \dot{z}xyw + \dot{w}xyz$

and so on, for any number of quantities.

For put $xy = \phi$, then $xyz = \phi z$.

Now F. $\phi z = \dot{\phi}z + \dot{z}\phi$; but $\phi = xy$, and conseq. $\dot{\phi} = \dot{x}y + \dot{y}x$, substitutes these values in the above expression, and we have,

F. $\phi z =$ F. $xyz = \dot{x}yz + \dot{y}xz + \dot{z}xy$.

Again, in the second example, put $xyz = \phi$, then

F. $xyzw =$ F. $\phi w = \dot{\phi}w + \dot{w}\phi$; but

$\phi = xyz$, and $\dot{\phi} = \dot{x}yz + \dot{y}xz + \dot{z}xy$ substitute these values in the expression,

F. $\phi w = \dot{\phi}w + \dot{w}\phi$; and we have,

F. $xyzw = \dot{x}yzw + \dot{y}xzw + \dot{z}xyw + \dot{w}xyz$ and the same rule will obviously obtain whatever may be the number of variables of which the product is composed.

To find the Fluxion of a Fraction. From the fluxion of the numerator multiplied into the denominator, subtract the fluxion of the denominator multiplied into the numerator, and divide by the square of the denominator.

$$\frac{x}{y} = \frac{\dot{x}y - \dot{y}x}{y^2}$$

For make $\frac{x}{y} = \phi$; then F. $\frac{x}{} = \dot{\phi}$.

Now $x = y\phi$, and $\dot{x} = \dot{y}\phi + \dot{\phi}y$;

Or, substituting $\frac{x}{y}$ for ϕ, we have

$$\frac{\dot{y}x}{y} + \dot{\phi}y = \dot{x};$$

$$\text{or, } \dot{y}x + \dot{\phi}y^2 = x\dot{y},$$

$$\text{whence } \dot{\phi} = \frac{\dot{x}y - \dot{y}x}{y^2}$$

$$\text{that is, F.} \frac{x}{y} = \frac{\dot{x}y - \dot{y}x}{y^2}$$

If the numerator of the fraction be a constant quantity, having its fluxion equal to 0, the fluxion of such fraction is equal to the fluxion of the denominator drawn into the numerator, and divided by the square of the denominator.

$$\text{That is, F.} \frac{a}{x} = \frac{a\dot{x}}{x^2}.$$

To find the fluxion of any integral power of a variable quantity. Multiply the given power by the index, then reduce its index by unity, and multiply by the fluxion of the simple quantity.

That is, F. $x^n = nx^{n-1}\dot{x}$.

This rule follows immediately from what has been shown with regard to the fluxion of a product; for it is obvious in the first place, that the number of terms in that result is always equal to the number of quantities; and that the number of variables in each term is always one less than that number. If, therefore, we suppose them all equal to each other, so that the product becomes a power of any one of them, then it is evident that its fluxion will assume the form above given; *viz.*

F. $xxx = \dot{x}xx + \dot{x}xx + \dot{x}xx = 3x^2\dot{x}$,
F. $xxxx =$ F. $x^4 = 4x^3\dot{x}$,
and generally,
F. $x^n = nx^{n-1}\dot{x}$.

To find the fluxion of a quantity having a fractional index.

Multiply the proposed quantity by the index, reduce that index by unity, and multiply by the fluxion of the simple quantity; that is,

F. $x^n = nx^{n-1}\dot{x}$, whether n be integral or fractional.

We have demonstrated this above, in the case where n is an integer, and have therefore in this place only to attend to the case in which n is fractional. Let, therefore, $n = \frac{p}{q}$, and make $\frac{p}{x^q} = z$ whence $x^p = z^q$, therefore $px^{p-1}\dot{x} = pz^{q-1}\dot{z}$. But $z^{q-1} = \frac{z^q}{z} = \frac{x^p}{\frac{p}{x^q}} = x^{p - \frac{p}{q}} = x\,(q-1)\frac{p}{q}$ substitute this for z^{q-1} $\frac{p}{x^p}$ we have $p\,x^{p-1}\dot{x} = q\,x\,(q-1)\frac{p}{q}\dot{z}$.

$$\text{or } \dot{z} = \frac{p}{q}\,\frac{x^{p-1}}{x^{(q-1)\frac{p}{q}}}\dot{x} = \frac{p}{q}\,x^{\frac{p}{q}-1}\dot{x}$$

Whence it appears, that the same rule has place both for integral and fractional indices, and consequently also for negative ones; that is,

F. $x^{-n} = -nx^{-n-1}\dot{x}$

For $x^{-n} = \frac{1}{x^n}$ and F. $\frac{1}{x^n} = -\frac{nx^{n-1}\dot{x}}{x^{2n}} = -nx^{-n-1}\dot{x}$.

To find the fluxion of the hyperbolic logarithm of a variable quantity.

Divide the fluxion of the quantity by the quantity itself, and it will be the fluxion required. Thus,

$$\text{F. h. log. } x = \frac{\dot{x}}{x}$$

For put $x = 1 + y$, then

h. log. $(1+y) = y - \frac{1}{2}y^2 + \frac{1}{3}y^3 - \frac{1}{4}y^4 +$ &c. as is shown under the article logarithms; therefore,

F. log. $(1+y) = \dot{y} - y\dot{y} + y^2\dot{y} - y^3\dot{y} = \dot{y}\,(1 - y + y^2 - y^3 + \text{\&c.})$

but $1 - y + y^2 - y^3 +$ &c. is derived from the conversion of $\frac{1}{1+y}$ into a series; therefore,

$$\text{F. log. } (1+y) = \dot{y} \times \frac{1}{1+y} = \frac{\dot{y}}{1+y}$$

But since $1 + y = x$, $\dot{y} = \dot{x}$; whence,

F. log. $(1+y) =$ F. log. $x = \frac{\dot{x}}{x}$.

The logarithm of one system is always convertible into any other system by means of a constant multiplier;

Therefore the fluxion of any log. of x, is equal to $\mathrm{M} \times \frac{\dot{x}}{x}$, where M is the modulus of the system. If it be the common, or Brigg's logarithm, then $\mathrm{M} = 0\ 43429$, &c. and for the hyperbola logarithm, $\mathrm{M} = 1$.

To find the fluxion of exponential quantities, that is, quantities which have their exponent a flowing or variable letter.

These are of two kinds, *viz.* when the root is a constant quantity, as e^x; and when the root is variable, as y^x.

In the former case, put the proposed exponential $e^x = z$, a single variable quantity; then take the logarithm of each, so shall $\log. z = x \times \log. e$; take the fluxions of these, so shall

$\dot{z} = \frac{\dot{x}}{z} \times \log. e$; hence $\dot{z} = z\,\dot{x} \times \log. e = e^x \dot{x} \times \log. e$, the fluxion of the proposed exponential e^x; and which, therefore, is equal to the proposed quantity, drawn into the fluxion of the exponent, and also into the log. of the root.

Also in the second case, put the exponential $y^x = z$; then the logarithms give $\log. z = x \times \log. y$, and the fluxions give $\frac{\dot{z}}{z} = \dot{x} \times \log. y + x \times \frac{\dot{y}}{y}$; hence $\dot{z} = z\,\dot{x} \times \log. y + \frac{zx\dot{y}}{y} =$ (by substituting y^x for z) $y^x\,\dot{x} \times \log. y + xy^{x-1}\,\dot{y}$, is the fluxion of the proposed exponential y^x; which therefore consists of two terms, of which the one is the fluxion of the proposed quantity, considering the exponent only as constant, and the other is the fluxion of the same quantity, considering the root as constant.

To find the fluxions of sines, cosines, &c.

Suppose we require the fluxion of the sine of the angle or arc denoted by z, we must suppose that by a motion of one of the legs including the angle, it becomes $z + \dot{z}$, then $\sin. (z + \dot{z}) - \sin. z$ is the fluxion of $\sin. z$. But according to the formulæ for the sines of sums of arcs, we have $\sin. (z + \dot{z}) = \sin. z \cos. \dot{z} + \sin. \dot{z} \cos. z$, the radius being assumed equal to unity. But the sine of an arc indefinitely small does not differ sensibly from that arc itself, nor its cosine differ perceptibly from radius; hence we have $\sin. z = \dot{z}$, and $\cos. \dot{z} = 1$; and therefore $\sin. (z + \dot{z}) = \sin. z + \dot{z} \cos. z$; whence $\sin. (z + \dot{z}) - \sin. z$, or $(\sin. z)^{\cdot} = \dot{z} \cos. z$, *viz.* the fluxion of the sine of an arc whose radius is unity, is equal to the product of the fluxion of the angle into the cosine of the same arc.

In like manner the fluxion of $\cos. z$, or $\cos. (z + \dot{z}) - \cos. z = \cos. z \cos. \dot{z} - \sin. z \sin. \dot{z} - \cos. z$, or since $\cos. (z + \dot{z}) = \cos. z \cos. \dot{z} - \sin. z \sin \dot{z}$; therefore, because $\sin. \dot{z} = \dot{z}$, and $\cos. \dot{z} = 1$, we have $\mathrm{F.} \cos. z = \cos. z - \dot{z} \sin. z - \cos. z = - \dot{z} \sin. z$, that is, the fluxion of the cosine of an arc, radius being 1, is found by multiplying the fluxion of the arc (taken with a contrary sine) by the sine of the same arc.

By means of these two formulæ, many other fluxional expressions may be found, *viz.*

$\mathrm{F.} \cos. mz = - mz \sin. mz.$

$\mathrm{F.} \sin. mz = + m\dot{z} \cos. mz.$

$$\mathrm{F.} \tan. z = \frac{\dot{z}}{\cos.^2 z}.$$

$$\mathrm{F.} \mathrm{cotan.} z. = - \frac{\dot{z}}{\sin.^2 z}.$$

$$\mathrm{F.} \sec. z = \frac{\dot{z} \sin. z}{\cos.^2 z}.$$

$$\mathrm{F.} \mathrm{cosec.} z = - \frac{\dot{z} \cos. z}{\sin.^2 z}.$$

$\mathrm{F.} \sin.^m z = m \sin.^{m-1} z\,\dot{z} \cos. z.$

$\mathrm{F.} \cos.^m z = - m \cos.^{m-1} z\,\dot{z} \sin. z.$

To find the second, third, &c. fluxion of a flowing quantity.

These fluxions differ in nothing except their order and notation, from first fluxions, being actually such to the quantities from which they are immediately derived; and therefore they may be found, in the same manner, by the general rules already delivered.

Thus, by the third rule, the first fluxion of x^3 is $3x^2\dot{x}$, and if $\dot{x}$ be supposed constant, cr if the root x be generated with an equable celerity, the fluxion of $3x^2\dot{x}$, or $3\dot{x} \times x^2$, will be $3\dot{x}, \times 2x\dot{x}^2 = 6\dot{x}x^2$, which is the second fluxion of x^3, and $6\dot{x}^3$ will be its third fluxion; but if the celerity with which x is generated be variable, either increasing or decreasing, then $\dot{x}$ being variable, will have its fluxion denoted by $\ddot{x}$, &c. In this case the fluxion of $3x^2 \times \dot{x}$ will be, by the first and third rules, $6x\dot{x} \times \dot{x} + 3x^2 \times \ddot{x} = 6x\dot{x}^2 + 3x^2\ddot{x}$, the second fluxion of x^3. And the third fluxion of x^3 obtained in like manner from the last, will be $6\dot{x} \times \dot{x}^2 + 6x \times 2\dot{x}\ddot{x} + 6x\dot{x} \times \ddot{x} + 3x^2\dddot{x} = 6\dot{x}^3 + 18\dot{x}\ddot{x}x + 3x^2\dddot{x}$. Thus also, if $\dot{y} = nx^{n-1}\dot{x}$ then $\ddot{y} = n \times n - 1 \times x^{n-2}\dot{x}^2 + n\ddot{x}x^{n-1}$; and if $z^2 = \dot{x}\dot{y}$, then $2z\ddot{z} = \dot{x}\ddot{y} + \dot{y}\ddot{x}$, &c.

If the function proposed were ax^n, we should find F. $ax^n = nax^{n-1}\dot{x}$; the factors na and $\dot{x}$ being regarded as constant in the first fluxion $nax^{n-1}\dot{x}$, to obtain the second fluxion it will suffice to make x^{n-1} flow, and to multiply the result by $na\dot{x}$; but F. $x^{n-1} = (n-1)x^{n-2}\dot{x}$; we have, therefore,

2nd F. $ax^n = n(n-1)ax^{n-2}\dot{x}^2$.
3d F. $ax^n = n(n-1)(n-2)$
$ax^{n-3}\dot{x}^3$.
4th F. $ax^n = n(n-1)(n-2)(n-3)$
$ax^{n-4}\dot{x}^4$
&c. = &c.
mth F. $ax^n = n(n-1)\,n-2) \cdots$
$(n-m+1)a^{n-m}\dot{x}^m$,

m being supposed not to exceed n, for it is manifest that in the case of n being integral, the function ax^n has only a limited number of fluxions, of which the most elevated is in the nth, and which of course is expressed by the formula,

nth F. $ax^n = n(n-1)(n-2) \cdots 3.2.1.a\dot{x}^n$ in which state it admits no longer of being put into fluxions, as it contains no variable quantity, or, in other words, its fluxion is equal to *o*.

Inverse Method.—In the direct method of fluxions our object is to find the fluxion of any flowing or variable quantity, for which the proper rules have been laid down in the preceding part of this article, and the operation is therefore always direct and easily accomplished; but in the inverse method, where it is required to find the fluents of given fluxions, the operation is much more difficult, as no rules can be laid down that will be sufficient for performing this in all cases; for though every flowing quantity has its peculiar fluxion, yet every fluxion has not its fluent, at least not without having recourse to infinite series, quadrature of curves, or other methods of approximation. There are, however, a few rules which may be useful.

1. When any power or root of the variable quantity is multiplied by the fluxion of that quantity.

Substitute the variable quantity instead of its fluxion, which will increase the index of the power or root by unity; then divide the quantity by the index thus increased, and it will be the fluent required. That is,

$$f.\ mx^n\dot{x} = \frac{m}{n+1}x^{n+1}$$

Thus, for example,

$$f.\ 3x^5\dot{x} = \frac{3}{6}x^6$$

$$f.\ 5x^{\frac{1}{2}}\dot{x} = \frac{10}{3}x^{\frac{3}{2}}$$

2. When the root under the vinculum is a compound quantity, and the index of the part or factor without the vinculum, increased by unity, is some multiple of that under the vinculum.

Put a single variable quantity for the compound root, and substitute its powers and fluxions instead of those of the quantity itself. Find the fluent of this simple fluxion, and then re-establish the value of the compound root, which will be the fluxion required.

For example, let it be proposed to find the fluent of $(a^2 + x^2)^{\frac{2}{3}}$ $x^3 \dot{x}$.

Put $a^2 + x^2 = z$, then $x^2 = z - a^2$; and $2 x \dot{x} = \dot{z}$, therefore $x^3 \dot{x} = \frac{1}{2} (z - a^2) \dot{z}$ $(a^2 + x^2)^{\frac{2}{3}} x^3 \dot{x} = \frac{1}{2} z^{\frac{2}{3}} (z - a^2) \dot{z} = \frac{1}{2} z^{\frac{5}{3}} \dot{z} - \frac{1}{2} a^2 z^{\frac{2}{3}} \dot{z}$

Now f. $(\frac{1}{2} z^{\frac{5}{3}} \dot{z} - \frac{1}{2} a^2 z^{\frac{2}{3}} \dot{z}) =$

$$\frac{3}{16} z^{\frac{8}{3}} - \frac{3}{10} a^2 z^{\frac{5}{3}} =$$

$$\frac{3}{16} (a^2 + x^2)^{\frac{8}{3}} - \frac{3}{10} a^2 (a^2 + x^3)^{\frac{5}{3}}$$

which may be farther reduced to

$$\frac{3}{16} (a^2 + x^2)^{\frac{5}{3}} \times (x^2 - \frac{3}{5} a^2).$$

3. When the fluxion of a quantity is divided by the quantity itself.

Then the fluent is equal to the hyperbolic logarithm of that quantity; or, which is the same thing, the fluent is equal to 2·30258509, &c. multiplied by the common log. of the same quantity.

Thus the fluent

of $\frac{\dot{x}}{x}$ or $x^{-1} \dot{x}$, is the hyp. log. of x;

of $\frac{2 \dot{x}}{x}$, is 2 × hyp. log. of x or hyp. log. of x^2;

of $\frac{\dot{x}}{a + x}$, is the hyp. log. of $a + x$;

of $\frac{3 x^2 \dot{x}}{a + x^2}$, is the hyp. log. of $a + x^3$;

of $\frac{\dot{x}}{\sqrt{x^2 \pm a^2}}$, is hyp. log. of $(x + \sqrt{x^2 \pm a^2})$;

of $\frac{x}{\sqrt{x^2 \pm 2 a x}}$, is hyp. log. $(x \pm a + \sqrt{x^2 \pm 2 a x})$;

of $\frac{2 a \dot{x}}{a^2 - x^2}$, hyp. log. of $\frac{a + x}{a - x}$;

of $\frac{2 a \dot{x}}{x \sqrt{a^2 \pm x^2}}$, is hyp. log. of

$$\frac{a - \sqrt{a^2 \pm x^2}}{a + \sqrt{a^2 \pm x^2}}.$$

Many fluents may be found by the direct method of fluxions, thus:

Take the fluxion again of the given fluxional expression, or the second fluxion of the fluent sought; into which substitute $\frac{\dot{x}^2}{x}$ for $\ddot{x}$ and $\frac{\dot{y}^2}{y}$ for $\ddot{y}$, &c. that is, make $x, \dot{x}, \ddot{x}$, as also $y, \dot{y}, \ddot{y}$, &c. in continual proportion. Then divide the square of the given fluxional expression by the second fluxion, just found, and the quotient will in many cases be the fluent.

Or the same rule may be delivered thus: In the given fluxion $\dot{F}$ write x for $\dot{x}$, y for $\dot{y}$, &c. and call the result G, taking also the fluxion of this quantity $\dot{G}$; then make $\dot{G} : \dot{F} = G : F$, so shall the fourth proportional F be the fluent, as before.

It may be proved whether this be the true fluent, by taking the fluxion of it again. If it agree with the proposed fluxion, it will show that the fluent is right; otherwise the method fails.

Thus, if it be proposed to find the fluent of $n x^{n-1} \dot{x}$. Here $\dot{F} = n x^{n-1} \dot{x}$; write first x for $\dot{x}$, and it is $n x^{n-1} x$, or $n x^n = G$; the fluxion of this is $\dot{G} = n^2 x^{n-1} x$; therefore $\dot{G} : \dot{F} = G : F$ becomes $n^2 x^{n-1} \dot{x} : n x^{n-1} \dot{x} = n x^n : x^n =$ F, the fluent sought.

To find Fluents by Series.

When the proposed fluxion is of any form not included in the preceding rules, nor given in the annexed table, there is no other method of obtaining the fluent but by a series, at least it will generally be found most convenient and direct, and will apply in all cases; the rule is as follows:

Expand the radical or fraction into a series by the binomial theorem, and multiply the several terms by the fluxion of the variable, then take the fluents of the several terms, which will be the fluent required.

It should be observed, however,

that the quantities must be so arranged that the resulting series may be a converging one, which may always be effected by placing the greatest terms foremost in the given fluxion.

EXAM. Required the fluent of $\dot{x}$ $(a^2-x^2)^{\frac{1}{5}}$. $(a^2-x^2)^{\frac{1}{3}} = a^{\frac{2}{3}} \times$

$$\left\{1-\frac{x^2}{5a^2}-\frac{2x^4}{25a^4}-\frac{6x^6}{125a^6}-,\&c.\right\}$$

$$\dot{x}\,(a^2-x^2)^{\frac{1}{6}}=a^{\frac{2}{3}}\times$$

$$\left\{\dot{x}-\frac{x^2\dot{x}}{5a^2}-\frac{2x^4\dot{x}}{25a^4}-\frac{6x^6\dot{x}}{125a^6}-,\&c.\right\}$$

Now taking the fluents of each term, we have,

$$f.\,\dot{x}\,(a^2-x^2)^{\frac{1}{6}}=a^{\frac{2}{3}}\times$$

$$\left\{x-\frac{x^3}{15a^2}-\frac{2x^5}{125a^4}-\frac{6x^7}{875a^6}-,\&c.\right\}$$

And the method is the same in all similar cases.

If the index of the proposed fluxion be fractional the series will be infinite; but when it is integral the series will terminate, and the fluent will be finite. Thus it will be found that,

$$f.\,\frac{x^m\dot{x}}{x-a}=\frac{x^m}{m}+\frac{ax^{m-1}}{m-1}+,\&c.+a^m\text{ h. log. }(x-a)$$

$$f.\,\frac{x^m\dot{x}}{x+a}=\frac{x^m}{m}-\frac{ax^{m-1}}{m-1}+,\&c.\pm a^m\text{ h. log. }(x+a)$$

m being any integer number.

FLY, in *Mechanics*, is a heavy weight applied to some part of a machine, principally in order to render its motion uniform, though it is sometimes employed for the purpose of increasing the effect, as in the coining engine. It regulates the motion, because its momentum is not easily disturbed.

FOCAL *Distance*, in the *Conic Sections*, is the distance of the focus from some fixed point; *viz.* from the vertex of the parabola, and from the centre in the ellipse and hyperbola.

FOCUS, is that point in the transverse axis of a conic section, at which the double ordinate is equal to the parameter, or to a third proportional to the transverse and conjugate axis.

These points are thus denominated from a remarkable property that obtains in all the three figures; *viz.* that if lines be drawn from the two foci, to any point in the curve, and a tangent be drawn at their point of meeting, then those two lines will form equal angles with the tangent. And since it is a known property in optics, that the angle of incidence is equal to the angle of reflection; therefore, all rays issuing from the one focus, and falling on the various points of the curve, will be reflected into the other focus, or into the line directed to the other focus, which is the case in the hyperbola, whence the denominated foci or *burning points*.

Properties relating to the foci of the conic sections may be enumerated as follows:

1. In the ellipse and parabola the transverse axis is equal to the sum of any two lines drawn from the two foci, to meet in any point of the curve. And in the hyperbola the transverse axis is equal to the difference of the same lines.

2. Again, the square of the distance between the foci is equal to the difference of the squares of the two axis, in the ellipse and parabola, but in the hyperbola it is equal to their sum.

3. In the parabola the focal distance is equal to ¼ the parameter, or ½ of the ordinate at the focus.

4. Also a line drawn from the focus to any point of the curve is equal to the focal distance *plus* the ordinate at that point.

FOCUS, in *Optics*, is a point wherein several rays concur, or are collected, after having undergone either refraction or reflection.

This point is thus denominated, because the rays being here brought together and united, their joint effect is sufficient to burn bodies exposed to their action; and hence this point is called the *focus*, or burning point. It must be observed, however, that the focus is not, strictly speaking, a point; for the rays are not accurately collected into one and the same place or point; owing to the different nature and refrangibility of the rays of light, to the imperfections in the figure of the lens and other similar impediments. The focus, therefore, is a small circle.

To find, by experiment, the focus of a convex spherical glass, being of a small sphere; apply it to the end of a scale of inches and decimal parts, of a convenient length, and expose it before the sun; upon the scale may be seen the bright intersection of the rays measured out: or, expose it in the hole of a dark chamber; and where a white paper receives the distinct representation of distant objects, there is the focus of the glass. For a glass of a pretty long focus, observe some distant object through it, and recede from the glass till the eye perceives all in confusion, or the object begins to appear inverted; then the eye is in the focus. For a plano-convex glass: make it reflect the sun against the wall; on the wall will then be seen two sorts of light, a brighter within, another more obscure; withdraw the glass from the wall, till the bright image be in its least dimensions; then is the glass distant from the wall about a fourth part of its focal length. For a double convex: expose each side to the sun in like manner; and observe both the distances of the glass from the wall: then is the first distance about half the radius of the convexity turned from the sun; and the second is about half the radius of the other convexity. The radii of the two convexities being thus known, the focus is then found by this rule: As the sum of the radii of both convexities is to the radius of either convexity, so is double the radius of the other convexity to the distance of the focus.

FORCE, in its most general and comprehensive sense, denotes whatever produces a change in the state of any body. The abstract force itself is unknown and inscrutable, and we speak of it only from the effects which it produces. Changes which are accompanied by motion or an alteration in the direction of motion, are said to be produced by *mechanical forces;* and those forces get different names according to their effects. Thus we say, the force of steam, the force of gunpowder, animal force, the force of impulse, the force of gravity, and a great many others. But while we speak thus, we merely mean that there are certain specific results arising from them in given cases; and when we say the force of steam, the force of gunpowder, or animal force, we are aware that the substance, or the animal, undergoes a change in itself at the same time that it appears to be the medium in operating a change upon something else.

All forces, however various, are measured by the effects which they produce in like circumstances, whether the effect be creating, accelerating, retarding, or deflecting motions. The result of some general and commonly observed force, is taken for unity, and with this any others may be compared; and their proportions represented by numbers or lines. Under this point of view they are considered by the mathematician; all else falls within the province of the metaphysician. When we say that a force is represented by a right line, AB, it is to be understood that it would cause a material point, situated at rest in A, to pass over the line AB, which is called the direction of the force, so as to arrive at B at the end of a given time, while another force would cause the same point to have moved a greater or less distance from A in the same time.

Mechanical forces may be reduced to two sorts; one of a body at rest, the other of a body in motion. The former is that which we conceive as residing in a body when it is supported by a plane, suspended by a rope, or balanced by the action of a spring, &c. being denominated pressure, tension, force; or *vis mortua*, *solicitatio*, *conatus movendi conumen*, and may always be estimated or measured by a weight, *viz*. the weight which sustains it. Thus, the ultimate standard to which all forces are referred is the gravitating or falling of bodies toward the earth.

This has been adopted because it is the most constant and uniform in its operation.

To mechanical forces may also be referred centripetal and centrifugal forces, though they relate to a body in motion, because these forces are homogeneous to weights, pressures, or tensions of any kind.

The force of a body in motion, is a power relating to that body, so long as it continues its motion; by means of which, it is able to remove obstacles lying in its way; to lessen, destroy, or overcome the force of any other moving body, which meets it in an opposite direction; or to surmount any the largest dead pressure or resistance, as tension, gravity, friction, &c. for some time; but which will be lessened or destroyed by such resistance as lessens or destroys the motion of the body. This is called *vis motrix*, moving force, or motive force, and by some late writers *vis viva*, to distinguish it from the *vis mortua*, spoken of before.

Concerning the measure of moving force, mathematicians have been divided into two parties. It is allowed on both hands, that the measure of this force depends partly upon the mass of matter in the body, and partly upon the velocity with which it moves: the point in dispute is, whether the force varies as the velocity, or as the square of the velocity.

Descartes, and all the writers of his time, assumed the velocity produced in a body as the measure of the force which produces it; and observing that a body, in consequence of its being in motion, produces changes in the state or motion of other bodies, and that these changes are in the proportion of the velocity of the changing body, they asserted that there is in a moving body a *vis insita*, an inherent force, and that this is proportional to its velocity; saying that its force is twice or thrice as great, when it moves twice or thrice as fast at one time as at another. But Leibnitz observed, that a body which moves twice as fast, rises four times as high, against the uniform action of gravity; that it penetrates four times as deep into a piece of uniform clay; that it bends four times as many springs, or a spring four times as strong to the same degree; and produces a great many effects which are four times greater than those produced by a body which has half the initial velocity. If the velocity be triple, quadruple, &c. the effects are 9 times, 16 times, &c. greater; and, in short, are proportional, not to the velocity, but to its square. This observation had been made before by Dr. Hooke, who has enumerated a great variety of important cases in which this proportion of effect is observed. Leibnitz, therefore, affirmed that the force inherent in a moving body is proportional to the square of the velocity; and the same principle was adopted by Wolfius, the Bernouillis, and many other mathematicians.

Leibnitz's first paper on this subject was published in the "Leipsic Acts," for the year 1686, and was answered by Catalan and Papin, from which time the dispute became very general, and was carried on for several years between Leibnitz, John and Daniel Bernouilli, Poleni, Wolfius, Gravesand, Camus, Muschenbroek, &c. on the one side; and by Pemberton, Eames, Desaguliers, Clarke, Mairan, Jurin, Maclaurin, Robins, &c. on the other, Newton having prudently avoided entering at all into the controversy.

It must be obvious to any one that this is a mere dispute about words; the one party taking one result of force as the measure, and the other party taking another. To do so is perhaps quite natural, although it is certainly not worthy of being the foundation of a philosophical controversy. It is true that a mathematical figure or quantity may be defined by any of its qualities; but this does not hold with regard to force, for in mathematical figures and quantities it is existence only that we have to consider, whereas forces are actions. We, for instance, say either a quadrilateral or a quadrangle, as the one quality is inseparable from the other; but when we speak of force as producing velocity and penetration, the one does not necessarily follow from the other, neither are they *qualities* of force. If we call the force of gunpowder that which impels a cannon-ball with a given velocity, we mean one kind of force, and if we speak of it as driving the same ball a certain distance into the earth, or elevating it a certain way into the air, we mean another; and though we have ascertained all the changes produced in the former, by altering the quantity and composition of the gunpowder, we have made no advance toward ascertaining the latter.

It is evident that a body, moving with the same velocity, has the same inherent force, whether this be employed to move another body, to bend springs, to rise in opposition to gravity, or to penetrate a mass of soft matter. Therefore these measures, which are so widely different, while each is agreeable to a numerous class of facts, are not measures of this something inherent in the moving body, which we call its force, but are the measures of its exertions when modified according to the circumstances of the case; or, to speak still more cautiously and securely, they are the measures of certain classes of phenomena consequent on the action of a moving body. It is in vain, therefore, to attempt to support either of them by demonstration. The measure itself is nothing but a definition. The Cartesian calls that *a double force which produces a double velocity in the body on which it acts*. The Leibnitzian calls that *a quadruple force which makes a quadruple penetration*. The reasonings of both in the demonstration of a proposition in dynamics may be the same, as also the result, though expressed in different numbers.

But the two measures are far from being equally proper; for the Leibnitzian measure obliges us to do continual violence to the common use of words. When two bodies moving in opposite directions meet, strike each other, and stop, all men will say that their forces are equal, because they have the best test of equality which we can devise. Or when two bodies in motion strike the parts of a machine, such as the opposite arms of a lever, and are thus brought completely to rest, we and all men will pronounce their mutual energies by the intervention of the machine to be equal. Now, in all these cases, it is well known that a perfect equality is found in the products of the quantities of matter and velocity. Thus a ball of two pounds, moving with the velocity of four feet in a second, will stop a ball of eight pounds moving with the velocity of one foot per second. But the followers of Leibnitz say, that the force of the first ball is four times that of the second.

Laplace has shown, by a very ingenious investigation, how we may experimentally be convinced of the proportionality of force to the velocity, or, at least, that since the difference must be, if any, extremely small, it is highly improbable that there should be any difference whatever.

It can moreover be shown, that if any considerable variation existed in this law, the relative motions of bodies on the earth's surface would be sensibly affected by the motion of the earth; that is, that the effect of a given force would vary very much, according as its direction coincided with, or was opposed to, the direction of the earth's motion.

The effects of the same apparent forces would vary likewise in different seasons of the year, as the velocity of the earth is about one thirtieth greater in the winter than in summer. Now, since no variation of this kind is discernible, we may safely infer the proportionality of force to velocity.

This law, and the law of *inertia*, which expresses the tendency of a body to persevere in a state of rest, or motion, may therefore be considered as derived from observation and experiment. They are the most simple and natural that can be imagined, and are sufficient to serve as a basis for the whole science of mechanics, and are accordingly adopted by all the most eminent authors of the present day.

The most general distinction of force is *mechanical* and *chemical;* the former refers to all those forces which change the shape of a body or its relative situation among other bodies, and the latter all those which alter its constitution or external structure. Thus, the forces which fell a tree, bring it home, and cleave it into billets, are all mechanical; but the force which converts it into charcoal is a chemical force; the former leave the internal structure of the wood the same as they found it; but under the operation of the latter the wood disappears, and a new substance is produced. A short distinctive definition therefore is, that mechanical forces alter the situation or form, and chemical forces the substance.

FORCE, is also distinguished into *motive* and *accelerative*, or *retardive, constant, variable*, &c.

Motive FORCE, otherwise called *momentum*, or force of percussion, is the absolute force of a body in motion, &c.; and is expressed by the product of the weight or mass of matter in the body multiplied by the velocity with which it moves.

Motive FORCE also denotes the force by which a system of bodies is put in motion, as it is the difference between the power or weight which produces the motion, and the resistance or weight to which it is opposed.

Accelerative FORCE, or retardive force, is that which respects the velocity or rate of motion only, accelerating or retarding it; and it is denoted by the quotient of the motive force, divided by the mass or weight of the body. Thus,
if m denote the motive force,
and b the body, or its weight,
and f the accelerating or retarding force, then is f as $\frac{m}{b}$.

Constant FORCE, is such as remains and acts continually the same for some determinate time. Such, for example, is the supposed force of gravity, which acts constantly the same upon a body, while it continues at the same distance from the centre of the earth, or from the centre of force, wherever that may be.

Constant or uniform forces produce uniformly accelerated motions, the laws of which will be found under the article ACCELERATION.

Variable FORCE, is that which is continually changing its effect and intensity, such as the force of gravity at different distances from the earth's centre. See the formulæ relating to variable forces, under the article ACCELERATION.

FORCES are farther distinguished into *central, centrifugal*, &c. which see under the several articles.

Animal FORCE, is that which results from the muscular power of men, horses, and other animals.

There are several curious as well as useful observations in Desagulier's "Experimental Philosophy," concerning the comparative forces of men and horses, and the best way of applying them. A horse draws with the greatest advantage when the line of direction is level with his breast; in such a situation, he is able to draw 200lb. for eight hours a-day, walking about 2½ miles an hour. But if the same horse be made to draw 240lb. he can work only six hours a-day, and cannot go quite so fast. On a carriage, indeed, where friction alone is to be overcome, a middling horse will draw 1000lb. But the best way to try the force of a horse, is to make him draw up out of a well, over a single pulley or roller; and, in that case, an ordinary horse will draw about 200lb. as before observed.

It is found, that five men are of equal force with one horse, and can, with equal ease, push round the horizontal beam of a mill, in a walk 40 feet wide; whereas, three men will do it in a walk only 19 feet wide.

The worst way of applying the force of a horse is, to make him carry or draw up hill: for if the hill be steep, three men will do more than a horse, each man climbing up faster with a burden of 100lb. weight, than a horse that is loaded with 300lb.; a difference which is owing to the position of the parts of the human body being better adapted to climb than those of a horse.

But, on the other hand, the best way of applying the force of a horse is the horizontal direction, in which a man can exert the least force: thus, a man that weighs 140lb. when drawing a boat along by means of a rope coming over his shoulders, cannot draw above 27lb. or exert above $\frac{1}{7}$th part of the force of a horse employed to the same purpose; so that, in this way, the force of a horse is equal to that of seven men.

The best and most effectual posture in a man is that of rowing; when he not only acts with more muscles at once, for overcoming the resistance, than in any other position, but also as he pulls backwards, the weight of his body assists by way of lever. See Desagulier's "Exp. Philos." v. i. p. 241, where several other observations are made relative to the force acquired by certain positions of the body; from which, that author accounts for most feats of strength and activity. See also a Memoir on this subject by M. De Lahire, in the "Mem. Roy. Acad." 1729; or in Desaguliers's "Exp." &c. p. 267, &c. who has published a translation of part of it with remarks.

FORMULA, or *Form*, in *Analysis*, is any general theorem or literal expression, and is said to be algebraical, logarithmic, trigonometrical, &c. according as it relates to either of these subjects.

Thus $x = a \pm \sqrt{(a^2+b)}$

$$x = \sqrt[3]{\left\{\frac{b}{2} + \sqrt{\left(\frac{b^2}{4} + \frac{c^3}{27}\right)}\right\}} +$$

$$\sqrt[3]{\left\{\frac{b}{2} - \sqrt{\left(\frac{b^2}{4} + c^3\right)}\right\}}$$

are algebraical formulæ, exhibiting the roots of all quadratic and cubic equations in the forms

$$x^2 - 2ax - b = 0$$

$$x^3 + ax - b = 0.$$

FOUNTAIN, or *Artificial* Fountain, in *Hydraulics*, a machine or contrivance by which water is violently spouted or darted up. It is also called a *Jet d'eau*.

There are various kinds of artificial fountains, but they are all formed by a pressure of one sort or another upon the water; *viz.* either the pressure or weight of a head of water, or the pressure arising from the spring and elasticity of the air, &c. When these are formed by the pressure of a head of water, or any other fluid of the same kind with the fountain, or jet, then will this spout up nearly to the same height as that head, abating only a little for the resistance of the air, with that of the adjutage, &c. in the fluid rushing through; but, when the fountain is produced by any other force than the pressure of a column of the same fluid with itself, it will rise to such a height as is nearly equal to the altitude of a column of the same fluid, whose pressure is equal to the given force that produces the fountain.

Circulating Fountain of *Hero*. This is so called because it was contrived by him, is represented (Plate III. fig. 2,) in this, the air being only compressed by the concealed fall of water, makes a jet, which, after some continuance, is considered by those who are unacquainted with the principle as a perpetual motion, because they imagine that the same water which fell from the jet rises again. The boxes C E and D Y X being close, we see only the bason A B W, with a hole at W, into which the water spouting at B falls; but that water does not come up again; for it runs down through the pipe W X into the box D Y X, from which it drives out the air, through the ascending pipe Y Z, into the cavity of the box C E, where, pressing upon the water that is in it, it forces it out through the spouting pipe OB, as long as there is any water in C E; so that this will play no longer than whilst the water contained in C E, having spouted out, falls down through the pipe W X into the cavity D Y X. The force of the jet is proportional to the height of the pipe W X, or of the boxes CE and DY above one another: the height of the water, measured from the bason A B W to the surface of the water in the lower box D Y X, is always equal to the height measured from the top of the jet to the surface of the water in the middle cavity at CE. Now, since the surface CE is always falling, and the water in DY always rising, the height of the jet must continually decrease, till it is shorter by the height of the depth of the cavity CE, which is emptying, added to the depth of the cavity DY, which is always filling; and when the jet is fallen so low, it immediately ceases. The air is represented by the points in this figure. To prepare this fountain for playing, which should be done unobserved, pour in water at W, till the cavity D X Y is filled; then invert the fountain, and the water will run from the cavity D X Y into the cavity CE, which may be known to be full, when the water runs out at B held down. Set the fountain up again, and, in order to make it play, pour in about a pint of water into the bason A B W; and, as soon as it has filled the pipe W X, it will begin to play, and continue as long as there is any water in CE. You may then pour back the water left in the bason A B W, into any vessel, and invert the fountain, which, being set upright again, will be made to play, by putting back the water poured out into A B W; and so on as often as you please.

FOUNTAIN *Pen*, is a pen contrived to hold a greater quantity of ink than usual, so as to supply the writer with it for a considerable time, without the trouble of taking a fresh supply.

FRACTION, in arithmetic and algebra, is an expression for the division of one quantity by another, when that division either is not, or cannot be expressed by a separate quantity or quotient. It is called a fraction or part, from the circumstance of dividing or separating into parts, being involved in it. The dividend is written over the divisor, the former being called the numerator, and the latter the denominator; and it follows, from the nature of division, that the value of the fraction is less, equal to, or greater than one, according as the numerator is less, equal to, or greater than the denominator. Also, since in numbers the quotient contains one for every time as many one's as are in the divisor is contained in the dividend, one in the numerator, is the same part of the integer number one's as there are one's in the divisor, thus $\frac{1}{8}$ means one eighth part of the integer number one.

Fractional expressions are usually distinguished into *proper*, *improper*, *simple*, *compound*, and *mixed*.

A *Proper* FRACTION, is that of which the numerator is less than the denominator, and consequently the fraction itself less than unity; as $\frac{3}{4}$, $\frac{5}{4}$, $\frac{6}{11}$, &c.

An *Improper* FRACTION has its numerator either equal to, or greater than its denominator; and the fraction is accordingly either equal to, or greater than unity, as $\frac{5}{5}$, $\frac{6}{5}$, $\frac{71}{19}$, &c.

A *Simple* FRACTION, is that which consists of a single numerator, and single denominator; and is either proper, or improper, as $\frac{5}{7}$, $\frac{11}{7}$, $\frac{16}{81}$, &c.

A *Compound* FRACTION, is a fraction consisting of two or more other fractions connected by the word *of;* thus $\frac{2}{3}$ of $\frac{3}{4}$, or $\frac{2}{5}$ of $\frac{7}{11}$ of $\frac{6}{9}$, &c. are compound fractions. This is nothing but another name for the product of two fractions.

A *Mixed* FRACTION, is that whose numerator and denominator are both fractions; thus $\dfrac{\frac{3}{4}}{7\frac{1}{2}}$ is a complex fraction.

These two distinctions, though frequently made by authors on arithmetic, are certainly improper, the former indicating an operation in multiplication, and the latter an operation in division; it is, therefore, improper to apply to them the denomination of fractions.

An integer and fraction together is called a mixed number, that is, $7\frac{3}{4}$, $9\frac{1}{9}$, &c. are mixed numbers.

For the several operations in Fractions, as ADDITION, SUBTRACTION, MULTIPLICATION, DIVISION, REDUCTION, &c. See the several articles.

FREEZING, *Congelation*, in *Philosophy*, the transformation of a fluid body into a firm or solid mass, by the action of cold; in which latter sense the term is applied to water when it freezes into ice; to metals when they resume their solid form after being melted by heat; or to glass, wax, pitch, tallow, &c. when they harden again after having been rendered fluid by heat. This is more a chemical than a mechanical operation; still, it is sometimes attended with mechanical effects, as in the bursting of vessels, the splitting of rocks, &c.

The process of congelation is always attended with the emission of heat, as is found by experiments on the freezing of water, wax, spermaceti, &c.; for in such cases it is always found that a thermometer dipped into the fluid mass keeps continually descending as this cools, till it arrive at a certain point, being the point of freezing, which is peculiar to each fluid, where it is rather stationary, and then rises a little, while the congelation goes on, at the same time the bulk of the body is expanded.

The prodigious power of expansion evinced by water in the act of freezing is nearly double that of the most powerful steam-engines, and exerted in so small a mass, seemingly by the force of cold, was thought a very material argument in favour of those who supposed that cold, like heat, is a positive substance.

Freezing *Rain*, or *Raining Ice*, a very uncommon kind of shower, which fell in the west of England, in December, 1672, of which we have various accounts in the Philosophical Transactions. This rain, as soon as it touched any thing above ground, as a bough, &c. immediately settled into ice; and, by multiplying and enlarging of the icicles, broke all down with its weight. The rain that fell on the snow immediately froze into ice, without sinking in the snow at all. It made an incredible destruction of trees, beyond any thing recorded in history.

FRICTION, the act of rubbing two bodies together, or the resistance in machines caused by the motion of the different parts against each other.

Friction arises from the roughness or asperity of the surface of the bodies moved; for such surfaces consisting alternately of small eminences and cavities, these act against each other, and prevent the free motion that would ensue, on a supposition of the two bodies being perfectly smooth.

Amontons, a member of the French academy, seems to have been the first who made experiments connected with this subject. He found that the resistance opposed to the motion of a body upon an horizontal surface was exactly proportional to its weight, and was equal to one-third of it, or more generally to one-third of the force with which it was pressed against the surface over which it was passed. He discovered also that this resistance did not increase with an increase of the rubbing surface, nor with the velocity with which it moved.

The experiments of M. Bulfinger authorized conclusions similar to those of Amontons, with this difference only, that the resistance of friction was equal only to one-fourth of the force with which the rubbing surfaces were pressed together.

This subject was also considered by Parent, who supposed that friction is occasioned by small spherical eminences in one surface being dragged out of corresponding spherical cavities in the other, and proposed to determine its quantity by finding the force which would move a sphere standing upon three equal spheres. This force was found to be to the weight of the sphere as 7 to 20, or nearly one-third of the sphere's weight. In investigating the phenomena of friction, M. Parent placed the body upon an inclined plane, and augmented or diminished the angle of inclination till the body had a tendency to move, and the angle at which the motion commenced he called the angle of equilibrium. The weight of the body, therefore, will be to its friction upon the inclined plane, as radius to the sine of the angle of equilibrium; and its weight will be to the friction on a horizontal plane, as radius to the tangent of the equilibrium.

Euler seems to have adopted [illegible] hypothesis of Bulfinger respe the ration of friction to the force of pression; and in two curious dissertations which he has published upon this subject, has suggested many important observations, to which Mr. Vince seems afterwards to have attended. He observes, that when a body is in motion, the effect of friction will be only one half of what it is when the body has begun to move; and he shows that if the angle of an inclined plane be gradually increased, till the body which is placed upon it begins to descend, the friction of the body at the very commencement of its motion will be to its weight or pressure upon the plane, as the sine of the plane's elevation is to its cosine, or as the tangent of the same angle is to radius, or as the height of the plane is to its length. But when the body is in motion the friction is diminished, and may be found by the following equation

$$\mu = \text{tan. } a \frac{m}{15625\, nn \text{ cos. } a},$$ in which

μ is the quantity of friction, the weight or pressure of the body being $= 1$, a is the angle of the plane's inclination, m is the length of the plane in 1000th parts of a rhinland foot, and n the time of the body's descent.

The Abbé Nollet and Bossut have distinguished friction into different kinds, that which arises from one surface being dragged over another, and that which is occasioned by one body rolling upon another. The resistance which is generated by the first of these kinds of friction, is always greater than that which is produced by the second; and it appears evidently from the experiments of Mushenbrock, Schoeber, and Meister, that when a body is carried along with an uniformly accelerated motion, and retarded by the first kind of friction, the spaces are still proportional to the squares of the times; but when the motion is affected by the second kind of friction, this proportionality between the spaces and the times of their description does not obtain.

The subject of friction has more lately occupied the attention of Professor Vince of Cambridge. He found that the friction of hard bodies in motion is an uniformly retarding force; and that the quantity of friction, considered as equivalent to a weight drawing the body backwards, is equal to

$$M \times \frac{M \times W \times S}{rt^2},$$ where M is the

moving force expressed by its weight, W the weight of the body upon the horizontal plane, S the space through which the moving force or weight descended in the time t and $r = 16{\cdot}087$ feet, the force of gravity. Mr. Vince also found that the quantity of friction increases in a less ratio than the quantity of matter or weight of the body, and that the friction of a body does not continue the same when it has different surfaces applied to the plane on which it moves, but that the smallest surfaces will have the least friction.

Notwithstanding these various attempts to unfold the nature and effects of friction, it was reserved for the Columb to surmount the difficulties which are inseparable from such an investigation, and to give an accurate and satisfactory view of this complicated part of mechanical philosophy. By employing large bodies and ponderous weights, and conducting his experiments on a large scale, he has corrected several errors which necessarily arose from the limited experiments of preceding writers: he has brought to light many new and striking phenomena, and confirmed others which were hither to but partially established. A few are as follow:

1. The friction of bodies of the same kind moving upon each other, is generally supposed to be greater than that of heterogeneous bodies; but Columb has shown that there are exceptions to this rule.

2. It was generally supposed that in the case of wood, the friction is greatest when the bodies are drawn contrary to the course of their fibres; but Columb has shewn that the friction is in this case sometimes the smallest.

3. The longer the rubbing surfaces remain in contact, the greater is their friction.

4. Friction is in general proportional to the force with which the rubbing surfaces are pressed together, and is commonly equal to between one-half and one-quarter of that force.

5. Friction is not generally increased by augmenting the rubbing surfaces.

6. Friction is not increased by an increase of velocity, at least it is not generally so; and even in some cases the friction decreases with an increase of celerity.

7. The friction of cylinders rolling upon an horizontal plane is in the direct ratio of their weights, and in the inverse ratio of their diameters.

From a variety of experiments on the friction of the axis of pulleys, Columb also obtained the following results:—When an iron axle moved in a brass bush or bed, the friction was $\frac{1}{6}$ of the pression; but when the bush was besmeared with very clean tallow, the friction was only $\frac{1}{11}$; when swine-grease was interposed, the friction amounted to $\frac{1}{8\cdot5}$; and when olive-oil was employed as an unguent, the friction was never less than $\frac{1}{8}$, or $\frac{1}{7\cdot5}$. When the axis was of green oak and the bush of guaiacum-wood, the friction was $\frac{1}{26}$ when tallow was interposed; but when the tallow was removed, so that a small quantity of grease only covered the surface, the friction was increased to $\frac{1}{17}$. When the bush was made of elm, the friction was in similar circumstances $\frac{1}{33}$ and $\frac{1}{20}$ which is the least of all. If the axis be made of box, and the bush of guaiacum-wood, the friction was $\frac{1}{23}$ and $\frac{1}{14}$, circumstances being the same as before. If the axle be of box-wood, and the bush of elm, the friction will be $_{29}$ and $\frac{1}{20}$; and if the axle be of iron, and the bush of elm, the friction will be $\frac{1}{20}$ of the force of pressure.

An accurate knowledge of the laws and quantity of friction is of the utmost consequence to the practical mechanic; and when he estimates the effect of any force acting through the medium of a train of machinery, it will be necessary to allow, upon an average, $\frac{1}{3}$ for friction.

FRUSTUM, in *Geometry*, is the part of a solid next the base, left by cutting off the top or segment, by a plane parallel to the base.

To find the Solid Content of the Frustum of a Cone or Pyramid.

Add into one sum the areas of the two ends and the mean proportional between them; then $\frac{1}{3}$ of that sum will be a mean area, or the area of an equal prism, of the same altitude with the frustum; and, consequently, that mean area being multiplied by the height of the frustum, the product will be the solid content of it. That is, if A denote the area of the greater end, a that of the less, and h the height; then $(A + a + \sqrt{Aa}) \times \frac{1}{3}h$ is the solidity.

Or if S, s, be the sides of the bottom and top of the frustum, and t, the tabular number for that figure, then

$(S^2 + sS + s^2) \times \frac{1}{3}th =$ solidity of Frust. of Pyram.

and $(D^2 + dD + d^2) \times \frac{1}{3}hp =$ solidity of Frust. of Cone, where D and d are the diameters, and $p = \cdot7854$ reversed.

The Curve Surface of the Zone or Frustum of a Sphere, is had by multiplying the circumference of the sphere by the height of the frustum. Hutton's Mensuration, p. 197.

And the Solidity of the same Frustum is found, by adding together the squares of the radii of the two ends, and $\frac{1}{3}$ of the square of the height of the frustum; then multiplying the sum by the said height and by the number 1·5708; that is, $(R^2 + r^2 + \frac{1}{3}h^2) \times \frac{1}{2}ph$ is the solid content of the spheric frustum, whose height is h, and the radii of its ends R and r, p being $= 3\cdot1416$.

FULCRUM, in *Mechanics*, the prop or support upon which a lever is sustained.

FUNCTION. A quantity is said to be a function of another quantity, when its value depends on that quantity and known quantities only; and it is said to be a function of several quantities, when its value depends on those quantities and known quantities only.

Thus, if $y = x^m$, $y = (a + bx)^m$, $y = a^x$, $y =$ log. x, &c.; or, in the equation

$x^n + (a + by)\,.x^{n-1} + (a + dy + ey^2)\,.x^{n-2} +$, &c. $= o$ y is said to be a function of x. In the former case y is called an explicit, and in the latter an implicit function of x. And if

$y = ax^m + bz^n$, $y = a^x + b^z$, $y =$ log. $x +$ log. z, &c. or in the equation

$$\begin{aligned} & x^n + (a + by)\,x^{n-1}, +, \text{\&c.} \\ & \quad + (a' + b'y)\,.z^{n-1} +, \text{\&c.} \\ & \quad + (\alpha + \beta y + \gamma y^2 \ldots \gamma y^p)\,.x^m z^{n-p-m} +, \text{\&c.} = o \end{aligned}$$

y is a function of x and z. a, b, &c. m, n, &c. being constant quantities.

Such being the meaning of the term, algebra, in general, might be considered as the science of functions, since in it the various relations of the values of quantities are the objects of investigation; and in this manner it was customary to regard functions, until Lagrange asserted that the theory of the expansion of functions, in series, contained the true principles of the differential or fluxional calculus; and by this theory demonstrated Taylor's celebrated theorem, which he justly regards as the fundamental principle of the calculus. He afterwards collected and generalized his ideas on this subject, and published them in a work entitled "Théorie des Fonctions Analytiques," which appeared in 1797; to this, in 1806, he added a commentary and supplement, under the title of "Leçons sur le Calcul des Fonctions." Since this, the investigation of the properties of the functions which result from the expansion of any function in a series, on attributing an increase to one or more quantities in that function, has been considered as the calculus of functions.

The calculus of functions has the same object as the differential calculus, taken in the most extended sense; but it is not subject to the difficulties which are met with in the principles and ordinary course of this calculus: it besides serves to connect the differential calculus immediately with algebra.

The chief object of the calculus of functions is to establish the principles of the fluxional or differential calculus, without having recourse to velocities, or to vanishing or infinitely small quantities.

Let fx denote any function of x; then if x be increased by an indeterminate quantity i, fx will become $f\,(x + i)$; which, according to Lagrange, may be expanded in a series $fx + pi + qi^2 +$, &c. where p, q, &c. are functions of x, independent of i.

To ascertain the manner in which the functions p, q, r, &c. are derived from fx, let us suppose that x is again increased by the indeterminate quantity o, which is independent of i: $f\,(x + i)$ then becomes $f\,(x + o + i)$; and it is evident that the result must be the same, whether we suppose that it is $x + o$ which is increased by i, or that it is x which is increased by $o + i$ or $i + o$; that is,

$$f\left\{(x + o) + i\right\} = f\left\{x + (i + o)\right\}$$

Now since $f\,(x + i) = fx + pi + qi^2 + ri^3$, &c. and that p, q, r, &c. are independent of i, we have

$$\begin{aligned} f\left\{x + (i + o)\right\} &= fx + p\,.(i + o) + q\,(i + o)^2 \\ &\quad + r\,(i + o)^3 + s\,(i + o)^4 + \text{\&c.} \\ &= fx + pi + qi^2 + ri^3 + si^4 + \text{\&c.} \\ &\quad + po + 2qoi + 3roi^2 + 4soi^3 + \text{\&c.} \\ &\quad + qo^2 + \text{\&c.} \\ &\quad + ro^3 + \text{\&c.} \\ &\quad + so^4 + \text{\&c.} \end{aligned}$$

and $f\,(x + o) = fx + po + qo^2 + ro^3 + so^4 +$ &c.

As p, q, &c. are functions of x, we may suppose that when x becomes $x + o$, they will become $p + p'o +$ &c. $q + q'o +$ &c. p', q', &c. being likewise functions of x, independent of o; hence

$$\begin{aligned} f\left\{(x + o) + i\right\} &= f\,(x + o) + (p + p'o + \text{\&c.})\,i + \\ &\quad (q + q'o + \text{\&c.})\,.i^2 + (r + r'o + \text{\&c.})\,i^3 + \text{\&c.} \\ &= fx + pi + qi^2 + ri^3 + si^4 + \text{\&c.} \\ &\quad + po + p'oi + q'oi^2 + roi^3 + \text{\&c.} \\ &\quad + qo^2 + \text{\&c.} + \text{\&c.} + \text{\&c.} \\ &\quad + ro^3 + \text{\&c.} + \text{\&c.} \\ &\quad + so^4 + \text{\&c.} \end{aligned}$$

And since this series must be identical with the former, the terms affected with the same powers of i and o must be the same, independent of any values of i and o; consequently we shall have,

$$2q = p', \ 3r = q', \ 4s = r', \text{ \&c.}$$

$$\text{or } q = \frac{p'}{2}, \ r = \frac{q'}{3}, \ s = \frac{r'}{4}, \text{ \&c.}$$

Let $f'x$ denote the co-efficient of i, or that part of the second term in the expansion of $f(x+i)$ which is independent of i; and in like manner let $f''x, f'''x, f^{iv}x$, &c. denote the co efficients of i in the expansions of $f'(x+i), f''(x+i), f'''(x+i)$, &c.

Thus we have $p = f'x$; now p' is the co-efficient of i in the expansion of p, when in p, x becomes $x+i$; that is, p' is the co-efficient of i in the expansion of $f'(x+i)$, or $p' = f''x$ and $\therefore q = \frac{f''x}{2}$; in the same manner, q' is the co-efficient of i in the expansion of $\frac{f''(x+i)}{2}$, or $q' = \frac{f'''x}{2}$; and $\therefore r = \frac{f'''x}{2.3}$; likewise $s = \frac{f^{iv}x}{2.3.4}$, &c. Substituting these expressions for p, q, r, &c. in the series $fx + pi + qi^2 + ri^3 +$, &c. we shall get this fundamental formula,

$$f(x+i) = fx + f'x \,.\, i + \frac{f''x}{2} \,.\, i^2 + \frac{f'''x}{2.3} i^3 + \frac{f^{iv}x}{2.3.4} \,.\, i^4, \text{ \&c.}$$

where $f'x, f''x, f'''x, f^{iv}x$, &c. are derived from $fx, f'x, f''x, f'''x$, &c.; in the same manner, namely, by taking the co-efficient of i in the expansions of $f(x+i)$, $f'(x+i)$, $f''(x+i)$, $f'''(x+i)$, &c.

Hence, if for every function of x, we can find the co-efficient of i in the expansion of $f(x+i)$, we may find the co-efficients of all the other powers of i.

fx is called the primitive function with regard to $f'x, f''x, f'''x$, &c. which are derived from it; and these are called derived functions with regard to fx.

$f'x$ is called the first function, $f''x$ the second function, $f'''x$ the third function, &c. of fx.

If a letter y be put for the function of x, and y', y'', y''', &c. denote the first, second, third, &c. functions of y; then when x becomes $x+i$, y will become

$$y + y'i + \frac{y''}{2} \,.\, i^2 + \frac{y'''}{2.3} i^3 +, \text{ \&c.}$$

We may here observe, that $f'x$, $f''x$, &c. y', y'', &c. are the same as the fluxional co-efficients

$$\frac{(fx)^{\cdot}}{\dot{x}}, \quad \frac{(fx)^{\cdot\cdot}}{\dot{x}^2}, \text{ \&c.}; \ \frac{\dot{y}}{\dot{x}}, \ \frac{\ddot{y}}{\dot{x}^2}, \text{ \&c.};$$

or as the differential co-efficients $\frac{dfx}{dx}$, $\mathrm{D} \,.\, fx$; $\frac{d^2fx}{dx^2}$, $\mathrm{D}^2 fx$, &c.; $\frac{dy}{dx}$, $\mathrm{D} \,.\, y$; $\frac{d^2y}{dx^2}$, $\mathrm{D}^2 y$, &c.; and that the above formula agrees with Taylor's theorem, that z flowing uniformly and becoming $z+v$, x becomes x

$$+ \dot{x} \,.\, \frac{v}{1.\dot{z}} + \ddot{x} \frac{v^2}{1.2\,\dot{z}^2} + \dddot{x} \frac{v^3}{1.2.3\,\dot{z}^3} +, \text{ \&c.}$$

We will now illustrate what has been said by applying it to the expansions of the functions x^m, a^x and log. x, when x becomes $x+i$. First let $fx = x^m$, then $f(x+i) = (x+i)^m$; and it is easy to show, by the first operations of algebra, that the two first terms of the expansion will be $x^m + mx^{m-1}i$, whether m be an integer or a fraction, positive or negative; consequently $f'x = mx^{m-1}$; again, the co-efficient of i, in $(x+i)^{m-1}$ expanded, is $(m-1) \,.\, x^{m-2}$; $\therefore f''x = m \,.\, (m-1) \,.\, x^{m-2}$: and in like manner $f'''x = m \,.\, (m-1) \,.\, (m-2) \,.\, x^{m-3}$, &c.

And since

$$f(x+i) = fx + f'xi + \frac{f''x}{2} i^2 +, \text{ \&c.}$$

$$(x+i)^m = x^m + mx^{m-1}i + \frac{m(m-1)}{2} x^{m-2} i^2 + \frac{m \,.\, (m-1) \,.\, (m-2)}{2.3} \,.\, x^{m-3} i^3, \text{ \&c.}$$

which is the well-known binomial theorem.

Next let $fx = a^x$, then $f(x+i) = a^{x+i} = a^x \,.\, a^i$; now $a^i = (1 + a - 1)^i = 1 + i \,.\, (a-1)$

$$+\frac{i.(i-1)}{2}.(a-1)^2+\frac{i.(i-1).(i-2)}{2.3}.(a-1)^3+, \&c. = 1 + \Big\{(a-1)-\tfrac{1}{2}(a-1)^2+\tfrac{1}{3}(a-1^3-, \&c.\Big\}\, i, \&c.$$

omitting the following terms, since it is only the co-efficient of i which is required; or if we put

$$A=(a-1)-\tfrac{1}{2}(a-1)^2+\tfrac{1}{3}(-1)^3-, \&c.$$

$$a^i=1+A\,i+, \&c.$$

$$\therefore a^{x+i}=a^x+A\,a^x.\,i, \&c.$$

consequently the co-efficient of i in a^{x+i} expanded is $A\,a^x$, or $f^I x = A\,a^x$; hence $f^{II}x$, being the co-efficient of i in $A\,a^{x+i}$, expanded is $A^2\,a^x$; and in the same manner, $f^{III}x = A^3\,a^x$, &c. Substituting these, we shall have

$$a^{x+i}=a^x+A\,a^x.\,i+\frac{A^2\,a^x}{2}.\,i^2+\frac{A^3\,a^x}{2.3}.\,i^3+, \&c. = a^x.(1+A\,i+\frac{A^2}{2}.\,i^2+\frac{A^3}{2.3}.\,i^3+, \&c$$

Here, if we divide by a^x, we get

$$a^i=1+A\,i+\frac{A^2}{2}.\,i^2+\frac{A^3}{2.3}.\,i^3+, \&c. \text{ or,}$$

$$a^x=1+Ax+\frac{A^2}{2}.\,x^2+\frac{A^3}{2.3}.\,x^3+, \&c.$$

which is the series for the expansion of a^x.

Suppose now $x=1$, then we have

$$a=1+A+\frac{A^2}{2}+\frac{A^3}{2.3}+, \&c.$$

Let e be the value of a when $A=1$; then

$$e=1+1+\frac{1}{2}+\frac{1}{2.3}+, \&c. = 2.7182818284590452.$$

Again, if $x=\frac{1}{A}$

$$a^{\frac{1}{A}}=1+1+\frac{1}{2}+\frac{1}{2\ 3}+, \&c. = e$$

$\therefore a = e^A$, or A is the logarithm of a to the base e.

If $l.x$ denote the logarithm of x to the base e, that is the Naperean or hyperbolic logarithm of x; and $L.x$ denote the logarithm of x to the base a; then

$$x=e^{l.x} \text{ and } x=a^{Lx}.$$

But $a^{Lx}=e^{A.L.x}=e^{l.a\,L\,x}$ $\therefore l.x = l.a.Lx.$

By which equation, the logarithm of a number being given, in any system of logarithms, it may be converted into the Naperean logarithm of that number, and *vise versâ.*

If $X=(x-1)-\tfrac{1}{2}(x-1)^2+\tfrac{1}{3}(x-1)^3+, \&c.$

Then

$$x^z=1+Xz+\frac{X^2}{2}.\,z^2+\frac{X^2}{3}.\,z^3+, \&c.$$

and if $z=\frac{1}{X}$, then

$$x^{\frac{1}{X}}=1+1+\frac{1}{2}+\frac{1}{2.3}+, \&c. = e = a^{\frac{1}{A}}$$

and $x=e^X$, or $x=a^{\frac{X}{A}}$ $\therefore l.x = X$, and $Lx=\frac{X}{A}$; that is

$$l.x=(x-1)-\tfrac{1}{2}(x-1)^2+\tfrac{1}{3}(x-1)^3-, \&c.$$

$$L.x=\frac{1}{A}.\Big\{(x-1)-\tfrac{1}{2}(x-1)^2+\tfrac{1}{3}(x-1)^3-, \&c.\Big\}$$

We will now proceed to the expansion of $L(x+i)$.

$$a^{L(x+i)}=x+i=x\Big(1+\frac{i}{x}\Big)$$

$$=a^{Lx}\times a^{L\left(1+\frac{2}{x}\right)}$$

$$=a^{Lx+L\left(1+\frac{i}{x}\right)}$$

$$\therefore L(x+i)=Lx+L\Big(1+\frac{i}{x}\Big)= Lx+\frac{1}{A}\Big\{\Big(\frac{i}{x}\Big)-\frac{1}{2}\Big(\frac{i}{x}\Big)^2+\frac{1}{3}\Big(\frac{i}{x}\Big)^3-, \&c.\Big\}$$

$$= Lx + \frac{1}{A}\left\{\frac{i}{x}\cdot i - \frac{1}{2x^2}\cdot i^2 + \frac{1}{3x^3}\cdot i^3 -, \&c.\right\}$$

Let us now proceed to find the derived function of the function of a quantity, which quantity is itself a function of x, and likewise the derived functions of the function of two or more functions of x.

First, let $y = fp$, p being a function of x; if p be increased by o, then fp will become $fp + f'p\,.\,o + \frac{f''p}{2}\,.\,o^2 +$, &c. But p being a function of x when x becomes $x+i$, p becomes $p + p'i + \frac{p''}{2}\,.\,i^2 +$, &c. $\therefore o = p'i + \frac{p''}{2}i^2 +$, &c. and if this be substituted for o, fp will become $fp + p'f'p\,.\,i +$

$$\left(\frac{p'^2 f''p + p''f'p}{2}\right).\,i^2 +, \&c.$$

$$\therefore y' = p'f'p$$

Now let z be a function of p and q, which are functions of x. Suppose that this function is represented by $f(p,q)$; and that $f'(p)$ denotes the first function of $f(p,q)$, considered as a function of p only, q being regarded as constant; and $f'(q)$ denotes the first function of $f(p,q)$, considered in like manner as a function of q only. It is evident that the result must be the same, whether we substitute $x+i$ for x, in p and q, at the same time, or first substitute it in p and then in q, in the result of the first substitution. Therefore substituting $x+i$ for x in p only, $f(p,q)$ will become $f(p,q) + p'f'(p)\,.\,i +$, &c.; and substituting $x+i$ for x in q only, in this expression it will become

$$f(p,q) \begin{matrix} + p'f'(p) \\ + q'f'(q) \end{matrix}\Big\} i +, \&c.$$

$$\therefore z' = p'f'(p) + q'f'(q)$$

and by taking the first function of this we get z'', and so on.

From this we likewise obtain y'' in the foregoing case; for

$$y'' = (p'f'p)' = p'(f'p)' + p''f'p = p'\,.\,p'f''p + p''f'p = p'^2 f''p + p''f'p$$

and taking the first function of this, we get y''' and so on.

If y be an implicit function of x, y' may be determined, thus:

Let the equation which determines the value of y be denoted by $F(x,y) = o$; then y must be some function of x, which we will denote by fx. Substituting this for y we shall have $F(x, fx)$, a function of x alone, which let be denoted by ϕx, and which is $= o$, whatever may be the value of x. Consequently it will be $= o$ when $x + i$ is substituted for x. Therefore we shall have

$$\phi x + \phi' x i + \frac{\phi'' x}{2}\,.\,i^2 + \&c. = o.$$

And this must be independent of any particular value of i, each of the co-efficients must be $= o$, or $\phi x = o$, $\phi' x = o$, $\phi'' x = o$, &c.

Since $\phi x = F(x, fx) = F(x, y)$, $\phi' x$ will be the first function of $F(x,y)$, y being considered as a function of x.

$$\therefore F'(x) + y'F'(y) = F'(x,y) = \phi' x = o$$

$$\text{and } y' = -\frac{F'(x)}{F'(y)}.$$

We shall get y'' by taking the first function of this, and so on.

The equations $F'(x,y) = o$, $F''(x,y) = o$, $F'''(x,y) = o$, &c. are called first, second, third, &c. derived equations of the primitive $F(x,y) = o$. And any equation formed by a combination of $F'(x,y) = o$, and $F(x,y) = o$, is called an equation of the first order; of $F''(x,y) = o$, $F'(x,y) = o$, and $F(x,y) = o$, an equation of the second order, and so on.

Having shown how the several derived functions of y may be found, when $y = f(p,q)$, p and q being each functions of the same quantity x, and consequently how $f,(p,q)$, may be expanded when in p and q, x becomes $x + i$; let us now consider how we shall find the expansion of a function of two quantities, which are independent of each other, when each is increased by an indeterminate quantity, for instance, of $f,(x,y)$, when x becomes $x + i$, and y becomes $y + o$. Here, as in the former case, we may suppose the changes in x and y to take place separately, that is, that y remains constant while x becomes $x+i$; and then that x remains constant while y becomes $y+o$. We shall thus have different derived functions of $f,(x,y)$; first, on the supposition that $f(x,y)$, is a function of x only, y being constant; and, secondly, that it is a function of y only, x being constant. Not to confound these different derived functions, we will denote those derived on the supposition of x being alone variable, thus $f^{\prime\prime}(x,y)$, $f^{\prime\prime\prime}(x,y)$; those derived on the supposition of y being alone variable, thus $f^{\prime/}(x,y)$, $f^{\prime//}(x,y)$, &c.; and those derived on the supposition partly of x variable, and partly of y variable, thus $f^{\prime\prime\prime}(x,y)$, $f^{\prime\prime\prime/}(x,y)$, $f^{/\prime//}(x,y)$, &c.

Thus we shall have in the first place $f(x+i,y)=f(x,y)+f^{\prime}(x,y)\,.\,i+\frac{f^{\prime\prime}(x,y)}{2}\,.\,i^2+\frac{f^{\prime\prime\prime}(x,y)}{2\,.\,3}\,.\,i^3+$ &c.; and substituting $y+o$ for y we shall have

$$f(x+i,\ y+o)=f(x,\ y+o)+f^{\prime},(x,y+o)\,.\,i+\frac{f^{\prime\prime}(x,y+o)}{}\,.\,i^2+\frac{f^{\prime\prime\prime}(x,y+o)}{2\,.\,3}\,.\,i^3,\ \&c.$$

$$=f(x,y)+f^{\prime}(x,y)\,.\,i+\frac{f^{\prime\prime}(x,y)}{2}\,.\,i^2$$
$$+\frac{f^{\prime\prime\prime}(x,y)}{2\,.\,3}\,.\,i^3+\&c.$$
$$+f^{/}(x,y)\,.\,o+f^{\prime/}(x,y)\,.\,i\,o+\frac{f^{\prime\prime/}(x,y)}{2}\,.\,i^2o+\&c.$$
$$+\frac{f^{//}(x,y)}{2}\,.\,o^2$$
$$+\frac{f^{\prime//}(x,y)}{2}\,.\,i\,o^2+\&c.$$
$$+\frac{f^{///}(x,y)}{2\,.\,3}\,.\,o^3+\&c.$$

in which the general form of the terms is

$$\frac{f^{m,n}(x,y)}{(1\,.\,2\,.\,3\,.\,.\,.\,.\,m)\,.\,(1\,.\,2\,.\,3\,.\,.\,.\,.\,n)}\,.\,i^m\,.\,o^n$$

where m and n denote the order of the function.

It is evdient, that we must have obtained the same result, if we had first supposed y to become $y+o$, and then x to become $x+i$; consequently, to obtain the derived function $f^{\prime/}(x,y)$, it will make no difference whether we first get $f^{\prime}(x,y)$, and then the first function of this relative to y, $f^{\prime/}(x,y)$; or, first get $f^{/}(x,y)$, and then the derived function of this relative to x, $f^{\prime/}(x,y)$; and the same of $f^{\prime\prime/}(x,y)$, $f^{\prime//}(x,y)$, &c. It therefore appears that the operations which are denoted by $f^{\prime/}$ $f^{\prime\prime/}$ &c. may be performed in any order without at all affecting the results. For example, let

$$f\,(x,y)=\frac{x^3}{y^2}\ \text{then}$$

$$f^{\prime}(x,y)=\frac{3x^2}{y^2},\quad\text{and}$$

$$f^{\prime/}(x,y)=-\frac{6x^2}{y^3};\ \text{or}$$

$$f^{/}(x,y)=-\frac{2x^3}{y^3},\ \text{and}$$

$$f^{\prime/}(x,y)=-\frac{6x^3}{y^3};\ \text{also}$$

$$f^{\prime\prime}(x,y)=\frac{6x}{y^2},\ \text{and}$$

$$f^{\prime\prime/}(x,y)=-\frac{12x}{y^3};$$

$$f^{//}(x,y,=\frac{6x^3}{y^4},\ \text{and}$$

$$f^{\prime//}(x,y)=\frac{18x^2}{y^4}:$$

and the first function of $f^{\prime/}(x,y)$ relative to x, is $-\frac{12x}{y^3}$, and relative to y is $\frac{18x^2}{y^4}$, and so on.

And this condition must have place in order that $F(x,y)+\phi(x,y)$ may $=f^{\prime/}(x,y)$, that is, if $F(x,y)=f^{\prime}(x,y)$, and $\phi(x,y)=f^{/}(x,y)$; then must $F^{/}(x,y)=\phi^{\prime},(x,y)$.

For example, $\frac{y-x}{x^2+y^2}$, may $=f^{\prime/}(x,y)$ for $\left(\frac{y}{x^2+y^2}\right)^{/}=\frac{x^2-y^2}{(x^2+y^2)^2}$ and $\left(\frac{-x}{x^2+y^2}\right)^{\prime},=\frac{x^2-y^2}{(x^2+y^2)^2}$

We may here just observe, that $f^{\prime}(x,y)$, $f^{/}(x,y)$, $f^{\prime\prime}(x,y)$, $f^{\prime\prime/}(x,y)$, $f^{//}(x,y)$, &c. correspond to the partial differentials $\frac{d\,(x,y)}{dx}$,

$\frac{d(x,y)}{dy}$, $\frac{d^2(x,y)}{dx^2}$, $\frac{d^2(x,y)}{dx,dy}$, $\frac{d^2(x,y)}{dy^2}$, &c. which therefore arise from the expansion of a function of two or more quantities, that are independent of each other, on attributing to each of those quantities an indeterminate increase.

We will now show how this theory may be applied to those purposes to which the differential or fluxional calculus is usually applied. But it will be necessary first to demonstrate the two following propositions:

1st. That y being a function x, i may be taken so small in $y + y' i + \frac{y''}{2} i^2 + \frac{y'''}{2.3} i^3 +$ &c. that any term shall exceed the sum of all the following terms by any quantity less than itself, provided that the quotient of any term divided by the preceding is in no case infinite.

In the series $y + y' i + \frac{y''}{2}.i^2 \ldots \frac{y^p}{2.8\ldots p}.i^p + \frac{y^{p+1}}{2.3..p.p+1} i^{p+1} + \frac{y^{p+2}}{2.3\ldots p.(p+1)(p+2)} i^{p+2}$, &c. let the consecutive terms $\frac{y^m}{1.2\ldots m} i^m$, $\frac{y^{m+1}}{1.2\ldots m.(m+1)} i^{m+1}$ be such, that the co-efficient of the latter, divided by that of the former, or $\frac{y^{m+1}}{(m+1).y^m}$ is not less than the co-efficient of any other term whatever in the series, divided by that of the preceding. Then if i be taken $< \frac{(m+1) y^m}{n y^{m+1}}$ (n any number) the sum of all the terms after any term, as $\frac{y^p}{2.3\ldots p} i^p$ will be

$$< \frac{i^p}{2.3\ldots p} \times \left\{ \frac{y^{p+1}}{p+1} . \frac{(m+1).y^m}{n y^{m+1}} + \frac{y^{p+2}}{(p+1).(p+2)} \left(\frac{(m+1).y^m}{n y^{m+1}}\right)^2 + \frac{y^{p+3}}{(p+1).(p+2).(p+3)} \left(\frac{(m+1).y^m}{n y^{m+1}}\right)^3 \text{\&c.} \right\}$$

and

$$\therefore < \frac{i^p}{2.3\ldots p} \times \left\{ \frac{y^{p+1}}{p+1} . \frac{(p+1).y^p}{n y^{p+1}} + \frac{y^{p+2}}{(p+1).(p+2)} . \frac{(p+1).y^p}{n y^{p+1}} . \frac{(p+2).y^{p+1}}{n y^{p+2}} + \frac{y^{p+3}}{(p+1).(p+2)(p+3)} \frac{(p+1).y^p}{n y^{p+1}} . \frac{(p+2).y^{p+1}}{n y^{p+2}} . \frac{(p+3).y^{p+2}}{n.y^{p+2}} + \text{\&c.} \right\}$$

since $\frac{(m+1).y^m}{y^{m+1}}$ is not $> \frac{(p+1) y^p}{y^{p+1}}$, or $\frac{(p+2) y^{p+1}}{y^{p+2}}$, or $\frac{(p+3 . y^{p+2}}{y^{p+3}}$, &c. or $< \frac{y^p}{2.3\ldots p}.i^p \times \left(\frac{1}{n} + \frac{1}{n^2} + \frac{1}{n^3} + \text{\&c.}\right)$ or $< \frac{y^p}{2.3\ldots p}.i^p \times \frac{1}{n-1}$ consequently $\frac{y^p}{2.3\ldots p}.i^p$ will exceed the sum of all the following terms by a quantity $> \frac{y^p}{2.3\ldots p}.i^p + \frac{n-2}{n-1}$. Now, if $n = 2$, or $i < \frac{(m+1).y^m}{2 y^{m+1}}$, $\frac{y^p}{2.3\ldots p} i^p$ will exceed the sum of the following terms by a quantity >0; and it is evident by taking n greater, and consequently i less, that $\frac{y^p}{2.3\ldots p} i^p$ may be made to exceed the sum of all the following terms by any quantity $< \frac{y^p}{2.3\ldots p} i^p$, provided that $\frac{y^{m+1}}{(m+1).y^m}$ is in no case infinite.

2d. That y and u being functions of x, $y = f x$, $u = F x$, x becoming $x + i$, and y and u becoming $y + y' i + \frac{y''}{2}.i^2 + \frac{y'''}{2.3} i^3 +$ &c., and $u + u' i + \frac{u''}{2}.i^2 + \frac{u'''}{2.3} i^3 +$ &c.; if $u = y$, and $u' = y'$, then i may be taken so small, that the value of another function of x, $v = \phi x$, or $v + v' i + \frac{v''}{2} i^2 + \frac{v'''}{2.3}.i^3 +$ &c. cannot when $v = u = y$ fall between the above values of y and u unless $v' = u' = y'$.

Let $D = f(x+i) - \phi(x+i)$ and

$$\Delta = \phi(x+i) - \mathrm{F}(x-i)$$

$$\mathrm{D} = (y' - v').\,i + \frac{y'' - v''}{2}.\,i^2 + \frac{y''' - v'''}{2.3}\,i^3 + \&c. = t'\,i + \frac{t''}{2}\,i^2 + \frac{t'''}{2.3}\,i^3 + \&c.$$

$$\Delta = (v' - u').\,i + \frac{v'' - u''}{2}.\,i^2 + \frac{v''' - u'''}{2.3}.\,i^3 + \&c. = z'\,i + \frac{z''}{2}.\,i^2 + \frac{z'''}{2.3}.\,i^3 + \&c.$$

if t be put for $y - v$, and z for $v - u$. Now we have shown that in these i may be taken so small, that the first term shall exceed the sum of all the following terms, and consequently that D shall have the same sign as $(y' - v').\,i$ and Δ the same sign as $(v' - u').\,i$, or $(v' - y').\,i$ (since $u' = y'$): that is, i may be taken so small that D and Δ shall have contrary signs; or, that if $\phi(x+i)$ is $< f(x+i)$, it shall likewise be $< \mathrm{F}(x+i)$, and if $\phi(x+i)$ is $> f(x+i)$, it shall likewise be $> \mathrm{F}(x+i)$; or, which is the same thing, i may be taken so small, that the value of $\phi(x+i)$ cannot fall between the values of $f(x+i)$ and $\mathrm{F}(x+i)$, unless $v' = u' = y'$.

In the same manner precisely it may be shown, that if $v = u = y$, $v' = u' = y'$, $v'' = u'' = y''$, &c. to $v^p = u^p = y^p$, and $u^{p+1} = y^{p+1}$, then the value of $\phi(x+i)$ cannot fall between the values of $f(x+i)$ and $\mathrm{F}(x+i)$ unless $v^{p+1} = u^{p+1} = y^{p+1}$.

To apply this to the contacts of curves, let x and y be the abscissa and ordinate of a curve whose equation is $y = f\,x$, t and u the abscissa and ordinate of another curve whose equation is $u = \mathrm{F}\,t$, the abscissas of the curves being measured in the same line.

Suppose that the curves have a common point, that is, that $u = y$; and for the sake of simplicity, let the abscissas in each be measured from the same point, or $t = x$, the equations will become $y = f\,x$, $u = \mathrm{F}\,x$. Let now another ordinate be drawn to each curve at the distance i from the former, that is, let x become $x + i$, y will become $y + y'\,i + \frac{y''}{2}\,i^2 + \frac{y'''}{2.3}\,i^3$, &c.; and u, $u + u'\,i + \frac{u''}{2}\,i^2 + \frac{u'''}{2.3}\,i^3 + \&c.$; then if $u' = y'$ the contact of the curves is such, that no other curve whose equation is $v = \phi\,x$ can pass between them unless $v = u = y$, and $v' = u' = y'$, if $u' = y'$ and $u'' = y''$ the contact is such, that no other curve whose equation is $v = \phi\,x$ can pass between them unless $v = u = y$, $v' = u' = y'$, $v'' = u'' = y''$, and in general if $u' = y'$, $u'' = y''$, &c. to $u^{p+1} = y^{p+1}$, the contact is such, that the other curve cannot pass between them unless $v = u = y$, $v' = u' = y'$, $v'' = u'' = y''$, &c. to $v^p = u^p = y^p$; this is evident from what has before been shown, for the value of the ordinate of the third curve cannot fall between the values of the ordinates of the other two, unless it have the above conditions.

Hence the contacts of curves admit of different degrees, and calling one in which $u' = y'$ a contact of the first order, one in which $u' = y'$, and $u'' = y''$, a contact of the second order, &c.; there are curves, which with a given curve admit only of contacts of a certain order.

For example, a right line can only have a contact of the first order, a circle can only have a contact of the second order, and a curve, whose equation is $y = a + a'\,x + a''\,x^2$, &c. $+ a^n x^n$ can only have a contact of the order n with a given curve. These different degrees of contact require a certain number of constant quantities, which may be called elements of the contact.

FUNICULAR *Machine* (from *funiculus, a rope*), is a term used to denote an assemblage of cords, by means of which two or many powers sustain one or many weights. This is classed by some authors amongst the simple mechanical powers, and is the simplest of them all.

In order to find the law of equilibrium in this machine, we must first reduce all the powers which meet at one point to a single power; which single power must act in the direction of the cord, this being evidently necessary for establishing an equilibrium: following this method, we may reduce all the powers which act on different points of the cord, to a system of powers which act on the same point. Then reduce these powers, which act all on the same point, to one equivalent power, and we shall ultimately arrive at two powers only, which ought to be equal and acting in contrary directions, in order to establish the perfect equilibrium.

FUSEE, in *Clock-work*, is a mechanical contrivance for equalizing the power of the main-spring of a watch; for as the action of a spring varies with its distance from the quiescent position, the power derived from the force of a spring requires to be modified according to circumstances, before it can become a proper substitute for a uniform weight, which is what it is intended to supply. In order, therefore, to correct this irregular action of the spring, the fusee on which the chain or catgut acts is made somewhat conical, so that its radius at every point may correspond with the strength of the spring.

G.

GAGE, is the name of various instruments used for measuring the state of exhaustion in the air-pump, variations in the barometer, the depth of the sea, the quantity of rain, &c.

The GAGE of the *Air Pump* is of a variety of forms, as the *Barometer Gage*, the *Syphon Gage*, the *Pear Gage*, &c.

Short Barometer GAGE is a tube about eight or nine inches in length, filled with mercury, and immersed with its aperture into a small quantity of the same fluid, contained in a glass vessel which forms the cistern. This gage is either placed under the receiver, upon the principal plate of the air-pump, or it is placed under a separate small receiver, on an auxiliary plate attached to some pumps for the purpose. As this gage is not equal to a whole barometer, it will not begin to show the state of exhaustion till after three-fourths of the air have been extracted, that is, when the elasticity of the remaining air is about one fourth of that of common air, the barometer itself being but about one-fourth of the usual height of the mercury; but after this the farther decrease of elasticity is exhibited by the sinking of the mercury, and a graduated scale attached to the instrument.

There is another gage, called the *Long Barometer Gage*, which works in a similar manner, and indicates the state of rarefaction on nearly the same principles.

Syphon GAGE differs from the short barometer gage only in this: that instead of terminating in a small cistern, the tube is bent, and rises upwards with its aperture, which by means of a brass tube is made to communicate with the inside of the pump, so that the ascending leg of the tube performs the office of a cistern.

Pear GAGE. This gage does not indicate the rate of rarefaction as it proceeds, but shows the ultimate state to which it was carried after the re-admission of the air. The gage is suspended in the receiver, and exhausted to the same degree; but when this is carried on as far as is intended, the open orifice of the gage is let down into a vessel of mercury, which upon the re-admission of the air is forced into the pear, and thus the ultimate state of rarefaction is determined.

GAGE of the *Barometer*, estimates the exact degree of the rise or fall of the mercury in the barometrical tube. It is obvious that when the mercury sinks in the tube it rises in the cistern, and *vice versa;* and as the distance between the divisions graduated on the scale, and the surface of the mercury in the cistern, is not truly shown by the numbers on the scale, errors must happen in determining the exact height of the mercury. To remedy this inconvenience, a line is cut upon a round piece of ivory, which is fixed near the cistern: this line is accurately placed at a given distance from the scale, as for instance, twenty-seven inches; and a small float of cork, with a cylindric piece of ivory fixed to its upper surface, (on which a line must be cut at the distance of two inches, exactly from the under surface of the cork) is left to play freely on the quicksilver, and the cylinder works in a grove made in the other piece; from this construction it appears, that if these marks are made to coincide, by raising or lowering the screw which acts on the quicksilver, then the divisions on the scale will express the true measure of the distance from the surface.

Gage of the *Condenser*, is a glass tube of a particular construction, adapted to the condensing engine, and designed to show the exact density and quantity of the air contained at any time in the condenser.

Sea Gage, is an instrument for finding the depth of the sea. It consists of a bottle having a brass cap, in which are small holes for the admission of the water. Into the centre of the cap there is screwed the gage tube, closed at the upper end, and having its under end immersed into a quantity of mercury in the bottle. A small quantity of treacle is placed upon the top of the mercury in the tube; and the whole apparatus is attached to an air-vessel, which acts as a float, and a weight which acts as a sinker; the weight being so contrived, that the machine, when it touches the bottom of the sea, is disengaged from it, and floated to the surface by the air-vessel. When the gage is let down, the water forces the mercury up in the gage-tube, to a height proportional to the depth of the sea; and the height to which it has been forced when at the bottom, is indicated by the trace left by the treacle. In small depths, the gage-tube may be made of glass, and the height of the treacle may be seen at once; but when the depth is great, the tube should be of iron, and the height may be ascertained by an inclosed rod. Capt. Scoresby found that the alteration of the specific gravity of a cube of wood, by the forcing of the water into its pores, was a very good approximation of the depth; and professor Leslie proposes a sea-gage on that principle.

Rain Gage. An instrument for ascertaining the quantity of rain that falls. It is composed of a large cylinder, open at top, and freely exposed to the weather, of such depth that none of the rain which falls into it shall escape by the surface. In the middle of this cylinder, is attached a smaller cylinder, in which there is a float, that shows the height of water by a rod when the cylinder is opaque, and by a scale on the cylinder itself, when that is of glass. The advantage of the smaller tube is, that the range of the scale is increased, as the square of the diameter. Thus, if the diameter of the cylinder which is exposed to the atmosphere, be three times that of the cylinder to which the scale is attached, a fall of one inch of rain will give a range of nine inches to the scale.

A rain-gage should be placed in an open situation, where it is neither exposed to violent gusts of wind, nor sheltered from the weather. In consequence of the absorption by the lower stratum of the atmosphere, a rain-gage on the top of a building often indicates a greater fall of rain than one upon the ground.

Tide Gage. This is an instrument used for determining the height of the tides. It consists of a glass tube, whose internal diameter was seven-tenths of an inch, lashed fast to a ten-foot fir rod, divided into feet, inches, and parts; the rod being fastened to a strong post fixed firm and upright in the water. At the lower end of the tube was an exceedingly small aperture, through which the water was admitted. In consequence of this construction, the surface of the water in the tube was so little affected by the agitation of the sea, that its height was not altered the tenth part of an inch, when the swell of the sea was two feet, and Mr. Bayley was certain, that with this instrument he could discern a difference of the tenth of an inch in the height of the tide.

GALAXY, *Via Lactea*, or *Milky Way*, in *Astronomy*, that long, luminous track or zone, which encompasses the heavens, forming nearly a great circle of the celestial sphere. It is inclined to the plane of the ecliptic at about an angle of 60°, and cuts it nearly at the two solstitial points. It traverses the constellations Cassiopeia, Perseus, Auriga, Orion, Gemini, Canis Major, and the Ship, where it appears most brilliant in southern latitudes; it then passes through the feet of the Centaur, the Cross, the southern Triangle, and returns towards the north by the Alter, the tail of the Scorpion, and the arc of Sagittarius, where it divides into two branches, passing through Aquila, Sagitta, the Swan, Serpentarius, the head of Cepheus, and returns into Cassiopeia. Its white appearance is attributed to an immense multitude of very small or distant stars.

GALLON, an English measure of capacity, being equal to 4 quarts or 8 pints.

	Cub. Inches.
The gallon, wine measure, contains	231
Ditto beer measure	282
Ditto dry measure	268⅘

GALVANISM, an interesting branch of science, thus named after its discoverer, professor Galvani of Bologna. Galvanism comprises all those apparently electrical phenomena arising from the chemical agency of certain metals with different fluids.

Galvani's experiments have been repeated by many eminent philosophers, both on the continent and in this country. None of them, however, added any thing new to what Galvani had himself discovered, excepting the celebrated Volta, whose improvement was so decided, that the science itself has nearly changed its name, taking that of Voltaism instead of Galvanism.

When we view, indeed, the numerous facts that have been added to the labours of Galvani, his discoveries form but a very small part of the whole mass, whereas a great many of them are due to Volta. Yet when we recollect again, that the investigation began with the former, and was in a great degree promoted by his own perseverance, we must ever consider him as a principal in this extensive field of research; and cannot, without injustice, deprive him of the honour which has been conferred upon him, of giving his own name to the science which he discovered and promoted.

Philosophy, however, is infinitely indebted to Signor Volta, it being to him that we owe, in a great measure, the rapid progress that has since been made in this interesting branch of philosophy.

He repeated the experiments of Galvani, and found that when two pieces of metal, of different kinds, were placed in different parts of an animal at the same time, that the metals were brought in contact, or were connected by a metallic arc; as often as the contact was made, convulsions were observed. He found that the greatest was produced when the metals were zinc and silver. When several pairs of metals were employed, having pieces of moist cloth between them, the effect appeared to increase as the number of pairs. This important discovery of accumulating the effects of this species of electricity was made by Volta in 1800, and hence has been denominated the *Voltaic pile*. The apparatus first made by Volta consisted of a certain number of pairs of zinc and silver plates separated from each other by pieces of wet cloth; the arrangement being as follows: zinc, silver, wet cloth; zinc, silver, wet cloth, and so on. The silver plates were chiefly silver coins, the plates of zinc and the pieces of cloth being of the same size. He found this pile much more powerful, when the pieces of cloth were moistened with a solution of common salt instead of pure water, and an apparatus consisting of forty pairs of plates he found to possess the power of giving a very smart shock, similar to that of a small electric jar; and that this effect took place as often as a communication was made between each end of the pile, and as long as the pieces of cloth remained moist.

An account of this discovery was communicated to the Royal Society, and published in the Philosophical Transactions. Since this time we have no account of any farther discoveries of Volta; but the science of Galvanism has been since considerably extended by the researches and experiments of philosophers in France, England, and other countries.

The first experiments made on the pile in this country were performed by Messrs. Nicholson and Carlisle, who observed, that on bringing the wires from each end of the column, in contact with a drop of water, bubbles of some elastic gas were disengaged, which on closer examination they found to be hydrogen gas; this discovery gave rise to a great variety of experiments, and to many interesting results, but, being chiefly chemical, we cannot enter into any detail of them in the present article.

The Galvanic energy evinced in the decomposition of bodies, which the experiments of Nicholson and Carlisle had first made known, was farther prosecuted by Mr. Cruikshank of Woolwich, to whom we are indebted for the invention of the Galvanic trough, which we have described under the article *Galvanic* BATTERY. This again led the way to other batteries of similar construction, but of a more powerful nature, by which it was found, that all the metals reduced into thin leaves were deflagrated with brilliant, though differently coloured, flames; and henceforth Galvanism, which had not before assumed any particular character, it being doubtful to what branch of science it properly belonged, was directed entirely to chemistry, and has since been the means of throwing great light on that interesting branch of human knowledge.

Subsequently, the Galvanic pile has been applied, by Sir H. Davy and others, to the successful decomposition of substances which were previously regarded as simple. These discoveries, brilliant as they are, rather belong to chemical than to mechanical science.

GUAGING, the art or act of measuring the capacities of all kinds of vessels, and thence ascertaining the quantity of liquor which they contain.

The most common, as well as the most difficult, problem in guaging is to find the capacity of casks, for which the following is the general rule.

Put B for the diameter at the bung, H for the diameter at the head, and L for the length of the cask; then, $(39B^2 + 25H^2 + 26BH) \times \frac{L}{114} = C$, the capacity of the cask in inches; which divide by 282 for ale gallons, and 231 for wine gallons.

Ex. Length 30 inches, being diameter 25, head diameter 20, how many ale and wine gallons?

$B = 25$, $H = 20$, $L = 30$. Then

$$(39 \times 25^2 + 25 \times 20^2 + 26 \times 25 \times 20) \times \frac{30}{114} = C;$$

that is

$$(24375 + 10000 + 15000) \times \frac{5}{19} = C;$$

$$\text{or,}\quad 49375 \times \frac{5}{19} = C;$$

$$\text{or,}\ \frac{256875}{19} = 13519\tfrac{14}{19} = \text{inches.}$$

Then $13519 \div 282 = 48$ ale gallons nearly

$13519 \div 231 = 58\frac{1}{2}$ wine gallons nearly.

GUAGING, or *Diagonal Rod*, is a rod or rule adapted for determining the contents of casks, by measuring the diagonal from the bung to the extremity of the opposite stave next the head, and, by this means, saving the labour of calculation. It is a square rule, having four sides or faces, being usually four feet long, and folding together by means of joints.

Upon one face of the rule is a scale of inches, for taking the measure of the diagonal; to these are adapted the areas, in ale gallons, of circles to the corresponding diameters, like the lines on the under sides of the three slides in the sliding rule, described below. And upon the opposite face are two scales, of ale and wine gallons, expressing the contents of casks having the corresponding diagonals; and these are the lines which chiefly constitute the difference between this instrument and the sliding rule; for all the other lines upon it are the same with those in that instrument, and are to be used in the same manner.

To use the Diagonal Rod.—Put the rod in at the bung-hole of the cask till its end arrive at the meeting of the head and opposite stave, which is found by pushing it to the farthest possible distance from the bung-hole, and note the inches and parts cut by the middle of the bung; then withdraw the rod, and look for the same inches and parts on the opposite face of it, and annexed to them are found the contents of the cask, both in ale and wine gallons.

GAUGING *Rule*, or *Sliding-Rule*, is a sliding-rule particularly adapted to the purposes of guaging. It is a square rule, of four faces or sides, three of which are furnished with sliding pieces running in grooves. The lines upon them are mostly logarithmic ones, or distances which are proportional to the logarithms of the numbers placed at their ends; which are used by applying a pair of compasses for taking off and applying the several logarithmic distances; but, instead of the compasses, sliding pieces are now added.

GAUSS'S *Theorem*, a theorem invented by Gauss, professor of mathematics at Strasburgh, for the solution of certain binomial equations; and for showing how the sines and cosines of angles may be obtained by the solution of equations. It may be briefly explained, thus:

Let $x^n - 1 = 0$, be a binomial equation, then it is obvious that the real root of this equation is $x = 1$.

And if now, according to the known theory of equations, we divide $x^n - 1 = 0$, by $x - 1 = 0$, we have

$$\frac{x^n - 1}{x - 1} = x^{n-1} + x^{n-2} + x^{n-3} \ldots . x + 1 = 0$$

which latter equation necessarily contains all the imaginary roots of the original equation $x^n - 1 = 0$.

Let these roots be a, a^2, a^3, a^4, &c. it being a property of these equations, that their roots are all powers of each other. It is also obvious that their sum is equal to -1, that is, to the co-efficient of the second term with its sign changed from + to −; also the product of them $= +1$.

Gauss's solution depends upon the subdivision of this period of roots into other less periods, so that the product, square, cube, &c. of these minor periods, shall produce the sum or product of all the roots. We shall give the solution of one case.

Let it be proposed to find the imaginary roots of the equation

$$x^5 - 1 = 0, \text{ or of}$$
$$x^4 + x^3 + x^2 + x\,1 = 0.$$

Let the four required roots be a, a^2, a^3, a^4

$$\text{also make} \begin{cases} p = a + a^4 \\ p' = a^2 + a^3 \end{cases}$$

then $pp' = a^3 + a + a^4 + a^2 = -1$
and $p + p' = a + a^2 + a^3 + a^4 = -1$

Therefore the values of p and p' are found from the quadratic equation.

$p^2 + \quad p = 1$
whence $p = -\tfrac{1}{2} + \tfrac{1}{2}\sqrt{5}$
and $p' = -\tfrac{1}{2} - \tfrac{1}{2}\sqrt{5}$

which are the numerical values of the cosines of 72° and 144°.

GEMINI, or *Twins*, one of the northern zodiacal constellations, denoted by the character ♊. See CONSTELLATION.

GENERANT, that which is generated, or supposed to be generated, by the motion of any point, line, or figure. The generant is always one dimension higher than the generating quantity, thus a line is generated by the motion of a point, a surface by a line, and a solid by a surface.

It is generally a theorem in geometry, that the measure of any generant is always equal to the product of the generating quantity, drawn into the path of the centre of gravity of the latter, whether its motion be rectilineal or rotatory.

GENERATING *Line* or *Figure*, in *Geometry*, is that line or figure, by the motion of which another figure or solid is supposed to be described or generated.

GEOCENTRIC, having the earth for its centre. Applied to the latitude, longitude, &c. of the celestial bodies, as seen from the earth.

GEODESY is properly that part of geometry which relates to the division of lands, or other surfaces; but it is now more commonly employed to denote those trigonometrical operations which have been executed for the purpose of measuring the lengths of degrees in different latitudes; or to the surveys of whole countries.

GEOGRAPHY, the doctrine or knowledge of the earth, or a description of the terrestrial globe, and particularly of its known habitable parts, with all its subordinate divisions, &c.

GEOMETRICAL, any thing relating to the science of geometry.

GEOMETRICAL *Analysis*. See ANALYSIS.

GEOMETRICAL *Line* or *Curve*, otherwise denominated *algebraical curve*, is that wherein the relation of the absciss to the ordinate may be expressed by an algebraical equation; being thus distinguished from a *transcendental* curve, in which no such relation has place between the absciss and ordinate. See CURVE.

GEOMETRICAL *Locus*. See LOCUS.

GEOMETRICAL *Progression* and *Proportion*. See PROGRESSION and PROPORTION.

GEOMETRICAL *Solution*, is that which is obtained from the simple principles of geometry.

GEOMETRICALLY, according to the principles of geometry.

GEOMETRY, originally implied the measuring of the earth, or that part of science which is now called *Geodesy*. In its modern and more extended sense, it may be termed the science of *magnitude*, *figure*, and *relation*. Being applied to the investigation of extension of one dimension—length; of two dimensions—surface; and of three dimensions—solidity or body; as also to the various properties and circumstances which arise from their nature and position, and the relations or ratios which they bear to each other.

Geometry is distinguished into several denominations, as *analytical*, *elementary*, *practical*, &c.

Elementary GEOMETRY, is that which treats of the properties and proportions of right lines, and right lined figures, as also of the circle and its several parts, and is either theoretical or practical.

Theoretical GEOMETRY, has for its object the establishing of geometrical truths.

Practical GEOMETRY, relates to the performance of geometrical operations, such as the construction of figures, and the drawing of lines in certain positions, as parallel, perpendicular, &c. to other given lines; in application of the truths established by the theory.

The *Higher* or *Transcendental* GEOMETRY, is that which treats of the higher order of curves and problems.

The origin of geometry is hidden in obscurity; but we can scarcely conceive any state of society, however rude, in which something like the first principles of geometry did not exist. As soon as man began to relinquish his wandering and savage life, and taste the pleasures of social intercourse; as soon as laws were framed to secure to each individual the result of his own industry and labour, the lands, which had before yielded spontaneously all that he required in his barbarous state, stood now in need of cultivation, in order to render their productions subservient to his more refined appetites, and to the necessities of his family, or the little society to which he belonged.

It is generally supposed that the Greeks derived their first knowledge of the sciences from the magi, or priests of Egypt, and it was probably known in that country long before the time of Thales, who is said to have acquired his knowledge, both of geometry and astronomy, from these people. For we are informed by Herodotus, that this philosopher predicted an eclipse, so memorable for the consequences which attended it; and as the prediction of an eclipse is very far from an elementary problem in astronomy, we may presume that the sciences had already attained a considerable degree of perfection at this time.

Soon after this period, that is, about 500 years B. C. Pythagoras, a friend and pupil of Thales, began to illuminate the world by his important discoveries. At this period also flourished Anaximander, and soon after Anaximenes, Anaxagoras, and Cleostratus; these were all eminent in astronomy and geometry. Œnopides, 480 B. C. was a learned geometer, author of several problems, and his contemporary, Zenodorus, is the first of the ancients whose works have been handed down to us; all before his time having been lost or destroyed. About this time also flourished Hippocrates of Chios, who distinguished himself by the celebrated quadrature of the lunes which bear his name, as well as by his discoveries connected with the problem of *doubling the cube*, which excited great interest amongst the ancient mathematicians of this period. It is said, that Apollo having afflicted the Athenians with a dreadful pestilence, to revenge an affront he had received from them, the oracle of the temple Delos being consulted on the means of appeasing his wrath, answered, *Double the alter*, which being a perfect cube, gave rise to this celebrated problem. The question at first appeared easy, but the mistake was soon discovered, and all the genius of the Greek mathematicians was unable to produce a complete solution. Hippocrates found, that if two geometrical mean proportionals could be inscribed between the side of the given cube, and double that side, the first of these two lines would be the side of the double cube. This discovery revived some hopes of obtaining the required solution, but it soon appeared that the difficulty was merely changed, and not in the least diminished, and that it still presented obstacles that were insurmountable. This did not, however, discourage other mathematicians from following up the pursuit; and several curious geometrical properties were the result of these investigations, the *Conchoid* of Nichomedes, the *Cissoid* of Diocles, and the *Quadratrix* of Dinostratus, owe their origin to the same source.

Passing over some geometricians and astronomers of less note, we come to Plato, who cultivated both astronomy and geometry with great assiduity, about 390 years B. C. The celebrated inscription that he caused to be placed over the door of his school, "Let no one enter here who is ignorant of geometry," is a proof of the high estimation in which he held the latter science. To this philosopher we owe the introduction of the conic sections into geometry, and his disciple Aristeus is said to have composed five books on these figures, of which the ancients have spoken with the greatest commendations, but, unfortunately, they have not been transmitted down to our time. Besides Aristeus, Plato numbered amongst his friends or scholars Eudoxus, Menechmus, and Dinostratus; the former of whom was very celebrated for his extensive knowledge in geometry as well as in astronomy; Menechmus, for his application of the conic sections to various problems; and the latter for the invention of the *Quadratrix*, as applicable to the problem of doubling the cube, which seems to have been the germ of what is now termed the geometrical analysis.

It was about ninety years from the time of Plato to that of Euclid, during which time all the sciences were considerably advanced and extended, and treatises on particular subjects appeared from time to time, in which all the propositions then known were collected and arranged in systematic order, which was the object of Euclid in in his celebrated "Elements," a work too well known to need any particular description. At that period the Grecian sciences were in their meridian splendour; Archimedes, one of the greatest geometers that ever appeared in any age or country, followed soon after the time of Euclid. His universal genius led him to the contemplation of almost every species of human knowledge. In geometry he discovered the ratio between the sphere and its circumscribing cylinder, found the quadrature of the parabola, and the solidity of its conoid; he invented the spiral which bears his name, and discovered its rectification; beside a great variety of other important and interesting geometrical propositions.

At the distance of about fifty years after Archimedes, Apollonius cultivated the mathematical sciences with the greatest possible success. Eratosthènes, about the same time, first attempted to measure the circumference of the earth; Ctesibius invented water pumps; Hero, of Alexandria, was much celebrated for his application of geometry to the practical purposes of mensuration, and to whom we are indebted for the invention of *Clepsydræ* or water-clocks. Pappus, Theon, and Diocles, all flourished during the fifth century; Eutocius early in the sixth; and about the same time Proclus and Marinus.

Immediately began that period so fatal to the sciences; which had for a long time taken refuge in the museum of Alexandria; but, destitute of support and encouragement, they could not fail to degenerate. Still, however, they preserved, at least by tradition or imitation, that ancient strict character impressed upon them by the early Greeks; till about the middle of the seventh century a tremendous storm arose, which threatened their total destruction. Filled with all the enthusiasm a militant religion inspires, the successors of Mohammed ravaged that vast extent of country which stretches from the east to the southern confines of Europe. All the cultivators of the arts and sciences, who from every nation had assembled in Alexandria, were driven away with ignominy; some fell beneath the swords of the conquerors; others fled into remote countries, to drag out the remainder of their lives in obscurity and distress.

From the ninth to the fourteenth century, the Arabs may be considered as the guardians and promoters of geometry, as well as the other sciences; at which time it was that they began to be again revived in Europe.

During the fifteenth century there appeared several translations of the ancient writings, and a general progress was made in the several sciences, and particularly in geometry. In the sixteenth century we have, besides a number of translations and commentaries, a few original works, highly creditable to their respective authors, by Vieta, Tartalea, Descartes, &c.; but the first who left the beaten paths of the ancients was Cavalerius, who published his "Indivisibles" in 1635. Here he considers surfaces as composed of an infinite number of lines, and solids, as made up of surfaces, &c. and assumes as a principle, that the ratio of these infinite sums of lines or planes, compared with the unit in each case, are the same as the superfices or solids that were to be measured. This work may be considered as the first link in the modern geometry, and it was soon followed by others of a similar kind, which after this become too numerous for particular detail. We can, therefore, only mention the names of Wallis, Pascal, Fermat, Roberval, Leibnitz, Newton, and the Bernouillis, as the principal promoters of geometry during the seventeenth century.

The most approved modern works on the elements of geometry are those of Euclid, as translated by Simson and Playfair; and the treatises of Leslie and Legendre.

GIBBOUS, denotes generally a convexity or protuberance; and hence the moon is said to be gibbous in the second and third quarters, being, during this period, more than half enlightened, and consequently convex or gibbous on both sides.

GIRT, in *Timber Measure*, is the circumference of a tree; and a quarter of this is called the quarter-girt. The common practical rule is to square the quarter-girt, and multiply by the length of the tree, for the content.

GLOBE, in *Geometry*, a round solid body, which may be conceived to be generated by the revolution of a semi-circle about its diameter: it is also called a sphere. See SPHERE.

GLOBE, or *Artificial Globe*, in *Geography* and *Astronomy*, is more particularly used to denote a globe of any convenient materal; on the surface of which is drawn a map, or representation of either the heavens or the earth, with the several circles of reference, which are conceived to be drawn upon them; that which has a map of the earth upon it, being called the

Terrestrial Globe, and that which has a map of the heavens the *Celestial*.

The principal parts, which are common to both globes, are, 1. The two poles whereon the globe is supported, representing those of the earth or heavens. 2. The meridian, which is divided into degrees, and passes through the poles. 3. The horizon, whose upper side represents the horizon, and is divided into several circles, one of which contains the twelve signs of the zodiac, subdivided into their degrees; another the calendar; a third the points of the compass; and a fourth, the quadrants divided into degrees, and numbered both ways for estimating azimuths and amplitudes. 4. A quadrant of altitude, divided into ninety degrees from the horizon. 5. The hour circles, divided into twice twelve hours, and fitted on the poles, with an index to point out the hour. A mariner's compass is sometimes added on the bottom of the frame; and a semicircle of position is sometimes fixed in the east and west points of the horizon.

On the surface are delineated, 1. The equinoctial line, divided into 360 degrees, commencing from the vernal intersections. 2. The ecliptic, divided into twelve signs, and these subdivided into degrees. 3. The zodiac. 4. The two tropics. 5. The polar circles. 6. Meridians at every hour, or 15° of latitude, beginning (in British globes) at Greenwich.

What else belongs to globes, either as to construction or description, is different as the globe is either celestial or terrestrial.

Very large globes have been made in different parts of Europe, as at Gottorp, Paris, &c.; but we believe they are all inferior in size to one erected at Pembroke College, Cambridge, under the direction of the late Dr. Long. The description of this machine, in the doctor's own words, is this:—" I have, in a room lately built in Pembroke Hall, erected a sphere of eighteen feet diameter, wherein thirty persons may sit conveniently; the entrance into it is over the south pole, by six steps: the frame of the sphere consists of a number of iron meridians, not complete semicircles, the northern ends of which are screwed to a large round plate of brass, with a hole in the centre of it; through this hole, from a beam in the ceiling, comes the north pole, a round iron rod, about three inches long, and supports the upper parts of the sphere to its proper elevation for the latitude of Cambridge: the lower part of the sphere, so much of it as is invisible in England, is cut off; and the lower or southern ends of the meridians, or truncated semicircles, terminate on, and are screwed down to, a strong circle of oak, of about thirteen feet diameter, which when the sphere is put into motion runs upon large rollers of lignum vitæ, in the manner that the tops of some windmills are made to turn round. Upon the iron meridians is fixed a zodiac of tin, painted blue, whereon the ecliptic and heliocentric orbits of the planets are drawn, and the constellations and stars traced: the Great and Little Bear, and Draco, are already painted in their places round the north-pole; the rest of the constellations are proposed to follow: the whole is turned round with a small winch, with as little labour as it takes to wind up a jack, though the weight of the iron, tin, and wooden circle is about a thousand pounds. When it is made use of, a planetarium will be placed in the middle thereof. The whole, with the floor, is well supported by a frame of large timbers."

Since this was written, in 1758, the constellations and chief stars visible at Cambridge have been painted in their proper places, upon plates of iron joined together, which form one concave surface. It is said that, since the death of Dr. Long, this curious structure appears to have been much neglected, and now exhibits strong tokens of decay; although the Doctor made ample provision for keeping it in constant repair.

GLOBULAR, relating to, or partaking of the nature of a globe; thus we say, *globular chart, globular sailing*, &c.

GNOMON, in *Astronomy*, is an instrument or apparatus for measuring the altitudes, declinations, &c. of the sun and stars. The gnomon is usually a pillar, or column, or pyramid, erected upon level ground, or a pavement. For making the more considerable observations, both the ancients and moderns have made great use of it, especially the former; and many have preferred it to the smaller quadrants, both as more accurate, more easily made, and more simply applied.

The elevation of the pole may be found by means of the gnomon, by finding the meridian height of the sun; for, this being given, we have the elevation of the equator, and consequently that of the pole. The meridian height of the sun may be found by taking the length of the shadow at mid-day. Then

As the length of the shadow
Is to the height of the gnomon;
So is radius
To tangent of sun's altitude.

If the gnomon be perfectly erect, the ground (or rather pavement) perfectly level, and a number of concentric circles be drawn from the gnomon as a centre, the meridian may also be found by observing the longest shadow; or, at a time when the variation in declination is very small, as for instance at the summer solstice, the points at which the shadow touches a circle in the morning and evening may be marked, and the arch between them bisected.

GNOMON, in *Dialling*, is the style, pin, or cock of a dial, the shadow of which points out the hours. This is always parallel to the earth's axis, and points to the north and south poles.

GNOMON, in *Geometry*, is the space included between the lines forming two similar parallelograms, of which the smaller is inscribed within the larger, so as to have one angle in each common to both.

GNOMONIC, or **GNOMONICAL** *Projection*, that which represents the circles of a hemisphere upon a plane touching it in the vertex, by lines or rays from the centre of the hemisphere to all the points of the circles to be projected. In this projection, all the great circles of the sphere are projected into right lines. Any lesser circle parallel to the plane of projection, is projected into a circle. And any lesser circle not parallel to the plane of projection, is projected into a conic section. The gnomonic projection is called the *horologiographic projection*, because it is the foundation of dialling.

GOLDEN *Number*, a number shewing what year of the Metonic, or lunar cycle, any given year is. To find the golden number, add 1 to the given year, and divide the sum by 19; what remains will be the golden number; unless 0 remain, for then 19 is the golden number.

GONIOMETER, an instrument for measuring solid angles, particularly in crystals.

GONIOMETRICAL *Lines*, are lines employed in measuring or determining the quantity of angles; such as sines, tangents, secants, &c.

GONIOMETRY, the art of measuring angles, whether on paper or on the earth, by means of instruments proper for the purpose. In the former case this is performed by means of a protractor, or a line of chords; and in the latter, by the theodolite, quadrant, or circumferentor.

GRADUATION, the art of dividing into degrees or other parts.

GRADUATION of *Mathematical Instruments*, is the process by which arches of quadrants, theodolites, and circular instruments are divided into degrees and other smaller subdivisions. The accuracy of this operation is of the highest importance in practical astronomy and surveying, and it has accordingly engaged the attention of many celebrated mechanics, and, latterly, very ingenious machines have been invented, which perform it with a wonderful degree of accuracy. The most celebrated of them are by Ramsden and Troughton.

GRAIN, a small weight, being the 480th part of an ounce troy-weight, or apothecaries weight; and very nearly the 7,000th part of a pound avoirdupois.

GRAPHOMETER, a mathematical instrument, otherwise called a semicircle, the use of which is to observe any angle whose vertex is at the centre of the instrument; and to measure it off in degrees, minutes, &c.

GRAVITATION, the act of tending to a centre; or the exercise of that power or tendency by which every body moves or is disposed to move toward every other body.

The astronomical discoveries of Kepler naturally suggested the following problems: Why do the planets and satellites describe *elliptic* orbits; the former about the sun, and the latter about their primaries? Why is the centre of motion in the focus, rather than in the centre of the ellipse? Why are equal areas described in equal times about the centre? And what can be the cause of that remarkable law which obtains between their distances and time of periodic revolution? On these subjects Kepler himself had formed certain hypotheses. Gravity, he says, is only a mutual and corporal affection between similar bodies. Heavy bodies do not tend to the centre of the world, but to that of the round body of which they form a part; and if the earth were not spherical, heavy bodies would not fall towards its centre, but towards different points; which conjecture has been since verified by experiment. The earth, it is found, is not a perfect sphere, and heavy bodies falling towards it have not their motion directed in a line to its centre, but in a line perpendicular to its surface. Kepler likewise suspected, that the attraction of the moon was the cause of the tides, and that the lunar irregularities arose from the action of the earth and sun.

Fermat, who preceded Kepler, affirmed, that the weight of a body was the sum of the tendencies of all its particles to all the particles of the earth; and Kepler was of opinion, that two bodies left alone in free space would approach each other, with velocities inversely proportional to their masses or quantities of matter.

Dr. Hook, at a meeting of the Royal Society, May 3, 1668, advanced the three following positions:

"1. That all the heavenly bodies have not only a gravitation of their parts to their own proper centres, but that they also mutually attract each other within their spheres of action.

"2. That all bodies having a simple motion will continue to move in a straight line, unless continually deflected from it by some extraneous force causing them to describe a circle, an ellipse, or some other curve.

"3. That this attraction is so much the greater as the bodies are nearer. As to the proportion in which those forces diminish by an increase of distance, I own I have not yet discovered it, although I have made some experiments to this purpose. I leave this to others, who have time and knowledge sufficient for the task."

Hook had before exhibited to the society a very neat illustration.—A ball suspended by a long thread from the ceiling, was made to swing round another ball laid on a table immediately below the point of suspension. When the impulse given to the pendent ball was nicely adjusted to its deviation from the perpendicular, it described a perfect circle round the ball on the table; but when the impulse was very great, or very small, it described an ellipse, having the other ball in its centre. He contended, that this was the operation of a deflecting force, proportioned to the distance from the other ball, adding, that although this illustrated the planetary motions in some degree, yet it was not perfect, as the planets describe ellipses, having the sun not in their centre but in their focus; are, therefore, not retained by a force proportional to the distance from the sun.

In the year 1666, Newton, having retired from Cambridge to the country on account of the plague, was led to meditate on the probable cause of the planetary motions, and upon the nature of that central force by which they are retained in their orbits; and it occurred to him, that probably the same force, or some modification of the same force, which caused a heavy body to descend to the earth, might extend to the moon, and be the means of retaining her in her orbit, by causing a constant deflection from her rectilinear path.

But before this conjecture could be submitted to the test of calculation, it was necessary to assume some conditional hypotheses relative to the modification of the force with respect to the distance. The supposition he made was, that the force of gravity decreases as the square of the distance increases. But when he first attempted to verify this conjecture, the requisite data, with regard to the distance of the moon in radii of the earth, and the measure of the terrestrial radius, were but imperfectly known; and the result which he obtained, was not exactly what he could have wished, so he at first abandoned his hypothesis. A few years afterwards, he was induced to renew his calculations, as in the interval more correct data had been obtained by the measure of a degree by Picard. This attempt succeeded.

Having established it as a principle, that the force of gravity decreases as the square of the distance increases, and supposing the distance of the moon to be 60 semi-diameters of the earth, that is, 60 times as far from the earth's centre as a heavy body at its surface, we shall have, by calling the force at the surface 1, as $60^2 : 1 :: 1 : \frac{1}{3600}$, for the force of the earth's gravity, at the distance of the moon; and consequently, as a heavy body at the surface falls 16·083 feet in one second of time, the same body placed at the distance of the moon, would only fall through $\frac{16\cdot083}{3600}$ feet in a second, or 16·083 feet in a minute; because the spaces fallen are as the square of the times. Hence it follows, supposing the moon to be retained in her orbit by this force, that her deflection from the tangent to her orbit at any point, ought to be $\frac{16\cdot083}{3600}$ feet in one second; or, which is the same, the versed sine of the arc described in one second, ought to be $\frac{16\cdot083}{3600}$ feet. Now the distance of the moon from the earth in semi-diameters being known, and the diameter of the earth itself being also known in feet, as well as the exact time of one lunar revolution, it is easy to find the circumference of the lunar orbit, and the measure of the arc which the moon describes in one second, and hence again the versed sine of that arc in feet; which, as observed above, ought to be $\frac{16\cdot083}{3600}$.

This doctrine of innate gravitation has been the source of disputes among philosophers. Some have been startled by the idea of giving a self-operating power to inert matter, and others have contended for the actual existence of such miracle. Some have had the caution to regard gravitation as a mere accident and not as a quality of matter; and others have gravely proceeded to investigate the cause. These latter have been roundly accused of impiety, as if this matter of gravitation were one of those hidden mysteries into which man must not be permitted to look. Newton himself endeavoured, however, to assign a cause, and he was followed by Boscovich, Bernoulli, and Le Sage. But their theories are so palpably wrong, that Professor Vince availed himself of their errors to publish an attempted demonstration that no mechanical cause could be assigned for gravitation. In our own days, however, Sir Richard Phillips has given a challenge to the philosophical world, and has published various proofs, that all the phenomena of matter arise from quantity multiplied by velocity, which in all cases is transferred from body to body, or from atom to atom. Thus Sir Richard contends, that the two motions of the earth, namely, its rotatory and orbicular motions, are general motions quite adequate to the production of the phenomena of planetary aggregation, or of the fall of bodies towards their centres.

"If," says Sir Richard, "the rotatory force acted by itself, it would project the component parts of the earth into space, with momenta of the parts, which would increase till their momenta exceeded the original power of rotation or projection. But the instant a rectilinear force like the orbicular motion is applied to the mass, 65·9 times greater than the velocity of the rotatory motion at the circumference, or at its maximum, then every dense body would seek the line of the orbicular force, or collapse, towards that line. And, as the collapsing force would vary according to a law represented by the distance from the plane of motion or the sine of the latitude; and as the rotatory force would be as the velocity; so the sine and cosine of the latitude would constantly represent these varying forces, and would produce an equal diagonal motion in the constant radius of the planet.

"A cube of 800 feet of atmospheric air, with a density ·00125, revolves at the equator with a momentum equal to the momentum of a cubic foot of water moving 31.42625 miles in 24 hours, or 23 inches per second, and of any metal of a density of 10, at the rate of 2.3 inches, or of a density of 15 at the rate of 1.5333. But, if we suppose a density of .001 as having no central force, then the cubic foot of water would *relatively* have a momentum of one-fourth more. And, at lat. 45, where the sine and cosine are *equal*, or each are .707, and the rotatory velocity is as 17600 to 25100 at the equator; the ratios would be 10.0025 for water, 1.00025 for metal at 10, and .0673 for metal at 15, nearly. If then we consider the initial velocity, as expressed by the velocity in the first quarter of a second, and in this we cannot be far from the truth, though the initial velocity will, in its turn, be determined by the relative densities of bodies; but taking 10 as a mean density, it will appear that 1.0025 feet is the initial with which metal at 10 descends to the earth, in a quarter of a second; and in a second is as the square of the time, which gives 16.004 inches per second, as the actual velocity of metal of the density of 10 to fall to the earth; and, then allowing for the resistance of air, we have to a fraction the actual velocity per second at which a body falls to the earth, deduced from this theory of the mundane forces.

"Of course, as the squares of the sine and cosine are constantly equal to the square of the radius, and vary as one another, the same results, *cæteris paribus*, will take place in all latitudes in a system of bodies in which the particular result takes place at latitude 45°.—By generating a series of bodies having no central force from .001 at 4050 miles from the centre, it would appear that at 1 mile from the earth's centre, a body will have no central force which has a density of 4.06; at ½ a mile 8.12; at ¼th of a mile 16.24; and at ⅛th of a mile 32.48; and we may hence conclude, that no terrestrial bodies exceed 32.48 in density, if any are so dense; and this probable result seems to confirm the foregoing calculations."

Gravitation, therefore, on this theory, is a local phenomenon generated by the *motions* of each planet, and not inherent in matter, or universal. The action of the sun on the planets, of primaries on their secondaries, and of planets on each other, Sir Richard ascribes to their action or transferred motion to the gazeous medium which fills space, which force would be diffused or radiated *inversely as the squares of the distances*. The primary motion of the sun round the centre of the momenta of the solar system is transferred or diffused by that law to the medium which occupies the space of the system, the current of which carries round the planets by a law compounded of their distances, densities, and bulks, the cubes of their distances being necessarily as the squares of their periodical times. Hence gravitation is a local effect, generated within each planet by its own general motions; and of the two planetary motions the orbicular is caused by the impression of the revolving medium on the rear of the planet, and the rotatory by the different force with which the same medium acts on the near and remote hemispheres.

GRAVITY, the tendency or inclination of bodies towards a centre; or the force real or supposed of which gravitation is the display.

Terrestrial GRAVITY, is that by which all bodies are continually urged towards the centre of the earth. It is in consequence of this that bodies are accelerated in their fall, and when at rest that they press the body, or that part of the body by which they are supported.

We measure the force of terrestrial gravity by the space through which a body, from rest and free to fall, passes in one second, or rather by the velocity that it has acquired at the end of that time. Now, a body thus circumstanced, in the latitude of London, has been found from repeated experiments to have acquired, in one second, a velocity of $32\frac{1}{6}$ feet. This is called the force of gravity, and it is by it all other forces are compared.

Two causes, the spheroidal figure of the earth, and the diminution of the rotatory motion which takes place toward the poles, tend to disturb the uniformity of gravitation; and, on both accounts, a diminution must take place in the force of gravity in the equatorial regions, and hence also in the velocities of falling bodies.

In any intermediate latitude, the elliptical figure of the earth diminishes the force of gravity, as we proceed from the poles to the equator, by $\frac{1}{230}$ of the whole force at the equator multiplied by the square of the cosine of the latitude; and the centrifugal force diminishes it in the same proportion, and hence, by a combination of the two causes, the diminution of gravity from the poles to the equator is equal to 0·00694 multiplied by the square of the cosine of the latitude, the force of gravity at the equator being taken as unity.

The most accurate instrument that we possess, for measuring the intensity of gravity in different situations, is the pendulum, the oscillations of which are accelerated or retarded by the slightest alteration in this force, which retardations or accelerations being accumulated by repeated oscillations, renders the minutest change obvious, by means of a corresponding retardation or acceleration in the clock to which the pendulum is attached. The same means might also be employed to demonstrate experimentally the diminution in gravity on the tops of mountains, though we are not aware of the experiment having ever been attempted.

Absolute GRAVITY, is that by which a body descends freely and perpendicularly in a vacuum or non-resisting medium.

Relative GRAVITY, is that by which a body descends, when the absolute gravity is constantly counteracted by a uniform but inferior force, such as in the descent of bodies down inclined planes, or in resisting mediums.

Specific GRAVITY, is the relative gravity of any body or substance, considered with regard to some other body which is assumed as a standard of comparison, and this standard, by universal consent and practice, is rain-water, on account of its being less subject to variation in different circumstances of time, place, temperature, &c. than any other body. By a fortunate coincidence, at least to English philosophers, it happens, that a cubic foot of rain-water weighs 1000 avoirdupoise ounces; and, consequently, assuming this as the specific gravity of rain-water, and comparing all other bodies with this, the same numbers that express the specific gravity of bodies will denote the weight of a cubic foot of each in avoirdupoise ounces.

In bodies of equal magnitudes the specific gravities are directly as the weights, or as their densities.

In bodies of the same specific gravities, the weights will be as the magnitudes.

In bodies of equal weights, the specific gravities are inversely as the magnitudes.

The weights of different bodies are to each other in the compound ratio of their magnitudes and specific gravities.

Hence it is obvious that of the magnitude, weight, and specific gravity of a body, any two of these being given, the third may be found.

Exam. 1. The weight of marble statue being 748 lbs. avoirdupoise, required the number of cubic feet, &c. which it contains; the specific gravity of marble being 2742.

Since a cubic foot weighs 2742 ounces, we have

as $2742 : 748 \times 16 = 1 : 4{\cdot}36$ feet.

Exam. 2. Required the weight of a block of granite whose length is 63 feet, and breadth and thickness each 12 feet; the specific gravity of granite being 3500.

Here $63 \times 12 \times 12 = 9072$ feet; then again as $1 : 9072 = 3500 : 31752000$ ounces;
or 885 ton, 18¾ cwt.

A body immersed in a fluid will sink, if its specific gravity be greater than that of the fluid; if it be less, the body will rise to the top, and be only partly immerged; and if the specific gravity of the body and fluid be equal, it will remain at rest in any part of the fluid in which it may be placed.

When a body is heavier than a fluid, it loses as much of its weight when immersed, as is equal to a quantity of the fluid of the same bulk or magnitude.

If the specific gravity of the fluid be greater than that of the body, then the quantity of the fluid displaced by the part immerged, is equal to the weight of the whole body. And hence, as the specific gravity of the fluid is to that of the body, so is the whole magnitude of the body to the part immerged.

The specific gravities of equal solids, are as their parts immerged in the same fluid.

The specific gravities of fluids, are as the weights lost by the same immerged solid.

To find the specific gravity of a body.—This may be done generally by means of the *hydrostatic balance,* which is contrived for the easy and exact determination of the weights of bodies, either in air, or when immersed in water, or other fluid, from the difference of which the specific gravity of both the solid and fluid may be computed.

1. *When the body is heavier than water.*—Weigh it both out of water and in water; then say,

As the weight lost in water
Is to the whole or absolute weight,
So is the specific gravity of water
To that of the body.

2. *When the body is lighter than water.*—In this case attach to it a piece of another body heavier than water, so that the mass compounded of the two may sink together. Weigh the denser body and the compound body separately, both out of the water and in it; and find how much each loses in the water by subtracting its weight in water from its weight in air; and subtract the less of these remainders from the greater. Then use this proportion:

As the last remainder
Is to the weight of the light body in air,
So is the specific gravity of water
To the specific gravity of the body.

3. *When the specific gravity of a fluid is required.*—Take some body of known specific gravity; weigh it both in and out of the fluid, and find the loss of weight in the fluid, by taking the difference of these two; then say,

As the whole or absolute weight
Is to the loss of weight,
So is the specific gravity of the solid
To the specific gravity of the fluid.

For the method of finding the specific gravity of fluids by means of an hydrometer. See *Hydrometer.*

The following table exhibits the specific gravity of several of the most common bodies.

SPECIFIC GRAVITIES OF BODIES.

METALS.		Gold, pure, melted, not hammered …………	19258
Brass, cast, not hammered	8396	—— the same hammered …	19369
—— cast, common ………	7824	—— guinea, present coin …	17622
Copper, not hammered ……	7788		

Iron, cast	7207
Steel, neither tempered nor hardened	7833
—— hardened, but not tempered	7840
Iron ore, prismatic	7355
Lead, molten	11352
—— ore of, cubic	7587
Mercury, solid or congealed	15632
—— fluent	13568
Nickel, molten	7807
Platina, crude, in grains	15602
—— purified, not hammered	19500
—— ditto, hammered	20337
Silver, virgin, not hammered	10477
—— ditto, hammered	10511
—— shilling, present coin	10534
Tin, pure Cornish, melted, and not hardened	7291
—— hardened	7299
Zinc, molten	7191

STONES.

Beryl	3549
Chrysolite, of the jewellers	2782
Crystal	2653
Diamond	3521
Emerald	2775
Garnet	4189
Ruby	4283
Sapphire	3994
Spar	2595
Topaz	4011
Agate onyx	2638
Flint	2594
Jasper	2816
Opal	2114
Pearl	2684
Pebble	2664
Stone, paving	2416

VARIOUS STONES, EARTHS, &c.

Alabaster	2730
Amber	1078
Ambergris	926
Basaltes, from Giant's Causeway	2864
Brick	2000
Chalk	2784
Gypsum, opaque	2168
Glass, green	2642
—— white	2892
—— bottle	2733
Granite, red Egyptian	2654
Hone, white razor	2876
Limestone	3179
Marble	2742
Phosphorus	1714
Porphyry	2765
Pyrites	4954
Slate	2672
Stone, common	2520
Sulphur	2033

LIQUORS, OILS, &c.

Alcohol, commercial	837
—— highly rectified	829
Beer	1034
Milk, woman's	1020
—— cow's	1032
Oil of turpentine	870
—— whale	923
Water, rain	1000
—— distilled	1000
—— sea	1026
Wine, port	997

RESINS, GUMS, &c.

Bees-wax	965
Bone of an ox	1659
Camphor	989
Gunpowder	930
Honey	1456
Ivory	1822
Tallow	945
Wax	897

WOODS.

Alder	800
Ash, the trunk of	845
Beech	852
Box, French	912
Cedar	596
Cherry-tree	715
Cork	240
Ebony	1331
Elder-tree	695
Elm, trunk of	671
Fir	550
Lignum-vitæ	1333
Logwood	913
Mahogany	1063
Maple	750
Oak, heart of, 60 years old	1170
Poplar	383
Vine	1327
Walnut	671
Willow	585
Yew, Dutch	788
—— Spanish	807

GASSES.

Atmospheric air	1·2
Azotic	1·182
Hydrogenous gas	0·1
Oxygenous	1·435

GREAT *Circle* of the sphere, are those which have the same centre as the sphere, dividing it into two equal hemispheres; of these, in astronomy, the principal are the equator, ecliptic, horizon meridians, and colures.

GUNNERY, the art of charging,

directing, and exploding all kinds of fire-arms, but more commonly restricted to the larger pieces, as cannons, mortars, &c.

To this art belongs the knowledge of the force and effect of gunpowder, the dimensions of the pieces, and the proportions of the powder and ball they carry, with the method of adjusting, pointing, loading, firing, moving, &c. Gunnery may be therefore divided into theoretical and practical, the former of which consists in computing the angles of elevation, the impetus of projection, the range of the ball, &c. from certain data previously established; we may also consider those experiments which have been made with a view to ascertain the velocity of projection with given charges, the resistance of the air, the deflection of the ball or of the gun, &c. as forming a part of theoretical gunnery.

GUNPOWDER, a mixture of nitre, charcoal, and sulphur, for the purpose of producing an explosive force by combustion.

The first person who is said to have been acquainted with the nature and effects of a combination of these three materials in a certain proportion to each other was Roger Bacon. In his treatise "De Secretis Operibus Artis et Naturæ et de Nullitate Magiæ," cap. vi. published at Oxford about 1216, he informs us, that from saltpetre and other ingredients, we are able to make a fire that shall burn at any distance we please, which "other ingredients," it appears from some of his other manuscripts, were sulphur and charcoal. It was not, however, for a considerable time after this date that gunpowder became generally known; indeed, it is very difficult, amongst the various contradictory accounts, to affix any precise date to this invention. Robins, in his "Gunnery," considers it as known before the time of Bacon, and that this philosopher mentions it not as a new discovery, but as the application of an old principle to military purposes, to which it had not been before employed.

The proportions of the above ingredients, according to the best modern practice, is as follows, viz. 75 parts of nitre or saltpetre, 15 of charcoal, and 10 of sulphur. The specific gravity of which composition at a mean, as stated by Count Rumford, is about 0·868, that of water being 1.

One method of ascertaining the strength of gunpowder is by a machine called an eprouvette, of which there are various constructions; but it commonly consists of a small strong barrel, in which a determinate quantity of the powder is fired, and the force of expansion is measured by the action excited on a strong spring or a great weight.

Another method often adopted, is to fire a very heavy ball from a short mortar with a given weight of the powder, and to find the range of projection. The French eprouvette for government-powder is a mortar of seven inches (French) in calibre, which with three ounces of powder should throw a copper globe of sixty pounds weight to the distance of 300 feet. No powder is admitted which does not answer this trial.

A third method, which unites accuracy with dispatch, is to suspend a small cannon as a pendulum, to fire with powder only, and to judge of the force of explosion by that of the recoil, which in this circumstance is a greater or less arc of a circle. That which Dr. Hutton employed on this principle, was a small cannon about one inch in the bore, the charge of which is two ounces of powder.

To determine the elasticity and quantity of the elastic vapour, produced from a given quantity of powder, Mr. Robins premises, that its elasticity is equally increased by heat and diminished by cold, as that of common air; and, consequently, its weight is the same with the weight of an equal bulk of air at the same elasticity and temperature. Hence, and from direct experiments, he concludes, that the elastic fluid produced by the firing of gunpowder is nearly three-tenths of the weight of the powder itself, which, expanded to the rarity of common air, is about 244 times the bulk of the powder.

Hence it would follow, that the mere conversion of confined powder into elastic vapour, would exert against the sides of the containing vessel an expansive force 244 times greater than the elasticity of common air, or in other words, than the pressure of the atmosphere. But to this is to be superadded all the increase of expansive power produced by the heat generated, which is certainly very intense, though its exact degree cannot be ascertained. Supposing it to be equal to the full heat of red-hot iron, this would increase the expansion of common air (and also of all gasses) about four times, which, in the present instance, would increase the 244 to nearly 1000; so that in a general way it may be assumed, that the expansive force of closely confined powder at the instant of firing is 1000 times greater than the pressure of common air: and this latter is known to press with the weight of fourteen pounds and a quarter on every inch; the force of explosion of gunpowder is 1000 times this, or 14750 pounds, or about six tons and a half on every square inch. This enormous force, however, is diminished in proportion as the elastic fluid dilates, being only half the strength when it occupies a double space, one-third of the strength when in a triple space, and so on.

Mr. Robins found that the strength of powder is the same in all variations of the density of the atmosphere, but not so in every state of moisture, being much impaired by a damp air, or with powder, which is become damp with careless keeping, or any other cause; so that the same powder which will discharge a bullet at the rate of 1700 feet in a second in dry air, will only propel it at the rate of 1200 feet when the air is fully moist; and a similar difference holds between dry and moist powder.

Gunpowder is reckoned to explode at about 6000 of Fahrenheit's thermometer, but if heated to a degree just below that of faint redness, the sulphur will mostly burn off, leaving the charcoal unaltered.

GUNTER'S *Chain*, the chain in common use for measuring land according to the true or statute measure; so called from the name of its inventor. The length of the chain is 66 feet, or 22 yards, or four poles of five yards and a half each; and it is divided into 100 links of 7·92 inches each: 10 square chains, or 100,000 square links make one acre.

GUNTER'S *Line*, a logarithmic line, usually marked upon scales, sectors, &c. It is also called the *line of lines*, or *of numbers;* being the logarithms graduated upon a ruler, and serving to solve problems instrumentally, in the same manner as logarithms do arithmetically. It is usually divided into a hundred parts, every tenth of which is numbered, beginning with 1, and ending with 10; so that if the first great division, marked 1, stand for one-tenth of any integer, the next division, marked 2, will stand for two-tenths; 3, three-tenths, and so on; and the intermediate divisions will, in like manner, represent one-hundredth parts of an integer. If each of the great divisions represent 10 integers, then will the lesser divisions stand for integers; and if the great divisions be supposed each 100, the subdivisions will be each 10. The following is the method of performing the common operations.

1. *Multiplication.*—From 1 extend the compass to the multiplier; and the same extent, applied the same way from the multiplicand, will reach to the product. Thus if the product of 4 and 8 be required, extend the compasses from 1 to 4; and that extent laid from 8 the same way, will reach to 32, their product.

2. *Division.*—The extent from the divisor to unity, will reach from the dividend to the quotient; thus, to divide 36 by 4, extend the compasses from 4 to 1, and the same extent will reach from 36 to 9, the quotient sought.

3. *To find a fourth proportional to three given numbers.*—Suppose the numbers 6, 8, 9, extend the compasses from 6 to 8; and this extent, laid from 9 the same way, will reach to 12, the fourth proportional required.

4. *To find a mean proportional between any two given numbers.*—Suppose 8 and 32; extend the compasses from 8, in the left-hand part of the line, to 32 in the right; then bisecting this distance, its half will reach from 8 forward, or from 32 backward, to 16, the mean proportional sought.

5. *Involution.*—Extend the compasses from 1 to the root; and this distance repeated as often as there are 1s in the exponent, will fall on the power.

6. *To extract any root of a number.*—Extend the compasses from 1 to the number; divide this distance into as many equal parts as there are 1s in the exponent; and one of these parts set the same way from 1, will show the root.

GUNTER'S *Quadrant*, is a kind of stereographic projection of the sphere on the plane of the equinoctial, the eye being supposed in one of the poles; so that the tropic, ecliptic, and horizon, form the arches of circles, but the hour circles are other curves, drawn by means of several altitudes of the sun, for some particular latitude every year. This instrument is used to find the hour of the day, the sun's azimuth, &c. and other common problems of the sphere or globe; as also to take the altitude of an object in degrees.

GUNTER'S *Scale*, is usually two feet long, and about an inch and a half broad, with various lines upon it, both natural and logarithmic, relating to trigonometry, navigation, &c. On the one side are the natural lines, and on the other the logarithmic ones. The former side is first divided into inches and tenths, and numbered from one to twenty-four inches, running the whole length near one edge. One half the length of this side consists of two plain diagonal scales, for taking off dimensions for three places of figures. On the other half of this side are contained various lines relating to trigonometry, as performed by natural numbers, and marked thus; viz.

Rumb, the rumbs, or points of the compass;
Chord, the line of chords;
Sine, the line of sines;
Tang., the tangents;
S. T., the semi-tangents; and at the other end of this half are,
Leag., leagues, or equal parts;
Rumb, another line of rumbs;
M. L., miles of longitude;
Chor., another line of chords.

Also, in the middle of this foot are L and P, two other lines of equal parts; and all these lines on this side of the scale serve for drawing or laying down the figures to the cases in trigonometry and navigation. On the other side of the scale are the following artificial or logarithmic lines, which serve for working or resolving those cases; viz.

S. R., the sine rumbs;
T. R., the tangent rumbs;
Numb., line of numbers;
Sine, sines;
V. S., the versed sines;
Tang., the tangents;
Meri., meridional parts;
E. P., equal parts.

The lines on the natural side of the Gunter, serve for the construction of figures, and the measuring of their sides and angles after they are constructed. The logarithmic side serves for the purposes of calculation.

GYRATION, the act of whirling a body, or system of bodies, about one common centre.

H

HALO, or CORONA, a luminous and sometimes coloured circle, appearing occasionally around the heavenly bodies, but more especially about the sun and moon. This phenomenon is at times accompanied with others, such as *parhelia*, or mock suns; *paraselenes*, or mock moons; *anthelia*, or glories. All these appearances are occasioned by the refraction, reflection, or inflection of light falling upon or passing near this concrete vapour floating in the atmosphere; but they have not yet received the satisfactory explanation which has been given to the rainbow, a phenomenon nearly related to them.

Halos are observed in an atmosphere somewhat foggy or misty, but not in a cloudy nor clear one: a thin haziness in the higher regions of the air in summer, and in the lower regions in winter, commonly occasions some appearance of the halo; the characters of which are various, according to circumstances. In summer, halos are considered by the farmers as forerunners of wind or rain.

It is remarkable, that the large halos surrounding the sun and moon are commonly of given diameters; namely, about 23° or 24°, or double that magnitude, or 47°.

HARDNESS, or RIGIDITY, that quality in bodies by which their parts so cohere as not to yield inward, or give way to an external impulse, without instantly going beyond the distance of their mutual attraction; and therefore are not subject to any motion, in respect of each other, without breaking the body.

HARMONICAL *Arithmetic,* is sometimes used for the application of numbers to the science of music.

HARMONICAL *Curve,* an ideal curve into which a musical chord is supposed to be inflected, when put into such a motion as to excite sound.

HEAT, in *Natural Philosophy,* is one of the effects indicated by an increase of temperature, and the sensation it produces on the organs of feeling.

Various hypotheses have been advanced, both by ancient and modern philosophers, to account for the phenomena of heat, some considering it as a body, *sui generis,* and others only as a quality common to all bodies.

Macquer, although he regards heat as a fluid, and the only essentially fluid body in nature, and consequently as the cause of fluidity in other bodies; and notwithstanding that he speaks of the great facility with which it penetrates other bodies, or separates from them, and infers from these and other facts, the infinite smallness of the integrant parts of fire, (meaning by this term the cause of heat, or what is now denominated caloric); yet when treating of the same, he expresses an opinion directly the reverse of this, and calls heat only a mode of existence of a material substance. He carries on the discussion through several pages with much ability; and amongst various other modes of argumentation, he institutes an analogy, which he sustains with considerable ingenuity between the communication of heat and the communication of motion, deducing from the known and acknowledged laws of the latter the similarity of the former, and maintaining consequently that heat is really nothing else than the motions of the parts of heated bodies.

"I have hitherto," says he, "been of opinion, I confess, with most natural philosophers, that heat was a particular kind of matter, so subtile as to penetrate all bodies, and to separate their parts when put in action by light or percussion, and that this being was the true matter of fire; but the above reflections have suggested a very different opinion. There certainly is a matter of fire; and it is pure light, which is a material substance whose existence cannot be questioned. But we cannot say the same of heat; the causes which excite it, and the effects which it produces, do not prove or even suppose the existence of a peculiar matter. They all concur, on the contrary, to indicate that it is only an accident, a modification of which some bodies are susceptible, and consisting merely in the intestine motion of their aggregant and constituent parts, which may be produced not only by the impulse of light, but also by all frictions and percussions of any bodies."

Count Rumford, from two considerations, infers the immateriality of heat; first, from his frequent failures, in attempting to discover that it had any weight; and, secondly, from the possibility, which he conceives he has demonstrated, of obtaining an inexhaustible supply of heat from a given quantity of insulated matter.

After ascertaining that when a hot body is cooled in tranquil air, a very small part of the heat, about a seventeenth, is communicated to the air, all the rest being communicated by radiation through the air to the surrounding solid bodies; and confirming, by various results, his hypothesis of the immateriality of heat, he proceeds to treat of the communication of heat amongst solid bodies, and concludes with an inquiry, whether the quantity of heat excited by the striking of the solar rays upon an opaque body, is always in proportion to the quantity of light that has disappeared? This question he answers in the affirmative; ascribing to light the power of producing heat. This last position, however, has been rendered doubtful, by Dr. Herschel's distinction in the solar rays, between those which produce heat and those which only illuminate. Sir Richard Phillips contends, that heat is the effect of atomic motion, and that the degree of temperature depends upon the velocity which is communicated to the maleculæ. And with regard to the apparently frigorific effect upon the skin, of steam at a very high temperature, he conceives that it arises from the very great distance to which the particles are separated from each other.

Latent HEAT, is a phrase used by Dr. Black, arising from the discovery that a large portion of heat sometimes disappears, or is absorbed by a body, without any increase of temperature. This commonly happens when a body from a solid becomes liquid, or from a liquid becomes aeriform; sometimes also, when two bodies of the same temperature combine, a great degree of cold is produced; but when a due portion of heat is acquired from the surrounding bodies, the temperature is restored. The heat which disappeers in these cases is called *latent.*

HEAVEN, among the ancients, denoted an orb or circular region of the universe. They assumed as many heavens as they observed celestial motions. These they supposed to be solid, as otherwise they could not sustain the bodies fixed in them; and spherical, that being the most proper form for motion. They had seven heavens for the Moon, Mercury, Venus, the Sun, Mars, Jupiter, and Saturn. The eighth was the fixed stars, which was denominated the firmament. Ptolemy adds a ninth, which he calls the *primum mobile:* after him two crystalline heavens were added by Alphonsus, to account for some irregularities in the motions of the other heavens. An empyrean heaven, the abode of the gods, surmounted the whole.

HEGIRA, the epoch or account of time used by the Mahomedans, who begin their computation from the day that Mahomet was obliged to abandon the city of Mecca, July 16, 622.

The years of the hegira are lunar ones, of 354 days; and therefore to reduce them to our Calendar, we must multiply the year of the hegira by 354, and divide the product by 365¼; adding 622, the date at which the Hegira commenced; the result will be the Julian year.

HELIACAL, as applied to the rising of a star, planet, &c. denotes its emerging out of the sun's rays, in which it was before hid. When applied to the setting of a star, it denotes the entering or emerging into the sun's rays.

HELICOID *Parabola,* or *Parabolic Spiral,* is a curve arising from a supposition of the axis of the common parabola being bent round into the periphery of a circle.

HELIOCENTRIC *Place* of a *Planet,* the place in the ecliptic in which the planet would appear if viewed from the centre of the sun.

HELIOCENTRIC *Latitude* of a *Planet,* is the angle which a line joining the centres of the sun and planet, makes with the plane of the ecliptic.

HELIOCOMETES, *Comet* of the *Sun,* is used to denote a large tail or column of light which follows the sun at his setting, much in the same manner as the tail of a comet.

HELIOMETER, an instrument for measuring with particular exactness the diameter of the sun, moon, and planets. It is a sort of telescope, consisting of two object-glasses, of equal focal distance, placed by the side of each other, so that the same eye glass serves for both. The tube of this instrument is of a conical form, larger at the upper end, which receives the two object-glasses, than at the lower, which is furnished with an eye-glass and micrometer. Hence two distinct images of an object are formed in the focus of the eye-glass, the distance of which depending upon that of the two object-glasses from one another, may be measured with the greatest accuracy.

HELIOSCOPE, a sort of telescope peculiarly suited for viewing the sun without prejudice to the eye. The same effect, however, is more commonly produced by coloured glasses.

HELISPHERICAL *Line*, in *Navigation*, denotes the rhumb line; and is thus called from its winding round and round the pole in the form of a *helix* or *spiral*.

HEMISPHERE, one-half of a globe or sphere, formed by a plane passing through the centre.

HEMISPHERE, in *Astronomy*, is particularly used to denote one-half of the sphere, as separated by the plane of the equator; that situated towards the north pole being called the northern, and the other the southern hemisphere.

The horizon also divides the sphere into two hemispheres, the lower and upper; the latter being that half which has the zenith for its vertex, the former having the nadir in its vertex.

A meridian, in like manner, divides the sphere into an eastern and a western hemisphere; and any other great circle divides it into two oblique hemispheres.

HEPTAGON, a figure having seven angles and seven sides. When the sides and angles are equal, it is called regular; when not, irregular.

Properties.

The angle at the centre $= \frac{360^\circ}{7}$ $= 51^\circ\frac{3}{7}$

The angle of the polygon $= 128\frac{4}{7}$

The area, when the side is 1, $=$ 3·6339126

And, therefore, when the side is s, it $= s^2 \times 3{\cdot}6339126$.

The heptagon does not admit of a geometrical construction; nor can its side be found, when the radius of the circle is given, by an equation lower than a cubic.

HEPTAGONAL *Numbers*, are those included in the formula $\frac{5n^2-3n}{2}$; being thus called, because all such numbers may be arranged so as to form an heptagonal figure.

It is a property of heptagonal numbers, that if they be multiplied by 40, and 3 added to the product, the sum will be a square.

HETEROGENEOUS, signifies something whose parts are of different kinds, in opposition to *homogeneous*.

HETEROGENEOUS *Bodies*, are of unequal density and composition.

HETEROGENEOUS *Light*, consists of rays having different degrees of refrangibility.

HETEROGENEOUS *Quantities*, are those which admit of no comparison as to greater or less. Thus, a surface and a solid are heterogeneous quantities, because we can in no way make comparison between, or say that a surface, however large, is greater than a solid however small. In this respect it has been said, that momentum and pressure are heterogeneous quantities.

HETEROGENEOUS *Surds*, are such as have different indices, as $\sqrt[3]{7}$, $\sqrt[6]{5}$, also the sums of two surds having the same index is heterogeneous with a single surd having that index kind, at least if the quantities under the radicals are prime to each other; thus $(\sqrt{7} \pm \sqrt{5})$ is heterogeneous with the square root of any single quantity.

HERETOSCII, such inhabitants of the earth as have their shadows at noon projected always the same way with regard to themselves, or always contrary ways with respect to each other.

HEXAEDRON, the CUBE, one of the five regular or Platonic bodies; and so called from its having six faces.—The square of the side or edge of a hexaedron, is one-third of the square of the diameter of the circumscribing sphere; and hence the diameter of a sphere is to the side of its inscribed hexaedron as $\sqrt{3}$ to 1.

In general, if l, s, and S, be put

to denote respectively the linear side, the surface, and the solidity of a hexaedron or cube, also r the radius of the inscribed sphere, and R the radius of the circumscribed one; then we have these general equations or relations:

1. $l = 2r = \frac{2}{3}R\sqrt{3} = \sqrt{\frac{1}{6}s}$
$= \sqrt[3]{s}$
2. $s = 24r^2 = 8R^3 = 6l^3$
$= 6\sqrt[3]{s^2}$
3. $S = 8r^3 = \frac{8}{9}R^3\sqrt{3} = l^3$
$= \frac{1}{6}s\sqrt{\frac{1}{6}s}$
4. $R = r\sqrt{3} = \frac{1}{2}l\sqrt{3} = \frac{1}{2}\sqrt{\frac{1}{2}s}$
$= \sqrt{\frac{3}{4}}\sqrt[3]{s}$
5. $r = \frac{1}{3}R\sqrt{3} = \frac{1}{2}l = \frac{1}{2}\sqrt{\frac{1}{6}}$
$= \frac{1}{2}\sqrt[3]{s}$

From which equations all those quantities may be found, if any one of them be given.

HEXAGON is a figure of six sides and angles. It is regular, when both sides and angles are equal; irregular, when these are unequal.

To describe a regular Hexagon on a given Line.—On the given line describe an equilateral triangle, and from the vertex as a centre, and with a radius equal to the given line describe a circle; the given line applied to the circumference will cut it in the angles of the Hexagon. To inscribe a hexagon on a circle: apply the radius to the circumference.

Propertus.

Angle at the centre = 60°
Angle at the circumference = 120°
Area to side 1 = 2.5980762
Area to any side $(s) = s^2 \times 2.5980762$

HIERO'S *Crown*. Hiero, king of Syracuse, having furnished a workman with a quantity of gold for making a crown, suspected that he had been cheated, by the workmen's using a greater alloy of silver than was necessary in the manufacture of it; so he applied to Archimedes for a discovery of the fraud. This celebrated mathematician was led to a method of detecting the imposture, and of determining precisely the quantities of gold and silver of which the crown was composed; by observing, whilst he was bathing, that as he immerged his body the water ran over the bath; from which he concluded, that the water which ran out, when his whole body was immerged, was equal in bulk to his body. It is said that he was so pleased with the discovery as to run about naked, crying out, "Ευρεικα! Ευρεικα!—I have found it."

On this he procured a ball of gold and another of silver, exactly of the weight of the crown, considering that if the crown were altogether of gold, the ball of gold would be of the same bulk as the crown, and would, when immersed in water, raise the water just as high as the crown would; but if it were wholly of silver, the ball of silver being immersed, would raise the water no higher than the crown immersed; and if the crown was of gold and silver mixed in a certain proportion, this proportion would be discovered by the height to which the crown would raise the water higher than the gold and lower than the silver: this, upon trial, he found to be the case; and hence, by a comparison of the quantities of water displaced by the three masses, he discovered the exact portions of gold and silver in the crown.—Suppose, for example, that each of them weighed 100 ounces, and that on immersing them severally in water, there were displaced 5 ounces of water by the golden ball, 9 ounces by the silver, and 6 ounces by the compound or crown; 5, 9, and 6 would be their comparative bulks, and 20 their sum. The difference between the gold and silver is 4; and the crown is 1 different from the gold, and 3 from the silver; wherefore,
As 4 : 3 = 100 : 75 ounces of gold; and
As 4 : 1 = 100 : 25 ounces of silver.

HIGH *Water*, that state of the tides when they have flowed to the greatest height, in which state they remain nearly stationary for about 15 or 20 minutes, when the water begins again to ebb.

The time of high water is always nearly the same in the same place at the full of the moon, and at all other times the time of high water depends upon the age of the moon. The rule for finding which, the age of the moon being given, is as follows, *viz.* Add $\frac{4}{5}$ths of the days of the moon's age, as so many hours, to the time of high water at the full of the moon, and the sum is the time of high water, answering to that day *nearly*. The time of high water at London, on the day of the full moon, is 3 o'clock in the afternoon.

HOGSHEAD, an English measure of 63 gallons.

HOMOGENEAL, or HOMOGENEOUS, is a term applied to various subjects to denote that they consist of similar parts, or of parts of the same nature and kind; in contradistinction to heterogeneous.

HOMOGENEAL *Light*, is that whose rays are all of one and the same colour, degree of refrangibility and reflexibility.

HOMOGENEAL *Numbers*, are those of the same kind and nature.

HOMOGENEAL *Surds*, are such as have one common radical part; as $\sqrt[3]{27}$, and $\sqrt[3]{3}$.

HOMOLOGOUS, literally signifies, *of the same name*, and is applied to the corresponding sides of similar figures, which are said to be homologous, or in proportion to each other.

Thus, the base of one triangle is homologous to the base of another similar triangle; and in similar triangles, the sides opposite to equal angles are homologous.

Equiangular, or similar triangles, have their homologous sides proportional.

All similar triangles, rectangles, and polygons, are to each other as the squares of their homologous sides.

HORARY *Circles*, hour lines or circles, marking the hours on globes, dials, &c.

HORARY *Motion*, the motion or space moved in an hour; the horary motion of the earth on its axis, or the apparent horary motion of the sun is 1°.

HORIZON, in *Geography* and *Astronomy*, a great circle of the sphere, dividing the world into two parts or hemispheres, the one upper and visible, the other lower and hid.

The horizon is either *rational* or *sensible*.

The *Rational*, *True*, or *Astronomical* HORIZON, which is also called simply and absolutely the horizon, is a great circle, whose plane passes through the centre of the earth, and whose poles are the zenith and nadir. It divides the sphere into two equal parts or hemispheres.

The *Sensible*, *Visible*, or *Apparent* HORIZON, is a less circle of the sphere, which divides the visible part of the sphere from the invisible. Its poles are likewise the zenith and nadir; and consequently the sensible horizon is parallel to the rational. These two horizons, though distant from each other by nearly the semidiameter of the earth, will appear to coincide, when continued to the sphere of the fixed stars, because the earth compared with this sphere is but a point.

The sensible horizon is divided into eastern and western.

The *Eastern* or *Ortive* HORIZON, is that part of the horizon wherein the heavenly bodies rise.

The *Western* or *Occidual* HORIZON, is that wherein the stars set.

By sensible horizon is also frequently meant a circle, which determines the segment of the surface of the earth, over which the eye can reach. In this sense we say, a spacious horizon, a narrow scanty horizon. It is manifest that the higher the spectator is raised above the earth, the farther this visible horizon will extend. On account of the refraction of the atmosphere, distant objects on the horizon appear higher than they really are, or appear less depressed below the true horizon, and may be seen at a greater distance, especially on the sea. Legendre says, that, from several experiments, he is induced to allow for refraction a 14th part of the distance of the place observed, expressed in degrees and minutes of a great circle.

HORIZON of a *Globe*, the broad circular ring, in which the globe is fixed. On this are several concentric circles, which contain the months and days of the year, the corresponding signs and degrees of the ecliptic, and the 32 points of the compass.

Artificial Horizon, is a contrivance intended to be employed in taking altitudes. It is usually made by a quantity of mercury in a trough, upon the top of which is laid a piece of plate glass; and the whole is fenced in by pannels of glass, which make equal angles with the plane of the mercury.

HORIZONTAL, relating to the horizon.

Horizontal *Dial,* is drawn on a plane parallel to the horizon, and having its style elevated according to the altitude of the pole of the place for which it is designed.

Horizontal *Distance*, is that estimated in the direction of the horizon.

Horizontal *Plane,* is that which is parallel to the horizon of the place, or not inclined to it.

HOROGRAPHY, the art of making dials, &c. to tell the hours.

HOUR, in *Chronology,* an aliquot part of a natural day, usually a 24th, but sometimes a 12th. An hour, with us, is a measure or quantity of time, equal to a 24th part of the natural day, or the duration of the 24th part of the earth's diurnal rotation. Fifteen degrees of the equator answer to an hour, though not precisely, yet near enough for common use. It is divided into 60 minutes, the minute into 60 seconds, &c. The *Jewish* or *Ancient* hours are twelfth parts of the artificial day or of the night. Hence, as it is only at the time of the equinoxes that the artificial day is equal to the night, it is then only that the hours of the day are equal to those of the night, or to the 24th part of the natural day.

HOUR *Circles,* are great circles of the sphere, meeting in the poles, and crossing the equator at right angles. They are drawn through every 15th degree of the equinoctial or equator, each answering to an hour.

Hour *Glass,* a popular kind of time-keeper, which indicates the hours by the running of sand, water, or mercury, from one vessel to another.

Hour *Lines* on a *Dial,* are those which are drawn for the purpose of pointing out the hours by the shadow of the style.

HUMIDITY, or *Moisture;* has been differently defined by modern writers, some considering it as a peculiar kind of fluidity, others defining it simply as a quality. It is obvious, however, that whatever definition is given of humidity, that it is only a relative mode. For quicksilver is not moist with respect to wood, glass, or iron, while it is so with regard to gold, tin, or lead, to the surfaces of which it will adhere and render them soft and moist. Even water itself, which wets almost every thing, and is the great standard of moisture and humidity, is yet not capable of wetting all things, for it stands or runs off in globular drops from the leaves of many plants, as also from oiled and varnished surfaces, the feathers of ducks; water therefore is no more moist, with reference to these substances, than quicksilver is with regard to iron or wood.

HYDRAULICS, the science or doctrine of the motion of liquids universally, the laws of that motion, and the effects resulting from it. The terms hydraulics and hydrodynamics are nearly synonymous.

Hydraulic Press. See *Press.*

HYDRODYNAMICS, that science which treats of the power or force of water, whether it acts by impulse or pressure; or that part of mechanics which relates to the motion of liquids or non-elastic fluids, and the forces with which they act upon other bodies, in which sense *hydrodynamics* bears the same relation to *hydrostatics* as *dynamics* does to *statics.*

HYDROGRAPHY, that part of geography which relates to seas or oceans.

HYDROMETER, an instrument for measuring the density, gravity, &c. of water, and other liquids.

Dr. Desaguliers contrived an hydrometer for determining the specific gravities of different waters to such a degree of nicety, that it would show when one kind of water was but the 40,000th part heavier than another. It consists of a hollow ball of about three inches of diameter, charged with shot to a proper degree, and having fixed in it a long and very slender wire, of only the 40th part of an inch in diameter, and divided into tenths of inches, each tenth answering to the 40,000th, as above. Fig. 7, plate III. is an hydrometer by Mr. Nicholson, adapted to finding specific gravity both of solids and fluids.

B is a hollow ball of copper, C a dish affixed to the ball by a short slender stem, D; E is another, affixed to the opposite side of the ball by a kind of stirrup. In the instrument actually made, the stem D is of hardened steel $\frac{1}{40}$th of an inch in diameter, and the dish is so heavy as in all cases to keep the stem vertical when the instrument is made to float in any liquid. The parts are so adjusted that the addition of 1000 grains in the upper dish C, will just sink it in distilled water (at the temperature of 60° of Fahrenheit's thermometer) so far that the surface shall intersect the middle of the stem D. Let it now be required to find the specific gravity of any fluid. Immerse the instrument in it, and by placing weights in the dish C, cause it to float, so that the middle of its stem D shall be cut by the surface of the fluid. Then, as the known weight of the instrument, added to 1000 grains, is to the same known weight added to the weight used in producing the last equilibrium, so is the weight of a quantity of distilled water displaced by the floating instrument, to the weight of an equal bulk of the fluid under examination. And these weights are in the direct ratio of the specific gravities. Again, let it be required to find the specific gravity of a solid body, whose weight is less than 1000 grains. Place the instrument in distilled water, and put the body in the dish C. Make the adjustment of sinking the instrument to the middle of the stem, by adding weights in the same dish. Subtract those weights from 1000 grains, and the remainder will be the weight of the body. Place now the body in the lower dish F, and add more weight in the upper dish, C, till the adjustment is again obtained. The weight last added will be the loss the solids sustain by immersion, and is the weight of an equal bulk of water. Consequently, the specific gravity of the solid is to that of water, as the weight of the body to the loss occasioned by the immersion.

HYDROMETRY, the mensuration of the density, gravity, force, &c. of water, and other fluids.

HYDROSTATIC *Balance*, an instrument for determining very accurately the specific gravity of bodies, whether fluid or solid: there are various constructions given to this instrument; that represented on plate IV. is one of the most accurate. VCG, is the stand or pillar of this hydrostatic balance, which is to be fixed on a table 1, 2, 3, 4. From the top A hangs, by two silk strings, the horizontal bar BB, from it is suspended, by a ring *l*, the fine beam of a balance *b*; which is prevented from descending too low on either side by the gently springing piece *txyz*, fixed on the support M. The harness is annulated at *o*, to show distinctly the perpendicular position of the examen, by the small pointed index fixed above it. The strings by which the balance is suspended, passing over two pulleys, one on each side the piece at A, go down to the bottom on the other side, and are passed over the hook at *v*; which hook, by means of a screw P, is moveable about one inch and a quarter, backward and forward, and therefore the balance may be raised or depressed so much. But if a greater elevation or depression be required, the sliding piece S, which carries the screw P, is readily moved to any part of the square brass rod V K, and fixed by means of a screw.

The motion of the balance being thus adjusted, the rest of the apparatus is as follows: H H is a

HYDROSTATIC BALANCE.

Plate IV.

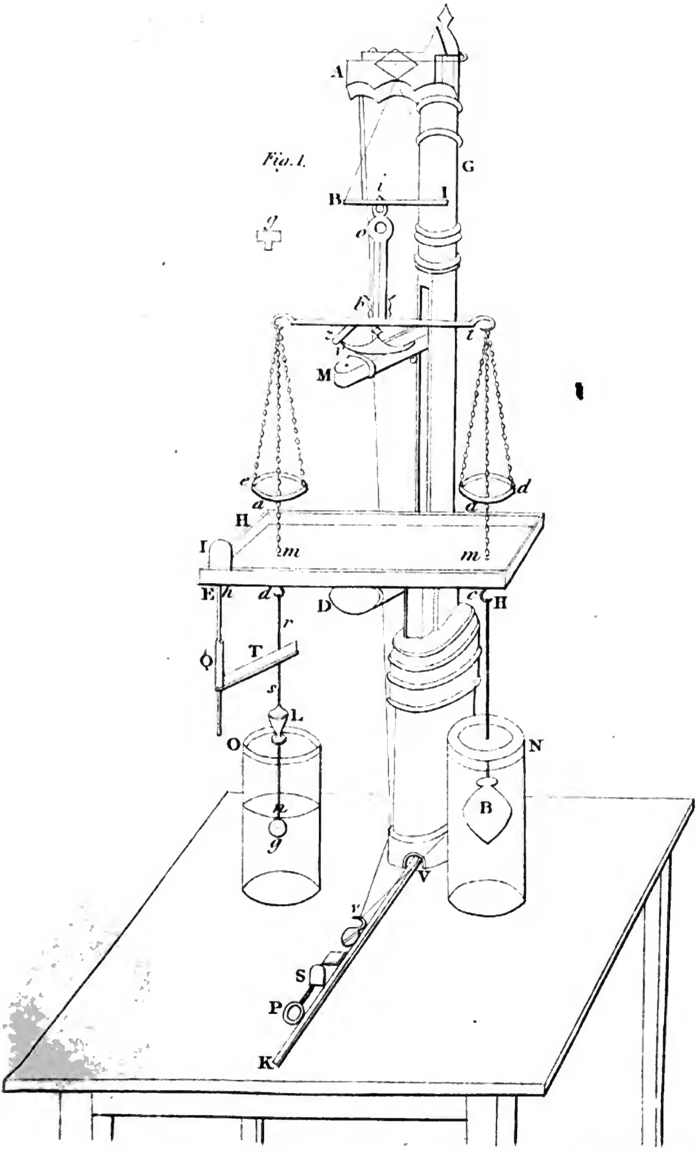

Fig. 1.

Neele & Son Sc. Strand.

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ASTOR, LENOX AND
TILDEN FOUNDATIONS
R L

small board, fixed upon the piece D, under the scales *d* and *e*, and is moveable up and down in a low slit in the pillar above C, and fastened at any part by a screw behind. From the point in the middle of the bottom of each scale hangs, by a fine hook, a brass wire *a d* and *a c*. These pass through two holes *m m* in the table. To the wire *a d* is suspended a curious cylindric wire *r s*, perforated at each end for that purpose: this wire *r s* is covered with paper, graduated by equal divisions, and is about five inches long. In the corner of the board at E is fixed a brass tube, on which a round wire *h l* is so adapted as to move neither too tight nor too free, by its flat head I. Upon the lower part of this moves another tube Q, which has sufficient friction to make it remain in any position required: to this is fixed an index T, moving horizontally when the wire *h l* is turned about, and therefore may be easily set to the graduated wire *r s*. To the lower end of the wire *r s* hangs a weight L; and to that a wire *p n*, with a small brass ball *g*, about one-fourth of an inch diameter. On the other side, to the wire *a c*, hangs a large glass bubble R by a horse-hair.

Let us first suppose the weight L taken away, and the wire *p n* suspended from S; and, on the other side, let the bubble R be taken away, and the weight F suspended at *c*, in its room. This weight F we suppose to be sufficient to keep the several parts hanging to the other scale in equilibrium; at the same time that the middle point of the wire *p n* is at the surface of the water in the vessel O. The wire *p n* is to be of such a size that the length of one inch shall weigh four grains.

It is evident, since brass is eight times heavier than water, that for every inch the wire sinks in the water, it will become half a grain lighter, and half a grain heavier for every inch it rises out of the water: consequently, by sinking two inches below the middle point, or rising two inches above it, the wire will become one grain lighter or heavier. Therefore, if when the middle point is at the surface of the water in equilibrio, the index T be set to the middle point *a* of the gradnated wire *r s*, and the distance on each side *a r* and *a s* contains 100 equal parts; then, if in weighing bodies the weight is required to the hundredth part of a grain, it may be easily had by proceeding in the following manner.

Let the body to be weighed be placed in the scale *d*. Put a weight into the scale *e*; and let this be so determined, that one grain more shall be too much, and one grain less too little. Then the balance being moved gently up or down, by the screw P, till the equilibrium be nicely shown at *o*; if the index T be at the middle point *a* of the wire *r s*, it shows that the weights put into the scale *e* are just equal to the weight of the body. By this method we find the absolute weight of the body: the relative weight is found by weighing it hydrostatically in water, as follows:

Instead of putting the body in the scale *e*, as before, let it hang with the weight F, at the hook *c*, by a horse-hair, as at R, supposing the vessel N of water were away. The equilibrium being then made, the index T standing between *a* and *r*, at the 36 division, shows the weight of the body put in to be 1095, 36 grains. As it thus hangs, let it be immersed in the water of the vessel N, and it will become much lighter: the scale *e* will descend till the beam of the balance rest on the support *z*. Then suppose 100 grains put into the scale *d* restore the equilibrium precisely, so that the index T stand at the 36 division above *a*; it is evident that the weight of an equal bulk of water would, in this case, be exactly 100 grains. In like manner, this balance may be applied to find the specific gravity of liquids.

Hydrostatic *Bellows*, is a machine for illustrating the upward pressure of fluids. It consists of two thick boards EF, CD, Plate III. fig. 8. about sixteen or eighteen inches diameter, covered or connected firmly with pliable leather round the edges to open and shut like common bellows, but without valves; a pipe A B, about three feet high, is fixed into the bellows at B. Now let water be poured into the pipe at A, and it will run into the bellows, gradually separating the boards by raising the upper one. Then, if several weights (three hundred weights, for instance) be laid upon the upper board, the water being poured in at the pipe till it be full, will sustain all the weights, though the water in the pipe should not weigh a quarter of a pound. For the narrower the pipe the better (beyond certain limits,) provided we make it long enough, the proportion being always this:

As the area of the orifice or section of the pipe,
To the area of the bellows' board, E F;
So is the weight of water in the pipe A G,
To the weight it will sustain on the board. For the fluid at B, the bottom of the tube, is pressed with a force varying as its altitude A B; and this pressure is communicated horizontally to all the particles in the space F E, and then distributed equally throughout the fluid in the bellows; consequently, the pressure upwards at F E is equal to the weight of a cylinder of the fluid whose base is F E, and altitude A B; while the actual weight of water borne up is only that of the cylinder, whose base is F E, and height B G; and hence no weights laid upon C D that do not exceed the weight of a cylinder, of the fluid whose base is E F and altitude A G, will disturb the equilibrium.

HYDROSTATIC *Paradox*, relates to the principle of the equal pressure of fluids in all directions, as stated in the preceding article. It is thus denominated, because, at first sight, it has a paradoxical appearance; it results, however, from the nature of fluids, and will be found illustrated under the article PRESSURE. What is usually denominated the hydrostatic paradox is this, that any quantity of water, however small, may be made to balance and support any quantity however great. Thus water in a pipe or canal open at both ends, always rises to the height at each, whether those ends be wide or narrow, equal or unequal.

HYDROSTATICS, that branch of mechanics which relates to the pressure and equilibrium of non-elastic fluids, and that of the weight, pressure, stability, &c. of solids immersed in them.

HYGROMETER is an instrument for measuring the degree of moisture or dryness of the atmosphere.

Hygrometers are constructed of a great variety of substances, and in a great variety of manners. The most simple is a piece of rock-salt, but though the moisture upon its surface indicate moisture in the air, it does not afford the means of measuring the degree. After this the most simple hygrometer is formed of a catgut, made fast at the one extremity. When moistened, the catgut shrinks in length, and when dry it expands; and thus by fixing to the moveable end an index, which ranges over a graduated scale, the degree is pointed out. Saussure has obviated some of the disadvantages of this by a hygrometer of which the following is a description: The principal piece in this hygrometer is a hair, which Saussure first causes to undergo a preparation, the design of which is to divest it of a kind of oiliness which is natural to it, and which secures it, to a certain point, from the action of humidity. This preparation is made at the same time upon a certain number of hairs forming a tuft, the thickness of which need not exceed that of a writing-pen, and contained in a fine cloth, serving them for a case. The hairs thus enveloped are immersed in a long-necked phial full of water, which holds in solution nearly a hundredth part of its weight of sulphate of soda, making this water boil near thirty minutes; the hairs are then passed through two vessels of pure water, while they are boiling; afterwards they are drawn from their wrapper, and separated, then they are suspended to dry in the air; after which there only remains to make choice of those which are the cleanest, softest, most brilliant, and most transparent. It is known that humidity lengthens the hair, and that the process of drying shortens it. To render both these effects more perceptible, Saussure attached one of the two ends of the hair to a fixed point, and the other to the circumference of a moveable cylinder, which carries at one of its extremities a light index or hand. The hair is bound by a counter-weight, of about three grains, suspended by a delicate silk, which is rolled in a contrary way about the same cylinder. In proportion as the hair lengthens or shortens, the weight causes the cylinder to turn in one or the other direction, and by a necessary consequence the little index turns likewise, the motions of which are measured on a circumference of a graduated circle, about which the index performs its revolution, as in common clocks. In this manner a very small variation in the length of the hair becomes perceptible, by the much more considerable motion that it occasions in the extremity of the index; and it will be easily conceived, that equal degrees of expansion, or of contraction in the hair, answers to equal arcs described by the extremity of the index. Now in order to give to the scale such a basis as may establish a relation between all hygrometers constructed upon the same principles, Saussure assumes two fixed terms, one of which is the extreme of humidity, and the other that of dryness; he determines the first by placing the hygrometer under a glass receiver, the whole interior surface of which he had completely moistened with water; the hair being saturated by this water, acts by its humidity upon the hair to lengthen it. He then again moistened anew the interior of the receiver, repeating it as often as it was necessary, and thus obtained the term of extreme humidity. To obtain the contrary limit of extreme dryness, he made use of a hot and well-dried receiver, under which he included the hygrometer, with a piece of iron-plate, likewise heated and covered with a fixed alkali. This salt, by exercising its absorbent faculty upon the remaining humidity in the surrounding air, causes the hair to contract itself, until it has attained the ultimate limit of its contraction. The scale of the instrument is then divided into a hundred degrees, the zero indicating the limit of extreme dryness, and the number one hundred that of extreme humidity. Other hygrometers, more simple, and perhaps as useful, have been contrived by Mr. Kater, Professor Lesslie and others.

HYGROMETRY, the art of measuring the degree of moisture and dryness of the atmosphere.

HYGROSCOPE, an instrument for showing the degree of moisture or dryness of the atmosphere, having the same relation to the hygrometer that the baroscope has to the barometer.

HYPERBOLA, is one of the conic sections, formed by the intersection of a plane and cone, when the plane makes a greater angle with the base of the cone, than that formed by the base and side of the cone.

Fig. 1. *Plate* HYPERBOLA. The equation $yy = \frac{\sin A}{\cos^2 \frac{1}{2} B} \left\{ cx \sin B + xx \sin (A + B - 180^\circ) \right\}$ shews that the hyperbola meets its axis A P in two points, one of which is at A, where $x = o$: the other at *a* where

$$x = - \frac{c \sin B}{\sin (A + B - 180^\circ)};$$

therefore supposing that A*a* is equal to $\frac{c \sin B}{\sin (A + B - 180^\circ)}$, the point *a* will be in the opposite hyperbola M′*am*′. The points A, *a*, are called the vertices of the hyperbola, the line A*a* (2*a*) is its axis, its middle point C is the centre; lastly, a right line B*b* = 2 C B = 2*b*, such that

$$\frac{bb}{aa} = \frac{\sin A \sin (A + B - 180^\circ)}{\cos^2 \frac{1}{2} B}$$

drawn perpendicular to the axis, and passing through the centre C, is called the second, less, or conjugate axis.

These values of a and b being substituted in the equation of the hyperbola, gives $yy = \frac{bb}{aa}(2ax = xx)$. This equation shews that the curve has two branches, A M and Am, equal and infinite in a positive direction. But if x be negative there will be no curve as long as $x < 2d$; if $x > 2a$ the ordinates become real, and the curve will have two other branches indefinitely extended.

It is easy to prove that these two branches are equal to those of the *positive* hyperbola M Am. For since, calling A P $(-x)$, $P'm' = P'M'$ (y), we have $\frac{a^2}{b^2}\ yy = -2ax + xx$, if we make $aP' = x'$ we shall have $x = 2a + x'$, and consequently $\frac{a^2y^2}{b^2} = 2ax' + x'x'$; an equation perfectly similar to that of the hyperbola M Am.

Since $yy = \frac{bb}{aa}\ (2ax + xx)$, we have $PM^2 : AP \times Pa = CB^2 : CA^3$.

Therefore in the hyperbola, the squares of the ordinates of the transverse axis are to the rectangles of their abscissæ, (that is of the distances to the two vertices,) as the square of the second or conjugate axis is to the square of first or transverse axis.

If we take the centre C for the origin of the abscissæ, then calling $CP = z$, we shall have $yy = \frac{bb}{aa}$ $(zz - aa,)$ this equation is somewhat more simple than the preceding one. It gives $zz = \frac{aa}{bb}\ (bb + yy)$; therefore if we draw M Q perpendicular on the conjugate axis CB, prolonged if necessary; and if we call the co-ordinates CQ, MQ, z and y, we shall have $MQ^2 = \frac{a^2}{b^2}\ (b^2 + CQ^2)$, or $yy = \frac{aa}{bb}\ (b^2 + z^2)$, for the equation to the co-ordinates of the second axis.

If $a = b$, the hyperbola is said to be equilateral, and its equations are $yy = 2ax + xx$, and $yy = zz - aa$, according as the origin of the abscissæ is at the vertex or at the centre; and the equation to its second axis becomes $yy = aa + zz$.

If, having drawn B A, we take on both sides of the centre C, C F $= Cf =$ B A, the points F, f, are the foci of the hyperbola. The double ordinate Dd passing through one of the foci, is called the parameter, and a line F M, or f M drawn from one of these points to any point of the curve is called the *Radius vector*.

This premised, the distance C F $= \sqrt{(aa + bb)}$. Hence $FA \times Fa =$

$$\left\{\sqrt{(aa + bb)} - a\right\}$$

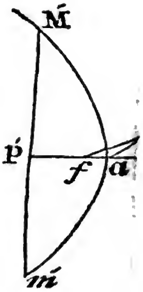
$$\times \left\{\sqrt{(aa + bb)} + a\right\} = bb.$$

Consequently the semi-conjugate axis of the hyperbola is a mean proportional between the two distances from one of the foci to the two vertices.

The ordinate $DF = \frac{b}{a}\sqrt{(a^2 + b^2 - a^2)} = \frac{bb}{a}$, therefore the parameter $p = Dd = \frac{2bb}{a} = \frac{4bb}{2a}$. It is therefore a third proportional to the transverse and conjugate axes. The parameter of the second axes is a line q, a third proportional to the conjugate and transverse axes.

If we introduce the expression of the parameter into the equations of the hyperbola $yy = \frac{bb}{aa}\ (2ax + xx)$; and $yy = \frac{bb}{aa}\ (zz - aa)$ we shall have $yy = \frac{p}{2a}(2ax + xx)$ and $yy = \frac{p}{2a}\ (zz - aa)$ similarly the equation $yy = \frac{aa}{bb}(bb + zz)$ which belongs to the second or conjugate axis becomes $yy = \frac{4b^2}{p^2}(bb + zz)$ $= \frac{2a}{p}(\frac{1}{2}ap + zz.$

Call $CF = Cf = c$, we shall have $FM = \sqrt{(yy + zz - 2cz + cc)} = \frac{cz}{a} - a$ and $fM = \frac{cz}{a} + a$; hence $fM - FM = 2a$. Therefore in the hyperbola the difference of the lines drawn from the two foci to


M′
P′
f
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any point of the curve is every where equal to the transverse axis.

From this we may deduce an easy method of describing an hyperbola, whose axis are $2a$ and $2b$. Take an interval $Ff = 2\sqrt{(aa+bb)}$, and make use of a rule fMO, longer, in proportion as you desire to have a greater proportion of the hyperbola: fix one extremity of it at one of the foci, to the point f, for example, so that it may revolve freely about that point. Take then a thread FMO equal in length to $f\mathrm{MO} - 2a$. Fasten one of the ends of this thread to the point O of the rule, and the other to the focus F. This done, draw the rule away from the axis as far as the thread FMO will allow, and then bring it again gradually towards the axis, taking care to keep the thread always extended by means of a point or style M which glides along the rule fMO. The curve described during this motion by the point M, will be an hyperbolical branch AM, since the difference of the radius-vectors will be every where equal to the transverse axis.

This same property will enable us to draw a tangent MT, fig. 2. to any point m of the hyperbola. For if we conceive the arc Mm to to be infinitely little, and draw the lines fM, fm, FM, Fm, we may prove much in the same manner as in the ellipse, that the angles fmM, MmF are equal, and consequently that if we bisect the angle fMF by the line MT, this line will be the required tangent.

This being premised, in the triangle fMF, we have $f\mathrm{M} : \mathrm{MF} = f\mathrm{T} : \mathrm{FT}$ and therefore by composition $f\mathrm{M} + \mathrm{FM} : f\mathrm{M} :: f\mathrm{T} + \mathrm{FT} : f\mathrm{T}$ or $\frac{2cz}{a} : \frac{aa+cz}{a} = 2c : \frac{aa+cz}{z} = \frac{aa}{z} + c$, therefore $f\mathrm{T} - c$ or $\mathrm{CT} = \frac{aa}{z}$, which gives this proportions CP : CA = CA : CT, by which it is easy to find the point T, and consequently to draw the tangent MT.

We may observe that CT being equal to $\frac{aa}{z}$, it is always positive as long as z is so. Therefore every tangent to the hyperbola cuts the axis in some point T, between A and C. But as the abscissa increases, the line CT diminishes so that it is infinitely little or nothing when the abscissa is infinitely great. Hence we see that through the centre C we may draw two right lines cX, Cx which will be the limits of the tangents to the hyperbola, these lines, whose position we shall soon determine, are called the *asymptotes* of the hyperbola.

The subtangent $\mathrm{PT} = z - \frac{aa}{z} = \frac{zz-aa}{z}$, and the tangent $\mathrm{MT} = \sqrt{\left\{\frac{bb}{aa}(zz-aa) + zz - 2a^2 + \frac{a^4}{z^2}\right\}} = \sqrt{\left(\frac{aa+bb}{aa}z^2 - bb - 2a^2 + \frac{a^4}{z^2}\right)}$. If we draw the normal MN we shall have the subnormal

$$\mathrm{PN} = \frac{\frac{bb}{aa}(zz-aa)}{\frac{zz-aa}{z}} = \frac{bbz}{aa},$$

and the normal

$$\mathrm{MN} = \sqrt{\left\{\frac{b^2z^2}{a^4} + \frac{b^2}{a^2}(z^2-a^2),\right\}}$$

The line $\mathrm{AT} = a - \mathrm{CT} = a - \frac{aa}{z}$; and if we draw AS parallel to MP, we shall have PT : PM = AT : AS, or $\frac{zz-aa}{z} : y = a - \frac{aa}{z} : \frac{ay}{z+a} = b\sqrt{\left(\frac{z-a}{z+a}\right)}$. Now if we suppose z infinite, the quantity $\frac{z-a}{z+a}$ will not differ from unity. Consequently we shall then have $\mathrm{AS} = b$. Hence it follows that if we draw AD and Ad perpendicular to CA, and each equal to the semi-conjugate axis b, the lines CD, Cd, drawn so as to pass through the points D, d, and the centre C, will be the asymptotes to the hyperbola MAM$'$, and if prolonged in a contrary direction, they will be those of the opposite hyperbola.

If the hyperbola is equilateral, the angle DCd, made by the asymptotes, is a right angle, for then $\mathrm{DA} = \mathrm{A}d = \mathrm{CA}$.

The hyperbola referred to its asymptotes has many properties; here follow the principle of them:

If though any point N (fig. 3.) of the asymptote we draw the straight line Nn parallel to the line Dd, we shall have CA : DA = CP: NP or $a : b = z : \frac{bz}{a}$, therefore NM $= \frac{bz}{a} - y$, and M$n = \frac{bz}{a} + y$. Consequently NM × M$n = \frac{b^2 z^2}{a^2} y^2 = bb =$ DA^2. Since NP$^2 = \frac{b^2 z^2}{a^2}$, and that MP$^2 = \frac{b^2 z^2}{z^2} - bb$, we have always NB > MP. Consequently the hyperbolic branch can never come in contact with its asymptote, however it perpetually approaches towards it, for as the abscissa increases, the difference between $\frac{b^2 z^2}{a^2}$ and $\frac{b^2 1^2}{a^2} - bb$ becomes less sensible; so that if we suppose z infinite, this difference vanishes.

Draw MQ and AL parallel to the asymptote Cd. It is easy to see that the triangles DLA and LCA are isosceles. Let then AL = DL $= m$, CQ $= x$, QM $= y$. If we draw MK, parallel and equal to CQ, because of the similar triangles DLA, NQM, M$'$Kn, we shall have the proportions MN : DA = QM : LA

Mn : DA = MK : DL^2

Therefore Mn × MN : DA^2 = QM × MK : AL^2. But Mn × MN = DA^2, hence $xy = mn$, an equation to the hyperbola referred to its asymptotes, in which $mm = \frac{1}{4}(aa + bb)$ is called the power of the hyperbola.

If two parallels Ff, Gg, (fig. 4.) terminated at the asymptotes cut an hyperbola in the points m, h, p, K, we shall have Gp × pg = Fm × mf. For if we draw MmN, PpQ, perpendicular to the axis, we shall have Fm : Mm = Gp : Pp and mf : mN : : pg : pQ therefore Fm × mf : Mm × mN : : Gp × pg : Pp × pQ. But Pp × pQ = bb = Mm × mN. Consequently Fm × mf = Cp × pg. And therefore also Kg × KG = fh × hF.

If we suppose that the points p, K coincide in a single point D, the line TDt will be a tangent to the point D, and we shall have Fm × mf = Dt × DT and fh × hF = Dt = DT = Fm × mf, therefore fh (hm + mF) = Fm (mh + hf); therefore fh = Fm, and consequently TD = Dt. But if we draw DE parallel to Ct, or an ordinate to the asymptote CT, the similar triangles TDE, TtC, will give TE = EC. Consequently to draw to the hyperbol a tangent to a point D, corresponding to the ordinate DE, we must take ET = EC and draw through the points T and D, the tangent TDt.

Since we always have fh = Fm, in whatever manner we draw the straight line Ff, the two parts Fm, fh intercepted between the curve and its asymptotes will always be equal.

Hence we derive an easy manner of describing an hyperbola between two given asymptotes CT, Ct, and passing through a given point m.

Draw through this point the right lines Ff, MN, &c. and take fh = Fn, nN = Mm, &c. and the points n, h, &c. will be in the hyperbola.

According to what we have seen a tangent TMt, terminated at the asymptotes is bisected at the point of contact M. If we draw MCM$'$, this line is called a *diameter;* the tangent TMt is its conjugate diameter. Its ordinates are the right lines mQm' parallel to the conjugate diameter TMt or DCd, and the parameter of any diameter is a line, a third proportional to that diameter and its conjugate. The line MCM = 2CM is also called the first or principal diameter, and the line TMt = 2TM = DCd = 2DC is the second conjugate diameter.

This premised, it is easy to see that a diameter bisects all its ordinates. For NQ : Qn = TM : Mt, and Nm = mn. Call then CM = m, CD = TM = n, CQ = x, Qm = y; and we shall have $m : n : : z :$ NQ $= \frac{nz}{m}$. But Nm × mn = TM^2. Hence $n^2 = \frac{n^2 z^2}{m^2} - yy$, and $yy = \frac{n^2}{m^2}(z^2 - m^2)$, an equation similar to that of the co-ordinates to the transverse axis.

This equation gives $z^2 = \frac{m^2}{n^2}(y^2$

$+ n^2)$. Consequently if we make $Cp = z$, $pm = y$, we shall have $yy = \frac{m^2}{n^2}(z^2 + n^2)$ for the equation to the co-ordinates of the second diameter C D; it is easy to perceive the analogy which this equation has to that of the co-ordinates to the conjugate axis.

Let now aCA be the transverse axis of the hyperbola, and suppose that BA represents the semi-conjugate axis; if we draw DE, TG, M PK, perpendicular to this axis, and ML, tK parallel to it, the triangles MTL, MtK, CDE, will be equal and similar. If we call $\mathrm{CP} = u$, $\mathrm{PM} = z$, $\mathrm{CE} = t\mathrm{K} = \mathrm{ML} = r$, $\mathrm{MK} = \mathrm{DE} = \mathrm{TL} = s$, and as before $\mathrm{CM} = m$, $\mathrm{TM} = u$, $\mathrm{CA} = a$, $\mathrm{AB} = b$, we shall have $\mathrm{TG} = z + s$, $\mathrm{CG} = u + r$, and $z + s : u + r :: b : a$, and consequently $az + as = bu + br$; besides TL : ML :: MP : PS, or $s : r :: z : \frac{rz}{s}$ and $\mathrm{PS} = \frac{u^2 - a^2}{u} = \frac{aazz}{b^2 u} = \frac{rz}{s}$. Therefore $r = \frac{aasz}{b^2 u}$; substituting this value in the equation $az + as = bu + br$ we have $(bu - as)\ (bu - az) = o$. Now $bu - az$ cannot be equal to zero; therefore $bu - as = o$. Hence $bu = as$, and consequently $az = br$. Therefore we have $\mathrm{CP} : \mathrm{DE} :: a : b :: \mathrm{CE} : \mathrm{MP}$. Consequently,

First, the triangles CED, CMP, are equal in surface.

Second, if we draw DM, we shall have DMC, or ½ CDTM = to the trapezium $\mathrm{DEMP} = (s + z)\left(\frac{u - r}{2}\right) = \frac{su + uz - sr - rz}{2} = \frac{su - rz}{2} = \frac{b}{2a}u^2 - \frac{a}{2b}zz = \frac{bbuu - aazz}{2ab} = \frac{a^2\ b^2}{2ab} = \frac{1}{2}\ ab$; therefore the parallelogram TT′ constructed on the conjugate diameters is equal to the rectangle of the axes.

Third, $\mathrm{DE} = \frac{b}{a}\mathrm{CP}$. Hence $\mathrm{DE}^2 = \frac{b^2}{a^2}\mathrm{CP}^2 = b^2 + \mathrm{PM}^2$, and $\mathrm{DE}^2 - \mathrm{PM}^2 = b^2$.

Fourth, $\mathrm{CE} = \frac{a}{b}\mathrm{MP}$, and $\mathrm{CE}^2 = \frac{a^2}{b^2}\mathrm{MP}^2 = \mathrm{CP}^2 - a^2$. Consequently $\mathrm{CP}^2 - \mathrm{CE}^2 = a^2$.

Fifth, $a^2 - b^2 = \mathrm{CP}^2 + \mathrm{PM}^2 - \mathrm{DE}^2 - \mathrm{CE}^2 = \mathrm{CM}^2 - \mathrm{CD}^2$. Therefore the difference of the squares of any two conjugate diameters is equal to the difference of the squares of the two axes. Hence also in the equilateral hyperbola the conjugate diameters are equal.

Hyperbolas receive various denominations from certain peculiarities in their construction, proportions, &c.; as *acute*, *ambigenal*, *equilateral*, &c.

Acute HYPERBOLA, is that whose asymptotes form an acute angle with each other.

Ambigenal HYPERBOLA, is that which has one of its infinite legs inscribed, or falling within an angle formed by its asymptotes, and the other circumscribed or falling without that angle. This is one of Newton's triple hyperbolas of the second order. See Newton's "Enumeration of the Curves of the Third Order."

Anguinal HYPERBOLA, a name given by Newton to four of his curves of the second order; *viz.* species 33, 34, 35, 36, expressed by the equation $xy^2 + ey = - ax^3 + bx^2 + cx + d$; being hyperbolas of a serpentine figure.

Conic HYPERBOLA, is that which arises from the section of a cone by a plane; called also the *common* or *Apollonian hyperbola.*

Conjugate HYPERBOLAS, are those formed or lying together, and having the same axis, but in a contrary order; *viz.* the transverse of each equal the conjugate of the other. See the definitions in the preceding article.

Equilateral or *Rectangular* HYPERBOLA, is that whose two axes are equal to each other, or whose asymptotes make a right angle. Hence the property or equation of the equilateral hyperbola is $y^2 = ax + x^2$, where a is the axis, x the absciss, and y its ordinate; which is similar to the equation of the circle, *viz.* $y^2 = ax - x^2$; differing only in the sign of the second term, and where a is the diameter of the circle.

Infinite HYPERBOLAS, or *Hyperbolas* of the *higher Kind,* are expressed or defined by general equations, similar to that of the conic or common hyperbola, but having general exponents, instead of the particular numeral ones, but so that the sum of those on one side of the equation is equal to the sum of those on the other side.

Obtuse HYPERBOLA, is that whose asymptotes form an obtuse angle.

Nodated HYPERBOLA, a term applied by Newton to a curve of this kind, which by turning round crosses itself.

Rectangular HYPERBOLA, the same as equilateral hyperbola.

HYPERBOLIC *Arc*, is an arc of the hyperbola, the length of which may be found thus:

Let t and c represent any semi-diameter and its conjugate, and y the ordinate which limits the arc, to be measured from the vertex, making $\frac{t^2+c^2}{c^4}=q$, and hyp. log.

$$\frac{y+\sqrt{(c^2+y^2)}}{c}=\text{A}.$$

$$\text{Also } \tfrac{1}{2}\left\{y\ \sqrt{(c^2+y^2)}-c^2\text{A}\right\}=\text{B}$$

$$\tfrac{1}{4}\left\{y^3\sqrt{(c^2+y^2)}-3c^2\,\text{B}\right\}=\text{C}$$

$$\tfrac{1}{6}\left\{y^5\sqrt{(c^2+y^2)}-5c^2\,\text{C}\right\}=\text{D}$$

&c. &c.

the length of the arc contained between the vertex and ordinate will be

$$1.\ \text{Arc}=c\times\left\{\text{A}+\frac{q}{2}\text{B}-\frac{q^2}{2.4}\text{C}+\frac{3q^3}{2.4.6}\text{D}-\frac{3.5q^4}{2.4.6.8}\text{E, \&c.}\right\}$$

$$2.\ \text{Arc}=y\times\left\{1+\frac{t^2y^2}{6c^4}-\frac{(t^4+4t^2c^2)\,y^4}{40c^8}+\frac{(t^6+4^4\,tc^2+8t^2c^4)\ y^4}{112c^{12}}-\frac{5t^8+24t^6c^2+48t^4c^4+64t^2c^6)y^8}{1152c^{10}},\ \text{\&c.}\right\}$$

$$3.\ \text{Arc}=y\left\{1+\frac{t^2y^2}{6c^4}\text{A}-\frac{t^2+4c^2}{c^4}.\ \frac{3y^2}{20}\text{B}\mp\frac{t^4+4t^2c^2+8c^4}{t^4+4c^4}.\ \frac{5y^2}{14c^4}\text{C}-\frac{5t^6+24t^4c^2+48t^2c^4+64c^6}{t^4+4t^2c^2+8c^4}.\ \frac{7y^2}{72c^4}\text{D, \&c.}\right\}$$

To which may be added the following approximation:

$$4.\ \text{Arc}=\frac{120c^2t+(19t^2+21c^2)4x}{120c^2t+(9t^4+21c^4)4x}+y\text{ nearly.}$$

HYPERBOLIC *Area*, is the area or space contained between the curve of the hyperbola and other lines; which, when it is cut off by a double ordinate perpendicular to the axis, may be found as follow:

Let t and c still represent the semi-diameter and its conjugate, x an absciss, $2y$ the double ordinate which cuts off the segment, and z its distance from the centre; then

1. Hyp. area $= zy - tc \times$ hyp. log. of $\frac{ty+cz}{tc}$ making $\frac{x}{2t+x}=q$, we have

$$2.\ \text{Hyp. area}=2xy\left\{\tfrac{1}{3}-\frac{q}{1.3.5}-\frac{q^2}{3.5.7}-\frac{q^3}{5.7.9}-,\ \text{\&c.}\right\}$$

$$3\ \cdot\ \text{area}=2xy\left\{\tfrac{1}{3}-\tfrac{1}{5}\text{A}q-\tfrac{1}{7}\text{B}q-\tfrac{1}{9}\text{C}q-,\ \text{\&c.}\right\}$$

where A, B, C, &c. represent the preceding terms.

To which may be added the following approximations; *viz.*

$$4.\ \text{Hyp. area}=\frac{4cx}{15t}\left\{4\sqrt{(2tx+\tfrac{3}{4}x^2)}+\sqrt{2tx}\right\}\text{nearly}$$

$$5.\ \cdot\ \cdot\ \text{area}=\frac{4cx}{75t}\left\{21\sqrt{(2tx+\tfrac{5}{7}x^2)}+4\sqrt{2tx}\right\}\text{nearly.}$$

HYPERBOLIC *Frustum* or *Zone*, is the space included between the curve and two lines parallel to each other, and perpendicular to the axis; the area of which is equal to the difference of two segments cut off by those lines. Or, the same notation remaining as in the article *Hyperbolic Area*, let z be the distance of a second double ordinate 2 Y from the former;

$$\left.\begin{array}{l}\text{then the}\\ \text{area of the}\\ \text{frustum}\end{array}\right\} = ZY - zy - tc \text{ hyp. log. } \frac{tY + cZ}{ty + cz}.$$

HYPERBOLIC *Logarithms*, or *Naperian Logarithms*, express the areas or spaces contained between the asymptote and curve of the hyperbola; but as this property is not peculiar to this system, they are now more commonly called Naperean logarithms, from the name of the illustrious inventor of this method of computation.

The logarithm of any number is the index of the power to which another number would have to be raised, in order to become equal to the first, that is, if $r^x = a$ then x is the logarithm of a, and r will be the radix of the system; and it is obvious that r may be assumed at pleasure. If we take $r = 10$, it will be the common, or Brigg's logarithmic system; and if $r =$ 2·7182818, &c. it gives the hyperbolic scale:

h. log. $a = (a-1) - \frac{1}{2}(a-1)^2 + \frac{1}{3}(a-1)^3 - \frac{1}{4}(a-1)^4 +$. &c. or

h. log. $(1+a) = a - \frac{1}{2}a^2 + \frac{1}{3}a^3 - \frac{1}{4}a^4 + \frac{1}{5}a^5 -$, &c.

This is the simplest form that the logarithmic series admits of, and therefore such as would be naturally adopted by any person, before he was aware of the advantages and disadvantages of different systems; the former of which, in the tables in present use, much more than compensates for the additional labour employed in their computation.

We have seen above (writing x instead of a) that hyp. log. $(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 -$, &c.

Now let it be required to find the fluent of $\frac{\dot{x}}{1+x}$, or of $\dot{x} \times \frac{1}{1+x}$. Expanding this into a series, we have

$$\dot{x} \times \frac{1}{1-x} = \dot{x}\,(1 - x + x^2 - x^3 + x^4 -, \&c.)$$

$$= \dot{x} - x\dot{x} + x^2\dot{x} - x^3\dot{x} + x^4 x,$$

the fluent of which, by the common rule, is

$x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 -$, &c. which is the form for hyp. log. of $(1+x)$; hence universally

$$\text{fluent of } \frac{\dot{x}}{1+x} = \text{hyp. log. } x.$$

And this rule is general for every quantity, of which the numerator is the fluxion of the denominator, or any multiple of that fluxion; these being all reducible to the same form.

HYPERBOLICUM *Acutum*, the solid generated by the infinite space contained between an hyperbolic leg and its asymptote, by revolving about the latter, which is equal to a finite quantity.

HYPERBOLOID, or HYPERBOLIC *Conoid*, is the solid formed by the revolution of an hyperbola about its axis.

Let a and c represent the semi-axis of the generating hyperbola, v the distance of its base from the centre: also let $A = \frac{a^2}{a^2 + c^2}$ be the semi-transverse of another hyperbola, whose semi-conjugate is c, the same with that of the former.

Then find, by the proper formula, the area of the frustum of this latter hyperbola, whose ends are distant from the centre by v and a, multiply this area by 3·1416 for the surface; that is,

1. Surface $= p \times \left\{ vY - ay - AC. \text{ hyp. log. of } \frac{AY + cv}{Ay + ac} \right\}$

where $p = 3{\cdot}1416$, Y and y the ordinates of the latter hyperbola.

2. Solidity $= \frac{1}{2}$ par.$^2 \times \frac{t + \frac{2}{3}a}{t + a}$

where $a =$ altitude, $r =$ radius of the base, t the transverse axis, and $p = 3{\cdot}1416$.

3. Solidity $= \frac{r^2 + d^2}{6} \times ap$, where d is the diameter, in the middle between the base and vertex.

Frustums of Hyperboloids.

Let D and d denote the semi-diameters of the two ends, a the altitude, t and c the transverse and conjugate axis, $p = 3{\cdot}1416$; then

1. Solidity $= \frac{1}{4} p a \left\{ D^2 + d^2 - \frac{a^2 c^2}{3 t^2} \right\}$

2. Solidity $= \frac{1}{6} p a \left\{ D^2 + 4 \delta^2 + d^2 \right\}$ where δ is the middle diameter.

Hyperbolic Spindle.

Let A = the generating area, D the greatest diameter, L the length of the spindle, $p = 3{\cdot}1416$; then

1. Solidity $\frac{1}{2} p \left\{ \frac{(L^2 + D^2)A}{D} - \frac{1}{3} L^3 \right\}$

2. Solidity $\frac{1}{6} p L \left\{ D^2 + \frac{(3A - LD)4C}{L} \right\}$

where C is the central distance.

HYPOTENUSE, or HYPOTHENUSE in *Geometry*, is that side of a right-angled triangle which is opposite to the right angle, the square of which is equal to the sum of the squares of the other two sides.

HYPOTHESIS, a proposition or principle which is supposed or taken for granted, in order to draw conclusions for the proof of a point in question.

I.

JACK, in *Mechanics*, an instrument employed in raising heavy timbers and other great weights.

ICOSAHEDRON in *Geometry*, one of the regular platonic bodies, comprehended under twenty equal triangular sides or faces.

To find the surface and solidity of an Icosahedron, the side of one of its equal faces being given.

Let s represent the given side; then will surface $= 5 s^2 \sqrt{3}$
$= 8{\cdot}66025403\, s^2$

solidity $= \frac{5}{6} s^3 \, \frac{7 + 3\sqrt{5}}{2}$
$= 2{\cdot}1816950\, s^3$

The radius of the sphere circumscribing an Icosahedron being given, to find its side or linear edge, surface, and solidity.

Let R represent the given radius, then will

side - $= R\sqrt{\left(\frac{10 - 2\sqrt{5}}{5}\right)}$
surface $= 2R^2 (5\sqrt{3} - \sqrt{15})$
solidity $= \frac{2}{3} R^3 \sqrt{(10 + 2\sqrt{5})}$

Or putting r to represent the radius of the inscribed sphere, we shall have

side - $= r\sqrt{(42 - 18\sqrt{5})}$
surface $= 2r^2 (7\sqrt{3} - 3\sqrt{15})$
solidity $= 10 r^3 (7\sqrt{3} - 3\sqrt{15})$

Or writing s for the side, we have radius circum. sphere
$= \frac{1}{2} s \sqrt{\left(\frac{5 + \sqrt{5}}{2}\right)}$

radius inscrib. sphere
$= \frac{1}{2} s \sqrt{\left(\frac{7 + 3\sqrt{5}}{6}\right)}$

IDES, in the *Roman Calendar*, commenced on the fifteenth day of March, May, July, and October, but on the thirteenth in all the other months.

JET *d'Eau*, a French word signifying a fountain that throws up water to some height in the air.

IGNITION, the application of fire to metals, till such time as they become red-hot without melting. Ignition takes place in gold, silver, and particularly in iron; but lead and tin melt before they are red-hot.

ILLUMINATION, that which results from, or the effect of, a luminous body.

IMAGE, in *Optics*, is the appearance of an object, made either by reflection or refraction. See LENS, MIRROR, REFLECTION, and REFRACTION.

IMAGINARY *Quantities*, or IMPOSSIBLE *Quantities*, in *Algebra*, are certain expressions that arise in various algebraical operations, to which no value either rational or irrational can be assigned, yet being substituted in the equation whence they were deduced, they are found to answer the conditions of the question.

When the result of any equation leads to an imaginary quantity, we may always infer that some condition which is impossible has entered into the great line from which that equation is deduced. Thus, if the result were $x = 4 \pm \sqrt{-4}$, it would be evident that some value had been assumed for x which it could not have; and this might be discovered by retracing the steps of the equation.

First, transposing 4, we have

$$x - 4 = \sqrt{-4}$$

Then squaring both sides gives us

$$x^2 - 8x + 16 = -4$$

Transposing and changing the signs, we have

$$8x - x^2 = 20.$$

In which $8x - x^2$ is the product of x by $8 - x$; and the conditions of the problem are that two numbers shall be found whose sum is 8 and product 20; which is evidently impossible, as the greatest product is 16.

It is an established principle in algebra, that $+a \times +a = a^2$, and $-a \times -a = a^2$; and hence, conversely, it follows that $\sqrt{a^2} = +a$, or $-a$; but this ambiguity has not place if we know how a^2 was generated, and have occasion to retrace the steps of our operation; that is, we cannot say that $\sqrt{-a \times -a} = \pm a$; nor that $\sqrt{+a \times +a} = \pm a$; for the square root of a^2, in both these cases, is determined; that is, when considered with regard to its generation it has only one root, whereas had its origin not been known, we must have given the ambiguous sign to the root a, and for this obvious reason, that we know not when a^2 is unconditionally assumed, whether it be the representative of $(+a)^2$, or $(-a)^2$; these being both expressed by the same symbol a^2. The restriction of which we have been speaking sometimes takes place in equations; thus, for example, suppose it were required to find the value of x in the equation $\sqrt{(12 + x)} = 6 + \sqrt{x}$, we soon find $x = 4$; but there is this limitation, that the square root of x, or of 4, must necessarily be -2, and not $+2$, as the latter supposition will not answer the conditions of the equation; and the reason is obvious, namely, that we first found $\sqrt{x} = -2$, and then $x = 4$; but, in retracing our steps, we must remember how this 4 was generated, and that it has not two roots, or has not the ambiguous sign belonging to it as the square root of 4 would have if unconditionally assumed: in fact, the ambiguity in the extraction of the square root arises only in those cases in which we are unacquainted with the generation of the quantity whose root is to be extracted; and there it must necessarily occur, as we have before agreed to represent both $(+a)^2$ and $(-a)^2$ by the same character a^2.

If therefore it be required to find the product of $\sqrt{-1} \times \sqrt{-1}$, we see immediately that it is equal to $\sqrt{(-1)^2} = \sqrt{1}$; but under this limitation, that the root can only be expressed by -1, and therefore this product may always be represented by -1, or by $-\sqrt{1}$; and it can never have any other form. If the product $\sqrt{-1} \times \sqrt{-1} \times \sqrt{-1}$ were required, this would on the same principles be represented by $-\sqrt{-1}$; and the fourth power of $\sqrt{-1}$ is equal to $+1$; but with this limitation, that the root of this quantity can only be -1, and not ± 1: hence then we have $(\sqrt{-1})^2 = -1$, $(\sqrt{-1})^3 = \sqrt{-1}$ and $(\sqrt{-1})^4 = 1$, $(\sqrt{-1})^5 = \sqrt{-1}$; and consequently, the 6th power will be the same as the 2d, the 7th the same as the 3d, the 8th the same as the 4th, and so on. And exactly the converse of these rules must be observed in division. These examples involve all the cases that can arise; for if the quantities to be multiplied or involved be $\sqrt{\quad a} \times \sqrt{-b}$, we have only to write these $\sqrt{a} \cdot \sqrt{-1} \times \sqrt{b} \cdot \sqrt{-1} = \quad \sqrt{ab}$, and $(\sqrt{-a})^2 \times (\sqrt{\quad -b})^2 = ab(\sqrt{\quad 1})^2 = -ab$; this product may be put under a simpler form, $\sqrt{-a^2} \times \sqrt{-b^2} = a\sqrt{-1} \times b\sqrt{-1} = ab(\sqrt{-1})^2 = \quad ab$. This shows the necessity of always separating the quantities into such factors that only -1 is found under the radical, for according to the common rules for the multiplication of surds, we should have $\sqrt{-a^2} \times \sqrt{-b^2} = \sqrt{(-a^2 \times -b^2)}\sqrt{a^2b^2} = ab$, which is a false result; but we have no right in this case to assume $\sqrt{a^2b^2} = a\ b$, because we know its generation, and that it arose from the product of two negative signs, and therefore its root must necessarily be $\quad ab$, and not $\pm ab$. These rules must be very carefully attended to, in operating upon radicals under which certain imaginary quantities enter; as it may frequently happen, that in consequence of quantities differently generated being expressed by the same symbol, we are led to infer their equality, when in fact no equality subsists between them.

Imaginary *Roots* of an *Equation*, are those values of the unknown quantity which contain some imaginary expression. Thus the roots of the equation $y^2 - 10y + 26$, are $5 \pm \sqrt{-1}$, and the three roots of the cubic equation $x^3 - 1 = 0$, are

$$x = \frac{1 + \sqrt{-3}}{2}, \frac{-1 - \sqrt{-3}}{2},$$

and 1; of which the latter is the only possible value of x, the two former being imaginary. Sometimes the root of an equation may be represented by imaginary expressions, when it is in fact equal to a real quantity, as is the case in the solution of cubic equations of the irreducible form, according to the method of Cardan.

IMMERSION, in *Astronomy*, is when a planet, comet, or other heavenly body approaches so near to conjunction with the sun, that it is enveloped in his rays, and lost to our observation. It also denotes the beginning of an eclipse, or of an occultation, when the body or any part of it first begins to disappear, either behind another body or in its shadow.

IMPACT, the single instantaneous blow or stroke communicated from one body in motion, to another either in motion or at rest. See Percussion.

IMPENETRABILITY, that quality of body which prevents it from being pierced.

IMPETUS, the product of the mass and velocity of a moving body, considered as instantaneous, in distinction from *momentum*, with reference to time, and *force* with reference to capacity of continuing its motion.

IMPOSSIBLE, in *Algebra*, the same as *Imaginary*.

Impossible *Forms* of *Equations*, are those that will admit of no solution in rational numbers, neither integral nor fractional; such as $2x^2 \pm 3y^2 = z^2$, $3x^3 \pm 7y^3 = z^3$, $3x^4 \pm 7y^4 = z^4$, &c.; which are all impossible forms of equations, admitting of no rational solution.

The most general method of determining impossible forms is by means of the linear forms of squares, cubes, &c.—Thus all square numbers are of one of the two forms, $4n$, or $4n + 1$; *viz.* all square numbers, are either exactly divisible by 4. or when divided by it as far as possible, they will leave 1; and therefore no numbers of the forms $4n + 2$, or $4n + 3$, can be squares; or $4n + 2$, and $4n + 3$ are impossible forms for square numbers. In the same manner, all square numbers are of one of the forms $5n$, or $5n \pm 1$; that is, square numbers, when divided by 5, can only leave the three remainders 0, 1, and -1; or, which is the same, 0, 1, and 4; and therefore $5n + 2$, and $5n + 3$, are impossible forms for squares. Again, all square numbers are of one of the forms $7n$, $7n + 1$, $7n + 4$, or $7n + 2$; and therefore $7n + 3$, $7n + 5$, $7n + 6$, are all impossible forms.

And the impossible forms of cubes are obtained in the same manner; thus we shall find that all cubes are of one of the forms $7n$, $7n + 1$, or $7n + 6$, and consequently all the other forms to modulus 7 are impossible forms for cubes; that is, no number of the form $7n + 2$, $7n + 3$, $7n + 4$, $7n + 5$, can be a cube number; in the same way it will be found that no number of the form $9n + 2$, $9n + 3$, $9n + 4$, $9n + 5$, $9n + 6$, $9n + 7$, can be a cube number; that is, if a number, when divided by 7, leaves a remainder 2, 3, 4, or 5; or if, when divided by 9, it leaves a remainder 2, 3, 4. 5, 6, or 7; then in neither case is the number a complete cube.

A similar mode of investigation may be pursued with all the higher powers, the only difficulty being in fixing upon a proper modulus; that is, such a number as shall have the most impossible forms belonging to it, and this requires a separate investigation. But almost every power has some modulus that renders it expressible in three forms; thus,

All 3d powers are of one of the forms	$7n$, or $7n \pm 1$
4th powers	$5n$, or $5n + 1$
5th powers	$11n$, or $11n \pm 1$
6th powers	$7n$, or $7n + 1$
8th powers	$17n$, or $17n \pm 1$
9th powers	$19n$, or $19n \pm 1$
10th powers	$11n$, or $11n + 1$

the 7th power is omitted, not being reducible to a similar form.

IMPULSE, a momentary force, such as that which arises from the sudden explosion of fired gunpowder, or the momentum of a moving body.

INCIDENCE, in *Mechanics* and *Optics*, is used to denote the direction in which a body, or ray of light, strikes another body; and is otherwise called inclination. In moving bodies their incidence is said to be perpendicular or oblique, according as their lines of motion make a straight line, or an angle at the point of contact.

Angle of Incidence, generally denotes the angle formed by the line of incidence, and a perpendicular drawn from the point of contact to a plane or surface on which the body or ray impinges.

Thus if a body impinges on the plane at a point, and a perpendicular be drawn, then the angle made by this perpendicular and the incident ray is generally called the angle of incidence, and the complement of this the angle of inclination. Dr. Wallis, however, and some other old authors, call the angle lathe the angle of incidence.

The writers on optics have demonstrated the following propositions:—

1. That the angles of incidence is always equal to the angle of reflection; or that the angles of inclination on the two sides of the perpendicular are equal.

2. That the angles of incidence and refraction are to each other, accurately, or very nearly, in a given ratio.

3. That from air to glass the sine of the angle of incidence is to the sine of the refracted angle, as 300 to 193, or nearly as 14 to 9; and on the contrary from glass to air, the angle of incidence to that of refraction, is as 193 to 300, or nearly as 9 to 14.

Axis of Incidence is the perpendicular at the point of impact, or the line separating the angle of incidence from that of reflection.

Line of Incidence, is the direction in which a ray is propagated.

Point of Incidence, is the point where the incident ray meets the reflecting or refracting body.

INCLINATION, denotes the mutual approach or tendency of two bodies, lines, or planes, towards each other, so that the lines of their direction make at the point of contact an angle of greater or less magnitude.

Inclination of a *Right Line* to a *Plane*, is the acute angle which such a right line makes with another right line, drawn in the plane through the point where the inclined line intersects it, and through the point where it is also cut by a perpendicular drawn from any point of the inclined lines.

Inclination of *Meridians*, in *Dialling*, the angle that the hour line on the globe, which is perpendicular to the dial plane, makes with the meridian.

Inclination of the *Axis of the Earth*, is the angle which it makes with the plane of the ecliptic.

Inclination of a *Planet*, is an arc or angle comprehended between the ecliptic, and the plane of a planet's orbit.

Inclination of two *Planes*, is the acute angle made by two lines drawn one in each plane, through a common point of section, and perpendicular to the same common section.

INCLINED *Plane* in *Mechanics*, a plane which makes with an horizontal plane any angle whatever, forming one of the simple mechanical powers. The inclination of the plane is measured by the angle formed by two lines drawn from the sloping and the horizontal plane, perpendicular to their common intersection.

The following are the principal mechanical properties of the inclined plane.

1. When a body is sustained upon an inclined plane, the sustaining power, or weight, will be to the weight of the body, as the sine of the plane's inclination is to the sine

of the angle, which the direction of the power makes with a perpendicular to the plane.

When the power acts in a direction parallel to the inclined plane, then the proportion becomes the power: the weight = sine of plane's inclination: radius; that is, the power: weight = height of the plane: its length.

2. If two bodies keep each other in equilibrio, by a string passing over the vertex of two differently inclined planes, the weights of the bodies will be to each other as the sines of the angles of inclinations of the opposite planes. That is, calling the one body P, and the other W.

P : W = sine inclination at W, to inclination at P; or if the bodies are equal, their tendencies to descend will be inversely as the lengths of the planes, because the sides of triangles are to each other as the sines of their opposite angles.

3. The velocity acquired by a body descending by the action of gravity down an inclined plane, is to the velocity of a body falling perpendicularly during the same time, as the height of the plane is to the length.

4. The force whereby a body descends down an inclined plane is to the absolute force of gravity, as the height of the plane is to its length; which being a constant ratio for the same plane, it follows that the force whereby the body descends is uniform, and consequently that it will produce a uniformly accelerated motion. The same is also true with regard to the retardation in bodies projected up inclined planes:

5. Hence the space descended down inclined planes is to the space perpendicularly described in the same time, as the height of the plane is to its length.

6. Consequently, the velocities acquired, and the spaces descended by bodies down different inclined planes in the same time, are as the sines of the elevation of the planes.

7. If from any point in an inclined plane a perpendicular to that plane be drawn, and produced, till it meet the perpendicular to the horizon; then the time of descent through the perpendicular to the horizon will be equal to that along the intercepted part of the plane.

8. Hence, in any right-angled triangle having its hypothenuse perpendicular to the horizon, a body will descend down any of its three sides in the same time. And therefore, if on the hypothenuse a semicircle be described, the times of descending down the chords will be all equal, and each equal to the time of falling freely through the diameter.

9. The time of descending down an inclined plane, is to the time in falling through its perpendicular height, as the length of the plane is to its height. Consequently the times of descending down different planes of the same height are as the lengths of the planes.

10. A body acquires the same velocity in descending down an inclined plane, as in falling perpendicularly through the height of the plane. Hence the velocities are always the same in planes of the same altitude, whatever may be their degrees of inclination.

11. If a body descended down any number of contiguous planes, it will ultimately acquire the same velocity, as a body which falls perpendicularly through the whole height, supposing no change in the velocity, while passing from one plane to another.

12. As this is true, whatever may be the number and magnitude of the planes, if we suppose each of them to be indefinitely small, they will form a curve line, which will thus become the path of the body. Whence it follows, that a body acquires the same velocity in descending down any curve, as in falling perpendicularly, though the same height.

INCOMMENSURABLE *Lines*, are such as have no common measure, the diagonal and side of a square are incommensurable, being to each other as $\sqrt{2}$ to 1; and consequently whatever number of parts the side of the square may be divided into, the hypothenuse will not be made up of any exact number of such parts.

INCOMMENSURABLE *Numbers*, or *Numbers Prime to each other*, are those which have no integral common measure greater than unity.

If numbers be incommensurable with each other, they are incommensurable also in power, that is no powers of such numbers can be commensurable with each other.

INCREMENT, or *Finite Difference*, is the finite increase of a variable quantity. Euler, who seems to have given a permanent form to this branch of analysis, employs the character Δ to denote; thus the increment of $x = \Delta x$, of $y =$ the increment Δy, &c.

INCREMENTS, *Method* of, called by the French *Calcul des Differences Finies*; a method of solving problems involving variable quantities, by increments, or parts which are finite, and though small, have some magnitude.

Direct Method.

The increment of a variable quantity, as x, is the difference between that quantity in its first state, and what it becomes after it has been increased by any certain finite quantity; which increase Euler denotes by Δx; hence

The increment of

$$\begin{aligned} x &= \Delta x \\ x^2 &= (x + \Delta x)^2 - x^2 \\ x^3 &= (x + \Delta x)^3 - x^3 \\ x^m &= (x + \Delta x)^m - x^m \end{aligned}$$

that is,

$$\begin{aligned} \Delta x &= \Delta x \\ \Delta (x^2) &= 2x \Delta x + \Delta x^2 \\ \Delta (x^3) &= 3x^2 \Delta x + 3x \Delta x^2 + \Delta x^3 \end{aligned}$$

$$\Delta (x^m) = m x^{m-1} \Delta x + \frac{m(m-1)}{1 . 2} x^{m-2} \Delta x^2 + \text{\&c.}$$

Whence it is obvious, that we may readily find the increment of any power of a variable quantity. Another very general rule relating to continued products of the form x, $(x + \Delta x)\ (x + 2\Delta x)\ (x + 3\Delta x)$ &c. is this.

We have

$$\Delta x \qquad = \Delta x$$

$$\Delta \left\{ x(x + \Delta x \right\} 2 \Delta x (x + \Delta x)$$

$$\Delta \left\{ x (x + \Delta x)\ (x + 2\Delta x) \right\}$$

$$3 \Delta x (x + \Delta x)(x + 2 \Delta x)$$

and generally the increment

$$\Delta \left\{ x(x + \Delta x)\ (x + 2\Delta x) \ldots\ldots (x + n \Delta x) \right\} = (n+1)\ (x + \Delta x)(x + 2\Delta x) \ldots\ldots (x + n \Delta x)$$

By means of these formulæ we may readily find the increment of any function of a variable quantity, which consists only of integral powers.

Suppose the increment of the function $u = x^3 + 3x^2 + 4x + 10$ were required.

Here, from the preceding formulæ we have

$$\Delta u = \Delta (x^3) + 3 \Delta (x^3) + 4 \Delta (x).$$

Now

$$\begin{aligned} \Delta (x^3) &= 3x^2 \Delta x + 3x \Delta x^2 + \Delta x^3 \\ 3 \Delta (x^2) &= 6x \Delta x + 3 \Delta x^2 \\ 4 \Delta (x) &= 4 \Delta x. \end{aligned}$$

Hence

$$\Delta u = 3x^2 \Delta x + x(3 \Delta x^2 + 6 \Delta x) + \Delta x^3 + 3 \Delta x^2 + 4 \Delta x.$$

And we may proceed in the same manner in all similar cases.

But the most general formulæ for finding the increment of any function of a variable quantity is that given by Dr. Taylor, the author of the theory; it is as this:

Let Y represent any function whatever of the variable quantity x, then if x be increased by any finite difference Δx, the value of Δ Y adopting the differential notation will be denoted by the following formula, which is commonly called **TAYLOR'S** *Theorem*, and is of the most extensive use in this theory, as well as in the differential calculus. That is, Y being any function of x, we shall have

$$\Delta Y = \frac{\Delta x\, dy}{1 . dx} + \frac{\Delta x^2\, d^2 y}{1.\ 2\ dx^2} + \frac{\Delta x^3\, d^3 y}{1.2.3\ dx^3} + \text{\&c.}$$

where the law of continuation is obvious, and therefore requires no farther developement; it must however be remarked, that when Δx is negatived, the terms of the above series must be taken plus and minus alternately.

In order then to find the increment of any function of a variable quantity, we must take the successive orders of its fluxions, by means of which the fluxional parts in both the numerator and denominator will disappear; and we shall have the value of ΔY, expressed in terms of x and Δx; and this expression will always be finite, unless the function be transcendental.

Let us propose the function $y = ax + bx^2 + x^3$, to find the value of ΔY, when x becomes $x + \Delta x$.

$$\begin{aligned} d\,y &= a\,dx + 2bx\,dx + 3x^2dx \\ d^2y &= \phantom{a\,dx +{}} 2b\,dx^2 + 6x\,dx^2 \\ d^3y &= \phantom{a\,dx + 2b\,dx^2 +{}} 6\,dx^3 \\ d^4y &= \phantom{a\,dx + 2b\,dx^2 + 6\,d{}} 0 \end{aligned}$$

Now these values being substituted in the general series, we shall have for the increment of y,

$\Delta y = \Delta x\,(a + 2bx + 3x^2) + \Delta x^2(b + 3x) + \Delta x^3.$

Again, given $y = ax + bx^2 - cx^3 + x^4$, to find the increment of y.

$$\begin{aligned} d\,y &= a\,dx + 2bx\,dx - 3cx^2dx \\ &\qquad + 4x^3dx \\ d^2y &= \quad + 2b\,dx^2 - 6cx\,dx^2 \\ &\qquad + 12x^2dx^2 \\ d^3y &= \quad - 6c\,dx^3 + 24x\,dx^3 \\ d^4y &= \qquad + 24\,dx^4 \\ d^5y &= \qquad 0 \end{aligned}$$

Whence making these substitutions in the general theorem, we have

$$\Delta y = \left\{ \begin{array}{l} \Delta x\,(a + 2b\,x - 3\,c\,x^2 + 4 \\ x^3) + \Delta x^2(b - 3cx + 6x^2) \\ + \Delta x^3(c + 4x) + \Delta x^4 \end{array} \right.$$

Inverse Method.

In this method the question is to find the integral or function, from its increment being given. It is attended with the very same difficulties as the inverse method of fluxions; for as in that, every fluent may be readily put into fluxions; so may the increment of any function be readily obtained; but as it is frequently difficult, and sometimes impossible, to find the fluent of a given fluxion; so there are many cases that will not admit of integration; we shall give some of the most usual and obvious rules, and which will apply to many examples.

First let us attend to the powers of a variable quantity x.

Since $\Delta x = \Delta(x)$; therefore reciprocally, $\int.\Delta x = x$. And if we suppose Δx, as constant (a supposition that has place in all that follows) we shall have,

$\int.\Delta x \times 1 = x$, or $\Delta x \int. 1 = x$,

$$\text{therefore } \int. 1 = \frac{x}{\Delta x}.$$

Since

$$\Delta(x^2) = 2\,x\,\Delta x + \Delta x^2,$$

therefore reciprocally,

$$\int.(2\,x\,\Delta x + \Delta x^2) = x^2,$$

or, which is the same,

$$\int. 2\,x\,\Delta x + \int.\Delta x^2 = x^2;$$

whence again, also,

$$\int. x \qquad + \int.\frac{\Delta x}{2} = \frac{x^2}{2\,\Delta x};$$

and hence by transposition,

$$\int. x = \frac{x^2}{2\,\Delta x} - \int.\frac{\Delta x}{2} = \frac{x^2}{2\,\Delta x} - \frac{\Delta x}{2} \times \int. 1 = \frac{x^2}{2\,\Delta x} - \frac{x}{2}$$

Again, since $\Delta(x^3) = 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3$; therefore reciprocally,

$\int.(3x^2\Delta x + 3x\,\Delta x^2 + \Delta x^3) = x^3$; or, which is the same,

$\int. 3x^2\Delta x + \int. 3x\,\Delta x^2 + \int.\Delta x^3 = x^3$; or, dividing by $3\,\Delta x$,

$$\int. x^2 + \Delta x \int x + \frac{\Delta x^2}{3}.\int. 1 = \frac{x^3}{3\,\Delta x};$$

whence again,

$$\int. x^2 = \frac{x^3}{3\,\Delta x} - \Delta x \int. x - \frac{\Delta x^2}{3}.\int. 1;$$

or, which is the same,

$$\int. x^2 = \frac{x^3}{4\,\Delta x} - \frac{x^2}{2} + \frac{x\,\Delta x}{6}.$$

In a similar manner, by continuing to suppose Δx, as constant, and substituting always for the quantities contained under the particular sum, their respective values, the following results, for the integrals of the successive powers of x, in which we have repeated the two preceding ones.

$$\begin{aligned} \int. 1 &= \frac{x}{\Delta x}; \\ \int.\Delta x &= \frac{x^2}{2\,\Delta x} - \frac{x}{2}; \\ \int.\Delta x^2 &= \frac{x^3}{3\,\Delta x} - \frac{x^2}{2} + \frac{x\,\Delta x}{6}; \\ \int.\Delta x^3 &= \frac{x^4}{4\,\Delta x} - \frac{x^3}{2} + \frac{x^2\,\Delta x}{4}; \\ \int.\Delta x^4 &= \frac{x^5}{5\,\Delta x} - \frac{x^4}{2} + \frac{x^3\,\Delta x}{3} \\ &\quad - \frac{x\,\Delta x^3}{30}; \end{aligned}$$

$$\int . \Delta x^5 = \frac{x^6}{6 \Delta x} - \frac{x^5}{2} + \frac{5 x^4 \Delta x}{12} - \frac{x^2 \Delta x^3}{12};$$

&c. = &c.

Where it is only necessary to observe, that if the proposed increment have any constant multiplier, the integral above found must have the same.

Hence we may find the integral of any function made up of the powers of x, affected or not with any constant co-efficients a, b, c, &c. For it is only necessary to find those of the different powers of x, and their sum will be the integral required.

EXAM. 1. Required the integral of the increment $a + bx + cx^2$.

$$\int . a = a \int . 1 = \frac{a x}{\Delta x}$$

$$\int . bx = b \int . x = \frac{b x^2}{2 \Delta x} - \frac{b x}{2}$$

$$\int . cx^2 = c \int . x^2 = \frac{c x^3}{3 \Delta x} - \frac{c x^2}{2} + \frac{c x \Delta x}{6}$$

And hence by addition,

$$\int . (a + b x + cx^2) = \frac{a x}{\Delta x} + \frac{b x^2}{2 \Delta x} - \frac{b x}{2} + \frac{c x^3}{3 \Delta x} - \frac{c x^2}{2} + \frac{c x \Delta x}{6}$$

When it is required to find the integral of a quantity of any of the following forms, (Δx being supposed constant,)

$(x + a)$
$(x + a)(x + 2a)$
$(x + a)(x + 2a)(x + 3a)$
$(x + a)(x + 2a)(x + 3a)(x + 4a)$

we arrive at them by taking the actual product of those quantities, and finding successively the increments of each of the terms: thus,

$$\int . (x + a) = \int . x + \int . a = \frac{x^2}{2 \Delta x} - \frac{x}{2} + \frac{a x}{\Delta x}$$

$$\int . (x + a)(x + 2a) = \int . (x^2 + 3ax + 2a^2) = \int . x^2 = \frac{x^3}{3 \Delta x} - \frac{x^2}{2} + \frac{x \Delta x}{6}$$

$$\int . 3 a x = \frac{3 a x^2}{2 \Delta x} - \frac{3 a x}{2}$$

$$\int . 2 a^2 = \frac{2 a^2 x}{\Delta x}$$

Whence $\int . (x + a)(x + 2a) = \frac{x^3}{3 \Delta x} - \frac{x^2}{2} + \frac{x \Delta x}{6} + \frac{3 a x^2}{2 \Delta x} - \frac{3 a x}{2} + \frac{2 a^2 x}{\Delta x}$.

Again, $\int . (x + a)(x + 2a)(x + 3a) = \int . (x^3 + 6 a x^2 + 11 a^2 x + 6 a^3) =$

$$\int . x^3 = \frac{x^4}{4 \Delta x} - \frac{x^3}{2} + \frac{x^2 \Delta x}{4}$$

$$\int . 6 a x^2 = \frac{6 a x^3}{3 \Delta x} - \frac{6 a x^2}{2} + \frac{6 a x \wedge x}{6}$$

$$\int . 11 a^2 x = \frac{11 a^2 x^2}{2 \Delta x} - \frac{11 a^2 x}{2}$$

$$\int . 6 a^3 x = \frac{6 a^3 x}{\Delta x}$$

Whence, by addition, we have

$$\int . (x + a)(x + 2a)(x + 3a) = \frac{x^4}{4 \Delta x} - \frac{x^3}{2} + \frac{x^2 \Delta x}{4} + \frac{2 a x^3}{\Delta x} - \frac{3 a x^2}{1} + \frac{a x \Delta x}{1} + \frac{11 a^2 x^2}{2 \Delta x} + \frac{6 a^3 x}{\Delta x} - \frac{11 a^2 x}{2} + \frac{6 a^3 x}{\Delta x}$$

and so on of other similar quantities.

And in a similar manner we may find the integral to any other quantity of these forms.

As the increments of any variable quantities, x and $x + a$, are both expressed by Δx, (the constant part a, having no increment,) so reciprocally, the integral of the increment Δx, may be x, or $x + a$; therefore when we have found the integral of any increment, we must add to it a constant quantity, which will be zero, if the integral needs no correction; but a real quantity, positive or negative, in other cases which must be determined from the nature of the problem.

Application.

The summation of series is founded generally on this principle: That if we have any series of quantities, as a, b, c, d, e, &c. which are derived from each by some certain and invariable law, each term may be considered as the increment of the sum of all those which precede it. Thus if

$$a + b + c + d + e = z$$
$$a + b + c + d + e + f = z'$$

then it is obvious that $z' - z = \Delta z = f$, and therefore conversely the integral of any one of those terms considered as an increment, will represent the sum of all the preceding part of the series.

Exam. 1. Required the sum of n terms of the natural series.

$$1 + 2 + 3 + 4 + 5 + \ldots n.$$

Here by writing x instead of n, the term next in order will be $x + 1$, which being the increment of the series, we shall have $\int.(x + 1) =$ the sum required.

Now $\int.(x+1) = \int.x + \int.1$; and $\int.x = \frac{x^2}{2\Delta x} - \frac{x}{2}$; and $\int.1 = \frac{x}{\Delta x}$

and since in this case $\Delta x = 1$, we have $\int.(x + 1) = \frac{x^2}{2} - \frac{x}{2} + \frac{x}{1} = \frac{x^2 + x}{2} = \frac{n^2 + n}{2}$ by writing again n instead of x; and this is the sum of n terms of the proposed series.

This example offers an easy illustration of what has been observed of the correction of an integral. Suppose, for example, that instead of the preceding series beginning at unity, it had commenced from any other term, as 7; the general law of formation would have been the same, and the increment would still have had the form $x + 1$; and consequently the integral, in the first instance, would be represented as above; *viz.*

$$\int.(x + 1) = \frac{x^2 + x}{2}$$

But here a correction of the integral is necessary, for from the nature of the series, when $x = 7$, the sum of the series is 7, this being the term at which the series commences; whereas without a correction, we should have the sum $= 28$; we must therefore write,

$$\int.(x + 1) = \frac{x^2 + x}{2} + c;$$ c being the correction, and since when $x = 7$; $\frac{x^2 + x}{2} + c = 7$; or $\frac{49 + 7}{2} + c = 7$ we find $c = -21$; after which the sum of any number of terms of the proposed series are readily obtained. Thus for example, let $n = 16$, then the sum of the series beginning with the term 7, becomes

$$\frac{16^2 + 16}{2} - 21 = 115$$ as required.

Exam. 2. Required the sum of the natural series of cubes, $1^3 + 2^3 + 3^3 + 4^3 +$, &c. n^3.

Here the general term is $(x + 1)^3$; and the integral of this, that is $\int.(x + 1)^3 = \int.x^3 + 3\int.x^2 + 3\int.x + \int.1$, is computed as follows:

$$\int.x^3 = \frac{x^4}{4\Delta x} - \frac{x^3}{2} + \frac{x^2 \Delta x}{4}$$

$$3\int.x^2 = \frac{3x^3}{3\Delta x} - \frac{3x^2}{2} + \frac{3x\Delta x}{6}$$

$$3\int.x = \frac{3x^2}{2\Delta x} - \frac{3x}{2}$$

$$\int.1 = \frac{x}{\Delta x}$$

And here since $\Delta x = 1$, we have $\int.(x + 1)^3 =$

$$\frac{x^4}{4} + \frac{x^3}{2} + \frac{x^2}{4} = \left(\frac{x^2 + x}{2}\right)^2.$$

Or by making $x = n$, the sum required is $\left(\frac{n^2 + n}{2}\right)^2$.

Hence the sum of consecutive cubes beginning at 1, is equal to the square of the sum of all their roots.

Thus $1^3 + 2^3 = (1 + 2)^2$
$1^3 + 2^3 + 3^3 = (1 + 2 + 3)^2$
$1^3 + 2^3 + 3^3 + 4^3 = (1 + 2 + 3 + 4)^2$ &c. &c.

INDEFINITE, that which is without any assigned limits; thus we say an indefinite line, meaning a line of any length. Some authors use the word indefinite nearly in the same sense as we commonly attach to the term infinite.

INDETERMINATE, is nearly the same as *Indefinite*.

Indeterminate *Analysis*, a very interesting branch of algebra, in which there is always a greater number of unknown quantities than independent equations; by which means the number of solutions is indefinite, though it commonly happens that certain restrictions are introduced, which limit the number of solutions, and even in some cases render the problem impossible. Diophantus, who is supposed to have flourished about the middle of the third century of the christian æra, is the first writer on the indeterminate analysis, at least his Arithmetic or Algebra is the earliest treatise we know of on the subject.

Solution of Indeterminate Problems. When amongst the several unknown quantities there are none that exceed the simple power, the problem is said to be of the *first degree.* If the second power enters, the problem is of the *second degree.* If the third enter, it is of the *third degree*, and so on, of the higher powers.

Indeterminate problems of the first degree, are all included in the general form,

$$ax \pm by \pm cz \pm, \&c. = d,$$

and they are all solved by means of the simple formula $ap - bq = 1$.

That is, by finding the values of p and q in the equation $ap - bq = \pm 1$, we may thence immediately determine the values of x, y, and z, in the above more general form, our first investigation must therefore be directed to the solution of the latter equation, the general rule for which is as follows:

Convert the fraction $\frac{a}{b}$ into a continued fraction, and then again to series of converging fractions, then the numerator and denominator of the fraction preceding $\frac{a}{b}$ will be the values of p and q.

Exam. Let it be required to find the values of p and q, in the indeterminate equation, $16p - 41q = 1$.

Here the fractions converging towards $\frac{16}{41}$, are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{7}{18}$, $\frac{16}{41}$, therefore $p = 18$, and $q = 7$, which give $16 . 18 - 41 . 7 = 1$, as required.

To find the general Values of x and y, in the Equation $ax - by = \pm c$.

In the first place, we must have either a and b prime to each other, or if they have a common measure c must have the same, for otherwise the equation will be impossible. In this latter case the whole equation may be divided by that common measure, and thus reduced to one in which a and b are prime to each other: it will, therefore, only be necessary to consider the quantities a and b as prime to each other. Also, we may suppose that we know the case $ap - bq = \pm 1$; it will, therefore, only be necessary in this place to show how the general values of x and y, in the equation $ax - by = \pm c$, may be deduced from the known case $ap - bq = \pm 1$.

In the first place it is obvious, that since $ap - bq = \pm 1$, we shall have $acp - bcq = \pm c$; but this furnishes only one solution; and in order to have the general values of x and y, we must substitute $x = mb \pm cp$, and $y = ma \pm cq$; which give $a (mb \pm cp) - b (ma \pm cq) = \pm c$; the ambiguous sign $\pm$, in the two values of x and y being $+$, when $ap - bq$ has the same sign with c, but $-$ when it has a contrary one.

Exam. Find the values of x and y, in the equation

$$9x - 13y = 10.$$

First, in the equation $9p - 13q = \pm 1$, we have $p = 3$, and $q = 2$, which gives $9p - 13q = +1$; and this being the same sign with 10, in the proposed equation, the general values of x and y are

$$\begin{cases} x = 13m + 3c; \\ x = 13m + 30; \end{cases} \text{and} \begin{cases} y = 9m + 2c, \text{or} \\ y = 9m + 20. \end{cases}$$

And by assuming here $m = -2$, $-1, 0, 1, 2$, &c. we have the following values of x and y.

$$\begin{array}{llllllll} m = & -2, & -1, & 0, & 1, & 2, & 3, & \&c. \\ x = & & 4, & 17, & 30, & 43, & 56, 69, & \&c. \\ y = & & 2, & 11, & 20, & 29, & 38, 47, & \&c. \end{array}$$

each of which values has the required conditions for

$$9 . 4 - 13 . 2 = 10$$
$$9 . 17 - 13 . 11 = 10$$
$$9 . 30 - 13 . 20 = 10$$
$$9 . 43 - 13 . 20 = 10$$
&c. &c. &c.

To find the general Values of x and y, in the Equation $ax + by = c$.

In the foregoing proposition, where the difference of two quantities was the subject of consideration, we found that the number of solutions was infinite, providing a and b were prime to each other; but in considering the sum of two quantities, as in the present case, the number of solutions is always limited, and in many cases the equation is impossible; it may, however, be demonstrated that the equation admits of at least one solution, if $c > ab - a - b$, a and b being prime to each other; and it is proposed in the present proposition to ascertain the exact number of possible solutions, that any equation of this kind admits of in integer number, and to point out more accurately the limits of possibility.

The solution of the equation $ax + by = c$ depends, like that in the foregoing proposition, upon the equation $ap - by = \pm 1$, though its connection with it is not so readily perceived.

Let $ap - bq = 1$; then we have also $a \cdot cp - b \cdot cq = c$;
and it is evident that we shall have the same result, if we make

$$x = cp - mb, \text{ and } y = cq - ma;$$

for this also gives

$$a(cp - mb) - b(cq - ma) + c;$$

assuming, therefore, for m such a value that $cq - ma$ may become negative, while $cp - mb$ remains positive we shall have

$$a(cp - mb) + b(ma - cq) = c;$$

and consequently $x = cp - mb$, and $y = ma - cq$; but if m cannot be so taken, that $cq - ma$ may be negative, while $cp - mb$ remains positive, it is a proof that the proposed equation is impossible in integer numbers. And, on the contrary, the equation will always admit of as many integral solutions as there may be different values given to m, such that the above conditions may obtain. And hence we are enabled to determine, *a priori*, the number of solutions that any proposed equation of the above form admits of: for since we must have $cp > mb$, and $cq < ma$, the number of solutions will always be expressed by the greatest integer contained in the expression $\left(\frac{cp}{b} - \frac{cq}{a}\right)$, as is evident; because m must be less than the first of those fractions, and greater than the second; and therefore the difference between the integral part of these fractions will express the number of different values of m, except when $\frac{cp}{b}$ is a complete integer; or, which is the same, we must consider $\frac{b}{b}$ as a fraction and reject it, but not $\frac{a}{a}$; the reason for which is obvious.

Exam. 1. Required the values of x and y, in the equation

$$9x + 13y = 2000,$$

and the number of possible solutions, in integers.

First, in the equation $9p - 13q = 1$, we have at once $p = 3$ and $q = 2$; therefore the number of solutions will be expressed by

$$\frac{2000 \times 3}{13} - \frac{2000 \times 2}{9} = 461 - 444 = 17.$$

And these are readily obtained from the formulæ

$$\left\{\begin{array}{l} x = cp \quad - mb \\ x = 6000 - 13m \end{array}\right. \text{ and } \left\{\begin{array}{l} y = ma - cq, \text{ or} \\ y = 9m \quad 4000; \end{array}\right.$$

in which, assuming $m = 445, 446$, &c. in order that $9m > 4000$, we shall have the following solutions; each of which is deduced from the preceding one, by adding successively 9 for the values of y, and subtracting 13 from those of x; thus

$x = 215, 202, 189, 176, 163, 150, 137$, &c.
$y = 5, 14, 23, 32, 41, 50, 59$, &c.

that is,
$$9 \cdot 215 + 13 \cdot 5 = 2000$$
$$9 \cdot 202 + 13 \cdot 14 = 2000$$
$$9 \cdot 189 + 13 \cdot 23 = 2000$$
$$9 \cdot 176 + 13 \cdot 32 = 2000$$
&c. &c.

INDEX, in *Arithmetic* and *Algebra*, is the same as exponent. See Exponent.

Index of a *Logarithm*, called otherwise the *Characteristic*, is the integral part which precedes the decimals, and is always one less than the number of integral figures in the given number. Or otherwise, the index is always equal to the number of places that the units place of the proposed number is from the first effective figure, and is accounted positive when the first figure is to the left of the unit's place, and negative when it is to the right.

INDIVISIBLES, in *Geometry*, are those small elements or principles into which any body or figure may be ultimately resolved.

This manner of considering magnitudes, is called the *method of in-*

divisibles, which is only the *method of exhaustions* of the ancients, a little disguised and contracted; from which conclusions are drawn as principles without the trouble of demonstration; and it must be acknowledged also frequently without that satisfaction which ought to accompany mathematical investigations. The extreme facility, however, which it affords in a variety of problems, has induced many modern mathematicians to adopt it in preference to the more strict but tedious method of Euclid, and the other ancient geometers.

INDUCTION, is a term used by mathematicians to denote those cases in which the generality of any law, or form, is inferred from observing it to have obtained in several cases. Such inductions, however, are very deceptive, and ought to be admitted with the greatest caution, as there are numerous cases in which a law may obtain for a considerable way, and ultimately fail when its uniformity is supposed certain. A remarkable instance of the failure of induction appears in many of those formulæ which have been given for prime numbers. Thus the formula $x^2 + x + 41$, by making successively $x = 1, 2, 3, 4$, &c. will give a series, the first forty terms of which are prime numbers, whence one might be induced to conclude the law to be universal; it fails however in the very next case, the forty-first term being a composite number.

INERTIÆ. See VIS *Inertiæ*.

INFINITE, is a term applied to quantities which are greater than any assignable quantities; also, quantities that are less than any assignable quantity, are said to be infinitely small.

Some authors use the character ∞ to denote infinity, or an infinite quantity, which symbol, when submitted to the usual rules of analysis, is found to possess several curious properties: thus a being an infinite quantity, we have $\frac{a}{\infty} = 0$; $\frac{a}{0} = \infty$; $0 \times \infty = a$. That is, if a finite quantity be divided by a quantity infinitely great, the quotient is 0, or a quantity infinitely small.

Arithmetic of INFINITES, is a term applied by Dr. Wallis, to a method invented by him for the summation of infinite series.

INFINITE *Decimal*, the same as circulating decimal.

INFINITE *Series*. See SERIES.

INFINITESIMAL, or *Infinitely small Quantity*, is that which is so small, as to be incomparable with any finite quantity whatever, or it is that which is less than any assignable quantity.

In the *Method of Infinitesimals*, the element by which a quantity increases or decreases is supposed to be infinitely small, and is generally expressed by two or more terms; some of which are infinitely less than the rest, which being neglected, as of no importance, the remaining terms form what is called the difference of the proposed quantity. The terms that are thus neglected are infinitely less than the other terms of the element, and are the same which arise in consequence of the acceleration or retardation of the generating motion, during the infinitely small instant of time in which the element was generated; so that the remaining terms express the element that would have been formed in that time, if the generating motion had continued uniform. These differences are, therefore, in exactly the same ratio to each other, as the generating motions or fluxions; and hence, though infinitesimal parts of the elements are neglected, the conclusions are accurately true, in consequence of what may be termed a compensation of errors.

INFLECTION, called also *diffraction*, and *deflection*, in *Optics*, is a property of light, by reason of which, when it comes within a certain distance of any body, it will be either bent from it or towards it; which is a kind of imperfect reflection or refraction.

Point of INFLECTION of a *Curve* in *Geometry*, is a point where the curve begins to bend or turn the contrary way.

From the nature of curvature it is evident, that while a curve is concave towards its axis, the fluxion of the ordinate decreases, or is in a decreasing ratio with regard to the fluxion of the absciss; and, on the contrary, this fluxion increases, or is in an increasing ratio to the fluxion of the absciss, when the curve is convex towards the axis; and hence it follows that these fluxions are in a constant ratio at the point of inflection or retrogression, where the curve is neither concave nor convex; that is, $\frac{\dot{x}}{\dot{y}}$, or $\frac{\dot{y}}{\dot{x}}$, is a constant quantity, the fluxion of which is therefore at this point $= 0$. Whence we draw this general rule.

Put the given equation of the curve into fluxions, from which equations of the fluxions find $\frac{\dot{x}}{\dot{y}}$, or $\frac{\dot{y}}{\dot{x}}$, and take again the fluxion of this fraction, and make it equal to 0; and from this last expression find also $\frac{\dot{x}}{\dot{y}}$ or $\frac{\dot{y}}{\dot{x}}$, and by equating these two, together with the general equation of the curve, x and y will be determined, being the absciss and ordinate answering to the point of inflection.

Let it be proposed, for example, to find the point of inflection in the curve, whose equation is

$$a x^2 = a^2 y + x^2 y, \text{ or } y = \frac{x^2}{a^2 + x^2}.$$

The fluxion of $a x^2 = a^2 y + x^2 y$, is $2 a x \dot{x} = a^2 \dot{y} + 2 x y \dot{x} + x^2 \dot{y}$; whence $\frac{\dot{x}}{\dot{y}} = \frac{a^2 + x^2}{2 a x - 2 x y}$. And now making again the fluxion of this quantity $= 0$, we have

$$2 x \dot{x} (a x - x y) = (a^2 + x^2) (a \dot{x} - y \dot{x} - x \dot{y});$$

whence $\frac{\dot{x}}{\dot{y}} = \frac{a^2 + x^2}{a^2 - x^2} \times \frac{x}{a - y}$.

And now equating these two expressions, we have

$$\frac{a^2 + x^2}{a^2 - x^2} \times \frac{x}{a - y} = \frac{a^2 + x^2}{2 x} \times \frac{1}{a - y}$$

which being reduced, gives $2 x^2 = a^2 - x^2$, or $3 x^2 = a^2$, and $x = a \sqrt{\frac{1}{3}}$; and this value of x substituted in the original equation of the curve, becomes $y = \frac{a x^2}{a^2 + x^2} = \frac{\frac{1}{3} a^3}{\frac{4}{3} a^2} = \frac{1}{4} a$, the ordinate at the point of inflection.

INSCRIBED *Figure*, is one which touches all the sides of another figure internally.

To inscribe a circle in any triangle or regular polygon: Bisect two of the angles, the intersection of the bisecting lines will be the centre of the circle, and its radius will be the perpendicular drawn from that point to any of the sides.

INSCRIBED *Hyperbola*, lies wholly within the angles of its asymptotes; as the common or conical hyperbola.

INSTANT, an infinitely small and indivisible portion of duration, being, with regard to time, what a point is in respect to a line.

INSULATED *Body*, in *Electricity*, a body that is supported by electrics, or non-conductors, whereby its electrical communication with the earth is interrupted.

INTEGER, or **INTEGRAL** *Number*, is an unit, or an assemblage of units.

INTEGRAL *Calculus*, is the reverse of the differential calculus, and corresponds with the inverse method of fluxions; the finding of an integral to a given differential being the same as finding the fluent of a given fluxion, and is performed in a similar manner, and by the same rules.

INTEGRANT *Parts of a Body*, those parts which are of the same nature with the whole.

INTENSITY, the power or energy of any quality or action.

INTERCALARY *Day*, the odd day which is inserted in the calendar every fourth year.

INTERCEPTED *Axis*, in the *Conic Sections*, is the same as **ABSCISS**.

INTEREST, is the allowance made for the loan or forbearance of a sum of money, which is lent for, or becomes due, at a certain time; this allowance being generally estimated at so much for the use of 100 pounds for a year. This rate is by law fixed not to exceed 5 pounds, or, in other words, the greatest rate of interest must not legally exceed 5 per cent. per annum.

The money lent, or forborne, is called the principal. The sum paid for the use of it the interest. The interest of 100 pounds for one year, is called the rate per cent. And the sum of any principal, and its interest together, the amount.

Interest is either *Simple* or *Compound*.

Simple interest is that which is allowed upon the original principal only for the whole time of the loan, or forbearance.

To find the Simple Interest of any Sum for any Period, and at any given Rate per cent.

The general method of calculation is as follows:

Let $r =$ the interest of £1 per annum,

$p =$ any principal or sum lent,

$t =$ the time of the loan or forbearance,

$i =$ the interest of the given principal for the time t,

$a =$ the amount of the principal and its interest for the given time.

Then it is obvious that $i = ptr$, and $a = p + ptr = p\,(1 + tr)$; from which theorem the following formulæ are readily deduced; viz.

$$1 \ldots a = p\,(1 + rt) = p + ptr$$

$$2 \ldots p = \frac{a}{1 + rt}$$

$$3 \ldots r = \frac{a - p}{pt}$$

$$4 \ldots t = \frac{a - p}{pr}$$

$$5 \ldots i = ptr$$

By means of which theorems all circumstances relating to the simple interest of money are readily computed.

Example.—Let it be required to find the interest and amount of £250 for 2½ years, at 5 per cent.

Here $r = {\cdot}05$, $t = 2{\cdot}5$, $p = 250$; therefore

The interest $i = ptr = 250 \times 2{\cdot}5 \times {\cdot}05 = £31{\cdot}5$

The amount $a = p \times ptr = 250 \times 31.5 = £281{\cdot}5$

2. Again, suppose it were required; what principal put out at 5 per cent., for 2½ years, would amount to £281 10*s*.

Here $a = 281{\cdot}5$, $t = 2{\cdot}5$, and $r = {\cdot}05$; therefore

$$p = \frac{a}{1 + rt} = \frac{281{\cdot}5}{1 + 2{\cdot}5 \times {\cdot}05} = \frac{281{\cdot}5}{1{\cdot}125} = £250.$$

3. Let it now be required to find at what rate of interest 250*l*. must be put out, to amount to 280*l*. 10*s*. in 2½ years.

$$\text{Here } r = \frac{a - p}{pt} = \frac{31{\cdot}5}{250 \times 2{\cdot}5} = 05,$$

which corresponds to 5 per cent.

4. In what time will 250*l*. amount to 281*l*. 10*s*. at 5 per cent. per annum.

$$\text{Here } t = \frac{a - p}{pr} = \frac{31{\cdot}5}{250 \times {\cdot}05} =$$

2·5, or 2½ years.

The common rules for finding simple interest, expressed in words, are as follow:

1. To find simple interest for years: multiply the principal by the rate per cent. and by the number of years, divide by 100.

2. To find simple interest for months: multiply the principal by the rate per cent. and by the months, divide by 1200.

3. To find simple interest for days: multiply by double the rate per cent. and by the number of days, divide by 73,000.

The reason of these rules will be seen from the following compound proportions.

Let $p =$ the principal, and r the rate per cent., and y, m, and d, the years, months, and days, then,

$$1.\ \left.\begin{array}{r}\text{As } 100 : p\\ 1 : y\end{array}\right\} = r : \text{the interest.}$$

$$\text{That is, } \frac{pyr}{100} = \text{the interest.}$$

$$2.\ \left.\begin{array}{r}\text{As } 100 : p\\ 12 : m\end{array}\right\} = r : \text{the interest.}$$

$$\text{That is, } \frac{pmr}{1200} = \text{the interest.}$$

$$3.\ \left.\begin{array}{r}\text{As } 100 : p\\ 365 : d\end{array}\right\} = r : \text{the interest}$$

That is, $\frac{p\,d\,r}{36500}$ = the interest;

which, multiplying by 2, is $\frac{p\,d\,2\,r}{73000}$.

When the rate is 5 per cent. the calculation is more simple, for dividing $\frac{p\,d\,r}{36500}$ by $r\ (=5)$ we have $\frac{p\,d}{7300}$ = the interest. That is, multiply the principal by the number of days, divide by 7300.

As the computations of interest are of great importance in all mercantile transactions, it has been found convenient to have extensive tables of interest computed, whereby the interest of any sum, for any time, and at all rates of interest is found by inspection; of these works the most approved are those of Baily, Morgan, Smart, and Booth.

COMPOUND *Interest* is that which arises not only from the original principal, but also from the interest added to the principal as it becomes due. By the usury laws it is not allowed to exact interest in this manner, but there are some cases in which it has to be computed. The nature of the computation is obvious. The interest, at the terms when it becomes due, has merely to be added to the principal and interest till the next term computed on the amount.

Hence it follows that, except for the sake of brevity, compound interest would require no other rule than what is given for simple interest, for it is only necessary to find the interest upon the given principal; then add this interest to the principal, and find the interest upon this, and so on, through the whole number of payments. But this would be attended with immense labour if the time was at all considerable, and, therefore, other methods of computation become necessary.

Now, in the first place, it is obvious that if we know the amount of 1*l.* for any period, the amount of any other principal p, will be equal to p times that; we may, therefore, limit our investigation to the simple case of finding the compound interest and amount of 1*l.* for any given number of years at any given rate per cent.

In order to do this, let r be the amount of 1*l.* for one payment, whether yearly, half-yearly, or quarterly. Then, as the amount is always proportional to the principal, we shall have,

as $1 : r = r : r^2 = r^2 : r^3$, &c.

that is, if the amount of 1*l.* for 1 payment be r

the amount of 1*l.* for 2 payments $= r^2$

the amount of 1*l.* for 3 payments $= r^3$

the amount of 1*l.* for n payments $= r^n$

and, consequently, the amount (a) of any principal (p) for (n) payments $= pr^n$, which being for the conveniency of calculation put into the logarithmic form, becomes

$$\log. a = \log. p + n \log. r.$$

Suppose, for example, the amount of 50*l.* for 15 years, at 5 per cent. were required, the payments being due yearly.

Since the interest of 100*l.* is 5*l.*; the interest of 1*l.* is ·05, and the amount of 1*l.* in one payment 1·05.

Hence by the above formula,

$$\begin{array}{rr} \log.\ 1{\cdot}05 = & 0{\cdot}021189 \\ \text{mult. by } 15 & 15 \\ \hline & 0{\cdot}317835 \\ \log.\ 50 = & 1{\cdot}698970 \\ \hline & 2{\cdot}016805 \\ \hline\hline \end{array}$$

The natural number to this logarithm is 103·946 or 103*l.* 18*s.* 10*d.* the amount; from which, if the principal be subtracted, it will leave the compound interest itself; or this formula may be expressed in words, thus:

Multiply the logarithm of the amount of 1*l.* for one payment by the number of payments, add the logarithm of the principal, and the sum will be the logarithm of the amount.

By this means the amount and compound interest of any sum, for any period and rate per cent. may be readily computed. But the same may be done still more expeditiously by means of the following table, which exhibits the amount of 1*l.* for any number of years not exceeding 25.

Yrs.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.	6 per cent.
1	1.03000	1.035000	1.040000	1.045000	1 050000	1 060000
2	1.060900	1.071225	1.081600	1.092025	1.102500	1.123600
3	1.092727	1.108718	1.124864	1.141166	1.157625	1.191016
4	1.125509	1.147523	1.169859	1.192519	1.215506	1.262477
5	1.159274	1.187686	1.216653	1.246182	1.276282	1.338226
6	1.194052	1.229255	1.265319	1.302260	1.340096	1.418519
7	1.229874	1.272279	1.315932	1.360862	1.407100	1.503630
8	1.266770	1.316809	1.368569	1.422101	1.477455	1.593848
9	1.304773	1-362899	1.423312	1.486095	1.551328	1.689479
10	1.343916	1.410599	1.480244	1.552969	1.628895	1.790848
11	1.384234	1.459970	1.539454	1.622853	1.710339	1.898299
12	1.425761	1.511069	1.601032	1.695881	1.795856	2.012196
13	1.468534	1.563956	1.665074	1.772196	1.885649	2.132928
14	1.512590	1.618695	1.731676	1.851945	1.979932	2 260904
15	1.557967	1.675349	1.800944	1.935282	2.078928	2.396558
16	1.604706	1.733986	1.872981	2.022370	2.182875	2.540352
17	1.652848	1.794676	1.947900	2.113377	2.292018	2.692773
18	1.702433	1.857489	2.025817	2.208479	2.406619	2 854339
19	1.753506	1.922501	2.106849	2.307860	2.526950	5.025599
20	1.806111	1.989789	2.191123	2.411714	2.653298	5.207135
21	1.860295	2.059431	2.278768	2.520241	2.785963	5.399564
22	1.916103	2.131512	2.369919	2.633652	2.925261	5 603537
23	1.973587	2.206114	2.464716	2.752166	3.071524	5.819750
24	2.032794	2 283328	2.563304	2.876014	3.225100	4.048935
25	2.093778	2.363245	2.665836	3.005434	3 386355	4.291871

Find the amount in the table for the given number of years and rate of interest, and multiply this amount by the principal for the amount required. Thus in the preceding example,

Amount by the table = 2·67893
Principal 50

103·94650 = 103*l*. 18*s*. 10*d*. as before.

This table is only computed for yearly payments, and, therefore, when the payments are different from these, or when the number of years or payments exceed 25, recourse must be had to the preceding rule.

Putting p = the principal,
r = the amount of 1*l*. for 1 payment,
n = the number of payments,
a = the amount.

Then, as before, $a = pr^n$. Hence

$$\text{I. } p = \frac{a}{r\,n}$$

$$\text{II. } n = \frac{\log. - \log. p}{\log. r}$$

Example. In what time will any sum, say 100*l*. double itself at 5*l*. per cent.

Here $a = 200$, $p = 100$, $r = 1.05$
Log. $a = 2.301030$, log. $p = 2.000000$, log. $r = 0.021189$. Hence

$$r = \frac{2301030 - 2.00000}{0.021189} = \frac{301030}{21139} =$$

14½ years nearly.

INTERIOR *Angle* of a *Polygon*, that which is formed internally by the meeting of any of the sides of the figure.

INTERPOLATION, in *Analysis*, is the method of finding any intermediate term in a series, its place or distance from the first term being given. This is commonly effected by means of the successive differences of the given terms, and is therefore sometimes called the *differential method*, though this latter expression is more commonly employed to denote the method of summing series by means of their differences.

This theory is of very extensive use, not only in pure analysis and geometry, but in various other bu-jects of mathematical inquiry and computation, and particularly in astronomy; we shall therefore endeavour to explain the principles upon which it is founded, and show its application in a few cases to practical operations.

First, then, let a, b, c, d, e, f, &c. represent any series of similar quantities, and let the difference between the first and second, the second and third, the third and fourth, &c. terms be taken; and these several remainders will form what is called the first order of difference; then again let the differences of these differences be taken in the same way; and the differences of these last again the same, and so on, which will give the following result, observing, that for the convenience of exhibiting the operation, we have only retained the first remainders in each successive subtraction.

Series a, b, c, d, e, f, &c.
1st diff. $a - b$
2d diff. $a - 2b + c$
3d diff. $a - 3b + 3c - d$
4th diff. $a - 4b + 6c - 4d + e$
5th diff. $a - 5b + 10c - 10d + 5e - f$
6th diff. $a - 6b + 15c - 20d + 15e - 6f + g$
&c. &c.

Now the co-efficients of these terms are respectively the same as those of the co-efficients of the binomial, and the order of their generation evidently follows the same law, and therefore we may conclude, with equal certainty, that the nth difference of any series of quantities, will be expressed by the formula

$$a - n\,b + \frac{n\,(n-1)}{1\,.\,2}\,c - \frac{n\,(n-1\,(n-2)}{1\,.\,2\,.\,3}\,d + \frac{n\,(n-1)\,(n-2)\,(n-3)}{1\,.\,2\,.\,3\,.\,4}\,e - \text{\&c.}$$

Now it is obvious that if the given quantities be such that any order of their differences become equal to 0, any one of those quantities may be accurately expressed in functions of the others; thus for example, suppose the fourth difference to become zero, that is,

$$a - 4b + 6c - 4d + e = 0$$

then will $c = \frac{-u + 4b + 4d - e}{6}$

and it is obvious that any other of these quantities might be expressed in a similar manner; and therefore if all those quantities but one be known, that one may be ascertained. Thus, by way of illustration, suppose we had the three squares $10^2 = 100$, $8^2 = 64$, and $7^2 = 49$, and the square of 9 was required; since the third differences of squares equal 0, we should have

$$9^2 = \frac{100 + 3\,.\,64 - 49}{3} = 81,$$ and the same is obviously true of any terms of which the differences vanish.

But if the differences do not vanish, then any intermediate term found by this method only approximates towards the true result, which is, however, sufficiently correct in a number of cases; thus in finding any particular logarithm of which those consecutive to it are given, the above formula may be successfully employed; for though, in fact, the differences of logarithms never become zero, yet their fourth differences are so small, that by considering them as 0, the error will not affect the truth of the result to 8 or 9 places of decimals.

Exam. Given the logarithms of 101, 102; 104, 105, to find the logarithm of 103.

Here, calling the log. of $101 = a$, of $102 = b$, $103 = c$, $104 = d$, and $105 = e$; and considering the fourth differences of these logs. $= 0$, we shall have from the formula

$$a - 4b + bc - 4d + e = 0$$

$$c = \frac{4\,(b + d) - (a + e)}{6}$$

Hence the following computation:

$$\begin{array}{rrl}
\text{log. } 101 = & 2{\cdot}0043214 & = a \\
\text{log. } 102 = & 2{\cdot}0086002 & = b \\
\text{log. } 104 = & 2{\cdot}0170333 & = d \\
\text{log. } 105 = & \underline{2{\cdot}0211893} & = e \\
 & 4{\cdot}0256335 & = b + d \\
 & \underline{4} & \\
 & 16{\cdot}1025340 & = 4\,(b + d) \\
\text{subt.} & \underline{4{\cdot}0255107} & = a + e \\
6) & \underline{12{\cdot}0770233} & \\
\text{log. } 103 = & \underline{\underline{2{\cdot}0128372}} & \text{as required.}
\end{array}$$

This doctrine is applied with great success in various astronomical operations, and is the means of saving, in many cases, immense laborious calculations. Thus, for example, in finding the places of some of the planets, whose motion is not very rapid, it will be sufficiently accurate to find their places by calculation for every fourth or fifth day, and then by means of the methods above described, their places for all the intermediate days may be found by interpolating between the known terms, which method will give a result much nearer the truth, than by proportional parts, because this supposes a uniformity both in motion and time, which is not correct.

Again, in computing the moon's place for any particular hour, supposing its place for every day at noon to be given, the method of interpolations may be applied with great success, the result having scarcely any sensible difference from those that arise from actual computation, and we may thus frequently avoid one of the most laborious of astronomical calculations.

By this means also the place of a comet, at any particular time, may be ascertained, from observations made on it prior to, and subsequent to, that precise period, as also the times of the equinoxes and solstices, which are determined much more accurately by this means than can be done by proportional parts, for in this we are obliged to suppose that the sun's declination increase and decrease proportionally to the distance of this body from the equinoxial point; which is evidently a false hypothesis. In fact, astronomy has derived more assistance from this theory than any other of the mathematical sciences, although it has been applied to other purposes with very great success.

INTERSCENDENT *Quantities*, in *Algebra*, are those that have radical indices; as $x^{\sqrt{2}}$, $x^{\sqrt[3]{3}}$, &c.

INTERSECTION, the cutting of one line or plane by another; the point or line in which they meet being called the point or line of intersection. When two lines or two planes intersect each other the vertically opposite angles are equal.

INTESTINE *Motion* of the Parts of a *Body*, is that which takes place amongst the component parts.

INTRADOS, the internal curve of the arch of a bridge.

INVERSE *Proportion*, proportion in which the product of two quantities being always the same, the one increases as the other diminishes.

INVERSE *Method* of *Tangents*, is the method of finding a curve belonging to a given tangent; being thus distinguished from the direct method, or that of finding the tangent from the properties of the curve.

INVERSION, INVERTENDO, is the inverting the terms of a proportion;

thus if . . . $a:b::c:d$
then by inversion $b:a::d:c$

INVESTIGATION, in *Mathematics*, denotes the tracing out the solution of any proposed problem, or an examination of its several properties, &c.

INVOLUTE *Curve*, in the *Higher Geometry*, is that which is traced out by the extremity of a thread, as it is folded or wrapped about another figure or curve, being the reverse of evolute. See EVOLUTE.

INVOLUTION, in *Algebra* and *Arithmetic*, is the raising a given number or quantity to any proposed power. In arithmetical and in simple algebraical quantities, this consists only in multiplying the quantity as many times by itself as is necessary to produce the given power, the number of operations being one less than the index of the power to be produced; thus

$a \times a = a^2$. 2d power
$a \times a \times a = a^3$. 3d power
$a \times a \times a \times a = a^4$. 4th power; &c.
So also
$2 \times 2 = 4$. 2d power of
$2 \times 2 \times 2 = 8$. 3d power of 2
$2 \times 2 \times 2 \times 2 = 16$. 4th power of 2, &c.

And in the same way we may proceed with compound algebraical quantities; but for binomical ones, it is best to make use of the binomical theorem, the general form of which is as follows: *viz.*

$$a + b)^n = a^n + n\,a^{n-1}\,b + \frac{n\,(n-1}{1\,.\,2}\,a^{n-2}\,b^2 + \frac{n\,(n-1)\,(n-2)}{1\,.\,2\,.\,3}\,a^{n-3}\,b^3 + \text{,\&c.}$$

Where

$$(a+b)^2 = a^2 + 2a\;b + b^2$$
$$(a+b)^3 = a^3 + 3a^2\,b + 3a\,b^2 + b^3$$
$$(a+b)^4 = a^4 + 4a^3\,b + 6a^2\,b^2 + 4a\,b^3 + b^4$$

JOINT *Lives*, in the *Doctrine* of *Annuities*, are such as continue the same time, or during the existence of both parties. See LIFE *Annuities*.

JOURNAL, in *Navigation*, is a register kept at sea by the master and other officers, in which is noticed every incident of the voyage; as the rate and course of sailing, the state of the weather, tides, currents, astronomical observations, latitude, longitude, &c. &c.

In all sea journals the day is divided into twice twelve hours, those before noon being marked A. M., for *ante meridian*, and those after noon by P.M., *post meridian*.

IRIS, the name of the rainbow.

IRIS *Marina*, the *Sea Rainbow*. This elegant appearance is generally seen after a violent storm, in which the sea-water has been much agitated. The celestial rainbow has great advantage over the marine one, in the brightness and variety of the colours, and in their distinctness one from the other; for in the sea rainbow there are scarcely any other colours than a dusky yellow on the part towards the sun, and a pale green on the opposite side; the other colours are not so bright or distinct as to be well determined: but the sea rainbows are more frequent and more numerous than the others. It is not uncommon to see twenty or thirty of them at a time at noon day. Observ. sur l'Asie, p. 292.

IRRATIONAL *Number*, the same as SURD.

IRREDUCIBLE *Case*, in *Algebra*, is an expression arising from the solution of certain equations of the third degree, which always appears under an imaginary form, notwithstanding it is in fact a real quantity; but the reduction of it to a rational or irrational finite expression, has at present resisted the united efforts of many of the most celebrated mathematicians of Europe. Every cubic equation may be reduced to the form $x^3 + a\,x = b$; and then, according to the common rule,

$$x = \sqrt[3]{\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}} + \sqrt[3]{\frac{b}{2} - \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}}$$

Now when a is negative, $\frac{a^3}{27}$ is also negative; and therefore when $\frac{b^2}{4} < \frac{a^3}{27}$, the quantity under the inferior radical, *viz.* $\sqrt{\frac{b^2}{4} - \frac{a^3}{27}}$ is imaginary, because we cannot extract the square root of a negative quantity; and this is what constitutes that which is generally called the *Irreducible Case*.

But though no direct analytical solution can be found in the irreducible case, other methods of determining the roots have been invented; such as by means of series, a table of sines and tangents, the usual approximation, &c. The two first methods exhibit the solution to the eye in as neat a manner as can be desired, but they are both commonly attended with great labour in the practical operation. Another method of solution is by a table computed for the purpose, the general principle of which is as follows:

Let $x^3 - a\,x = b$ be any cubic in the irreducible case; this may be reduced to a dependent equation $y^3 - y = d$; and since in the first $\frac{a^3}{27} > \frac{b^2}{4}$, so in this last $\frac{1}{27} > \frac{d^2}{4}$, or $d^2 < \frac{4}{27}$, or d cannot exceed ·38491. Whence again it follows, that in equations falling under the irreducible case, when they are reduced to the form $y^3 -$

$y = d$, the value of y cannot exceed 1·1540, nor be less than 1. On this principle, therefore, has been constructed a table of all the values of y to five places between these limits, and those of d to eight places, whereby the solution of such equations is obtained by one or two simple operations, true to eight or nine places of figures. See Rees's Cyclopedia, article IRREDUCIBLE *Case;* also Leybourn's Math. Repository, No. 11; and for the solution by sines and tangents, see CUBIC *Equations.*

The reason why Cardan's rule fails in this particular case is, that we make two suppositions in order to arrive at the solution, which are inconsistent with each other. Let, for example, $x^3 - ax = b$, be an equation falling under the irreducible case, having $\frac{a^3}{27} > \frac{b^2}{4}$. Now the suppositions are $m^3 + n^3 = b$, $mn = \frac{a}{3}$, or $m^3 n^3 = \frac{a^3}{27}$; and it is to be shown that these two conditions cannot have place at the same time, under the conditions of the problem, at least only when $m^3 = n^3$, or when the equation has two equal roots, for since $m^3 + n^3 = b$, the greatest product $m^3 n^3$ is when $m^3 = n^3$, or when they are each equal to $\frac{1}{2}b$, and consequently when their product $= \frac{1}{4}b^2$. Now we suppose $m^3 n^3 = \frac{a^3}{27}$, and since $\frac{a^3}{27}$ is greater than $\frac{b^2}{4}$ by hypothesis, it is obvious that we make an impossible supposition, and consequently the result of it must necessarily be imaginary. But if $m^3 = n^3$ then $\frac{b^2}{4} = \frac{a^3}{27}$, and the solution may be obtained. It may be further remarked, that when an equation falls under the irreducible case, it will have three real roots; and conversely, that every equation having three real roots, unless two of them are equal, must according to Cardan's rule fall under the irreducible case, as may be shown thus.

Let $x^3 - ax = b$, be an equation having three real roots p, q, r, of which let r be the greatest; then from the known theory of equations

$$\begin{aligned} p + q + r &= 0 \\ pr + qr + pq &= -a \\ pqr &= -b \end{aligned}$$

Whence 1st, $p + q = -r$

2d, $r(p+q) + pq = -a$; or $(p + q^2) - pq = a$

3d, $(p+q)pq = b$.

Now in the case of equal roots we should have $p = q$; but this is not the case we are considering, having shown above that the solution will obtain on that supposition. Since then the roots are unequal, let p be greater than q, and make $p = q + d$, and we shall have, by substituting this value of p in the second and third equations.

$$\begin{aligned} (2q + d^2) - (q^2 + dq) &= a \\ (2q + d) \times (q^2 + dq) &= b \\ 3q^2 + 3qd + d^2 &= -a \\ 2q^3 + 3q^2 d + qd^2 &= b \end{aligned}$$

and it remains to be shown, that with these values of a and b, $\frac{1}{27}a^3$ is necessarily greater than $\frac{1}{4}b^2$.

$$\text{Or } \tfrac{1}{27}\left\{3q^2 + 3dq + d^2\right\}^3 > \tfrac{1}{4}\left\{2q^3 + 3q^2 d + d^2 q\right\}^2$$

$$\text{or } \left\{q^2 + dq + \tfrac{1}{3}d^2\right\}^3 > \left\{q^3 + \tfrac{3}{2}q^2 d + \tfrac{1}{2}d^2 q\right\}^2$$

which is obvious by simply expanding those quantities, the former being

$p^6 + 3p^5 r + 4p^4 r^2 + 3p^3 r^3 + \frac{4}{3}p^2 r^4 + \frac{1}{3}pr^5 + \frac{1}{27}p^6$;

the latter $p^6 + 3p^5 r + 3\frac{1}{4}p^4 r^2 + 3p^3 r^3 + \frac{1}{4}p^2 r^4$.

Therefore when the three roots of a cubic equation $x^3 - ax = b$ are real, $\frac{a^3}{27}$ is greater than $\frac{b^2}{4}$, and consequently the solution of it falls under the irreducible case. Hence then it appears, that when a cubic equation has only one real root, that root will be found by the formula of Cardan; but when all three roots are real, no one of them can be obtained by that method; which is the only analytical solution that has yet been discovered, and probably the only one the equation admits of.

IRREGULAR, that which deviates from the usual form or rules; thus, in geometry, a polygon which has not all its sides and angles equal, is called an irregular polygon.

ISAGONE, is used by some old authors to denote a figure having equal angles.

ISOCHRONAL, or ISOCHRONOUS, is applied to such vibrations of a pendulum as are performed in equal times. Of which kind are all the cycloidal vibrations or swings of the same pendulum, whether the arcs it describes be longer or shorter; for when it describes a shorter arc, it moves so much the slower; and when a long one, proportionally faster.

ISOMERIA, is a term used by Vieta to denote the operation of freeing an equation from fractions.

ISOPERIMETRICAL *Figures*, in *Geometry*, are those which have equal perimeters.

ISOPERIMETRY, in *Mathematics*, is a branch of the higher geometry, which treats of the properties of isoperimetrical figures; *viz.* of surfaces contained under equal perimeters, of solids under equal surfaces, of curves of equal lengths, &c. Of the foregoing heads, the two first may be considered as containing the elements of the science, which relates principally to the *maxima et minima* of different surfaces and solids, when bounded by figures of equal perimeters, but of a greater or less number of sides, and posited in a different order. The other head, which relates to the *maxima et minima* of curves, are problems of another kind and of the most difficult nature, which have engaged the attention and exercised the talents of many of the greatest mathematicians of modern times.

The problems in which it is required to find, among figures of the same or different kinds, those which within equal perimeters shall comprehend the greatest surfaces, and those solids which under equal surfaces shall contain the greatest volume, had long engaged the attention of mathematicians before the invention of fluxions; and different methods had been devised for the solution of them by Descartes, Fermat, Sluze, Hudde, and others; which were all supplanted by the simplicity and generality of the new analysis: after which time the elements of the science seems to have been lost sight of by mathematicians, who were all engaged in the solution of the higher order of isoperimetrical problems.

Simpson was the first who condescended to treat of the more elementary parts of this science, by giving in his "Geometry" a very interesting chapter on the *maxima et minima* of geometrical quantities, and some of the simplest problems concerning isoperimeters. The next who treated the subject in an elementary manner, was Simon L'Huillier of Geneva, who, in 1782, published his treatise "De Relatione mutua Capacitatis et Terminorum Figurarum," &c. His principal object in the composition of that work, was to supply the deficiency in this respect, which he found in most of the Elementary Courses, and to determine with regard to both the most usual surfaces and solids, those which possess the minimum of contour with the same capacity; and, reciprocally, the maximum of capacity with the same boundary. Legendre has also considered the same subject in a manner somewhat different from either Simpson or L'Huillier, in his "Elemens de Géométrie;" and Dr. Horsley has a paper on the same subject in the Phil. Trans. vol. lxxv., for 1775. These, however, principally relate to the elementary propositions of isoperimetry; but what relates to the higher geometry, forms the highest order of problems, which as we before observed have called into action the talents, and excited the passions, of some of the ablest mathematicians of Europe; having led to a dispute which, for want of competent and impartial judges, remained undecided for many years, and has since been termed the "war of problems," on account of the great interest it excited, and the determined and able manner in which each party supported its opinions and contested those of its opponents: a brief sketch of which will be found under the article **Brachystochrone**, as well as the solution of that celebrated problem. As to the rest, the reader who is desirous of farther information, may consult the papers mentioned in that article; as also the essays of several able mathematicians who have introduced the subject into their respective works.

Simpson, in his "Tracts," has a chapter entitled, "An Investigation of a General Rule for the Resolution of Isoperimetrical Problems of all Orders." He has also given the solution of several isoperimetrical problems, in his "Doctrine of Fluxions." Maclaurin has likewise a chapter on the same subject, in his "Treatise of Fluxions." To these may also be added, Emerson, Le Seur, Bossut, and Lacroix; each of whom has introduced this doctrine into their respectiveworks: but the two writers who have most contributed in bringing to perfection the theory of isoperimetry, are Euler and Lagrange, the former having, beside several memoirs in the Acta Petro. a tract on this subject, entitled "Methodus inveniendi Lineas Curvas Proprietate Maximi Minimive gaudentes;" which, with a very few exceptions, is what it was intended to be, a complete treatise, containing essentially all the requisite methods of solutions, with great variety and abundance of examples and illustrations: there were still, however, some defects in this work, for want of a better algorithm, or more compendious process of establishing the theorems, and certain supplemental formulæ; which defects have been finally removed by Lagrange, in his admirable and refined "Calculus of Variations;" and a small treatise on the same subject has lately been published by Mr. Woodhouse, in which are combined the history and progress of the science, with such observations and remarks as seem most calculated to render it instructive and familiar to the English student.

ISOSCELES *Triangle*, is a triangle of two equal legs or sides. The angles at the base of an isosceles triangle are equal, and if the sides be produced, the angles under the base are also equal. If the line be drawn perpendicular to the base, it will bisect the base and the vertical angle; or if it be drawn to bisect the base, it will be perpendicular to it.

JUNO, the name of one of the new planets, situated between the orbits of Mars and Jupiter, was discovered by Mr. Harding, at the observatory of Lilienthal, near Bremen, on the evening of the 1st of September, 1804.

This planet is of a reddish colour, and is free from that nebulosity which surrounds Pallas. Its diameter, and its mean distance, are less than those of the other new planets. It is distinguished from all other planets by the great eccentricity of its orbit; and the effect of this is so extremely sensible, that it passes over that half of its orbit which is bisected by its perihelion, in half the time that it employs in describing the other half. From the same cause, its greatest distance from the sun is double the least distance, the difference between the two distances being about 127 millions of miles.

Though there is no nebulous appearance around the planet Juno, yet it appears, that it must have an atmosphere more dense than that of any of the old planets of the system. A very remarkable variation in its brilliancy has been observed. This is attributed chiefly to changes that are going on in its atmosphere, though it is not improbable that these changes may arise from a diurnal rotation performed in 27 hours. The following are its elements, as calculated by Buckhardt.

Annual revolution	4yrs, 128 days
Mean longitude, 31st Dec. 1804, noon . .	$1^s\ 12°\ 17'\ 23''$
Place of ascending node	$5^s\ 21°\ 6'\ 0''$
Place of perihelion in 1805	$1^s\ 29°\ 49'\ 28''$

Eccentricity . . .	0.25096
Inclination of orbit .	21° 0′
Ditto	13° 4′
Mean distance from the sun in English miles	253,000,000
Mean distance . . .	2.057
Diameter in English miles, according to Schroeter	1425
Apparent mean diameter, as seen from the earth, according to Schroeter	3″.057

JUPITER, the largest planet in the solar system, and the fifth in order from the sun with regard to the old planets; but the ninth, including the new. Jupiter has four satellites, which were first discovered by Galileo.

This planet is also remarkable for the faint appearance of stripes, or belts, with which it is surrounded. These differ much at different times, and even at the same time, when viewed by telescopes of different powers. They appear usually of an uniform tint, but when viewed to the greatest advantage they seem to consist of a number of curved lines; and are now generally supposed to have some connection with the atmosphere of the planet.

Jupiter is, after Venus, the most brilliant of all the planets; and sometimes surpasses even Venus in brightness. He performs his sidereal revolution in 4332^d 14^h 18′ 41″, 0; or in 11·802 Julian years; but this period is subject to some inequality. He performs his synodical revolution in about 399 days. His mean distance from the sun is 5·203 that of the earth being considered as unity, which makes his mean distance above 485 millions of miles. The eccentricity of his orbit is ·0482; half the major axis being taken as unity. His mean longitude at the commencement of the present century was in 3^s 22° 36″, 1; and the longitude of his perihelion was, at the same time, in 0^s 11° 8′ 35″, 1; but the line of the apsides has an apparent motion according to the order of the signs, of 56″, 7 in a year, or of 1° 34′ 33″, 8 in a century. His orbit is inclined to the plane of the ecliptic in an angle of 1° 18′ 51″, 5; which is observed to decrease nearly 22″, 6 in a century. His orbit at the commencement of the present century, crossed the ecliptic, in 3^s 8° 25′ 34″, 2; but the place of the nodes has an apparent motion in longitude, according to the order of the signs of 34″·3 in a year, or 57′ 12″·4 in a century. The rotation on his axis is performed in 9^h 55′ 49″ 7; and his axis forms an angle of 86° 54½ with the plane of the ecliptic. His diameter is equal to 91,522 miles, and consequently he is above 1331 times larger than the earth; his polar diameter is to his equitorial as ·9287 to 1, or nearly as 13 to 14. The mass of Jupiter, compared with that of the sun as unity, is $\frac{1}{1067{\cdot}09}$ and its density to that of the sun is ·909501 to 1. The light and heat received by this planet from the sun, supposing it to be inversely as the square of the distance, is ·037 of that received by the earth. A body which weighs one pound at the earth's surface, would, if removed to equatorial regions of Jupiter, weigh 2·231 pounds. As seen from the earth, the motion of Jupiter appears sometimes to be retrograde; the mean arc which he describes in this case is about 9° 54′; and its mean duration is about 121 days. This retrogradation commences or finishes when the planet is not more distant than 115° 12′ from the sun. His mean apparent equatorial diameter is 38″·2; being greatest when in opposition, at which time it is equal to 47″·6.

K.

KALEIDOSCOPE, an optical instrument invented by Dr. Brewster, of Edinburgh, though a partial application of the same contrivance had previously been made. This instrument consists of two pieces of plane mirror, which make with each other an angle which is some aliquot part of a circle. They are inserted in a tube, at the bottom of which are placed two plane discs of glass, the outward one obscured. Then, if leaves, threads, bits of coloured glass, or any other substances, be placed between the discs, they are, by the reflection of the mirrors, formed into a regular stellar figure, which varies as the machine is turned. The kaleidoscope made a great noise at the time of its invention, but it is now seldom mentioned.

KEPLER's *Laws*, named after John Kepler, a celebrated German astronomer of the sixteenth century; a term used by astronomers to denote certain analogies between the distances of the planetary bodies, and their times of periodic revolution, as also between the rate of motion of any revolving body, whether primary or secondary, and its distance from the central body about which it revolves; to which may also be added, the figure of the planetary orbits, and the position of the central body; these, in order in which they were discovered, stand as follows:

1. Equal areas are described in equal times. That is, if a line be supposed to join the central and revolving body, this line passes over or describes equal areas in equal times, whether the planet be in its aphelion, perihelion, or in any other part of its orbit.

2. The planets all revolve in elliptic orbits, situated in planes passing through the centre of the sun; the latter body occupying one of the foci of the ellipse.

3. The squares of the times of revolution of the several planetary bodies, are as the cubes of their respective distances from the sun.

It is impossible in this place to enter into a history of these discoveries, and the difficulties and prejudices which Kepler had to encounter in bringing them to perfection; but the reader may see this subject fully illustrated in Dr. Small's "Account of the Astronomical Discoveries of Kepler."

KEPLER's *Problem*, is the determining the true from the mean anomaly of a planet, or the determining its place in its elliptic orbit answering to any given time; and so named from the celebrated astronomer Kepler, who first proposed it.

The general state of the problem is this: to find the position of a right line, which, passing through one of the foci of an ellipse shall cut off an area, which shall be in any given proportion to the whole area of the ellipse; which results from this property, that such line sweeps areas that are proportional to the times.

KEYSTONE of an *Arch*, the middle voussoir, or that immediately over the centre of the arch.

L.

LAMINÆ, in *Physics*, are extremely thin plates, of which solid bodies are supposed to be made up. These are indeed rather ideal than real; but such a confirmation is frequently supposed for the sake of simplifying the solution in a great variety of physical problems.

LARBOARD, the left-hand side of a ship when a person stands on board with his face towards the head of the vessel.

LATITUDE, in *Geography* or *Navigation*, is the distance of a place from the equator, reckoned on an arch of the meridian, intercepted between its zenith and the equator.

North **LATITUDE**, is that which falls in the northern hemisphere;

viz. between the equator and the north pole.

South LATITUDE, is that which falls in the southern hemisphere, or between the equator and south pole.

Parallels of LATITUDE, are small circles of the sphere supposed parallel to the equator; and are thus called because they show the latitude of places by their intersection with the meridians, all places falling under the same circle being in the same latitude.

The quadrant, or meridian, intercepted between the equator and either pole is divided into ninety degrees, and numbered both ways from the equator to the poles; and the latitude of any place is equal to the measure of the arch intercepted between the equator and that place, and which is said to be north or south, according as it is situated towards the north or south pole.

The latitude of a place, and the elevation of the pole above the horizon of that place, are terms frequently used indifferently the one for the other, being in fact equal to each other.

The knowledge of the latitude of places is of the greatest importance in geography, navigation, and astronomy; it may, therefore, be proper to state some of the best methods of determining it both by sea and land; one of these is by finding the elevation of the pole, which is always equal to the latitude of the place, as shown above, and this is done by observations on the pole-star or any other circumpolar stars, thus:

Either draw a true meridian line, or find the time when the star is on the meridian both above and below the pole, then at these times, with a quadrant or other instrument, take the altitude of the star, or take the same when the star comes upon your meridian line, which will be the greatest and least altitude of the star, half the sum of which will be the true elevation of the pole, or the latitude of the place.

A second method is by means of the declination of the sun, or a star, and one meridian altitude of the same, thus:

Having with a quadrant or other instrument observed the zenith distance of the luminary, or else its altitude, and taken its complement; then to this zenith distance add the declination when the luminary and place are on the same side of the equator, or subtract it when on different sides, and the sum or difference will be the latitude sought. But note, that all altitudes observed must be corrected for refraction and the dip of the horizon, and for the semidiameter of the sun, if that is the luminary observed.

LATITUDE, in *Astronomy*, as of a star or planet, is its distance from the ecliptic, being an arch of latitude of a circle of latitude, reckoned from the ecliptic towards the poles, either north or south. Hence, the astronomical latitude is quite different from the geographical, the former measuring from the ecliptic, and the latter from the equator, so that this latter answers to the declination in astronomy, which measures from the equinoctial. The sun has no latitude, being always in the ecliptic; but all the stars have their several latitudes, and the planets are continually changing their latitudes, sometimes north, and sometimes south, crossing the ecliptic from the one side to the other; the points, in which they cross the ecliptic, being called the nodes of the planet, and in these points it is that they can pass over the face of the sun or behind his body, viz. when they come both to this point of the ecliptic at the same time.

Circle of LATITUDE, is a great circle passing through the poles of the ecliptic, and consequently perpendicular to it, in the same manner as the meridians are perpendicular to the equator, and pass through its poles.

LATITUDE of the *Moon, North ascending*, is when she proceeds from the ascending node towards her northern limit, or greatest elongation. *North descending*, is when the moon returns from her northern limits towards the descending node. *South descending*, is when she proceeds from the descending node towards her southern limit. *South ascending*, is when she returns from her southern limit towards her ascending node. And the same is to be understood of the other planets.

Heliocentric LATITUDE, of a *Planet*, is its latitude or distance from the ecliptic, as it would appear from the sun.

Geocentric LATITUDE of a *Planet*, is its latitude as seen from the earth.

The latitude of a star is altered only by the aberration of light, and the secular variation of latitude.

Difference of LATITUDE, is an arc of the meridian between the parallels of latitude of two places.

To find the difference of latitude. When the two latitudes are of the same name, either both north or both south, subtract the less latitude from the greater, to give the difference of latitude; but if they are of different names, then their sum will be the difference of latitude.

Middle LATITUDE, is the middle point between two latitudes or places, and is found by taking half the sum of the two.

LATUS *Rectum*, in the *Conic Sections*, is the same as *Parameter*.

LATUS *Transversum*, in the *Hyperbola*, the same as transverse axis.

LATUS *Primarium*, is the diameter of the cone at the vertex of any conic section.

LEAGUE, a measure of three miles; a nautical league is equal to a twentieth part of a degree; other leagues vary in the same proportion, as the miles by which they are computed.

LEE, in *Navigation*, the quarter towards which the wind blows; thus, if the wind is east, that is blowing towards the west, then the west is leeward.

LEEWAY of a *Ship*, is the angle made by the point of the compass steered upon, and the real line of the ship's way occasioned by contrary winds, &c.

LEIBNITZIAN *Philosophy*, is a system formed and published by its author in the last century, partly in emendation of the Cartesian, and partly in opposition to the Newtonian philosophy. In this philosophy the author retained the Cartesian subtile matter, with the vortices and universal plenum; and he represented the universe as a machine that should proceed for ever, by the laws of mechanism, in the most perfect state, by an absolute inviolable necessity. After Newton's philosophy was published in 1687, Leibnitz printed an Essay on the Celestial Motions in the Act. Erud. 1689, where he admits the circulation of the ether with Des Cartes, and of gravity with Newton; though he has not reconciled these principles, nor shown how gravity arose from the impulse of this ether, nor how to account for the planetary revolutions in their respective orbits. His system is also defective, as it does not reconcile the circulation of the ether with the free motions of the comets in all directions, or with the obliquity of the planes of the planetary orbits, nor does he resolve other objections to which the hypothesis of the vortices and plenum is liable.

Soon after the period just mentioned, the dispute commenced concerning the invention of the method of fluxions, which led Mr. Leibnitz to take a very decided part in opposition to the philosophy of Newton. From the goodness and wisdom of the Deity, and his principle of a sufficient reason, he concluded, that the universe was a perfect work, or the best that could possibly have been made; and that other things, which are evil or incommodious, were permitted as necessary consequences of what was best: that the material system, considered as a perfect machine, can never fall into disorder or require to be set right; and to suppose that God interposes in it, is to lessen the skill of the author, and the perfection of his work. He expressly charges an impious tendency on the philosophy of Newton, because he asserts that the fabric of the universe and course of nature could not continue for ever in its present state, but in process of time would require to be re-established or renewed by the hand of its first framer. The perfection of the universe, in consequence of which it is capable of continuing for ever, by mechanical laws, in its present state, led Mr. Leibnitz to distinguish between the quantity of motion and the force of bodies; and whilst he owns, in opposition to Des Cartes, that the former varies, to maintain that the quantity of force is for ever the same in the universe; and to measure the forces of bodies by the squares of their velocities.

Leibnitz proposes two principles as the foundation of all our knowledge; the first, that it is impossible for a thing to be, and not to be at the same time, which, he says, is the foundation of speculative truth; and, secondly, that nothing is without a sufficient reason why it should be so, rather than otherwise; and by this principle, he says, we make a transition from abstracted truths to natural philosophy. Hence he concludes that the mind is naturally determined, in its volitions and elections, by the greatest apparent good, and that it is impossible to make choice between things perfectly like, which he calls indiscernibles; from whence he infers, that two things perfectly like could not have been produced even by the Deity himself: and one reason why he rejects a vacuum, is because the parts of it must be supposed perfectly like to each other. For the same reason, too, he rejects atoms, and all similar parts of matter; to each of which, though divisible *ad infinitum*, he ascribes a *monad* (Act Lipsic 1698, p. 435) or active kind of principle, endued with perception. The essence of substance he places in action or activity, or, as he expresses it, in something that is between acting and the faculty of acting. He affirms that absolute rest is impossible, and holds that motion, or a sort of *nisus*, is essential to all material substances. Each monad he describes as representative of the whole universe from its point of sight; and yet he tells us in one of his letters, that matter is not a substance, but a *substantiatum* or *phenomene bien fonde*.

LEMMA, in *Mathematics*, a previous proposition, laid down in order to clear the way for some following demonstration, and prefixed either to theorems in order to render their demonstration less perplexed and intricate, or to problems to make their resolution more easy and short. Thus, to prove a pyramid one third of a prism, or parallelopiped of the same base and height with it, the demonstration of which, in the ordinary way, is difficult and troublesome, this lemma may be premised, which is proved in the rules of progression, viz. that the sum of a series of square numbers in arithmetical progression, beginning from 0, as 1, 4, 9, 16, 25, 36, &c. is always subtriple of the sum of as many terms, each equal to the greatest; or is always one-third of the greatest term multiplied by the number of terms.

LEMNISCATE, in the *Higher Geometry*, the name of a curve in the form of a figure of 8.

LENS, a piece of glass or other transparent substance, having its two surfaces so formed that the rays of light in passing through it have their direction changed, and made to converge or diverge from their original parallelism, or to become parallel after converging or diverging. Lenses receive particular denominations according to their form; as *convex*, *concave*, *plano convex*, *plano concave* lenses, and *meniscuses*.

A *Convex* Lens, is one which is thickest in the middle. If only one side is convex and the other plane, it is called a *plano convex lens*, but if it be convex on both sides, it is called a *convexo convex*, or *double convex lens*. *Concave lens*, is that which is thinnest in the middle; but it is also divided into *plano concave* and *concavo concave*, as in the former case. And when the lens is concave on one side, and convex on the other, it is called a *meniscus*.

In every lens the right line perpendicular to the two surfaces, is called the *axis* of the lens; the points where the axis cuts the surface, are called the *vertices* of the lens; also the middle point between them is called the *centre*, and the distance between them the *diameter*.

Lenses are either blown or ground. *Blown Lenses*, are small globules of glass, melted in the flame of a lamp by a blow-pipe, or otherwise. *Ground Lenses*, are such as are ground to the required form by means of machinery for this purpose.

Optical Properties of Lenses.—Parallel rays refracted at a convex spherical surface of a denser, or a concave of a rarer medium, into which they pass, are made to converge. But if refracted at a concave spherical surface of a denser, or convex of a rarer medium, they are made to diverge. When the ray passes out of a denser medium into a rarer, and the surface of the medium into which they are refracted is spherically concave, the rays still converge as before.

Hence, if rays are made to traverse a double convex lens, they must converge more rapidly than through a plano convex one, for they are first refracted at the convex surface of a denser medium on entering the lens, and are again refracted at the concave surface of the rarer medium in passing again into the air, and must, therefore, have a double degree of convergency.

By a similar reasoning, it may be shown, that rays in passing out of a rarer medium into a denser, the surface of that into which they are refracted being spherically concave, or in passing from a denser medium into a rarer, the surface of that into which they are refracted being spherically convex, will, in both cases, be made to diverge. Hence, again, if rays are made to traverse a double concave lens, they must diverge more rapidly than in passing through a plano concave lens, for in this case they are first refracted at the concave surface of a denser medium on entering the lens, and are again refracted at the convex surface of a rarer medium, in passing again into the air, and must, therefore, have a double degree of convergency.

To find the Foci of Lenses.—The focus of a convex spherical lens, is distant from the vertex a diameter and a half of the convexity nearly. In a plano convex lens, the distance of the focus from the vertex is equal to a diameter of the convexity if the segment do not exceed 30°. Or the general rule is, as 107 : 193 = the radius of convexity : the refracted ray, taken to its concourse with the axis. In double convex glasses of the same sphere, the focus is distant from the vertex about the diameter, if the segment do not exceed 30°. But when the two convexities are unequal; that is, when they are segments of different spheres, then the rule is, as the sum of the radii of both convexities: the radius of either convexity alone = the radius of the other convexity : the distance of the focus from the vertex.

It must be observed, however, that the rays which fall near the axis of any lens, are not united so near the vertex as those which fall farther off; nor will the distance be so great in a plano convex lens, when the convex side is towards the object, as when the plane side is towards it. And hence it follows, that in viewing any object with a plano convex lens the convex side should always be turned outwards; and the same remark has place in burning with such a glass.

LEVEL, an instrument employed in ascertaining a horizontal line, of which there are various sorts; as the

Air LEVEL, which shews the line of level by means of a bubble of air inclosed with some fluid in a glass tube of an indeterminate length and thickness, and having its two ends hermetically sealed. When the bubble fixes itself at a certain mark, made exactly in the middle of the tube, the case or ruler in which it is fixed is then level. When it is not level the bubble will rise to one end. This glass tube may be set in another of brass, having an aperture in the middle, where the bubble of air may be observed. The liquor, with which the tube is filled, is oil of tartar, that not being so liable to freeze as common water, or so subject to rarefaction and condensation as spirit of wine.

Plumb Level, shows the horizontal line by means of a line perpendicular to that described by a plummet or pendulum. This instrument consists of two legs or branches, joined together at right angles, whereof that which carries the thread and plummet is about a foot and a half long; the thread is hung towards the top of the branch. The middle of the branch where the thread passes is hollow, so that it may hang free every where: but towards the bottom, where there is a little blade of silver, whereon is drawn a line perpendicular to the telescope, the said cavity is covered by two pieces o brass, making, as it were, a kind of case, lest the wind should agitate the thread; for which reason the silver blade is covered with a glass, to the end that it may be seen when the thread and plummet play upon the perpendicular. The telescope is fastened to the other branch of the instrument, and is about two feet long; having a hair placed horizontally across the focus of the object-glass, which determines the point of the level. The telescope must be fitted at right angles to the perpendicular. It has a ball and socket, by which it is fastened to the foot.

Water Level, that which shows the horizontal line by means of a surface of water or any other fluid; founded on this principle, that water always places itself level or horizontal. The most simple kind is made of a long wooden trough or canal; which being equally filled with water, its surface shows the line of level. The water-level is also made with two cups fitted to the two ends of a straight pipe, about an inch diameter, and three or four feet long, by means of which the water communicates from the one cup to the other; and this pipe being moveable on its stand, by means of a ball and socket, when the two cups show equally full of water, their two surfaces mark the line of level. This instrument, instead of cups, may also be made with two short cylinders of glass, three or four inches [illegible] fastened to each extremity of the pipe with wax or mastic. The pipe is filled with common or coloured water, which shows itself through the cylinders, by means of which the line of level is determined; the height of the water, with respect to the centre of the earth, being always the same in both cylinders. This level, though very simple, is yet very commodious for levelling small distances.

Where works of moderate extent are carried on, and where the perfect level of each stratum of materials is not an object of importance, the common bricklayer's level, made thus, ⊥, having a plumb suspended from the top, and received in an opening at the junction of the perpendicular with the horizontal piece, will answer well enough. The principle on which this acts is, that as all weights have a tendency to gravitate towards the centre of the earth, so as the plumb-line is a true perpendicular any line cutting that at right angles must be a horizontal line at the point of intersection.

But the most complete level that has ever yet been invented, is the spirit-level of the late Mr. Ramsden, and improved by Troughton. The level is surmounted by a telescope, and the whole fixed on a stand resembling that of the improved theodolite, the adjustment of the instrument being effected by means of the screws, and the error being taken by a very fine scale attached to the tube. We have had occasion to see a level of this kind employed in levelling twenty miles along the bank of a navigable river, across and back again; and notwithstanding the number of observations, and the effect of refraction, the last observation was but an inch or two different from the first.

LEVELLING, the finding a line parallel to the horizon at one or more stations, to determine the height or depth of one place with respect to another, for laying out grounds even, regulating descents, draining morasses, conducting water, &c.

Two or more places are on a true level when they are equally distant from the centre of the earth.

Also one place is higher than another, or out of level with it, when it is farther from the centre of the earth; and a line equally distant from that centre in all its points, is called the line of true level. Hence, because the earth is round, the line must be a curve, and make a part of the earth's circumference, or at least be parallel to it, or concentrical with it.

The line of sight given by the operations of levels, is a tangent, or a right line perpendicular to the semi-diameter of the earth at the point of contact, rising always higher above the true line of level, the farther the distance is, is called the apparent line of level. The difference, it is evident, is always equal to the excess of the secant of the arch of distance above the radius of the earth.

The common methods of levelling are sufficient for laying pavements of walks, or for conveying water to small distances, &c.; but in more extensive operations, as in levelling the bottoms of canals, which are to convey water to the distance of many miles, and such like, the difference between the true and the apparent level must be taken into the account.

Now the difference between the true and apparent level, at any distance, may be found by a well-known property of the circle, to be equal to the square of the distance between the places, divided by the diameter of the earth; and consequently it is always proportional to the square of the distance.

Now the diameter of the earth being nearly 7958 miles, if we first take the distance = 1 mile, then the excess becomes 7.962 inches, or almost eight inches, the height of the apparent above the true level at the distance of one mile. Hence, proportioning the excesses in altitude according to the squares of the distances, the following table is obtained, showing the height of the apparent above the true level for every 100 yards of distance on the one hand, and for every mile on the other.

Distance.	Dif. of Level.	Distance.	Dif. of Level.	
Yards.	Inches.	Miles.	Ft.	In.
100	0.026	¼	0	0½
200	0.103	½	0	2
300	0 231	¾	0	4½
400	0.411	1	0	8
500	0.643	2	2	8
600	0 925	3	6	0
700	1.260	4	10	7
800	1.645	5	16	7
900	2.081	6	23	11
1000	2.570	7	32	6
1100	3.110	8	42	6
1200	3.701	9	53	9
1300	4.344	10	66	4
1400	5.038	11	80	3
1500	5.784	12	95	7
1600	6 580	13	112	2
1700	7.425	14	130	1

By means of this table of reductions, we can now level to almost any distance at one operation, which the ancients could not do but by a great multitude; for, being unacquainted with the correction answering to any distance, they only levelled from one twenty yards to another, when they had occasion to continue the work to some considerable extent.

This table will answer several useful purposes. Thus, first to find the height of the apparent level above the true, at any distance. If the given distance is in the table, the correction of level is found on the same line with it. Secondly, To find the extent of the visible horizon, or how far can be seen from any given height, on an horizontal plane, as at sea, &c. Suppose the eye of an observer, on the top of a ship's mast at sea, is at the height of 130 feet above the water, he will then see about 14 miles all around. Or from the top of a cliff by the sea-side, the height of which is 66 feet, a person may see to the distance of near 10 miles on the surface of the sea. Also, when the top of a hill or the light in a light-house, or such like, whose height is 130 feet, first comes into the view of an eye on board a ship, the table shows that the distance of the ship from it is 14 miles, if the eye is at the surface of the water; but if the height of the eye in the ship is 80 feet, then the distance will be increased by near 11 miles, making, in all, about 25 miles in distance. Thirdly, Suppose a spring to be on one side of a hill, and the house on an opposite hill, with a valley between them, and that the spring seen from the house appears by a levelling instrument to be on a level with the foundation of the house, which suppose is at a mile distance from it, then is the spring eight inches above the true level of the house; and this difference would be barely sufficient for the water to be brought in pipes from the spring to the house, the pipes being laid all the way in the ground. Fourthly, If the height or distance exceed the limits of the table, then, first, if the distance be given, divide it by 2, or by 3, or by 4, &c. till the quotient come within the distances in the table; then take out the height answering to the quotient, and multiply it by the square of the divisor, that is, by 4, or 9, or 16, &c. for the height required.

All that has been previously stated has been said without any regard to the effect of refraction in elevating the apparent places of objects. But as the operation of refraction in incurvating the rays of light proceeding from objects near the horizon is very considerable, it can by no means be neglected, when the difference between the true and apparent level is estimated at considerable distances. It is now ascertained that, for horizontal refractions, the radius of curvature of the curve of refraction is about seven times the radius of the earth; in consequence of this, the distance at which an object can be seen by refraction, is to the distance at which it could be seen without refraction, nearly as 14 to 13. By reason of this refraction too, it happens, that it is necessary to diminish by $\frac{1}{7}$ of itself, the height of the apparent above the true level, as given in the preceding table of reductions.

To find the height H of a mountain, its angle of apparent elevation E, the arc A of a great circle of the earth included between the foot of the mountain and the place of the observer, and the apparent angle C made at the top of the mountain between the plumb-line and the apparent first place of the observer on the earth's surface, M. Lambert gave this theorem, R being the radius of the earth:

$$R + H = \frac{R \sin. (90 + E - \frac{1}{14} A)}{\sin. (C - \frac{1}{14} A)};$$

whence H is immediately found.

Levelling is either simple or compound; the former is when the level points are determined from one station, whether the level be fixed at one of the points or between them; and the latter, or compound levelling, is nothing more than a repetition of several such simple operations.

For the practical operations in levelling, the reader is referred to the treatises on this subject by De Lahire, Picard, and Lefebvre.

Levelling *Staves*, instruments used in levelling, serving to carry the marks to be observed, and at the same time to measure the height of those marks from the ground. They usually consist of two mahogany staves, ten feet long, in two parts, that slide upon one another to about $5\frac{1}{10}$ feet, for the greater convenience of carriage. They are divided in 1000 equal parts, and numbered at every tenth division by 10, 20, 30, &c. to 1000; and on one side the feet and inches are also sometimes marked. A vane slides up and down upon each set of these staves, which by brass springs will stand at any part. These vanes are about ten inches long and four inches broad; the breadth is first divided into three equal parts, the two extremes are painted white, the middle space divided again into three equal parts, which are less; the middle one of them is also painted white, and the two other parts black; and thus they are suited to all the common distances. These vanes have each a brass wire across a small square hole in the centre, which serve to point out the height correctly, by coinciding with the

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Lever I.

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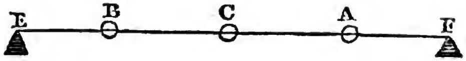

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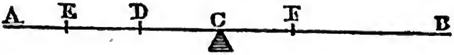

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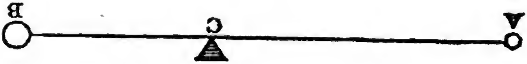

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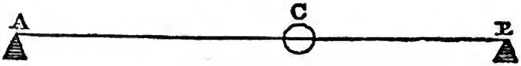

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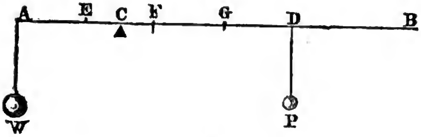

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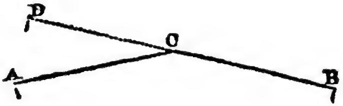

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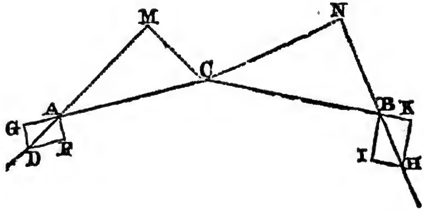

horizontal wire of the telescope of the level.

LEVER, a straight bar of iron, wood, &c. supposed to be inflexible, supported on a fulcrum or prop, about which all the parts are moveable. It is generally considered as the first of the mechanical powers, being the simplest of them all. Its principal use is in raising great weights to small heights, or in moving heavy blocks of stone, &c. to short distances.

The power and weight are supposed to act at right angles to the lever when horizontal, and to turn it in opposite directions.

When two weights balance each other upon a straight lever, the pressure upon the fulcrum is said to be equal to the sum of the weights (including that of the lever.) If there be more than one fulcrum, the pressure upon all the fulcrums is equal to the weight. Equal forces acting perpendicularly upon the equal arms of a straight lever, exert an equal effect.

If two equal weights act together upon a straight lever, the effort to put it in motion, round any fulcrum, will be the same as if they acted together at the middle point between them.

Let A and B be two equal weights, acting perpendicularly upon the lever F B, whose fulcrum is F (*Fig*. 1. *plate* Lever.) Bisect A B in C, make C E = C F, and at E suppose another fulcrum to be placed. Then, since the two weights A and B are supported by E and F, and these fulcrums are similarly situated with respect to the weights, each sustains an equal pressure; and, therefore, the weight sustained by E is equal to half the sum of the weights. Now let the weights A and B be placed at C, the middle point between A and B, and consequently the middle point between E and F; then, since E and F support the whole weight C, and are similarly situated with respect to it, the fulcrum E supports half the weight; that is, the pressure upon E is the same, whether the weights are placed at A and B, or collected in C, the middle point between them; and, therefore, the effort to put the lever in motion round F, is the same on either supposition.

If a weight be formed into a cylinder A B (*Fig*. 2.) which is every where of the same density, and placed parallel to the horizon, the effort of any part A D, to put the whole in motion round C, is the same as if this part were collected at E, the middle point of A D. For the weight A D may be supposed to consist of pairs of equal weights, equally distant from the middle point. What is here affirmed of weights, is true of any forces which are proportional to the weights, and act in the same directions.

Two weights, or two forces, acting perpendicularly upon a straight lever, will balance each other, when they are reciprocally proportional to their distances from the fulcrum.

1. When the weights act on *contrary* sides of the fulcrum. Let x and y be the two weights, and let them be formed into the cylinder A B, (*Fig*. 3.) which is every where of the same density. Bisect A B in C, then this cylinder will balance itself upon the fulcrum C. Divide A B into two parts in D, so that $AD : DB = x : y$; and the weights of A D and D B will be respectively x and y; bisect A D in E and D B in F; then, since A D and D B keep the lever at rest, they will keep it at rest when they are collected at E and F; that is, x, when placed at E, will balance y, when placed at F; and $x : y = AD : BD = \frac{AD}{2} : \frac{BD}{2} = \frac{AB - BD}{2} : \frac{AB - AD}{2} = CB - BF : AC - AE = CF : CE$.

2. When the two forces act on the *same* side of the centre of motion. Let A B, (fig. 4.) be a lever whose fulcrum is C, A and B two weights acting perpendicularly upon it; and let $A : B = BC : AC$, then these weights will balance each other, as appears by the former case. Now suppose a power sufficient to sustain a weight equal to the sum of the weights A and B, to be applied at C, in a direction opposite to that in which the weights act; then will this power supply

the place of the fulcrum; and the centre of motion may be conceived to be at A or B. Let B be the centre of motion; then we have a straight lever whose centre of motion is B, and the two forces A and A + B, acting perpendicularly upon it at the points A and C, sustain each other; also, A : B = B C : A C; therefore A : A + B = B C : B A. Hence,

1. If two weights, or two forces, acting perpendicularly on the arms of a straight lever, keep each other in equilibrio, they are inversely as their distances from the centre of motion. For the weights will balance when they are in that proportion, and if the proportion be altered by increasing or diminishing one of the weights, its effort to turn the lever round will be altered, or the equilibrium will be destroyed.

2. Since A : B = B C : A C when there is an equilibrium upon the lever A B, whose fulcrum is C, by multiplying extremes and means, A × A C = B × B C.

3. When the power and weight act on the same side of the fulcrum, and keep each other in equilibrio, the weight sustained by the fulcrum is equal to the difference between the power and the weight.

4. In the *common balance*, the arms of the lever are equal; consequently, the power and weight, or two weights, which sustain each other, are equal. In the false balance, one arm is longer than the other; therefore the weight, which is suspended at this arm, is proportionally less than the weight which it sustains at the other.

5. If the same body be weighed at the two ends of a false balance, its true weight is a mean proportional between the apparent weights. Call the true weight x, and the apparent weights, when it is suspended at A and B, a and b respectively; then $a : x =$ A C : B C, and $x : b =$ A C : B C; therefore $a : x = x : b$.

6. If a weight C, (fig. 18.) be placed upon a lever which is supported upon two props A and B in an horizontal position, the pressure upon A : the pressure upon B = B C : A C. For if B be conceived to be the fulcrum, we have this proportion, the weight sustained by A : the weight C = B C : A B; in the same manner, if A be considered as the fulcrum, then the weight C : the weight sustained by B = A B : C A; therefore, ex æquo, the weight sustained by A : the weight sustained by B = B C : A C.

7. If a given weight P, (fig. 5.) be moved along the graduated arm of a straight lever, the weight W, which it will balance at A, is proportional to C D, the distance at which the given weight acts. When there is an equilibrium, W × A C = P × D C; and A C and P are invariable; therefore $W \propto DC$.

If two forces, acting upon the arms of any lever, keep it at rest, they are to each other inversely as the perpendiculars drawn from the centre of motion to the directions in which the forces act.

1. Let two forces, A and B, (fig. 6.) act *perpendicularly* upon the arms C A, C B, of the lever A C B whose fulcrum is C, and keep each other at rest. Produce B C to D, and make C D = C A; then the effort of A to move the lever round C, will be the same, whether it be supposed to act perpendicularly at the extremity of the arm C A, or C D; and on the latter supposition, since there is an equilibrium, A : B = C B : C D; therefore A : B = C B : C A.

2. When the directions A D, B H, (fig. 7.) in which the forces act, are not perpendicular to the arms, take A D and B H, to represent forces; draw C M and C N at right angles to those directions; also draw A F perpendicular, and D F parallel to A C, and complete the parallelogram G F; then the force A D is equivalent to the two A F, A G, of which A G acts in the direction of the arm, and therefore can have no effect in causing, or preventing any angular motion in the lever about C. Let B H be resolved, in the same manner, into the two B I, B K, of which B I is perpendicular to, and B K in the direction of the arm C B; then B K will have no effect in causing, or preventing any angular motion in the lever about C; and since the lever

Lever II.

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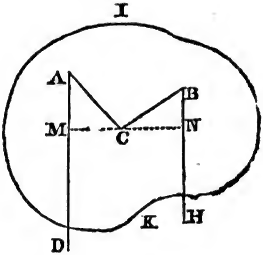

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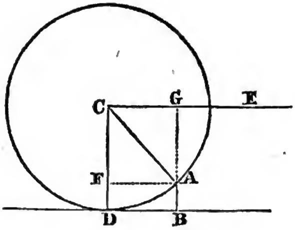

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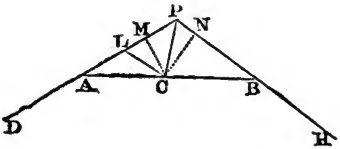

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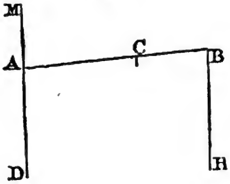

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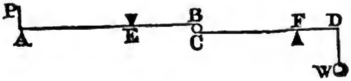

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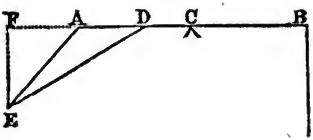

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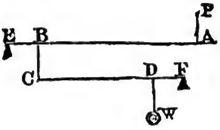

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is kept at rest, A F and B I, which produce this effect, and act perpendicularly upon the arms, are to each other, by the first case, inversely as the arms; that is, $AF : BI = CB : CA$, or $AF \times CA = BI \times CB$. Also, in the similar triangles A D F, A C M, $AF : AD = CM : CA$, and $AF \times CA = AD \times CM$; in the same manner, $BI \times CB = BH \times CN$; therefore $AD \times CM = BH \times CN$, and $AD : BH = CN : CM$. Hence,

1. Let a body I K, (fig. 8.) be moveable about the centre C, and two forces act upon it at A and B, in the directions A D, B H, which coincide with the plane A C B; join A C, C B; then this body may be considered as a lever A C B, and drawing the perpendiculars C M, C N, there will be an equilibrium, when the force acting at A : the force acting at B $= CN : CM$.

2. The effort of the force A, to turn the lever round, is the same, at whatever point in the direction M D it is applied; because the perpendicular C M remains the same.

3. Since $CA : CM$ rad. : sin. C A M, $CM = \frac{CA \times \text{sin. } CAM}{\text{rad.}}$, and, in the same manner, $CN = \frac{CB \times \text{sin. } CBN}{\text{rad.}}$; therefore, when there is an equilibrium, the power at A : the weight at B $= \frac{CB \times \text{sin. } CBN}{\text{rad.}} : \frac{CA \times \text{sin. } CAM}{\text{rad.}} = CB \times \text{sin. } CBN : CA \times \text{sin. } CAM$.

4. If the lever A B C (fig. 9.) be straight, and the directions A D, B H, parallel, $A : B = BC : AC$; because, in this case, sin. C A M = sin. C B H.

Hence, also, $A \times AC = B \times BC$.

5. If two weights balance each other upon a straight lever in any one position; they will balance each other in any other position of the lever; for the weights act in parallel directions, and the arms of the lever are invariable.

6. If a man balanced in a common pair of scales, press upwards by means of a rod, against any point in the beam, except that from which the scale is suspended, he will preponderate. Let the action upwards take place at D, (fig. 10.) then the scale, by the re-action downwards, will be brought into the situation E; and the effect will be the same as if D A, A E, D E constituted one mass; that is, drawing E F perpendicular to C A produced, as if the scale were applied at F; consequently the weight, necessary to maintain the equilibrium, is greater than if the scale were suffered to hang freely from A, in the proportion of $CF : CA$.

7. Let A D (fig. 11.) represent a wheel, bearing a weight at its centre C; A B an obstacle over which it is to be moved by a force acting in the direction C E; join C A, draw C D perpendicular to the horizon, and from A draw A G, A F, at right angles to C E, C D. Then C A may be considered as a lever whose centre of motion is A, C D the direction in which the weight acts, and C E the direction in which the power is applied; and there is an equilibrium on this lever when the power : the weight $= AF : AG$. Supposing the wheel, the weight, and the obstacle given, the power is the least when A G is the greatest; that is, when C E is perpendicular to C A, or parallel to the tangent at A.

8. Let two forces acting in the directions A D, B H, (fig. 12.) upon the arms of the lever A C B, keep each other in equilibrio; produce D A and H B till they meet in P; join C P, and draw C L parallel to P B; then will P L, L C represent the two forces, and P C the pressure upon the fulcrum.

For, if P C be made the radius, C M and C N are the sines of the angles C P M, C P N, or C P L, P C L; and $PL : LC =$ sin. P C L : sin. L P C $= CN : CM$; therefore P L, L C, represent the quantities and directions of the two forces, which may be supposed to be applied at P, and which are sustained by the re-action of the fulcrum; consequently, C P represents the quantity and direction of that re-action, or P C represents the pressure upon the fulcrum.

9. In a combination of straight levers, A B, C D, (figs. 13, 14) whose centres of motion are E and

F, if they act perpendicularly upon each other, and the directions in which the power and weight are applied be also perpendicular to the arms, there is an equilibrium when $P : W = EB \times FD : EA \times FC$. For the power at A : the weight at B, or $C = EB : EA$; and the weight at C : the weight at $D = FD : FC$; therefore, $P : W = EB \times FD : EA \times FC$.

By the same method we may find the proportion between the power and the weight, when there is an equilibrium, in any other combination of levers.

LEVITY, in *Philosophy*, the opposite to gravity, or that supposed quality of certain bodies which gives them a power of ascent, being thus opposed to gravity, by which they have always a tendency to descend.

LEYDEN *Phial*, in *Electricity*, is a glass phial or jar, coated both within and without with tin-foil, or some other conducting substance, which may be charged and employed in a variety of useful and entertaining experiments.

LIBRA, the *Balance*, one of the zodiacal constellations, denoted by the character ♎.

LIBRA also denotes the ancient Roman pound, which was equal to about 5040 of our Troy grains.

LIBRATION of the *Moon*, an apparent irregular motion of that body about her own axis, whereby we see a little more than one-half of the lunar disc; or rather, it is in consequence of our seeing more than one-half of it, that the moon appears to have such a motion; for although the term libration, agrees perfectly well with the appearances observed, still it must not be understood in a positive sense, the appearance itself arising from a totally different cause from that which the word seems to indicate. In order to illustrate this, let us conceive a visual ray drawn from the centre of the earth to the centre of the moon: the plane drawn through the latter centre perpendicularly to this ray, will cut the lunar globe according to the circumference of a circle which is, with respect to us, the apparent disc. If the moon had no real rotatory motion, its motion of revolution solely would discover to us all the points of its surface in succession: the visual ray would therefore meet that surface successively in different points, which to us would appear to pass, the one after the other, to the apparent centre of the lunar disc. The real rotatory motion counteracts the effects of this apparent rotation, and brings back constantly towards us the same face of the lunar globe. Suppose, now, that the rotation of the moon is sensibly uniform; that is to say, that it does not partake of any periodical inequalities (this supposition is at least the most natural which we can make, and it is conformable to observations); then one of the causes which produce the libration will become evident; for the motion of revolution partaking of the periodical inequalities, is sometimes slower, sometimes more rapid: the apparent rotation which it occasions cannot, therefore, always exactly counterbalance the actual rotation, which remains constantly the same; and these two effects will surpass each other by turns. The points of the lunar globe ought, therefore, to appear turning sometimes in one direction, sometimes in another, about its centre; and the resulting appearance is the same as if the moon had a little vibratory balancing from one side to the other of the radius vector drawn from its centre to the earth. It is this which is named the libration in longitude.

LIBRATION of the *Earth*, is a term applied by some astronomers to that motion, whereby the earth is so retained within its orbit, as that its axis continues constantly parallel to the axis of the world.

LIFE *Annuities*, periodical payments which depend on the continuance of some particular life or lives; and they may be distinguished into lives, to commence immediately; and annuities, to commence at some future period, called reversionary annuities.

The first writer who attempted the developement of a rational theory of life annuities was Van Hudden, which was farther extended by De Witt, the celebrated Dutch pensionary. In 1692, Dr. Halley gave in the Phil. Trans. "An Estimate of the Degrees of Mortality of Mankind, drawn from curious tables of the births and funerals at the city of Breslaw." De Moivre took up the subject where it was left by Halley, and, in 1724, published his "Annuities on Lives," founded on the principle of an equal decrement of life. In 1742, T. Simpson published his tract "On the Doctrines of Annuities and Reversions;" in which the tables were computed from the absolute observations on the bills of mortality. The science remained in this state, without much improvement, till the publication of Dr. Price's treatise, in 1769, entitled "Observations on Reversionary Payments," &c. Dr. Price saw the necessity of more accurate observations on the mortality of human life; in order to determine with more correctness the value of annuities, and to show more forcibly the futility and extravagance of the schemes that were issued by those societies. And his fourth edition appeared (1783) enriched with several valuable tables of annuities on single and joint lives, at different rates of interest, deduced not only from the probabilities of living as observed at Northampton, but also from the probabilities of living, as observed in the kingdom of Sweden at large.

The value or present worth of an annuity for any proposed life or lives, it is evident, depends on two circumstances; the interest of money, and the chance or expectation of the continuance of life. Upon the former only depends the value or present worth of an annuity certain, or that which is not subject to the continuance of a life or other contingency; but the expectation of life being a thing not certain, but only possessing a certain *chance*, it is evident that the value of the certain annuity, as stated above, must be diminished in proportion as the expectancy is below certainty: thus, if the present value of an annuity certain be any sum, as suppose £100, and the value and expectancy of the life be one-half, then the value of the life annuity will be only half of the former, or £50; and if the value of the life be only one-third, the value of the life annuity will be but one-third of £100; that is, £33 6*s.* 8*d.*; and so on. The measure of the value or expectancy of life, depends on the proportion of the number of persons that die, out of a given number, in the time proposed, thus, if fifty persons die out of one hundred, in any proposed time, then, half the number only remaining alive, any one person has an equal chance to live or die in that time, or the value of his life for that time is one-half; but if two-thirds of the number die in the time proposed, or only one-third remain alive, then the value of any life is one-third; and if three-quarters of the number die, or only one-quarter remain alive, then the value of any life is but one-quarter; and so on. In these proportions, then, must the value of the annuity certain be diminished, to give the value of the like life annuity. It is plain, therefore, that in this business it is necessary to know the value of life at all the different ages, from some table of observations on the mortality of mankind, which may show the proportion of the persons living, out of a given number, at the end of any proposed time; or from some certain hypothesis, or assumed principle. Now various tables and hypotheses of this sort were given by the writers on this subject: but the same table of probabilities of life will not suit all places; for long experience has shown that all places are not equally healthy, or that the proportion of the number of persons that die annually is different for different places. It may not, therefore, be improper to insert here a comparative view of two of the principal tables that have been given of this kind, where the first column shows the age, and the other columns the number of persons living at that age, out of 1003 born, or of the age 0, in the first line of each column.

TABLE of the Decrease of Life, at all Ages, from 1 Year to 79, as deduced from the Bills of Mortality in London and Northampton.

Ages.	London.	Northampton.	Ages.	London.	Northampton.	Ages.	London.	Northampton.	Ages.	London.	Northampton.
0	1000	1000	20	325	447	40	214	317	60	96	176
1	680	738	21	321	440	41	207	310	61	92	169
2	546	628	22	316	433	42	201	303	62	87	162
3	492	585	23	310	426	43	194	296	63	83	155
4	452	562	24	305	419	44	187	289	64	78	148
5	426	544	25	299	412	45	180	282	65	74	141
6	410	530	26	294	405	46	174	275	66	70	134
7	397	518	27	288	398	47	167	268	67	65	127
8	388	510	28	283	391	48	159	261	68	61	120
9	380	504	29	278	384	49	153	254	69	56	113
10	373	498	30	272	378	50	147	247	70	52	106
11	367	493	31	266	372	51	141	239	71	47	99
12	361	488	32	260	366	52	135	232	72	43	92
13	356	481	33	254	360	53	130	225	73	39	85
14	351	480	34	248	354	54	125	218	74	35	78
15	347	475	35	242	348	55	120	211	75	32	71
16	34.	470	36	236	342	56	116	204	76	28	64
17	338	465	37	230	336	57	111	197	77	25	58
18	334	459	38	224	330	58	106	190	78	22	52
19	329	453	39	218	324	59	101	183	79	19	46

TABLE showing the Value of an Annuity on One Life, at the several Rates of 3, 4, and 5 per Cent.

Age.	Years value at 3 per cent.	Years value at 4 per cent.	Years value at 5 per cent.	Age.	Years value at 3 per cent.	Years value at 4 per cent.	Years value at 5 per cent.
6	18·8	16·2	14·1	34	14·2	12·4	11·0
7	18·9	16·3	14·2	35	14·1	12·3	10·9
8	19·0	16·4	14·3	36	13·9	12·1	10·8
9	19·0	16·4	14·3	37	13·7	11·9	10·6
10	19·0	16·4	14·3	38	13·5	11·8	10·5
11	19·0	16·4	14.3	39	13·3	11·6	10·4
12	18·9	16·3	14.2	40	13·2	11·5	10·3
13	18·7	16·2	14·1	41	13·0	11·4	10·2
14	18·5	16·0	14·0	42	12·8	11·2	10·1
15	18·3	15·8	13·9	43	12·6	11·1	10·0
16	18·1	15·6	13·7	44	12·5	11·0	9·9
17	17·9	15·4	13·5	45	12·3	10·8	9·8
18	17·6	15·2	13·4	46	12·1	10·7	9·7
19	17·4	15·0	13·2	47	11·9	10·5	9·5
20	17·2	14·8	13·0	48	11·8	10·4	9·4
21	17·0	14·7	12·9	49	11·6	10·2	9·3
22	16·8	14·5	12·7	50	11·4	10·1	9·2
23	16·5	14·3	12·6	51	11·2	9·9	9·0
24	16·3	14·1	12·4	52	11·0	9·8	8·9
25	16·1	14·0	12·3	53	10·7	9·6	8·8
26	15·9	13·8	12·1	54	10·5	9·4	8·6
27	15·6	13·6	12·0	55	10·3	9·3	8·5
28	15·4	13·4	11·8	56	10·1	9·1	8·4
29	15·2	13·2	11·7	57	9·9	8·9	8·2
30	15·0	13·1	11·6	58	9·6	8·7	8·1
31	14·8	12·9	11·4	59	9·4	8·6	8·0
32	14·6	12·7	11·3	60	9·2	8·4	7·9
33	14·4	12·6	11·2	61	8·9	8·2	7·7

TABLE for an Annuity upon the Longer of Two given Lives.

Age of Younger.	Age of Elder	Value at 3 per cent.	Value at 4 per cent.	Value at 5 per cent.
10	10	23·4	19·9	17·1
	15	22·9	19·5	16·8
	20	22·5	19·1	16·6
	25	22·2	18·8	16·4
	30	21·9	18·6	16·2
	35	21·6	18·4	16·1
	40	21·4	18·3	16·0
	45	21·2	18·2	15·9
	50	20·9	18·0	15·8
	55	20·7	17·8	15·7
	60	20·4	17·6	15·5
	65	20·1	17·4	15·3
	70	19·8	17·2	15·1
	75	19·5	16·9	14·8
15	15	22·8	19·3	16·7
	20	22·3	18·9	16·4
	25	21·9	18·6	16·2
	30	21·6	18·3	16·0
	35	21·3	18·1	15·9
	40	21·1	17·9	15·7
	45	20·9	17·8	15·6
	50	20·7	17·6	15·4
	55	20·4	17·4	15·3
	60	20·1	17·2	15·2
	65	19·8	16·9	15·0
	70	19·4	16·6	14·7
	75	18·9	16·3	14·4
20	20	21·6	18·2	15·8
	25	21·1	17·9	15·5
	30	20·7	17·6	15·3
	35	20·4	17·4	15·1
	40	20·1	17·2	15·0
	45	19·9	17·0	14·9
	50	19·6	16·8	14·7
	55	19·4	16·6	14·5
	60	19·1	16·3	14·3
	65	18·7	16·0	14·1
	70	18·2	15·7	13·8
	75	17·7	15·3	13·5
25	25	20·3	17·4	15·1
	30	19·8	17·0	14·9
	35	19·4	16·7	14·7
	40	19·2	16·5	14·5
	45	18·9	16·3	14·3
	50	18·7	16·1	14·2
	55	18·4	15·9	14·0
	60	18·0	15·6	13·8
	65	17·6	15·3	13·6
	70	17·2	15·0	13·3
	75	16·7	14·6	12·9
30	30	19·3	16·6	14·5
	35	18·8	16·2	14·2
	40	18·4	15·9	14·0
	45	18·1	15·6	13·8
	50	17·8	15·4	13·6
	55	17·4	15·1	13·4
	60	17·0	14·8	13·2
	65	16·6	14·5	12·9
	70	16·1	14·1	12·6
	75	15·6	13·7	12·2
35	35	18·3	15·8	13·8
	40	17·8	15·4	13·5
	45	17·4	15·1	13·3
	50	17·1	14·8	13·1
	55	16·7	14·5	12·9
	60	16·3	14·2	12·7
	65	15·8	13·8	12·4
	70	15·3	13·4	12·0
	75	14·8	13·0	11·6
40	40	17·3	15·0	13·3
	45	16·8	14·6	13·0
	50	16·3	14·2	12·7
	55	15·9	13·9	12·4
	60	15·4	13·5	12·1
	65	14·9	13·1	11·8
	70	14·5	12·7	11·4
	75	14·0	12·3	11·0
45	45	16·2	14·2	12·8
	50	15·7	13·8	12·5
	55	15·2	13·4	12·1
	60	14·7	12·9	11·7
	65	14·1	12·5	11·4
	70	13·6	12·0	11·0
	75	13·1	11·6	10·6
50	50	15·0	13·3	12·1
	55	14·5	12·9	11·7
	60	13·9	12·4	11·3
	65	13·3	12·0	10·9
	70	12·8	11·5	10·5
	75	12·3	11·0	10·1
55	55	13·6	12·4	11·3
	60	13·0	11·9	10·9
	65	12·4	11·3	10·5
	70	11·8	10·8	10·0
	75	11·3	10·3	9·5
60	60	12·2	11·2	10·5
	65	11·5	10·6	10·0
	70	10·9	10·1	9·5
	75	10·3	9·5	9·0
65	65	10·7	10·0	9·4
	70	10·0	9·4	8·9
	75	9·3	8·7	8·3
70	70	9·2	8·6	8·2
	75	8·4	7·9	7·6
75	75	7·6	7·2	6·9

The uses of these tables may be exemplified in the following problems:

1. To find the probability or proportion of chance, that a person of a given age continues in being a proposed number of years.

Thus, suppose the age be 40, and the number of years proposed 15; then, to calculate by the table of probabilities for London, in table 1, against 40 years stands 214; and against 55 years, the age to which the person must arrive, stands 120; which shows that, of 214 persons who attain to the age of 40, only 120 of them reach the age of 55, and consequently 94 die between the ages of 40 and 55. It is evident therefore that the odds for attaining the proposed age of 55, are as 120 to 94, or as 9 to 7 nearly.

2. To find the value of an annuity for a proposed life.

This problem is resolved from table 2, by looking against the given age, and under the proposed rate of interest; then the corresponding quantity shows the number of years' purchase required. For example, if the given age be 36, the rate of interest 4 per cent. and the proposed annuity 250*l.* Then in the table it appears that the value is 12·1 years purchase, or 12·1 times 250*l.*; that is, 3025·1*l.*

After the same manner, the answer will be found in any other case falling within the limits of the table. But as there may sometimes be occasion to know the values of lives computed at higher rates of interest than those in the table, the two following practical rules are subjoined; by which the problem is resolved independent of tables.

1. When the given age is not less than 45 years, nor greater than 85, subtract it from 92; then multiply the remainder by the perpetuity, and divide the product by the said remainder added to 2½ times the perpetuity; so shall the quotient be the number of years' purchase required. Where note, that by the perpetuity is meant the number of years purchase of the fee-simple; found by dividing 100 by the rate per cent. at which interest is reckoned. Thus, let the given age be 50 years, and the rate of interest 10 per cent. Then subtracting 50 from 92, there remains 42; which multiplied by 10, the perpetuity gives 420; and this divided by 67, the remainder increased by 2½ times 10 the perpetuity, gives 6·3 nearly, for the number of years purchase. Therefore, supposing the annuity to be 100*l.*, its value in present money will be 630*l.*

2. When the age is between 10 and 45 years, take eight-tenths of what it wants of 45, which divide by the rate per cent. increased by 1·2; then if the quotient be added to the value of a life of 45 years, found by the preceding rule, there will be obtained the number of years purchase in this case. For example, let the proposed age be 20 years, and the rate of interest 5 per cent. Here taking 20 from 45, there remains 25; eight-tenths of which is 20; which divided by 6·2, quotes 3·2; and this added to 9·8, the value of a life of 45, found by the former rule, gives 13 for the number of years purchase that a life of 20 ought to be valued at.

3. To find the value of an annuity for the longest of two lives; that is, for as long as either of them continue in being.

In table 4, find the age of the youngest life, or the nearest to it, in col. 1, and the age of the elder in col. 2: then against this last is the answer in the proper column of interest.

Exam. So, if the two ages be 15 and 40, then the value of the annuity upon the longest of two such lives,

is 21·1 years purchase, at 3 per ct.
or 17·9 .... at 4 ......
or 15·7 .... at 5 ......

Note. In the last two problems, if the younger age, or the rate of interest, be not exactly found in the tables, the nearest to them may be taken; and then, by proportion, the value for the true numbers will be nearly found.

LIGHT, that which renders objects perceptible to our sense of seeing. This is, perhaps, one of the most interesting subjects that falls under the contemplation of the philosopher; at the same time it must be acknowledged to be one that is as little understood, and upon which opinions are as much divided, as any of the most abstruse subjects of philosophical inquiry. Some consider light as a fluid, *per se;* while others consider it merely as a phenomenon, and attribute it to a sort of pression, or vibration propagated from the luminous body through a subtle ethereal medium. But notwithstanding the imperfection of our knowledge, with regard to the nature and cause of light, repeated experiments and observations have made us acquainted with several of its properties; such as its *motion, inflection, reflection, refraction*, &c.

Motion of Light.—The ancients considered light as propagated from the sun and other luminous bodies instantaneously; but the observations of the moderns have shown that this was an erroneous hypothesis, and that the motion of light is not instantaneous, but that it takes up about sixteen minutes and a half of time to pass over a space equal to the diameter of the earth's orbit, which is near 190 millions of miles in length, or at the rate of near 200,000 miles per second; a conclusion which is placed beyond every possibility of doubt by the aberration of the stars.

LIGHTNING. See *Thunder.*

LIKE *Quantities*, in *Algebra*, those which consist of the same letter and power.

LIMB, the outermost border, or graduated edge, of a quadrant, astrolabe, or such like mathematical instrument. The word is also used for the arch of the primitive circle, in any projection of the sphere in plano.

Limb also signifies the outermost border or edge of the sun and moon; as the upper limb or edge; the lower limb; the preceding limb, or side; the following limb. Astronomers observe the upper or lower limb of the sun or moon, to find their true height, or that of the centre, which differs from the others by the semi-diameter of the disc.

LIMIT, is a term used by mathematicians for some determinate quantity, to which a variable one continually approaches, and may come nearer to it than by any given difference, but can never go beyond it.

LIMITS *of the Roots of an Equation.*—By finding the roots of an equation, is to be understood the finding two such numbers that one shall be greater, and one less than the root required; by which means an approximation is evidently made towards the true root, and the nearer these limits approach towards each other, so much the more accurate will be the approximation.

LIMITED *Problem*, that which admits of but one, or of a certain number of solutions, in contradistinction to an unlimited or indeterminate problem, which may admit of an infinite number of solutions.

LINE, in *Geometry*, is, according to Euclid's definition, that which has length without thickness. Lines are either right or curved:

A *Right* or *Straight* LINE is that which lies all in the same direction between its extremes or ends.

A *Curve* LINE, is that which continually changes its direction.

Curve LINES are again divided into *algebraical, geometrical*, and *mechanical*, or *transcendental.*

An *Algebraical* or *Geometrical* LINE, is that which may be expressed, that is, the relation between its absciss and ordinate by an algebraical equation. And such lines are divided into orders, according to the dimensions of the equations by which they are represented.

Mechanical and *Transcendental* LINES, are those which cannot be expressed by finite algebraical equations. See *Transcendental.*

Besides the above distinctions, lines receive other denominations according to their absolute or relative positions, as *parallel, perpendicular, oblique, tangential*, &c. for which see the respective terms.

LINES have again other distinguishing appellations, as they are introduced into the different sciences of astronomy, geography, dialling, perspective, &c.; as LINE of the *Apsides;* of the *Nodes; Horizontal, Hour, Equinoctial,* &c. *Lines;* each of which will be found illustrated under the respective articles.

LINEAR *Number,* is that which relates to length only, or to one dimension, as the number which expresses the side or perimeter of a geometrical figure. More correctly it is a number which is not considered as a product; the product of two numbers denoting a surface, and that of three numbers a solid.

LINEAR *Problem,* that which can be solved by means of right lines only.

LOCAL *Problem,* is used by some writers to denote a problem that admits of an indefinite number of solutions.

LOCUS, in *Geometry,* denotes a line, by which a local problem is solved. Thus let it be required to find a point P so situated that if two lines be drawn from it to the two given points A and B, the included angle A P B shall be equal to a given angle.

Here it is obvious, that if on A B a segment of a circle be described capable of containing the given angle, any point P in the periphery of that segment will answer the required condition, because all angles in the same segment are equal; and, therefore, in this case that segment is the *locus geometricus.*

If the locus which the equation or problem requires be a right line, it is called a *locus ad rectum;* if a circle, *locus ad circulum;* if a parabola, *locus ad parabolum;* and the same for the ellipse, the hyperbola, &c. The first of these are also called *plane loci,* and the latter *solid loci.*

The following are some of the simplest cases of the higher order of loci:

1. The base and sum of the sides of a plane triangle being given, the locus of its vertex is an *ellipse.*

2. The base and difference of the sides of a plane triangle being given, the locus is an *hyperbola.*

3. The locus of that point, from which, if perpendiculars be drawn to three right lines given in position, and such that the sum of their squares shall be equal to a given space, is an *ellipse.*

And the same is true whatever be the number of lines given in position.

4. If a triangle, given in species, have two of its angles upon a straight line given by position, and the side adjacent to those angles passing through a given point, the locus of the angle opposite that side is an *hyperbola.*

5. When the base of a triangle is given, and one of the angles at the base double the other, the locus of the vertex is an *hyperbola.*

6. The locus of the angles of a parallelogram, formed by drawing tangents at the vertices of any two conjugate diameters of an ellipse, is also an *ellipse concentric with the former.*

LOG, an instrument used on shipboard for determining the rate of the vessel, of which there are different constructions; see Robertson's and other Treatises on Navigation.

LOGARITHMIC *Curve,* or LOGISTIC *Curve,* is a curve having its abscisses in arithmetical progression, and its corresponding ordinates in geometrical progression, so that the abscisses may be considered as the logarithms of the respective ordinates, from which property the curve has derived its name. The subtangent of this curve is always equal to the same constant quantity *a* the modulus of the logarithms; and the area between any two ordinates is equal to the rectangle of the constant subtangent, and the difference of the ordinates.

This curve greatly facilitates the conception of logarithms, and affords a very obvious proof of the very important property of their fluxions, or very small increments; namely, that the fluxion of a number, is to the fluxion of its logarithm, as the number is to the subtangent.

LOGARITHMIC or *Logistic Spiral,* is a curve having similar properties to the above, but differently constructed; thus, divide the quadrant of a circle into any number of equal parts in the points; and from the radii cut off toward the centre parts continually proportional, then the curve passing through the points of section will be the logarithmic spiral. Here the several areas are as the logarithms of the ordinates; and hence the denomination of the curve.

LOGARITHMS, are a series of exponents denoting the powers to which a given *root* or *modulus* must be raised, in order to produce all the natural numbers; and thus a logarithm stands not for a *number*, but for a number of multiplications. Thus if r be the *modulus* or root and $r^x = a$, $r^y = b$, $r^z = c$, then is x the logarithm of a, y the logarithm of b, z the logarithm of c, &c. The modulus r may be taken any number except 1, which will not answer, as all its powers are $=1$.

By help of logarithms, multiplication is performed by addition; division by subtraction; involution by multiplication; and evolution by division, as will appear immediately from the latter of the preceding definitions. For since $r^x = a$, $r^y = b$, x and y being the logarithms of a and b, it is obvious, from the principles of *algebra*, that the logarithm of the product ab, is equal to the sum of the logarithms of a and b; the logarithm of the quotient a divided by b, is equal to the difference of the logarithms of a and b; the logarithm of the nth power of a, is equal to n times the logarithm of a; and the logarithm of the nth root of a is equal to the nth part of the logarithm of a.

In order to form some idea of the formation of logarithms, let N be any number, r the *radix* of the system of logarithms, and $r^x = \mathrm{N}$. Then x is the logarithm of N. Now, first, in order to find an analytical expression for N in terms of x and r, r^x must be converted into a series, for which purpose it may be put under the form

$$r^x = (\overline{1 + r - 1})^x = 1 + x(r-1) + \frac{x(x-1)}{1\,.\,2}(r-1)^2 + \&c.$$

$$= 1 + x\left\{(r-1) - \tfrac{1}{2}(r-1)^2 + \tfrac{1}{3}(r-1)^3 - \&c.\right\}$$

$$+ \frac{x^2}{1\,.\,2}\left\{(r-1)^2 - (r-1)^3 + \&c.\right\} = 1 + \mathrm{A}x + \mathrm{A}'x^2 + \mathrm{A}''x^3 + \&c.$$

that is, by writing

$$\mathrm{A} = (r-1) - \tfrac{1}{2}(r-1)^2 + \tfrac{1}{3}(r-1)^3 - \&c.$$

$$\mathrm{A}' = (r-1)^2 - (r-1)^3 + \&c.$$

$$\mathrm{A}'' = \&c.$$

where A, A′, A″, &c. are constant but unknown; but the law by which they are connected with each other may be thus obtained:

Let x be increased by any indeterminate quantity, then $r^{x+z} = 1 + \mathrm{A}(x+z) + \mathrm{A}'(x+z)^2 + \&c.\ \mathrm{A}^{(n-1)}(x+z)^n$; that is, by simply writing $x+z$ for x in the preceding series, or expanding the powers of $x+z$, and stopping at the first two terms, we have

$$\begin{array}{lll} r^{x+z} = 1 & + \mathrm{A} & (x+z) \\ & + \mathrm{A}' & (x^2 + 2xz + \&c.) \\ & + \mathrm{A}'' & (x^3 + 3x^2z + \&c.) \\ & + \mathrm{A}(^{n-1}) & (x^n + nx^{n-1}z + \&c. \\ & + \mathrm{A}(^n) & (x^{n+1}) + (n+1)x^nz + \&c. \end{array}$$

Again, $r^{x+z} = r^x \times r^z =$

$$(1 + \mathrm{A}x + \mathrm{A}'x^2 + \mathrm{A}''x^3 + \&c.) \times z$$
$$(1 + \mathrm{A}z + \mathrm{A}'z^2 + \mathrm{A}''z^3 + \&c.$$

the actual multiplication of which gives

$$\begin{array}{l} r^{x+z} = 1 + \mathrm{A}(x+z) + \mathrm{A}'x^2 + \\ \qquad \mathrm{A}''x^3 + \&c. \\ \qquad \mathrm{A}^2xz + \mathrm{A}'.\mathrm{A}x^2z + \&c. \end{array}$$

Whence, by comparing the corresponding terms in two expansions, we have

$$2\mathrm{A}' = \mathrm{A}^2, \text{ or } \mathrm{A}' = \frac{\mathrm{A}^2}{1.2};\quad 3\mathrm{A}'' = \mathrm{A}.$$

$$\mathrm{A}' = \frac{\mathrm{A}^3}{2}$$

$$\text{therefore} \qquad \mathrm{A}'' = \frac{\mathrm{A}^3}{1.2.3}$$

$$\left.\begin{array}{l}\text{in the same}\\ \text{way}\end{array}\right\} \mathrm{A}''' = \frac{\mathrm{A}^4}{1.2.3.4}$$

$$\text{and generally } \mathrm{A}(^{n-1}) = \frac{\mathrm{A}^n}{1.2.3\ \&c.\,n}$$

$$\mathrm{A}\,(^n) = \frac{\mathrm{A}^{n+1}}{1.2.3.\ \&c.\ n+1}$$

And consequently

$$r^x = \mathrm{N} = 1 + \mathrm{A}x + \frac{\mathrm{A}^2}{1.2}x^2 + \frac{\mathrm{A}^3}{1.2.3}x^3 + \&c.$$

which is the analytical

expression for any number in terms of the radix r, and logarithm x; but the reverse of this, by which the logarithm is expressed in terms of its number and radix is the formula, which is more particularly applicable in the present inquiry.

This may be found as follows:

In the preceding article we found

$$r^x = N = 1 + Ax + \frac{A^2}{1.2}x^2 + \frac{A^3}{1.2.3}x^3 + \&c.$$

where

$$A = (r-1) - \tfrac{1}{2}(r-1)^2 + \tfrac{1}{3}(r-1)^3 - \&c.$$

and if now we make

$$B = (N-1) - \tfrac{1}{2}(N-1)^2 + \tfrac{1}{3}(N-1)^3 - \&c.$$

we shall have on the same principles

$$N^z = 1 + Bz + \frac{B^2}{1.2}z^2 + \frac{B^3}{1.2.3}z^3 + \&c.$$

But

$$N^z = r^{xz} = 1 + Axz + \frac{A^2}{1.2}x^2z^2 + \frac{A^3}{1.2.3}x^3z^3 + \&c.$$

whence by comparing the co-efficients of z in both series we have

$$Ax = B;\ \frac{A^2x^2}{1.2} = \frac{B^2}{1.2};\ \frac{A^3x^3}{1.2.3} = \frac{B^3}{1.2.3}\ \&c.$$

each of which gives the same result, *viz.* $Ax = B$; whence we obtain immediately

$$x = \frac{B}{A} = \frac{(N-1) - \frac{1}{2}(N-1)^2 + \frac{1}{3}(N-1)^3 - \&c.}{(r-1) - \frac{1}{2}(r-1)^2 + \frac{1}{3}(r-1)^3 - \&c.}$$

which is the analytical expression, for the logarithm of any number N, in terms of itself and the radix of the system; that is, writing a instead of N,

$$\log.\ a = \frac{(a-1) - \frac{1}{2}(a-1)^2 + \frac{1}{3}(a-1)^3 - \&c.}{(r-1) - \frac{1}{2}(r-1)^2 + \frac{1}{3}(r-1)^3 - \&c.}$$

Or,

$$\log.\ 1 \pm a = \frac{\pm a - \frac{1}{2}a^2 \pm \frac{1}{3}a^3 - \frac{1}{4}a^4 \pm \&c.}{(r-1) - \frac{1}{2}(r-1)^2 + \frac{1}{3}(r-1)^3 - \&c.}$$

This, however, must only be considered as a simple algebraical method of expressing a logarithm; for it does not always answer the purposes of calculation; thus if a be any number greater than unity, it is obvious that the series in the numerator will either converge very slowly, or otherwise will diverge, and the same with regard to the denominator, supposing r to be equal to 10, as it is in the common system; in fact, the terms of the series are larger, the more remote they are from the beginning, and consequently no number of them can exhibit either exactly, or nearly, the true sum. Let us therefore investigate the method of submitting these to calculation; in order to which we will repeat again our last series, *viz.*

$$\log.\ 1 \pm a = \frac{\pm a - \frac{1}{2}a^2 \pm \frac{1}{3}a^3 - \frac{1}{4}a^4 \pm \&c.}{(r-1) - \frac{1}{2}(r-1)^2 + \frac{1}{3}(r-1)^3 - \&c.}$$

and here, since the denominator is always a constant quantity, when the radix of the system is given, we may make $M = (r-1) - \frac{1}{2}(r-1)^2 + \frac{1}{3}(r-1)^3 - \&c.$ which renders the above expression still more simple, as in that case it becomes barely

$$\log.\ 1 + a = \frac{1}{M} \times \left\{ a - \tfrac{1}{2}a^2 + \tfrac{1}{3}a^3 - \tfrac{1}{4}a^4 - \&c. \right\}$$

Or, taking a negative

$$\log.\ 1 - a = \frac{1}{M} \times \left\{ -a - \tfrac{1}{2}a^2 - \tfrac{1}{3}a^3 - \tfrac{1}{4}a^4 - \&c. \right\}$$

Whence again by subtraction

$$\log.\ \left(\frac{1+a}{1-a}\right) = \frac{2}{M}\left\{ a + \tfrac{1}{3}a^3 + \tfrac{1}{5}a^5 + \tfrac{1}{7}a^7 + \&c. \right\}$$

$$\text{Now } a = \frac{1 + \frac{a-1}{a+1}}{1\ \frac{a-1}{a+1}}$$ if therefore

we substitute in the foregoing expression $\frac{a-1}{a+1}$, instead of a, it becomes

$$\log. a = \frac{2}{M} \times \left\{ \left(\frac{a-1}{a+1}\right) + \tfrac{1}{3}\left(\frac{a-1}{a+1}\right)^3 + \tfrac{1}{5}\left(\frac{a-1}{a+1}\right)^5 + \&c. \right\}$$

which series must necessarily converge because the denominator of each of the fractions is greater than its denominator; still, however, when a is a number of any considerable magnitude, the decrease in the terms will be so slow as to render the formula useless for the purposes of calculation.

The limits of this article will not, however, admit of our entering into an investigation of the series suited to the several cases that may arise, but for the sake of reference it will be useful to give some of the most useful formula for this purpose, which are as follows; *viz.*

1. $$\text{Log. } a = \frac{1}{M} \times \left\{ (a-1) - \tfrac{1}{2}(a-1)^2 + \tfrac{1}{3}(a-1)^3 - \&c. \right\}$$

2. $$\text{Log. } a = \frac{1}{M} \times \left\{ \left(\frac{a-1}{a}\right) + \tfrac{1}{2}\left(\frac{a-1}{a}\right)^2 + \tfrac{1}{3}\left(\frac{a-1}{a}\right)^3 - \&c. \right\}$$

3. $$\text{Log. } a = \frac{2}{M} \times \left\{ \left(\frac{a-1}{a+1}\right) + \tfrac{1}{3}\left(\frac{a-1}{a+1}\right)^3 + \tfrac{1}{5}\left(\frac{a-1}{a+1}\right)^5 + \&c. \right\}$$

4. $$\text{Log. } \frac{a}{b} = \frac{1}{M} \times \left\{ \left(\frac{a \backsim b}{b}\right) - \tfrac{1}{2}\left(\frac{a \backsim b}{b}\right)^2 + \tfrac{1}{3}\left(\frac{a \backsim b}{b}\right)^3 - \&c. \right\}$$

5. $$\text{Log. } \frac{a}{b} = \frac{1}{M} \times \left\{ (\frac{a \backsim b}{a} - \tfrac{1}{2}\left(\frac{a \backsim b}{a}\right)^2 + \tfrac{1}{3}\left(\frac{a \backsim b}{a}\right)^3 - \&c. \right\}$$

6. $$\text{Log. } \frac{a}{b} = \frac{2}{M} \times \left\{ \left(\frac{a \backsim b}{a+b}\right) + \tfrac{1}{3}\left(\frac{a \backsim b}{a+b}\right)^3 + \tfrac{1}{5}\left(\frac{a \backsim b}{a+b}\right) - \&c. \right\}$$

7. $$\text{Log. } a = \log. (a-1) + \frac{1}{M} \times \left\{ \frac{1}{a} + \frac{1}{2a^2} + \frac{1}{3a^3} + \frac{1}{4a^4} + \&c. \right\}$$

8. $$\text{Log. } a = \log. (a-1) + \frac{1}{M} \times \left\{ \frac{1}{a-1} - \frac{1}{2(a-1)} + \frac{1}{3(a-1)^3} \&c. \right\}$$

9. $$\text{Log. } a = \log. (a-2) + \frac{2}{M} \times \left\{ \frac{1}{a-1} + \frac{1}{3(a-1)^3} + \frac{1}{5(a-1)^5} + \&c. \right\}$$

To the above may be added the following, which will be found useful on many occasions.

10. $$\text{Log. } a = \frac{1}{M} \times \left\{ (a - a^{-1}) - \tfrac{1}{2}(a^2 - a^{-2}) + \tfrac{1}{3}(a^3 - a^{-3}) - \&c. \right\}$$

11. $$\text{Log. } (a+z) = \log. a + \frac{1}{M} \times \left\{ \frac{z}{a} - \tfrac{1}{2}\,\frac{z^2}{a^2} + \tfrac{1}{3}\,\frac{z^3}{a^5} - \tfrac{1}{4}\,\frac{z^4}{a^4} + \&c. \right\}$$

12. $$\text{Log. } (a-z) = \log. a - \frac{1}{M} \times \left\{ \frac{z}{a} + \tfrac{1}{2}\,\frac{z^2}{a^2} + \tfrac{1}{3}\,\frac{z^3}{a^3} + \tfrac{1}{4}\,\frac{z^4}{a^4} + \&c. \right\}$$

13. $$\text{Log. } (a \pm z) = \log. a \pm \frac{2}{M} \times \left\{ \left(\frac{z}{a+z}\right) + \tfrac{1}{3}\left(\frac{z}{a+z}\right)^3 + \tfrac{1}{5}\left(\frac{z}{a+z}\right) + \&c. \right\}$$

14. $$\text{Log. } a = \frac{m}{M} \times \left\{ (\sqrt[m]{a} - 1) - \tfrac{1}{2}(\sqrt[m]{a} - 1)^2 + \tfrac{1}{3}(\sqrt[m]{a} - 1)^3 - \&c. \right\}$$

We have assumed the series which constitutes the denominator in our first expression or known quantity, which we have represented by M.; it will however be proper, before we conclude this article, to offer a few remarks upon the absolute value of this series, according to any given radix. First then, since log. $1 + a =$

$$\frac{a - \frac{1}{2}a^2 + \frac{1}{3}a^3 - \frac{1}{4}a^4 + \&c.}{(r-1) - \frac{1}{2}(r-1)^2 + \frac{1}{3}(r-1)^3 - \&c.}$$

the denominator and numerator of this fraction are totally independent of each other, and therefore r may be assumed at pleasure, and the value of the whole denominator computed for any particular magnitude assigned to this letter. Or otherwise, the whole denominator may be taken equal to any quantity, and the value of r itself determined by computation. The latter method, at first sight, appears the most eligible, for by assuming the whole denominator equal to unity it disappears entirely, and the expression becomes log. $1 + a = a - \frac{1}{2}a^2 + \frac{1}{3}a^3 - \frac{1}{4}a^4 + \&c.$; there are however inconveniences attending this system, that do not appear upon a slight view of the subject, but which are, notwithstanding, very evident upon a further investigation. In the case in which the whole denominator is assumed equal to unity, the value of r, the radix of this particular system is found to be 2·7182818284, &c., and the fraction $\frac{1}{M}$ becomes $= 1$. These constitute what are called hyperbolic logarithms, and which are treated of under that article in the present work; we shall therefore enter no farther upon the subject in this place, than is necessary to show the defect of this system for general purposes, when compared with that now in common use, a defect which is by no means compensated by the trifling advantage attending their computation. In the common system, the radix r is assumed equal to 10, the same as the radix of our scale of notation; and hence arises a most important advantage, which is, that the logarithm of all numbers expressed by the same digits, whether integers, decimals, or mixed of the two, have the same decimal part; the only alteration being in the index or characteristic of the logarithm. For the radix being 10; 0, 1, 2, 3, &c. will be logarithms of 1, 10, 10^2, &c. that is, $10^0 = 1$; $10^1 = 10$, $10^2 = 100$, &c.; and therefore to multiply or divide a number by any power of 10, we have only to add or subtract the number expressing that power from the integral part of the logarithm, and the decimal part will still remain the same, by which means the tables of logarithms are much more contracted than they could be with any other radix, for in the hyperbolic system, or in any other which has not its radix the same as that of the scale of notation, every particular number requires a particular logarithm, and this circumstance would either swell the tables to an unmanageable size, or if they were kept within the present limits, frequent computations would become necessary; so that in either way it is clear that the advantages of the present logarithms much more than counterbalance the extra trouble in computing them. This in fact only consists in multiplying the hyperbolic logarithm by a constant factor, *viz.* the reciprocal of the foregoing constant denominator represented above by $\frac{1}{M}$, the value of which when $r = 10$ is $\frac{1}{2{\cdot}30258509}$, &c. $=$ ·43429448, &c. Hence it is obvious that different systems of logarithms are connected together by constant multipliers, and by means of which a logarithm may always be converted from one scale to another. Thus the hyperbolic logarithm of a number is converted into a common logarithm by multiplying the former by ·4342944, and the latter is transformed into the former by multiplying it by 2·30258509.

Logistic LOGARITHMS, are certain logarithms of sexagesimal numbers or fractions used in astronomical calculations.

LONGITUDE, in *Astronomy*, is the angular distance of any star or celestial body from the vernal equinoctial point, that is, if a great circle pass through a star perpendicular to the ecliptic, the arc of the ecliptic intercepted between the intersection of it with this circle, and the vernal equinoctial point will be the longitude of the star.

Longitude, in *Geography* and *Navigation*, is the measure of the angle included between the meridian of any place, the longitude of which is required, and a certain fixed meridian from which the longitude is reckoned; or it is the number of degrees, minutes, &c. intercepted between a certain fixed point of the equator, and the intersection of the meridian of the place with the same circle.

Hence it appears that all places situated under the same meridian have the same longitude, and therefore, when only the longitude of a place is given, it may be situated any where within that circle. Or if only the latitude be given, the place may be situated any where in that parallel of latitude.

Now navigators can always find the latitude of the ship by observations on the sun, circumpolar stars, &c.; and it is obvious, therefore, how highly important it is for them to possess a similar easy method of finding their longitude, which would fix their position on the globe to the greatest possible degree of accuracy. But unfortunately this is not the case, and the problem of finding the longitude, though now much simplified, is still very embarrassing, and the result cannot ultimately be depended upon to the same degree of accuracy as in ascertaining the latitude. The only correct method is by ascertaining the difference in the time, between the place from which the longitude is reckoned, and that whose longitude is to be found; which difference of time converted into degrees, &c. will give the longitude required.

Since the earth performs one diurnal revolution in 24 hours, or, which is the same, since the sun makes one apparent revolution in that time, it is obvious that he passes over 15° every hour, and therefore, if one place is situated 15° to the westward of another, the sun will pass over the meridian of that place one hour later than at the former; and if it be 15° to the eastward, it will pass over it one hour sooner, and so on in proportion for any other difference in time, whence it is obvious that if the difference in time be given, the difference in longitude becomes known.

Most nations assume for a fixed meridian that which passes over their metropolis, or some other remarkable place. Thus the English reckon their longitude from the meridian of the observatory at Greenwich; the French from the meridian of Paris, &c.; and as we before observed, the great object is to ascertain the difference in time between these fixed places and that whose longitude is sought, that is, the time which elapses between the sun's passing the meridians of the two places. Now if a time-piece could be so accurately constructed as neither to gain nor lose, or if it either gained or lost, provided the rate of its deviation was constant, this would be all that is required for the complete solution of this problem. For supposing the watch to be set to the true Greenwich time, then whenever it is 12 by the watch, the sun will be upon the meridian of Greenwich, therefore, if the sun passes the meridian of any place when it is 11 o'clock by the watch, it shows the place to be 15° to the east, or that the place was in 15° east longitude; and if it is one o'clock by the watch, the place would then be in 15° west longitude, and so on for any other time.

Another method of finding the longitude is by astronomical observation, that is, by measuring the distance between the moon and certain fixed stars, which in consequence of the rapid motion of the moon, varies every instant; and therefore this distance being accurately ascertained by proper instruments, the exact corresponding time at Greenwich also becomes known by means of tables computed for that purpose, and published every year in the Nautical Almanack. Hence having the time at Greenwich, and the time at the ship, the difference in time between the two places, and consequently the difference in longitude becomes known.

This method of finding the longitude at sea, being then allowed on all hands to be far preferable to any other at present known, any improvement in it, either by means of new tables, or in instruments for taking the distances, will always be considered of the utmost importance; and though much has been already done towards bringing it to perfection, yet much more remains to be done; and which, it is hoped, may yet be effected by perseverance, for which the honours and rewards held out by the government will, no doubt, act as a powerful stimulus.

Degrees of LONGITUDE. It is obvious that the length of a degree of longitude varies with the parallels of latitude, being every where as the cosine of the latitude. The following table shows the length of a degree of longitude, corresponding to every degree of latitude from the equator to the pole, as expressed in English miles.

Deg. of Latitude	Deg. Lon. Eng. Miles	Deg. of Latitude	Deg. Lon. Eng. Miles	Deg. of Latitude	Deg. Lon Eng. Mil
0	69.20	30	59.93	60	24.60
1	69.19	31	59.32	61	33 55
2	69.16	32	58.69	62	32.49
3	69.11	33	58.04	63	31 42
4	69.03	34	57.37	64	30.34
5	68.94	35	56.69	65	29.25
6	68.82	36	55.98	66	28.15
7	68.69	37	55.27	67	27.04
8	68.53	38	54.53	68	25.92
9	68.35	39	53.78	69	24.80
10	68.15	40	53.10	70	23.67
11	67.93	41	52.23	71	22.53
12	67.69	42	51.43	72	21 38
13	67.43	43	50.61	73	20.23
14	67.14	44	49 79	74	19.07
15	66.84	45	48.93	75	17.91
16	66.52	46	48.07	76	16.74
17	66.18	47	47.19	77	15.57
18	65.81	48	46.30	78	14 39
19	65.43	49	45.40	79	13.20
20	65.27	50	44.48	80	12 02
21	64.60	51	43.55	81	10 83
22	64.06	52	42.60	82	9.63
23	63.70	53	41.65	83	8.43
24	63.22	54	40.68	84	7.23
25	62.72	55	39.69	85	6.03
26	62.20	56	38 70	86	4.83
27	61.66	57	37.69	87	3.62
28	61 10	58	36 67	88	2.42
29	60.52	59	35.64	89	1.24

LONGITUDE *Stars*, is a term frequently used to denote those fixed stars which have been selected for the purpose of finding the longitude by lunar observations; these are as follows:—α *Arietis*, a small star without the zodiac, about 22° to the right-hand of the Pleiades. —*Aldebaran*, in the Bull's-eye, a large conspicuous star, lying about half-way between the Pleiades and the star which forms the western shoulder of Orion.—α *Pegasi*, a star about 44° to the right of α Arietis, being nearly in a line with this latter star and the Pleiades.—*Pollux*, a little to the northward of Aldebaran, being the left-hand one of two bright stars in the constellation Gemini.—*Regulus*, about

38° S. E. of Pollux, being the southernmost of four bright stars to the N. E. of Aldebaran.—*Spica Virginis*, a white sparkling star, about 54° S. E. of Regulus.—*Antares*, lying to the right-hand of Regulus, and about 45° from Spica Virginis.—*Fomahault*, lying about 45° to the south of *α Pegasi*.—*α Aquilæ*, a star about 47° to the westward of *α Pegasi*.

LUNAR, any thing relating to the moon; thus we say *Lunar Cycle*, *Lunar Month*, *Lunar Year*, &c. *See* CYCLE, MONTH, YEAR, &c.

LUNAR *Distance*, in *Navigation*, is a popular term used to indicate the problem of finding the distance of the moon from the sun or some fixed star, for the purpose of ascertaining the longitude.

In this problem the apparent distance of the moon from the sun or a star, as found by observation, is given, and also their apparent altitudes or zenith distances, to find their true distance as seen from the earth's centre.

Since the observed altitude of any celestial object is affected by refraction and parallax, the effects of which are always produced in a vertical direction, it is obvious that the observed distance of any two bodies will also be affected by the same causes. With regard to the fixed stars the parallax vanishes, so that their places are changed by refraction only; but in observations of the moon particularly, the effect of parallax is very sensible, on account of her proximity to the earth; for which reason the distance of the moon, and any celestial object, is, for the most part, considerably different from the observed distance.

It may also be remarked, that since the refraction of the sun, at the same altitude, is always greater than his parallax, his true place will be lower than his apparent place; and because the moon's parallax, at any given altitude, is always greater than the refraction at that altitude, her true place will be higher than her apparent place.

This being premised, let Z M (Plate v. fig. 1.) be the observed zenith distance of the moon, and Z *m* her true zenith distance; M *m* being the difference between the moon's refraction and her parallax in altitude. Also let Z S be the observed zenith distance of the sun or star, and Z *s* its true zenith distance: S *s* being the difference between the sun's refraction and parallax, or the refraction of a star. Then since in the triangle Z S M, the three sides Z S, Z M, S M, are given, the vertical angle S Z M may be found by the known case of oblique-angled triangles. And because in the triangle Z *sm*, the two sides Z *s*, Z *m*, and the included angle *s* Z *m*, are also known, the true distance *sm* may also be found in a similar manner from the common rules of spherical trigonometry.

But this method, though direct and obvious, requires three separate statings, or analogies, for obtaining the true distance; it may, however, be rendered more commodious in practice, by incorporating the analytical formulæ for finding the angle Z and the side *sm*, into a single expression; which when converted into logarithms gives the following rule, using the altitudes instead of the zenith distances.

Take the difference of the apparent altitudes of the moon and star, or moon and sun, and half the difference of their altitudes; also take half the sum and half the difference of the apparent distance and difference of the apparent altitudes. Then to the log. sines of this half sum and half difference, add the log. cosines of the true altitudes (as corrected for semi-diameters, refraction, parallax, and dip, by means of the tables calculated for these purposes), and the complements of the log. cosines of the apparent altitudes, and take half the sum.

From this half sum take the log. sines of half the difference of the true altitudes, and find the remainder amongst the log. tangents; which being found, take out the corresponding log. cosine without taking out the arch, which is unnecessary. Lastly, subtract this log. cosine from the log.

sine of half the difference of the true altitudes, increased by 10 in the index, and the remainder will then be the log. sine of half the true difference. Thus, for example, let there be proposed the following data, to find the true distances; viz.

Apparent distance ☽ and ⊙	51° 28′ 35″
Apparent altitude ☽'s centre	12 30
Apparent altitude ⊙'s centre	24 48
True altitude ☽'s centre	13 20 42
True altitude ⊙'s centre	24 45 57

Apparent altitude of ⊙	24 48
Apparent altitude of ☽	12 30
Differen apparent altitudes	12 18
True altitude of ⊙	24 45 57
True altitude of ☽	13 20 42
	2)11 25 15
½ difference true altitude	5 42 37½
Apparent distance	51 28 35
Different apparent altitude	12 18
	2)63 46 35
½ sum	31 53 17½
	2)39 10 35
½ difference	19 35 17½

Then by the foregoing rule we have the following computation:

Log. sine	31° 53′ 17½″	9·7228488
Log. sine	19 35 17½	9 5253755
Co. log. cos	12 30	0 0104185
Log. cos	13 20 42	9·9881119
Co. log. cos	24 48	0·0420206
Log. cos	24 45 57	9·9580990
		2)39·2468743
		19·6234371
Log. sine	5 42 37	8·9978159
Log. tan. of an arch		10·6256212
Corresponding log. cosine		9·3625337
sine	25 34 54½	9.6352822
	2	
distance	51 9 49	

This is the direct method of determining the true distance, independently of any other tables than those of common logarithms, and what are found in the Nautical Almanack; but as this is the most laborious operation connected with the longitude problem, various other rules have been devised, which, by the help of certain tables, render the operation much more simple and expeditious. The most approved of these rules may be seen by consulting Mackay on the Longitude; see also Maskelyne's "Mariner's Guide," the Requisite Tables, and the other works referred to in the preceding part of this article, by Borda, Caille, Delambre, Robertson, Mendoza Rioes, &c.

LUNATION, the time between one new moon and another; consisting of 29 days, 12 hours 44 minutes, $3\frac{11}{60}$ seconds.

LUNE, Lunula, in *Geometry*, is the space included between the arcs of two unequal circles, forming a sort of crescent, or half-moon; the area of which may, in many cases, be as accurately determined as that of any rectilinear figure. The lune was the first curvilinear space of which the quadrature was ascertained, and this is said to have been first effected by Hippocrates of Chios; though others say it was discovered by Œnopides of Chios. However this may be, the former geometer has generally had the honour of the discovery attributed to him, and the figure still bears his name, being commonly denominated the lune of Hippocrates, the construction of which is as follows:

On the diameter of a semicircle describe a right-angled triangle, of which the angular point will necessarily fall in the circumference. Then on each of the sides describe a semicircle; the two figures contained between them and the first semicircle will be lunes; and the area of them will be equal to the area of the right-angled triangle. For the semicircles, when the two sides are equal to that upon the diameter; and if the segments between the sides and the lunes which are common to both be taken away, the remaining lines will be equal to the remaining triangle.

M.

MACHINE, any thing that is used to augment or to regulate moving forces or powers; or it is any instrument employed to regulate motion so as to save either time or force.

Machines are classed under different denominations, according to the agents by which they are put in motion, the purposes they are intended to effect, or the art in which they are employed; as *Electric, Hydraulic, Pneumatic, Military, Architectural*, &c.

Maximum Effect of Machines, is the greatest effect which can be produced by them. In all machines that work with a uniform motion, there is a certain velocity, and a certain load of resistance, that yields the greatest effect, and which are therefore more advantageous than any other. A machine may be so heavily charged, that the motion resulting from the application of any given power will be but just sufficient to overcome it, and if any motion ensue it will be very trifling, and therefore the whole effect very small. And if the machine is very lightly loaded, it may give great velocity to the load; but from the smallness of its quantity the effect may still be very inconsiderable, consequently between these two loads there must be some intermediate one that will render the effect the greatest possible. This is equally true in the application of animal strength as in machines.

1. The maximum effect of a machine is produced when the weight or resistance to be overcome is just $\frac{4}{9}$ of that which the power when fully exerted is just able to balance, or of that resistance which is necessary to reduce the machine to rest; and the velocity of the part of the machine to which the power

is applied should be one third of the greatest velocity of the power.

2. The moving power and the resistance being both given; if the machine be so constructed that the velocity of the point to which the power is applied be to the velocity of the point to which the resistance is applied, as nine times the resistance to four times the power, the machine will work to the greatest possible advantage.

3. This is equally true when applied to the strength of animals; that is, a man, horse, or other animal will do the greatest quantity of work, by continued labour, when his strength is opposed to a resistance equal to $\frac{4}{9}$ of his natural strength, and his velocity equal to $\frac{1}{3}$ of his greatest velocity when not impeded.

Now, according to the best observations, the force of a man at rest is on an average about 70lbs.; and his greatest velocity, when not impeded, is about 6 feet per second, taken at a medium. Hence the greatest effect will be produced when the resistance is equal to about $31\frac{1}{9}$lbs. and his uniform motion 2 feet per second.

The strength of a horse at a dead pull is generally estimated at about 420lbs. and his greatest rate of walking 10 feet per second; therefore the greatest effect is produced when the load $= 186\frac{2}{3}$lbs. and the velocity $\frac{10}{3}$, or $3\frac{1}{3}$ feet per second.

4. A machine driven by the impulse of a stream, produces the greatest effect when the wheel moves with one third of the velocity of the water.

MACULÆ, in *Astronomy*, dark spots appearing on the luminous faces of the sun, moon, and even some of the planets. In which sense maculæ stand contradistinguished from faculæ, which are luminous spots.

The solar maculæ are dark spots of an irregular changeable figure observed in the face of the sun. Many of these maculæ appear to consist of heterogeneous parts; of which the darker and more dense are called by Hevelius nuclei.

Various hypotheses have been invented to account for these spots, some considering them as dark clouds floating in the solar atmosphere, others as real excavations, which have not the property of propagating light as the other parts of this luminary, the depth of some of them having been estimated at more than 4000 miles, and their orifices much exceeding this in diameter; some of them, indeed, from the angle they subtend, must exceed in size the whole globe of the earth.

Dr. Herschel conjectured that the variations in the state of the weather, in different years, may arise from the greater or less number of maculæ of the solar disc.

MAGELLANIC *Clouds*, the name given to three permanent whitish appearances, resembling the milky way, near the south pole.

MAGIC *Circle* of *Circles*, is an invention of Dr. Franklin's, founded on the same principles and possessing similar properties to the magic square of squares, by the same author. This consists of eight concentric circles and eight radii, in the circumferences of which all the natural numbers from 12 to 75 are so posited, that the sum of the number in each circumference, together with the central number 12, is equal to 360; and the numbers in each radius, including always the central number, also is equal to 360. Besides the above, these circles possess several other curious properties.

MAGIC *Lantern*, a well-known optical instrument, by means of which small painted figures, on the wall of a dark room, are magnified to any size at pleasure.

This machine is represented on (plate V. fig. 2.); A B C D is a common lantern, to which is added the tube *b k l c*, *b h c* is the lens that throws the light of the candle or lamp *a*, augmented by the concave mirror *o*, on the object *d e*, and *k l* is the lens which magnifies the image *f g* on the white wall. Now, it is obvious that if the tube *b n k l m c* be contracted, and the glass *k l* be brought nearer the object *d e*, the image *f g* will be enlarged; and on the contrary, if the tube be protracted, the image of the object will be diminished.

Magic *Square*, is a square divided into cells, in which the natural numbers from 1 to the proposed square are so posited that the sum of each row, whether taken horizontally, vertically, or diagonally, is equal to a certain given number.

De Lahire gives the following rule for filling up the cells in any square consisting of an odd number of units, *viz*.

22	47	16	41	10	35	4
5	23	48	17	42	11	29
30	6	24	49	18	36	12
13	31	7	25	43	19	37
38	14	32	1	26	44	20
21	39	8	33	2	27	45
46	15	40	9	34	3	28

Place the least term 1, in the cell immediately under the middle or central one, and the rest of the terms, in their natural order, in a descending diagonal direction, till they run off either at the bottom or on the side. When the number runs off at the bottom, carry it to the uppermost cell, which is not occupied, of the same column that it would have fallen in below, and then proceed, descending diagonally again as far as you can, or till the numbers either run off at bottom or side, or are interrupted by coming at a cell already filled. When any number runs off at the right hand side, then bring it to the farthest cell on the left hand of the same row or line it would have fallen in towards the right hand; and when the progress diagonally is interrupted, by meeting with a cell already occupied by some other number, then descend diagonally to the left from this cell till an empty one is met with, where enter; and thence proceed as before. This rule, with reference to the above square, will be readily understood by the ingenious reader without farther explanation.

Magic *Square of Squares*, is an extension given to magic square, by Dr. Franklin.

MAGNET, or *Loadstone*, a ferruginous stone, found commonly in iron mines, which possesses the property of attracting iron, of pointing itself in a certain direction, and of communicating the same property to bars of iron or steel. Indeed, what is usually called a magnet, is merely a bar to which this property has been communicated.

The Poles of a Magnet, are those points in which all the power appears to be concentrated.

The Magnetical Meridian, is a vertical circle, which intersects the horizon, in the points to which the magnetical needle when at rest directs itself.

The Axis of a Magnet, is a right line which passes from one pole to another.

The Equator of a Magnet, is a line perpendicular to the axis, and exactly between the poles.

The distinguishing and characteristic properties of a magnet, are 1. Its attractive and repulsive power. 2. Its directive power. 3. Its dip or inclination to a point above or below the horizon. 4. Its power of communicating its own properties to certain other bodies.

Phenomena of the Magnet.—The principal ones are these. 1. A magnet, when freely supported either by a thread or in a light vessel on water, will place itself in a direction nearly coinciding with the poles of the earth. 2. This direction of the needle is not the same in all parts of the world, nor in the same place at different times. 3. A needle, which is not magnetised, being exactly balanced, will, if touched by a magnet so as to communicate that property to it, have its equilibrium destroyed, one of its extremities dipping considerably below the horizontal plane. 4. The centres of action of a magnet are at a very small distance from its extremities, and the law of attraction from these centres are reciprocally as the squares of the distances. 5. When a magnetic needle is put out of its natural line of direction, the force with which it tends to regain that position varies as the sine of the angle subtended by its natural direction, and that in which it is placed. 6. In every magnet there are two poles, of which the one points northwards, the other southwards; and if the magnet be divided into any number of pieces, the two poles will be found in each piece. The poles of a magnet may be found by holding a very fine short needle over it, for where the poles are the needle will stand upright, but no where else. By the following method also, the situation of the poles, and the direction of the (supposed) magnetic effluvia in passing out of the stone, may be exhibited to the sight: let there be strewed about the poles of the magnet, on every side, some iron or steel filings on a sheet of white paper; these will be affected by the magnet, and so disposed as to show the course and direction of the magnetic particles in every part. Thus, in the middle of each pole, it appears to go nearly straight on; towards the sides it proceeds in lines more and more curved, till at last the curve-lines, from both poles exactly meeting and coinciding, form numberless curves on each side, nearly of a circular figure. If the table on which the paper rests receives a few gentle knocks, so as to shake the filings a little, otherwise the action of the magnet will not be sufficient to dispose properly those particles which lie at a considerable distance. 7. The magnetic poles, in different parts of the globe, are differently inclined towards a point under the horizon. 8. These poles, though contrary to each other, help mutually towards the magnet's attraction, and suspension of iron. 9. If two magnets be spherical, one will turn or conform itself to the other, as either of them would do to the earth; and after they have so conformed or turned themselves, they have a tendency to approach or join each other; but if placed in a contrary position, they repulse each other. 10. If a magnet be cut through the axis, the segments or parts which before were joined, will repel each other. 11. If the magnet be cut perpendicular to its axis, the two points which were before conjoined, will become contrary poles 12. Iron receives virtue from the magnet by application to it, or barely from an approach towards it, though it do not touch it; and the iron receives its virtue variously according to the parts of the stone it is made to touch, or even approach to. 13. If an oblong piece of iron be any way applied to the stone, it receives virtue from it only lengthways. 14. A needle touched by a magnet will turn its end the same way, towards the poles of the world, as the magnet itself does. 15. Neither the magnet, nor needles touched by it, conform their poles exactly to those of the world, but have usually some variation from them; and this variation is different in different places, and at divers times in the same places. 16. A magnet will take up much more iron, when armed or capped, than it can alone. 17. A strong magnet at the least distance from a smaller or a weaker, cannot draw to it a piece of iron adhering actually to such a smaller or weaker stone; but if it touch it, it can draw it from the other; but a weaker magnet, or even a small piece of iron, can draw away or separate a piece of iron contiguous to a larger or stronger magnet. 18. In north latitudes the south pole of a magnet will raise up more iron than its north pole. 19. A plate of iron only, but no other body interposed, can impede the operation of the magnet, either as to its attractive or directive quality. 20. The power, or virtue of a magnet, may be impaired by lying long in a wrong position, as also by rust, wet, &c. and may be quite destroyed by fire, lightning, &c. 21. A wire being touched from end to end with one pole of a magnet, the end at which you begin will always turn contrary to the pole that touched it: and if it be again touched the same way with the other pole of the magnet, it will then be turned the contrary way. 22. If a piece of wire be touched in the middle with only the pole of the magnet, without moving it backwards or forwards; in that place will be the pole of the wire, and the two ends will be the other pole.

23. If a magnet be heated red hot, and again cooled, either with its south pole towards the north, in a horizontal position, or with its south pole downwards in a perpendicular position, its poles will be changed. 24. Hard iron tools well tempered, when heated by a brisk attrition, as filing, turning, &c. will attract thin filings, or chips of iron, steel, &c. and hence we observe that files, punches, augers, &c. have a small degree of magnetic virtue. 25. The iron bars of windows, &c. which have stood a long time in an erect position, grow permanently magnetical; the lower ends of such bars being the north pole, and the upper end the south pole. 26. Tongs and fireforks, by being often heated, and set to cool again, in a posture nearly erect, have gained this magnetic property. Sometimes iron bars, by long standing in a perpendicular posititon, have acquired the magnetic virtue in a surprising degree. A bar about ten feet long and three inches thick, supporting the summer beam of a room, was able to turn the needle at eight or ten feet distance, and exceeded a loadstone of three pounds and a-half weight; from the middle point upwards it was a north pole, and downwards a south pole. 27. A needle well touched, it is known, will point nearly north and south; but if it have one contrary touch of the same stone, it will be deprived of its faculty; and by another such touch it will have its poles interchanged. 28. A magnet acts with equal force in vacuo as in the open air. The smallest magnets have usually the greatest power in proportion to their bulk. A large magnet will seldom take up above three or four times its own weight, while a small one will often take up more than ten times its weight. A magnet worn by Sir Isaac Newton in a ring, and which weighed only 3 grains, would take up 746 grains, or almost 250 times its own weight. A magnetic bar made by Mr. Canton, weighed 10 oz. 12 dwts. took up more than 79 ounces; and a flat semicircular steel magnet weighed 1 oz 13 dwts. took up an iron wedge of 90 ounces.

Armed **MAGNET**, denotes one that is capped, cased, or set in iron, or steel, to make it take up a greater weight, and also more readily to distinguish its poles.

Artificial **MAGNET**, is a bar of iron or steel impregnated with magnetism, and possessing all the properties of the magnet itself, and commonly in a much higher degree. There are various methods of communicating the power of a magnet to bars of iron and steel, whereby these last become themselves magnets, and capable of again transmitting their power to others, and so on. There are also several ways of making artificial magnets without the assistance of any magnet whatever. One is as follows: Procure a dozen bars, six of soft steel and six of hard; the former to be each three inches long, quarter of an inch broad, and the twentieth of an inch thick, with two pieces of iron, each half the length of one of the bars, but of the same breadth and thickness, and the six hard bars to be each 5½ inches long, half an inch broad, and three-twentieths of an inch thick, with two pieces of iron half the length, but the whole breadth and thickness of one of the hard bars; and let all the bars be marked with a line quite around them at one end. Then take up an iron poker and tongs, or two bars of iron, the larger they are, and the longer they have been used, the better; and fixing the poker upright between the knees, hold to it, near the top, one of the soft bars, having its marked end downwards by a piece of sewing silk, which must be pulled tight by the left hand that the bar may not slide; then grasping the tongs with the right hand a little below the middle, and holding them nearly in a vertical position, let the bar be stroked by the lower end from the bottom to the top, about ten times on each side, which will give it a magnetic power sufficient to lift a small key at the marked end, which end, if the bar were suspended on a point, would turn towards the north, and is therefore called the north pole; and the unmarked end is for the same reason called the south pole: four of the soft bars being impregnated after this manner, lay two of them parallel to each other, at a quarter of an inch distance, between the two pieces of iron holding them, a north and south pole against each piece of iron: then take two of the four bars already made magnetical, and place them together so as to make a double bar in thickness, the north pole of one even with the south pole of the other; and the remaining two being put to these, one on each side, so as to have two north and two south poles together, separate the north from the south poles at one end by a large pin, and place them perpendicularly with that end downwards on the middle of one of the parallel bars, the two north poles towards its south end, and the two south poles towards its north end; slide them three or four times backward and forward the whole length of the bar; then removing them from the middle of this bar, place them on the middle of the other bar as before directed, and go over that in the same manner; then turn both the bars the other side upwards, and repeat the former operation; this being done, take the two from between the pieces of iron, and placing the two outermost of the touching bars in their stead, let the other two be the outermost of the four to touch these with; and this process being repeated till each pair of bars have been touched three or four times over, which will give them a considerable magnetic power. Put the half dozen together after the manner of the four, and touch them with two pair of the hard bars placed between their irons, at the distance of about half an inch from each other; then lay the soft bars aside, and with the four hard ones let the other two be impregnated; holding the touching bars apart at the lower end near two-tenths of an inch; to which distance let them be separated after they are set on the parallel bar, and brought together again before they are taken off: this being observed, proceed according to the method described above, till each pair has been touched two or three times over; but as this vertical way of touching a bar will not give it quite so much of the magnetic virtue as it will receive, let each pair be now touched once or twice over, in their parallel position, between the irons, with two of the bars held horizontally, or nearly so, by drawing at the same time the north end of one from the middle over the south end, and the south of the other from the middle over the north end of a parallel bar; then bringing them to the middle again without touching the parallel bar, give three or four of these horizontal strokes to each side. The horizontal touch, after the vertical, will make the bars as strong as they possibly can be made, as appears by their not receiving any additional strength, when the vertical touch is given by a great number of bars, and the horizontal by those of a superior magnetic power.

MAGNETISM, that quality or constitution of a body whereby it is rendered magnetical.

MAGNETISM of the *Earth*, is that property of the terrestrial globe, from which the magnetism of the ordinary magnets, the direction of the magnetic needle, and other phenomena are derived; and upon which they necessarily depend. This is obvious, since almost all the phenomena, which may be exhibited with a usual magnet, may be also exhibited with the earth, as far as it can be tried. The phenomena of the compass and the dipping-needle in different parts of the world, and the magnetism naturally acquired by soft iron, when properly situated, are exactly imitated with a common magnet, or a terella, or globular magnet; the only deviation being that which relates to the magnetic attraction of iron, which, if it exists, is not perceptible in the earth. If the earth attracted iron in consequence of its magnetism, the action of gravity still having its independent effect, iron would be more accelerated in its fall than any other substance, and the weight of it would be greater as it approached towards the magnetic poles; but neither of these circumstances has yet been shown to have place, and we are, therefore, ignorant at present whether or not the earth differs from the common magnet, even in this particular; for if such anomaly did attain with regard to iron bodies, it is reasonable to suppose that it must exist in a very small degree, which it would be extremely difficult to detect by any experiment that is within our power to perform.

Laws of MAGNETISM are certain results, which, from numerous and accurate experiments, are found to obtain in all natural and artificial magnets; the principal of which are briefly stated under *Phenomena* of the MAGNET.

MAGNIFYING, is used to denote the apparent enlargement of an object, by means of a convex lens, or some other optical instrument, particularly the microscope. See *Lens, Microscope, Telescope,* &c.

MAGNIFYING *Glass*, is a popular term for any convex glass or lens which has the property of magnifying.

MAGNITUDE, is used to denote the extension of any thing, whether it be in one direction, as a line; in two directions, as a surface; or in three directions, which constitute a body or solid.

Geometrical MAGNITUDES, may be conceived to be generated by motion, as a line by the motion of a point, a surface by the motion of a line, and a solid by the motion of a surface.

Apparent MAGNITUDE of a body, is that which is measured by the angle which that body subtends at the eye; at least this is what is always to be understood by this expression in the science of optics, though in reality the apparent magnitude depends not only on the visual angle, but also upon the supposed distance of the object we are viewing.

The mind judges of the magnitude of distant objects on two principles; viz. with reference to the optic angle, and the distance of the object from the eye, the latter arising out of our experience, which shows that distance diminishes that angle; and therefore, without being aware of the deduction, we always make a certain compensation agreeably to the supposed distance. If we see a man, or any other known object, at a distance, it conveys to our mind the idea of a certain magnitude, which we attribute to it entirely independent of the angle which it subtends; but if the object is unknown, then both the distance and the angle are considered in forming an idea of its magnitude; it is thus that we sometimes deceive ourselves with regard to the size of an object if we are mistaken as to its distance; thus a small bird on the branch of a tree may appear to be a larger bird at a greater distance, or the contrary; thus also a fly imperfectly seen in the corner of a window may have the appearance of a crow flying in the open air; these are illusions which commonly happen, and which most persons will recollect to have experienced at one time or other.

It is on this principle that some writers have accounted for the different apparent magnitudes of the sun and moon, and the apparent distance of two or more known stars, when seen near the horizon; and when they have a greater angle of elevation. When we see the moon, for example, at a considerable elevation, there being no intervening objects between that luminary and the eye wherewith to make a comparison of its distance, we intuitively suppose it nearer than when we observe it in the horizon; because there are then numerous objects, many of them at great distances, and the moon being evidently beyond them all, we thence suppose its distance greater than in the former case, and the optic angle being still nearly the same, we attribute to it a greater magnitude, in the horizon than in the zenith, because we suppose its distance then to be the greatest: hence also the apparent figure of the heavens, which instead of having the form of a perfect concave hemisphere, the eye being in the centre, it has always the appearance of being considerably flattened in the upper part, or being a segment considerably less than a hemisphere. This, however, is a subject that has much engaged the attention of astronomers and philosophers, from the earliest period to the present time; and various hypotheses have been advanced to account for it, none of which are, perhaps, perfectly satisfactory.

MALLEABILITY, the property of a solid that is hard and ductile, and which may, therefore, be beaten, forged, and extended under the hammer without breaking; as is the case with all metals not excepting quicksilver, but of these gold possesses this property in the highest degree.

MANOMETER, an instrument intended to measure the rarefaction and condensation of elastic fluids in confined circumstances, whether occasioned by variation of temperature or by actual destruction, or generation of portions of elastic fluid. It is sometimes called *manoscope*.

MAP, a plane figure representing the surface of the earth; or a part of it, according to the laws of perspective. Or a map is a projection of the globe, or of any part of it, on a plane surface, representing the forms and dimensions of the several countries and rivers, with the situation of mountains, cities, and other remarkable places.

Universal MAPS, are those which exhibit the whole surface of the earth, or the two hemispheres.

Particular MAPS, are those which exhibit some particular region, or country.

Each of these kinds of maps are called geographical or land maps, in contradistinction to hydrographical or sea-maps, representing only the sea or sea-coasts, and properly called *Charts*, which see.

In the construction of maps particular attention must be paid to the following circumstances: viz. 1. That all the places have their just situation with regard to the principal circles of the earth. 2. That the magnitudes and forms of the several countries have the same proportion as on the surface of the earth. 3. That the several places have the same distance and situation with regard to each other as on the earth itself.

The first maps are said to have been constructed by Anaximander, about 400 years before Christ, but the maps of Ptolomy are the first in which meridians are drawn, as also parallels for the more exact determination of places, which was a great improvement; all maps before his time being merely laid down according to their measured distances, without reference either to latitude or longitude. Little improvement was made in the construction of maps from the time of Ptolomy to that of Mercator in the sixteenth century, since which time numerous other authors have treated on the subject, and the greater part of the globe is now represented by maps of very accurate construction.

MARCH, the third month of the year amongst the moderns, but the first of the Roman year; it consists of 31 days.

MARS, in *Astronomy*, is one of the planets in our system, the fourth in order from the sun, and consequently the next above our earth. The character by which it is represented is ♂, a rude representation of a man holding a sphere.

This planet, which is known in the heavens by his red and fiery appearance, performs his revolution in his orbit in $686^d\ 23^h\ 30'\ 39''$, or in 1 881 Julian years. His mean distance from the sun is 1·524, the distance of the earth being taken as unity, which makes his mean distance 142 millions of miles. The eccentricity of his orbit is ·093, the semiaxis major being 1. His mean longitude for 1800 was $2^s\ 4^\circ\ 7'\ 2''{\cdot}3$; the longitude of his perihelion being then $11^s\ 2^\circ\ 24'\ 23''{\cdot}9$; but the line of his apsides has an apparent motion according to the order of the signs of $1'\ 5''{\cdot}9$ in a year, or $1^\circ\ 51'\ 3''{\cdot}5$ in a century. The orbit of Mars at the same time crossed the ecliptic in $1^s\ 18^\circ\ 1'\ 18''$, but the place of the nodes has an apparent motion in longitude, according to the order of the signs, of $26''{\cdot}8$ in a year, or $44'\ 41''{\cdot}5$ in a century. The rotation on his axis is performed in $1^d\ 0^h\ 39'\ 21''{\cdot}3$; and his axis is inclined to the ecliptic in an angle of $59^\circ\ 41'\ 49''{\cdot}2$. His mean diameter is equal to 4398 miles; consequently, he is rather more than $\frac{1}{5}$th the size of our earth. His mass, compared with that of the sun considered as unity, is $\frac{1}{2546320}$. The proportion of light and heat, received by him from the sun, is ·43; that received by the earth being considered as unity. He has a very dense but moderate atmosphere; and he is not accompanied by any satellite. As viewed from the earth, the motion of Mars appears sometimes retrograde. The mean arc which he describes in this case is 16° 12′; and its mean duration is about 73 days. This retrogradation commences, or finishes, when the planet is not more than 136° 48′ from the sun. Mars changes his phases somewhat in the same manner as the moon does from her first to her third quarter, according to his various positions with respect to the earth and the sun; but he never becomes cornicular, as the moon does when near her conjunctions. His mean apparent diameter is 9″·7; which augments in proportion as the planet approaches its opposition, when it is equal to 29″ 2. His parallax is nearly double that of the sun. At the poles of this planet there has been observed bright spots, which are, however, variable in their size and figure; and from the observations of Dr. Herschel, it seems probable that they proceed from snows accumulated in his polar regions during their long winter, these having been observed to decrease and increase, according as they are directed to or from the sun.

MASS, the quantity of matter in any body, which is always proportional to, and may be truly estimated by its weight, whatever be its figure or magnitude.

MATHEMATICAL, relating to mathematics.

MATHEMATICS, the science of magnitudes or quantities; or of the ratios, or relations of magnitudes or quantities. The subjects which fall to be considered under the different departments of Mathematics are spaces, distances, numbers, motions, velocities, and, in short, whatever is susceptible of being measured.

Mathematics are naturally divided into two classes; the one comprehending what we call *pure* and *abstract;* and the other, the *compound* or *mixed*.

Pure MATHEMATICS relate to magnitudes generally, simply, and abstractedly; and are, therefore, founded on the elementary ideas of quantity. Under this class is included *arithmetic*, or the art of computation; *geometry*, or the science of mensuration, and comparisons of all kinds of extensions; *analysis*, or the calculations and comparisons of magnitudes in general; and lastly, *geometrical analysis*, or the combination of elementary geometry and analysis.

Mixed MATHEMATICS, are certain parts of *physics*, which are by their nature susceptible of being submitted to mathematical investigation; we here borrow from incontestible experiments, or suppose in bodies, some principal and necessary quality, and then by a methodical and demonstrative chain of reasoning deduce, from the principles established, conclusions as evident and certain, as those which pure mathematics draw immediately from axioms and definitions. To this class belong *mechanics*, or the science of equilibrium, and motion of solid bodies. *Hydrodynamics*, in which the equilibrium and motion of fluids are considered. *Astronomy*, which relates to the motion of the celestial bodies. *Optics*, or the theory of the effects of light; and lastly, *acoustics*, or the theory of sound.

Mathematics are otherwise divided into practical and speculative.

Practical MATHEMATICS, is the application of these principles to the various uses of society.

Speculative MATHEMATICS, is that by which we merely contemplate the properties of things, without at all considering their practical application.

It is indeed impossible to fix the origin of mathematics with any thing like chronological precision, as there is every reason to suppose that it must be dated back to the remotest ages. When mankind began to relinquish their wandering and savage life, and general laws, or conventions, were established for its maintenance; when by common consent it was agreed that every individual should provide for his own subsistence, without seizing what was in the possession of another; necessity and self-preservation, the two great springs to physical exertion, first gave rise to the most useful arts. Huts were built; iron was forged; the lands were divided; and the courses of the stars were observed. With regard to the last, it would seem at first sight to have been a sort of amusement, and not in any degree connected with the economy of society; but upon a nearer and more accurate view it will appear no less useful than either of the former. It was seen that the earth yielded spontaneously many of the most common of its productions, and with which man in his savage state was enabled, or rather under the necessity of subsisting; but it was soon discovered that there were others of far greater utility, which required labour and cultivation to bring to perfection, and that it was necessary to regulate this by the proper seasons of the year; thus the ground was sown, the harvest was reaped, and the fruits of the year were preserved, till the bounty of nature furnished a farther supply.

All these observations and performances, at first extremely rude and unskilful, were connected with mathematics by a secret tie, though for a long time they had no other guide than experience and blind custom. The assiduous labour required in hunting or fishing, and the other business of the field, did not allow man to ascend to general and abstruse ideas, but confined the limits of their thoughts and actions to those of their physical wants. At length perhaps arose a commanding genius, who, collecting together the traditionary knowledge, observations, and remarks of his predecessors, formed them from the pure efforts of his own genius, into a rude system, sufficient at least to show his fellows, that man was a being formed to hold a much more elevated rank in the scale of creation than had been before imagined. It was then that man beheld with new eyes the magnificent spectacle which nature exhibited on all sides to his senses and imagination; he learned to examine things, and to compare them with each other; ideas acquired from physical objects were transported as it were into an intellectual world; the phenomena of nature were studied with discriminating attention, and the mind was impressed with a desire to know the causes by which they were produced. Geometry, confined at first to the art of measuring the fields, was extended to other purposes, and gave rise to loftier and more difficult problems. Astronomy was enriched by regular observations, and by several instruments adapted to increase their number, and to give them the requisite degree of accuracy and connection. Machines were invented, in which a skilful combination of the several powers of wheels and levers was employed for transporting the heaviest loads: in a word, all the parts of mathematics successively advanced; and their progress would in all probability have been more rapid and considerable had not fanaticism and ambition too frequently obscured the flame of genius for a long series of ages; but, as a fire concealed beneath the embers, it resumed its lustre in happier times, and burst forth with that flood of light which now illuminates so great a portion of the earth.

MATTER, in *Philosophy*, is understood to signify whatever is extended and capable of making resistance; hence, because all bodies, whether solid or fluid, are extended, and do resist, we conclude that they are made up of matter. That matter is one and the same thing in all bodies, and that all the variety we observe arises from the various forms and shapes it puts on, seems very probable, and may be concluded from a general observation of the procedure of nature in the generation and destruction of bodies. Thus, for instance, water rarified by heat becomes vapour; a great collection of which forms clouds; these condensed, descend in the form of hail or rain; part of this collected on the earth constitutes rivers; another part mixing with the earth enters into the roots of plants, and supplies matter to, and expands itself into various species of vegetables. In each vegetable it appears in one shape in the root, another in the stalk, a third in the flowers, and so on. Thus, from hemp and flax we have thread; thence our various kinds of linen; these degenerate into rags, which receive from the mill the various forms of paper, &c.

Dr. Berkeley argues against the existence of matter, and endeavours to prove that it is a mere *ens rationis*, and has no existence out of mind. Dr. Priestley maintains that matter is not that inert substance that it has been supposed to be; that powers of attraction or repulsion are necessary to its very being, and that no part of it appears to be impenetrable to other parts. Accordingly he defines matter to be a substance, possessed of the property of extension and of powers of attraction or repulsion, which are not distinct from matter and foreign to it, as it has been generally imagined, but absolutely essential to its very nature and being; so that when bodies are divested of these powers they become nothing at all. In another place, Dr. Priestley has given a somewhat different account of matter; according to which it is only a number of centres of attraction and repulsion; or, more properly, of centres not divisible, to which divine agency is directed; and as sensation and thought are not incompatible with these powers, solidity or impenetrability, and consequently a *vis inertiæ* only having been thought repugnant to them, he maintains that we have no reason to suppose that there are in man two substances absolutely distinct from each other. But Dr. Price suggested a variety of objections against this hypothesis of the penetrability of matter, and against the conclusions that are drawn from it. The *vis inertiæ* of matter, he says, is the foundation of all that is demonstrated by natural philosophy concerning the laws of the collision of bodies. This, in particular, is the foundation of Newton's philosophy, and especially of his three laws of motion. Solid matter has the power of acting on other matter by impulse; but unsolid matter cannot act at all by impulse; and this is the only way in which it is capable of acting, by any action that is properly its own. If it be said that one particle of matter can act upon another without contact and impulse, or that matter can, by its own proper agency, attract or repel other matter which is at a distance from it, then a maxim hitherto universally received must be false, that "nothing can act where it is not." Newton, in his letters to Bentley, calls the notion, that matter possesses an innate power of attraction, or that it can act upon matter at a distance, and attract and repel by its own agency, an absurdity into which he thought no one could possibly fall. And in another place he expressly disclaims the notion of innate gravity, and has taken pains to show that he did not take it to be an essential property of bodies. By the same kind of reasoning pursued, it must appear that matter has not the power of attracting and repelling; that this power is the power of some foreign cause, acting upon matter according to stated laws; and consequently that attraction and repulsion not being actions, much less inherent qualities of matter, as such, it ought not to be defined by them. And if matter has no other property, as Dr. Priestley asserts, than the power of attracting and repelling, it must be a nonentity; because this is a property that cannot belong to it. Besides, all power is the power of something; and yet if matter is nothing, the very idea of it is a contradiction. If matter be not solid extension, what can it be more than mere extension? Further, matter that is not solid, is the same with pore; and therefore it cannot possess what philosophers mean by momentum or force of bodies, which is always in proportion to the quantity of matter in bodies, void of pore. Sir R. Phillips considers the test of matter to be the exhibition of power when moved; the gaseous form as its general character; and the solid as an accident, or condensation of gaseous atoms. The entire extension of space is filled, he remarks, with gas, and only relative points with solidity. Admitting of no attractive principle, he conceives that primary atoms have plain or fitting surfaces, and that there are three varieties, cubes, tetrahedrons, and prisms, which, in the gaseous form, constitute the three primary gases, as oxygen, nitrogen, and hydrogen; their condensation and primary combinations forming solids and other gases. To these he also refers the three colours, three tones, and the two electricities and magnetism. Motion being essentially combined with gas, and gas existing every where, we have, in consequence, motion and power every where, and hence the universal energy of nature. No motion, says he, is lost, because action and reaction are equal; and every action or motion of one body is therefore only transferred to another body, action being the communication of motion, and re-action the reception of it. Thus, when gases are fixed in solids, extra-motion, called heat, is transferred to adjoining gas and diffused around; and when solids are converted into gas, extra motion or heat is absorbed, or transferred to atoms of the solid which become gaseous. And in the motion of solids, as when a large body at the short arm of a *solid* lever moves a small one on the long arm, the greater motion of the small body serves merely to transfer its motion to more of the atoms of the gas through which it moves. So also in a *gaseous* lever, as in the earth and moon, whose action and reaction are equalized by motion, the moon, in this case, moving through more gas, and to it transferring its excess of velocity.

MAXIMA et MINIMA, in *Analysis* and *Geometry*, are the greatest and least value of a variable quantity, and the method of finding these greatest and least values, is called the *Methodus de Maximis et Minimis;* which forms one of the most interesting inquiries in the modern analysis. This subject was considered geometrically by some of the most ancient mathematicians, particularly by Apollonius, in the 5th book of his Conics; and there are still a few problems of this kind, which succeed better by the geometrical than by the analytical method; their number, however, are very limited compared with those which may be elegantly performed by analysis.

The method of maxima et minima, according to the analytical doctrine, first arose at the beginning of the seventeenth century, after the invention of Des Cartes for expressing the properties of curve lines by means of algebraical equations, and classing them into different orders, according to the degree of the equation which expressed the relation between the absciss and ordinate. Besides the method of Des Cartes, we have also those of Fermat, Hudde, Huygens, Sluze, and some others.

Fermat's Method.—The principle upon which Fermat formed his operation consisted in this; that when the ordinate of a curve was the greatest possible, if we augmented the variable quantity x (which represents the absciss) by an indefinitely small quantity e, the ordinate corresponding to this absciss will be equal to the former, or will approach towards equality indefinitely near; or, which is the same, the increase or decrease of an ordinate when it approaches indefinitely near its maximum or minimum is nothing; and therefore these two ordinates may be considered as equal, whence an equation is obtained, from which cancelling the like quantities and all those powers of e beyond the first, because they are indefinitely small with regard to the others, and dividing the other terms by e the value of x will be obtained; that renders the function a maximum or a minimum.

For example: required the value of x in the equation $y^3 = ax^2$

$-x^3$, which renders the whole function a maximum.

Making as before $x = x + e$, we have

$$ax^2 - x^3 = ax^2 - x^3 + 2axe + ae^2$$
$$- 3ex^2 - 3e^2x - e^3;$$
$$\text{or, } 0 = +2axe - 3ex^2$$

by suppressing those powers of e above the first; whence

$$3ex^2 = 2axe, \text{ or } x = \tfrac{2}{3}a$$

Des Cartes's Method.—This consisted in making two of the roots of the equation equal to each other, in which case two of the ordinates of the curve became equal, and thus indicated the maximum or minimum state.

Hudde's Method.—This consisted in making two of the roots of the proposed equation equal to each other, and for which he gave the following rule, viz. Multiply each term of the equation, arranged according to the powers of x, by the terms of an arithmetical progression, viz. the first by the first, the second by the second, &c. and the equation thus obtained will indicate the maximum or minimum required.

Let us take for example the equation above, $ax^2 - x^3 = y^3$.

Arranging this equation according to the powers of x, and supplying the deficient term, we have

$$x^3 - ax^2 \pm 0x - y^3 = 0$$

$$\text{Arith. prog. } \underline{\quad 3.\ 2,\ 1,\ 0 \quad}$$
$$3x^3 - 2ax^2 = 0$$
$$3x^3 = 2ax^2, \text{ or } x = \tfrac{2}{3}a,$$

as above.

Again; let $x^2 - ax + y^2 - 2by + b^2$ be proposed.

Writing this,

$$x^2 - ax + (y^2 - 2by + b^2) = 0$$

$$\text{Arith. prog. } \underline{\quad 2.\ 1,\ 0 \quad}$$
$$2x^2 - ax = 0, \text{ or } x = \tfrac{1}{2}a.$$

This rule, though not so general as could be wished, is extremely simple and ingenious.

Huygen's Method.—Huygens, and also Sluse, arrive at the final equation at once by the following simple rule. Multiply each of the terms in which x is found, by its exponent, rejecting all those into which it does not enter, divide the result by x, and make the whole equal to zero; and the equation thus arising will give the value of x required.

For example; required the value of x in the equation

$$3ax^2 - x^3 = y^3.$$

Multiplying each of those terms by the exponent of x in them, we have $6ax^2 - 3x^3$; then dividing by x

$$6ax - 3x^2 = 0, \text{ or } 3x^3 - 6ax = 0,$$

or,

$$x^2 - 2ax + a^2 = a^2, \text{ or } x = a \pm a;$$

that is, $x = 0$; or $2a$.

This rule differs in no respect from our fluxional operation, except that we divide by $\dot{x}$ instead of x.

Fluxional, or Differential Method.—1. The fluxion of a quantity when it is a maximum or a minimum, is equal to 0. This is obvious, because at that point it admits of no increase or decrease.

2 If a quantity be a maximum or minimum, any power or root of that quantity must evidently also be a maximum or minimum. For the power or root of a quantity will increase or decrease as long as the quantity itself increases or decreases, and no longer.

3. Any constant multiple, or part of a quantity which is a maximum or a minimum, must also be a maximum or a minimum. For the multiple, or part of a quantity will increase or decrease as long as the quantity itself increases or decreases, and no longer; therefore when its fluxion is made equal to 0, the constant multiplier may be neglected.

4. The fluxion of a constant quantity $= 0$. For this admitting of no increase or decrease has no fluxion

Whatever is said here respecting fluxions is also true of differentials. We subjoin one or two examples:

1. To divide a given number (a) into two such parts x, and that $x^m y^n$ may be a maximum.

Since $x + y = a$, and $x^m y^n =$ a maximum, the fluxion of each $= 0$, the former because it is constant, and the latter because it is a maximum; whence

$$\dot{x} + \dot{y} = 0$$

$$my^n x^{m-1}\dot{x} + nx^m y^{n-1}\dot{y} = 0;$$

from the first we have $-\dot{x} = +\dot{y}$;

and substituting this in the second gives

$my^n x^{m-1}\dot{x} - nx^m y^{n-1}\dot{x} = 0$, or
$my^n x^{m-1} = nx^m y^{n-1}$, or
$my = nx$; whence

$$y = \frac{n}{m}x.$$

Consequently $x + \frac{n}{m}x = a$, or

$$x = \frac{ma}{m+n}, \text{ and } y = \frac{na}{m+n}.$$

If $m = n$, then the two parts are equal.

Hence to divide a quantity (a) into three parts, x, y, z, so that xyz may be a maximum; the three parts must be all equal.

2. To divide a given number (a) into two such parts, x and y, that the sum of their alternate quotients may be a maximum.

Here we must have $x + y = a$,

and $\frac{x}{y} + \frac{y}{x} = a$ maximum.

Now since the first is constant, and the latter a maximum, we have $\dot{x} = -\dot{y}$, and

$$\frac{\dot{x}y - \dot{y}x}{y^2} + \frac{\dot{y}x - \dot{x}y}{x^2} = 0.$$

Or substituting for $\dot{x}$, its equal $-\dot{y}$, this becomes

$$-\frac{y\dot{y} + x\dot{y}}{y^2} + \frac{x\dot{y} + y\dot{y}}{x^2} = 0, \text{ or}$$

$$\frac{x\dot{y} + y\dot{y}}{x^2} = \frac{y\dot{y} + xy}{y^2}.$$

Whence we have $\frac{1}{x^2} = \frac{1}{y^2}$, or $y^2 = x^2$, or $x = y$; that is, each of the required quantities is equal to $\frac{1}{2}a$.

In these two problems the resulting equations are only of the first degree, and consequently admit of only one solution; but it may happen that the resulting equation is of higher dimensions, and therefore contain several roots, and it then becomes necessary to have some means of distinguishing which of those roots give the greatest, and which the least result; and for this Lacroix gives the following general rule.

Rule. Let y represent any function of x; find the value of x in the equation $\frac{\dot{y}}{\dot{x}} = 0$, which value substitute for x in the equation $\frac{\ddot{y}}{x^2} = 0$; then if the result be negative, y is a maximum; if positive, a minimum; and if zero, neither a maximum nor a minimum, unless also $\frac{\dddot{y}}{\dot{x}^3} = 0$, and then it will depend upon the sign of $\frac{\ddddot{y}}{\dot{x}^4}$ and so on; and the same process being observed with regard to each of the roots of the fluxional equation, the number of maxima and minima will be obtained.

Let us illustrate the preceding rule by an example.

1. Find $y = x^4 - 8x^3 + 22x^2 - 24x + 10$, a max. et min.

Here $\frac{\dot{y}}{\dot{x}} = 4x^3 - 24x^2 + 44x - 24 = 0$

Where $x = 1, 2$, and 3.

And it is required to find which of these roots answer to the maxima, and which to the minima.

Now $\frac{\ddot{y}}{\dot{x}^2} = 12x^2 - 48x + 44$.

And here making $x = 1, 2, 3$; the results are respectively $+, -, +$; therefore the root 2 answers to the maximum, and the other two to the minima.

2. Let there now be proposed the function

$y = x^5 - 7x^4 + 19x^3 - 25x^2 + 16x + 10$

Here $\frac{\dot{y}}{\dot{x}} = 5x^4 - 28x^3 + 57x^2 - 50x + 16 = 0$

And the roots of this equation are 1, 1, 2, 1 $\frac{3}{5}$.

Now $\frac{\ddot{y}}{\dot{x}} = 20x^3 - 84x^2 + 114x - 50$

which $= 0$, when $x = 1$; therefore the root 1 gives neither a maximum nor a minimum, unless $\frac{\dddot{y}}{\dot{x}^3} = 0$; which upon trial does not obtain.

But by assuming $x = 2$, in this equation, the result is -4; and consequently this value of x answers to a maximum.

And by submitting the other root $1\frac{2}{3}$, to the same test, a similar result will be obtained.

We will add another example, with which we must conclude this article.

3. To find when the function

$$y = x^3 - 18x^2 + 96x - 20$$

becomes a maximum or a minimum.

Here $\frac{\dot{y}}{\dot{x}} = 3x^2 - 36x + 96 = 0$, in which equation the roots are $x = 4$, $x = 8$.

Now $\frac{\ddot{y}}{\dot{x}^2} = 6x - 36$.

Here the root 8 gives $\frac{\ddot{y}}{\dot{x}^2}$ *positive.*

And the root 4 gives $\frac{\ddot{y}}{\dot{x}^2}$ *negative.*

Therefore the former answers to the minimum and the latter to the maximum.

If the fluxional equation has no real root, then it follows that the proposed function admits of neither a maximum nor a minimum; but increases or decreases *ad infinitum.*

MAXIMUM *Effect* of *Machines.* See MACHINES.

MAY, the fifth month of the year, consisting of 31 days.

MEAN, is a middle state between two extremes; thus we say, *Mean Distance, Motion, Time,* &c.; *Arithmetical, Geometrical,* and *Harmonical Mean.*

Arithmetical MEAN, is half the sum of any two quantities: thus, $\frac{a+b}{2}$ is arithmetical mean between a and b.

Geometrical MEAN, is the square root of the product of any two quantities; that is, $\sqrt{ab}$ is the geometrical mean between a and b.

Harmonical MEAN, is double a fourth proportional to the sum of two quantities and the quantities themselves.

It is obvious, that the arithmetical mean is greater than the harmonical, and the harmonical greater than the geometrical.

If more than two mean proportionals are required between two quantities, they may be found analytically as follows: *viz.* a and b being the two quantities, we shall have

1 mean $\sqrt{ab}$,
2 means $\sqrt[3]{a^2b}$, $\sqrt[3]{ab^2}$,
3 means $\sqrt[4]{a^3b}$, $\sqrt[4]{a^2b^2}$, $\sqrt[4]{ab^3}$
4 means $\sqrt[5]{a^4b}$, $\sqrt[5]{a^3b^2}$, $\sqrt[5]{a^2b^3}$, $\sqrt[5]{ab^4}$,
5 means $\sqrt[6]{a^5b}$, $\sqrt[6]{a^4b^2}$, $\sqrt[6]{a^3b^3}$, $\sqrt[6]{a^2b^4}$, $\sqrt[6]{ab^5}$,
&c. &c. &c.

MEASURE, denotes any certain quantity, with which other homogeneous quantities are compared.

Geometrical MEASURES are of different kinds, as lines, (straight and curved,) surfaces, capacities, and angles.

MEASURE of an *Angle,* is the number of degrees, minutes, &c. contained in the arc of a circle comprised between the two lines forming that angle, its angular point being the centre.

Arithmetical MEASURES, are commonly used to denote numbers which divide other numbers without remainder. When a number measures two or more numbers, it is called a *common measure.*

MEASURE of a *Line,* is its length compared with some determinate line; as a foot, yard, &c.

MEASURE of a *Surface,* is the number of square units contained in it, whether that unit be a foot, a yard, mile, or other quantity.

MEASURE of a *Solid,* is the number of cubic units contained in it; as inches, feet, miles, &c.

MEASURE of a *Number,* is such a number as will divide another number without a remainder.

MEASURE of a *Ratio,* is its logarithm in any system of logarithm; thus, the measure of the ratio 2 : 3, or $\frac{2}{3}$, is the logarithm of $\frac{2}{3}$, or the log. of 2 *minus* the log. of 3.

MEASURE, in *Mechanics,* is used in various ways; as the measure of the *mass*, *momentum*, *velocity*, &c.: for which see the respective articles.

Measures, in *Commerce*, are of various denominations, according to the nature of the things of which the quantity is to be determined.

MECHANICS, that branch of practical mathematics which considers motion and moving powers, their nature, law, effects, &c. This term, in a popular sense, is applied equally to the doctrine of the equilibrium of powers, more properly called *statics*, and to that science which treats of the generation and communication of motion, which constitutes *dynamics*, or mechanics, strictly so called.

This science is divided into practical and rational mechanics, the former of which relates to the mechanical powers; *viz.* the lever, balance, wheel and axis, pulley, wedge, screw, inclined plane, and funicular machine: and the latter, to the theory of motion; showing when the forces or powers are given, how to determine the motion that will result from them; and conversely, when the circumstances of the motion are given, how to trace the forces or powers from which they arise.

The practical uses of the several mechanical powers were undoubtedly known to the ancients, but they were almost wholly unacquainted with the theoretical principles. Vitruvius, in his tenth book, enumerates several ingenious machines, which had then been in use from time immemorial. We find, that for raising or transporting heavy bodies, they employed most of the means which are at present commonly used for that purpose, such as the crane, the inclined plane, the pulley, &c.; but with the theory or true principles of equilibrium, they seem to have been unacquainted till the time of Archimedes. This celebrated mathematician, in his book of "Equiponderants," considers a balance supported on a fulcrum, and having a weight in each scale; and taking, as a fundamental principle, that when the two arms of the balance are equal, the two weights supposed to be in equilibrio are also of necessity equal, he shows, that if one of the arms be increased, the weight applied to it must be proportionally diminished. Hence he deduces the general conclusion, that two weights suspended to the arms of a balance of unequal length, and remaining in equilibrio, must be reciprocally proportional to the arms of the balance; and this is the first trace any where to be met with, of any theoretical investigation of mechanical science. Archimedes also farther observed, that the two weights exert the same pressure on the fulcrum of the balance, as if they were directly applied to it; and he afterwards extended the same idea to two other weights, suspended from other points of the balance; then for two others, and so on; and hence, step by step, advanced towards the general idea of the centre of gravity, a point which he proved to belong to every assemblage of small bodies, and consequently to every large body, which might be considered as formed of such an assemblage. This theory he applied to particular cases, and determined the situation of the centre of gravity in the parallelogram, triangle, trapezium, parabola, parabolic trapezium, &c. &c. To him we are also indebted for the theory of the inclined plane, the pulley and the screw, besides the invention of a multitude of compound machines; of which, however, he has left us no description, and therefore little more than their names remain.

From the time of Archimedes till the commencement of the 16th century, the theory of mechanics appears to have remained in the same state in which it was left by this prince of Grecian science, little or no additions having been made to it during so many ages; but about this time Stevinus, a Flemish mathematician, made known directly, without the introduction of the lever, the laws of equilibrium of a body placed on an inclined plane; he also investigated, with the same success, many other questions in statics, and determined the conditions of equilibrium between several forces concurring in a common point, which comes, in fact, to the famous proposition relating to the parallelogram of forces; but it does not appear that he was at all aware of its consequences and application. In 1592, Galileo composed a treatise on statics, which he reduced to this single principle; *viz*. It requires an equal power to raise two different bodies to heights having the inverse ratio of their weights; that is, whatever power will raise a body of two pounds to the height of one foot, will raise a body of one pound to the height of two feet. On this simple principle he investigated the theory of the inclined plane, the screw, and all the mechanical powers; and Descartes afterwards employed it in considering the statical equilibriums of machines in general, but without quoting Galileo, to whom he had been indebted for the first idea. To Galileo we are also indebted for the theory of accelerated motion; and its complete coincidence with the observed phenomena of nature may be considered as one of the greatest steps made at one time in the science of physics.

Torricelli, a pupil of Galileo, prosecuted the subject after his master, and added several curious propositions concerning projectiles, to those which the latter had previously investigated. Huygens considered the motion of bodies along given curves, and demonstrated that the velocity of a heavy body, which descends along any curve, is the same at every instant in the direction of the tangent, as it would have acquired by falling freely from a height equal to the corresponding vertical absciss. Then applying this principle to the inverted cycloid, the axis of which is vertical, he found that a heavy body, from whatever part of the cycloidal arc it falls, always arrives at the lowest point of that arc in the same space of time. This very remarkable proposition includes what is commonly called the *isochronism* of the cycloid, and would alone have been sufficient to establish the fame of a geometrician. In 1661, Huygens, Wallis, and Sir Christopher Wren, all discovered the true laws of percussion separately, and without any communication with each other, a proposition which Descartes had previously attempted, but failed in giving it a general solution. However, before the discovery of the fluxional calculus, there were many curious and interesting mechanical properties, which the ancient geometry was incompetent to investigate, and which could never have been brought to light but by the assistance of this modern branch of analysis.

MECHANICAL, relating to the theory of mechanics, or that which is performed by means of machines; it is also used in mathematics, to denote a method not strictly scientific, as when the solution of a problem is obtained by means of any other instrument beside the ruler and compasses, or by any art or contrivance not strictly geometrical.

MECHANICAL *Powers*, is a term under which is classed all those simple machines which are separately employed for the purposes of raising great weights, overcoming great resistances, &c.; and from the combination of which the most complex engines are constructed.

Authors have differed as to the number of the mechanical powers, some reckoning only *three*, others *six*, and others again *seven: viz*. the *lever*, *wheel and axle*, *pulley*, *inclined plane*, *screw*, *wedge*, and *funicular machine*.

MEDIUM, in *Philosophy*, denotes that space or region through which a body passes in its motion towards any proposed point; being used in contradistinction to a vacuum, which is a simple void space. Thus, *air*, *water*, *glass*, &c. are mediums of different densities, and possessed of different powers of refraction, resistance, &c.

Ethereal MEDIUM, is a suppositious subtle medium, supposed to occupy every part of space, and in which the planetary motions are performed without resistance, and by means of which light is reflected, inflected, and refracted; heat is propagated and increased; in short, all the great operations of nature are supposed to be carried on through the agency of this universal medium.

MENISCUS, a lens or glass, which is convex on the one side and concave on the other; and is otherwise called a *lune* or *lunula*.

The rule for finding the focus of a meniscus is as follows:

As the difference between the concavity and convexity is to either of them, so is the diameter to the focal length, or distance of the focus from the meniscus; that is, if D and d be the diameters of the two sides, and f the focal distance, then

$$D - d : D = d : f$$

Where,

if $D = 2d$, we shall have $f = D$
if $D = 3d$, then . . . $f = \frac{1}{2} D$
if $D = 5d$, then . . . $f = \frac{1}{4} D$
&c. &c.

And, lastly, if $D = d$, then f is infinite; and therefore a ray falling parallel to the axis, will also continue parallel to it after refraction.

MENSURATION, that branch of mathematics which treats of the measurement of the extensions, capacities, solidities, &c. of bodies.

It is to the Greeks that we seem to be indebted for the fundamental principles of mensuration; which, in its earliest state, was scarcely distinguishable from geometry: and, notwithstanding the perfection which Euclid attained in the latter science, the theory of mensuration had not in his time advanced beyond what related to right-lined figures; and this, so far as regards surfaces, might all be reduced to that of measuring a triangle. The mensuration of solid bodies, however, was of a more varied and complex nature, and gave the celebrated geometrician a greater scope for the exercise of his superior talents; and here still confining himself to bodies bounded by right-lined plane superfices, he was able to perform all that can be done even at this day. With regard to curvilinear figures, he attempted only the circle and the sphere; and if he did not succeed in these, he failed only where there was no possibility of success: but the ratio that such surfaces and solids have to each other, he accurately determined. After Euclid, Archimedes took up the theory of mensuration, and carried it to a much greater extent. He first found the area of a curvilinear space, unless indeed we except the lunules of Hippocrates, which required no other aid than that of the pure geometrical elements. Archimedes found the area of the parabola to be two-thirds of its circumscribing rectangle, which, with the exception above stated, was the first instance of the quadrature of a curvilinear space. He likewise determined the ratio of spheroids and conoids, to their circumscribing cylinders; and has left us his attempt at the quadrature of the circle. He demonstrated that the area of a circle is equal to the area of a right-angled triangle, of which one of its sides about the right angle is equal to the radius, and the other equal to the circumference; and thus reduced the quadrature of the circle to that of determining the ratio of the circumferenee to the diameter, a problem which has engaged the particular attention of the most celebrated mathematicians of all ages, but which remains at present, and in all probability ever will remain, the *desideratum* of geometers; and, at the same time, a convincing and humiliating proof of the limitted powers of the human mind. But though Archimedes failed in establishing the real quadrature of the circle, it is to him we are indebted for the first approximation towards it. He found the ratio between the diameter of a circle, and the periphery of a circumscribed polygon of 96 sides, to be less than 7 to 22, or less than 1 to $3\frac{10}{70}$; but the ratio between the diameter, and periphery of an inscribed polygon of the same number of sides, he found to be greater than 1 to $3\frac{10}{71}$; whence a *fortiori* the diameter of a circle is to its circumference, in a less ratio than 1 to $3\frac{1}{71}$, or less than 7 to 22. Having thus established this approximate ratio between the circumference and diameter, he readily ascertained that of the area of the circle of its circumscribed square, which he found to be nearly as 11 to 14; or rather he made the former depend upon the latter. Archimedes also determined the relation between the circle and ellipse, as well as that of their similar parts; besides which figures he has left us a treatise on the spiral, a description of which will be given under that article.

Some advances were successively made in geometry and mensuration, though but little novelty was introduced into the mode of investigation, till the time of Cavalerius; till his time the regular figures circumscribed about the circle, as well as those inscribed, were always considered as being limited both as to the number of their sides, and the length of each. He first introduced the idea of a circle being a polygon of an infinite number of sides, each of which was of course indefinitely small; solids were supposed to be made up of an infinite number of sections indefinitely thin.

Since that period various improvements have been made both in the theory and the practice of mensuration; and though the former be, along with all the mathematical sciences, rather on the decline in this country, the latter has been very much facilitated and improved by the invention of machines and instruments.

MERCURY, in *Astronomy*, is the nearest planet to the sun, denoted by the character ☿ : its mean distance being ·387, that of the earth being considered as unity; which makes that distance above thirty-six millions of miles.

He performs his sidereal revolution in 87^d 23^h 15′ 43″·9, and his mean synodical revolution in about 116 days. The eccentricity of his orbit is ·2055; half the major axis being taken equal to unity. His mean longitude, at the commencement of the present century, was in 5^s 3° 56′ 27″·0. The longitude of his perihelion was, at the same time, in 2^s 14° 21′ 46″·9; but the line of the apsides has a sidereal motion, according to the order of the signs, equal to 9′ 43″·6 in a century. Which motion, if referred to the ecliptic, will (owing to the precision of the equinoctial points) be equal to 55″·8 in a year; or to 1° 33′ 13″·6 in a century. The orbit of Mercury is inclined to the plane of the ecliptic in an angle of 7° 0′ 9″·1; but this angle is subject to a small increase of about 18″·2 in a century. His orbit, at the commencement of the present century, crossed the ecliptic in 1^s 15° 57′, 30″·9: having a sidereal motion, to the westward every century, of 13′ 2″·2; but if referred to the ecliptic, the place of the nodes will (on account of the precession of the equinoxes) fall more to the eastward by 42″·3 in a year, or 1° 10′ 27″·8 in a century. The rotation on his axis is accomplished in 1^d 0^h 5′ 28″·3, but the inclination of its axis is not known. The diameter of Mercury is about 3123 miles: which, compared with the earth's diameter considered as unity, is about ·3944; his apparent diameter being 6″·9. His mass, compared with that of the sun considered as unity, is $\frac{1}{2025810}$; and the proportion of light and heat received from the sun is about 6·68 times greater than on our planet. As seen from the earth, Mercury never appears at any great distance from the sun, either in the morning or the evening; his elongation, or angular distance, varying only from 16° 12′ to 28° 48′. His course sometimes appears retrogade, the mean arc, which it describes in this case, being about 13° 30′; and its mean duration is about 23 days; but there is great difference in this respect. This retrogradation commences or finishes when he is about 18° distant from the sun. Mercury changes his phases, like the moon, according to his various positions with regard to the earth and sun: but this cannot be discovered without the aid of a telescope. This planet is sometimes seen to pass over the sun's disc: which can only happen when he is in his nodes, and when the earth is in the same longitude; consequently this phenomenon can take place only in the month of May or November.

MERIDIAN, a great circle of the sphere passing through the poles of the world and zenith and nadir, crossing the equator at right angles, and dividing the sphere into an eastern and western hemisphere.

Meridian, in *Geography*, a corresponding terrestrial circle in the plane of the former, and which therefore passes through the poles of the earth.

All places situated under the same meridian have their noon or midnight at the same time; but under different meridians it will arrive sooner or later, according as they are situated to the eastward or westward of each other; *viz.* the sun will be upon that meridian soonest which is most to the eastward, and that at the rate of an hour for every 15 degrees.

First **Meridian** is that from which all the others are reckoned, which being totally arbitrary has been variously chosen by different geographers. Ptolomy makes his first meridian pass through the most western of the Canary Islands, others have chosen Cape Verd, some the Peak of Teneriffe, others the Island of Fero, &c.; but most nations now consider that the first meridian which passes over their metropolis, or their principal observatory. Thus the English reckon from the meridian of London, or rather of Greenwich; the French from that of Paris; the Spanish from Madrid, &c.

Meridian of a *Globe*. See **Globe**.

Meridian *Line*, is a north and south line, the exact determination of which is of the greatest importance in all cases relating to astronomy, geography, dialling, &c. because on this all the other parts have their dependance. The most celebrated meridian line is that on the pavement of the church of St. Petronio in Bologna, which was drawn to the length of one hundred and twenty feet by the celebrated Cassini.

To draw a Meridian Line.—Knowing the south quarter pretty nearly, observe the altitude of some star on the eastern side thereof, not far from the meridian: then, keeping the quadrant firm on its axis, so as that the plummet may still cut the same degree, only directing it to the western side of the meridian, wait till the star has the same altitude as before. Lastly, bisect the angle formed by the intersection of the two planes wherein the quadrant is placed at the time of the two observations, and the line of section is a meridian line.

Meridian *Line* on a dial, is the same as the 12 o'clock hour line.

Meridian *Line* on the Gunter's scale, is a line divided unequally towards 87°, corresponding to the meridian in Mercator's chart.

Magnetic **Meridian**, a great circle passing through the magnetic poles.

Meridian *Altitude*, the altitude of any of the heavenly bodies when they are upon the meridian.

MERIDIONAL *Distance*, in *Navigation*, is the same as **Departure**, or distance between the meridians of two places.

Meridional *Parts*, *Miles*, or *Minutes*, in *Navigation*, are the parts of the increased meridians according to the Mercator chart.

The parts of the enlarged meridians in the Mercator chart increase in proportion to the cosines of the latitudes to radius, or as the radius to the secants, or since the radius is constant they increase simply as the secants; and therefore the whole enlarged meridian for any parallel of latitude will be proportional to the sum of all the secants to that place, which for common purposes are found near enough by adding together the secants of every angle in minutes; that is, the meridional parts.

for $1' = \text{sec. } 1'$
for $2'' = \text{sec. } 1' + \text{sec. } 2'$
for $3''' = \text{sec. } 1' + \text{sec. } 2' + \text{sec. } 3$
&c. &c.

If the earth be considered as a spheroid, the method of computation is a little more varied, though it is attended with the same accuracy as in the former case.

METEOR, denotes all the various phenomena of the atmosphere, while others apply it exclusively to denote those luminous bodies which appear at a considerable height above the earth. These sometimes surpass the moon in splendour, and sometimes equal her in apparent size. They generally proceed in this hemisphere from north to south with vast velocity, frequently breaking into several smaller ones, sometimes vanishing with a report, and sometimes not. These luminous appearances no doubt constitute one part of the ancient prodigies, blazing stars or comets, which last they sometimes resemble in being attended with a train; but frequently they appear with a round and well defined disk. The first of which we have any accurate account was observed in 1719. Its height was computed at about 70 miles from the surface of the earth. The height of others has also been computed, and found to be various; though in general it is supposed to be beyond the limits assigned to our atmosphere, or where it loses its refractive power. The most remarkable appeared about nine in the evening of the 18th of August, 1783. It was seen to the northward of Shetland, and took a southerly direction for an immense space, being observed as far as the southern provinces of France; and one account says that it was seen at Rome also. During its course it frequently changed its shape; at one time appearing in the form of one ball, at another of two or more; sometimes with a train, and sometimes without one. It passed over Edinburgh nearly in the zenith, and had then the appearance of a well defined round body, extremely luminous, and of a greenish colour; the light which it diffused on the ground giving likewise a greenish cast to objects. After passing the zenith, it was attended by a train of considerable length, which continually augmenting, at last obliterated the head entirely; so that it looked like a wedge, flying with the obtuse end foremost. The motion was not apparently swift, by reason of its great height; though in reality it must have moved with great rapidity, on account of the vast space it travelled over in a short time. In other places its appearance was very different. At Greenwich we are told, that two bright balls parallel to each other led the way, the diameter of which appeared to be about two feet: and were followed by an expulsion of eight others, not elliptical; seeming gradually to mutilate, for the last was small. Between each two balls a luminous serrated body extended, and at the last a blaze issued which terminated in a point. Minute particles dilated from the whole. The balls were tinted first by a pure bright light, then followed a tender yellow, mixed with azure, red, green, &c.; which, with a coalition of bolder tints, and a reflection from the other balls, gave the most beautiful rotundity and variation of colours that the human eye could be charmed with. The sudden illumination of the atmosphere, and the form and singular transition of this bright luminary, tended much to make it awful: nevertheless the amazing vivid appearance of the different balls, and other rich connective parts not very easy to delineate, gave an effect equal to the rainbow in the full zenith of its glory.

Besides those, there are other fireballs much smaller and nearer the surface of the earth, rolling upon it, or falling upon it, exploding with violence, as is the case with those which appear in the time of thunder, and frequently produce mischievous effects. One of these is mentioned by some authors as falling in a serene evening in the island of Jamaica; exploding as soon as it touched the surface of the ground, and making a considerable hole in it. Another is mentioned by Dr. Priestley, as rolling along the surface of the sea, then rising and striking the topmast of a man of war, exploding, and damaging the ship. In like manner we hear of an electrified cloud at Java in the East Indies; whence, without any thunderstorm, there issued a vast number of fireballs, which did incredible mischief. This last phenomenon points out to us the true origin of balls of this kind, *viz.* an excessive accumulation of electricity in one part, or a violent tendency to circulate, when at the same time the place where the motion begins is at so great a distance, or meets with other obstacles of such a nature, that it cannot easily get thither. Urged on, however, by the vehement pressure from behind, it is forced to leave its place, but being equally unable to displace the great quantity of the same fluid, which has no tendency to move the same way with itself, it is collected into balls, which run hither and thither, according as they meet with conductors capable of leading them, into some part of the circle. This is even confirmed by an experiment related at the end of Dr. Priestley's fifth volume on Air. He relates, that a gentleman having charged, with a very powerful machine, a jar, which had the wire supporting the knob of a considerable length, and passed through a glass tube, a globe of fire was seen to issue out of it. This globe gradually ascended up the glass tube till it came to the top of the knob, where it settled, turning swiftly on its axis, and appearing like a red-hot iron ball of three quarters of an inch diameter. On continuing to turn the machine, it gradually descended into the jar; which it had no sooner done, than there ensued a most violent explosion and flash, the jar being discharged and burst at the same time.

METEOROLOGY, the science which treats of meteors; the state of the weather. This science, though of the utmost importance in a country where the climate is so variable as in England, is yet in its infancy. The only means by which it can be brought to perfection are careful and long continued observations.

MICROMETER, is an instrument fitted to telescopes in the focus of the object-glass for measuring small angles or distances, as the apparent diameters of the planets, &c. Various forms have been given to this instrument by different authors; but under all the forms the principle of operation is the same, which is, that it moves a fine wire parallel to itself, in the plane of the picture of an object, formed in the focus of the telescope; and with such accuracy as to measure with the greatest precision its perpendicular distance from a fixed wire in the same plane, by which means the apparent diameters of the planets and other small angles are exactly determined.

Divided Object-Glass MICROMETER. This instrument consists of a convex lens divided into two equal parts, by a plane which passes through its axis; and the segments are moveable on a graduated line, perpendicular to that axis.

MICROSCOPE, an instrument for magnifying small objects, by means of a proper adjustment and combination of lenses or mirrors.

Microscopes are divided into *single, compound, reflecting, solar,* &c.

Single or *Simple* MICROSCOPE, is one which consists of a single lens. In this, if the angle which subtends at the centre of the eye, when at a proper distance for distinct vision, be less than a certain limit, that is, if it be less than $4'$, it will only appear to the eye like a physical point; and if we endeavour to increase the image by bringing it nearer to the eye, the extreme rays will diverge so much as to render the object indistinct; and if the extreme rays be sloped, there will then be too little light to make the object distinctly visible; the microscope is, therefore, introduced in order to supply these defects of natural vision, which is effected by placing the object in the principal focus of a glass spherule or lens, whose focal length is short; in which case it may be distinctly seen, the visual angle, as well as the quantity of light admitted into the eye, being increased, and the parts which before appeared only as a physical point, may now be subjected to examination; the visual angle thus formed, being to that formed at the naked eye, at the least distance of distinct vision, as that distance is to the focal length of the glass.

Exam. If the focal length of the glas be $\frac{1}{50}$ of an inch, and at the least distance of distinct vision eight inches, the visual angle of the object when viewed through the glass: the visual angle when it is seen with the naked eye $= 8 : \frac{1}{50} = 400 : 1$.

Double or *Compound* MICROSCOPE, is one which consists of a combination of two or more lenses.

Reflecting MICROSCOPE, magnifies the object by reflection.

Solar MICROSCOPE, is a single convex lens used in the same manner as in a magic lantern; only the object is illuminated by the solar rays entering into a hole in the shutter of a dark room; in other respects, there is no difference in the principles of these two instruments.

MICROSCOPIC *Objects*, those things which are too small to be seen distinctly by the naked eye.

MIDDLE *Latitude*, is half the sum of two given latitudes when they are both in the same hemisphere, or half their difference if they are in different hemispheres; in which latter case it will always be of the same name as the greater.

MILE, a long measure, which the English, Italians, and some other nations use to express the distance between places; the same as the French use the word *league*.

The mile is of different lengths in different countries. The geographical, or Italian mile, contains 1000 geometrical paces, *mille passus*, whence the term mile is derived. The English mile consists of 8 furlongs, each furlong of 40 poles, and each pole of 16½ feet; so that the mile = 8 furlongs = 320 poles = 1760 yards = 5280 feet.

	Geomet. Paces.
Mile of Russia	750
——— Italy	1000
——— England	1200
——— Scotland and Ireland	1500
Old league of France	1500
Small league, ibid	2000
Mean league of France	2500
Great league, ibid	3000
Mile of Poland	3000
——— Spain	3428
——— Germany	4000
——— Sweden	5000
——— Denmark	5000
——— Hungary	6000

MILL, is properly an engine for grinding corn and other substances; but the same term is also used to denote a variety of machines, particularly if the first mover be either wind or water.

MINE, in *Fortification*, is a subterraneous passage, which is dug for the purpose of exploding or blowing up the works of fortified places. The passage leading to the powder is called the *gallery*, and the place where the powder is confined the *chamber*, the line drawn from the centre of the chamber perpendicular to the nearest surface is called *the line of least resistance*, and the part blown up the *excavation*.

MINUTE, the sixtieth part of a degree; or, in time, the sixtieth part of an hour.

MIRAGE, an optical phenomenon. It often happens at sea, that a ship seen at a distance appears as if painted in the sky, and not supported by the water. A similar effect was observed by the French in the course of their march through the Desert. The villages seen at a distance, seemed to be built on an island in the middle of a lake. In proportion as they approached, the apparent surface of the water became narrower, and ultimately disappeared entirely; while the same illusion was repeated on a village at a little farther distance.

This phenomenon has been variously accounted for by different philosophers. Monge ascribes the effects to a diminution of density in the lower stratum of the atmosphere. This, in the Desert, is produced by the increase of heat, arising from that communicated by the rays of the sun to the sand with which this stratum is in immediate contact. At sea it takes place when, by particular circumstances, such as the action of the wind, the lower stratum of the atmosphere holds in solution a greater quantity of water than the other strata. In this state of things, the rays of light which come from the lower part of the heavens, having arrived at the surface that separates the less dense stratum from those above it, do not pass through that stratum, but are reflected, and paint in the eye of the observer an image of the heavens; which appearing to him to be below the horizon, he takes it for water when the phenomenon occurs on land. And if at sea, he thinks he sees in the heavens all those objects which float on that part of the surface occupied by the image of the heavens.

MIRROR, a speculum, or polished body, which exhibits the images of objects by reflection.

Mirrors are either *plane, concave*, or *convex*.

Plane MIRRORS, are those in which the reflecting surface is a plane; such are our looking-glasses, or any other plain polished surface.

Convex MIRROR, is that whose reflecting surface is convex; such are those ornamental glasses commonly displayed in parlours and drawing-rooms.

Concave MIRROR, is that whose reflecting surface is concave; and in both these cases they are commonly segments of spheres: these are also called BURNING *Mirrors*, which see.

The ancient mirrors were commonly made of some kind of metal, which when well polished reflect very powerfully; but since the invention of glass, and the application of quicksilver and tinfoil to it have become generally known, it has been universally employed for those plane mirrors used as ornaments in houses; but in making reflecting telescopes, they have been found much inferior to metallic ones. It does not seem, however, that the same superiority belongs to the metallic burning mirrors, considered merely as burning speculums; since the mirror with which Mr. Macquer melted platina, though only twenty-two inches diameter, and made with quicksilver, produced much greater effects than M. Villette's metal speculum, which was of a much larger size.

A few of the most remarkable properties of reflecting mirrors are as follow:

1. If rays proceeding from a radiant point fall upon a reflecting plane surface, the focus of the reflected rays will be in the line perpendicular to the reflecting surface, and as far on the other side of it

2. If rays proceeding from a radiant point, fall upon any point of a reflecting spherical surface, the focus of the reflected rays will be on the other side of the centre, and the two rays will make equal angles with a radius at the point of impact.

3. If, on a spherical surface, a ray fall parallel to the axis, the reflected ray will bisect the secant.

4. Wherever the rays, which come from all the points of an object, meet again in so many other points, after they have been made to converge by reflection, they will there make a picture of the object upon any white surface.

5. In a convex speculum, the image always appears erect and on the other side of the glass, as in a plain speculum; and so it does in a concave one, when the object is between the glass and the principal focus. But, in a concave speculum, when the object is farther from it than the principal focus, the image is on the same side of the glass, and inverted in every respect.

MODULUS, in the *Theory of Logarithms*, is that number which connects any system of logarithms with the hyperbolic system, or that number by which the hyperbolic number of a logarithm must be multiplied, or by the reciprocal of which it must be divided, in order to transform it to another system; and this modulus is, therefore, always the reciprocal of the hyperbolic logarithm of the radix of that system to which the modulus belongs.

The modulus of the hyperbolic system is 1, this being the reciprocal of the hyperbolic logarithm of 2·71828182, the radix of this system; and the modulus of the common logarithmic system is·43429448, its reciprocal 2·3025809, which is the hyperbolic logarithm of 10, the radix of this system.

It is shown, under the article *Logarithms*, that log. $a =$

$$\frac{(a-1) - \frac{1}{2}(a-1)^2 + \frac{1}{3}(a-1)^3 \text{ \&c.}}{(r-1) - \frac{1}{2}(r-1)^2 + \frac{1}{3}(r-1)^3 \text{ \&c.}}$$

where r is the radix and may be assumed at pleasure, and the reciprocal of this whole denominator is called the modulus. In the hyperbolic system, the whole denominator is assumed 1, which makes

$r = 2{\cdot}71828182$; and in the common system, r is assumed 10, and the whole series becomes $= 2{\cdot}3025809$, which is the reciprocal of the modulus; and since in the former case we have

$$\text{hyp. log. } a = (a-1) - \tfrac{1}{2}(a-1)^2 + \tfrac{1}{3}(a-1)^3, \text{ \&c.}$$

it follows also, that in any other

$$\text{hyp. log. } r = (r-1) - \tfrac{1}{2}(r-1)^2 + \tfrac{1}{3}(r-1)^3 \text{ \&c.}$$

whatever be the value of r; therefore, in every system, the modulus is the reciprocal of hyp. log. of the radix.

MODULUS is also used in the theory of numbers, to denote a measure of comparison; thus we say, all odd squares are of the same form to *modulus* 4; meaning that all odd squares divided by 4, will leave the same remainder.

MOMENT, an indefinite small portion of time, having the same relation to duration as a point has to a line.

MOMENT, in the *Modern Analysis*, is the same as *Infinitesimal*, *Increment*, or *Decrement;* for an explanation of which see those terms.

MOMENTUM, in *Mechanics*, is the same with impetus, or quantity of motion, and is generally estimated by the product of the velocity and mass of the body. This is a subject which has led to various controversies between philosophers, some estimating it by the mass into the velocity, as stated above, while others maintain that it varies as the mass into the square of the velocity. But this difference seems to have arisen rather from a misconception of the term, than from any other cause. Those who maintain the former doctrine, understanding momentum to signify the momentary impact; and the latter, as the sum of all the impulses till the motion of the body is destroyed.

MONOMIAL, in *Algebra*, is a quantity consisting only of one term; as $a\,x$, $3\,b\,x^2$, &c.

MONSOON, a periodical wind, which blows half the year one way, and the other half the contrary way. The regions where those monsoons are most regular and conspicuous is on the southern shores of Asia, where they blow from the south during the summer, and from the north during winter. The former is caused by the rarefaction of the heated air over the Asiatic continent when the sun is on the north side of the line; and the latter by the condensation over the same when the sun is on the south side of the line.

MONTH, the twelfth part of the year. It is so called from the moon, by whose motions it was regulated, being properly the time in which the moon passes through the zodiac. The lunar month is either *illuminative, periodical,* or *synodical.—Illuminative.* The interval between the first appearance of one new moon and that of the next following. As the moon appears sometimes sooner after one change than after another, the quantity of the illuminative month is not always the same. The Turks and Arabs reckon by this month. *Periodical.* The time in which the moon runs through the zodiac, or returns to the same point again; the quantity of which is $27^d\ 7^h\ 43^m\ 8^s$.—*Synodical* (called also a lunation). The time between two conjunctions of the moon with the sun, or between two new moons; the quantity of which is $29^d\ 12^h\ 44^m\ 3^s\ 11'''$.

The ancient Romans used lunar months, and made them alternately of twenty-nine and thirty days. They marked the days of each month by three terms; viz. *Calends*, *Nones*, and *Ides*.

Solar MONTH, is the time in which the sun runs through one entire sign of the ecliptic; the mean quantity of which is $30^d\ 10^h\ 29^m\ 5^s$, being the twelfth part of $365^d\ 5^h\ 49^m$, the mean solar year.

Astronomical or *Natural* MONTH, is that measured by some exact interval corresponding to the motion of the sun or moon; such are the lunar and solar months above mentioned.

Civil or *Common* MONTH, is an interval of a certain number of whole days, approaching nearly to the quantity of some astronomical month. These may be either lunar or solar.

The *Civil Lunar* MONTH consists alternately of twenty-nine and thirty days. Thus will two civil months be equal to two astronomical ones, abating for the odd minutes; and so the new moon will be kept to the first day of such civil months for a long time together. This was the month in civil or common use among the Jews, Greeks, and Romans, till the time of Julius Cæsar.

The *Civil Solar* MONTH, consisted alternately of thirty and thirty-one days, excepting one month of the twelve, which consisted only of twenty-nine days; but every fourth year of thirty days. And this form of civil months was introduced by Julius Cæsar. Under Augustus, the sixth month, till then from its place called Sextilis, received the name Augustus, now August, in honour of that prince; and to make the compliment still the greater, a day was added to it, which made it consist of thirty-one days, though till then it had only contained thirty days; to compensate for which, a day was taken from February, making it consist of twenty-eight days, and twenty-nine every fourth year. And such are the civil or calendar months now used through Europe.

MOON, in *Astronomy*, one of the heavenly bodies, the only satellite attendant on our earth, about which she revolves as a centre, illuminating us by her reflected light in the absence of the sun.

The motions of the moon are exceedingly eccentric and irregular. She performs her mean *sidereal revolution* in $27^d\ 7^h\ 43^m\ 11''\cdot5$; but this period is variable, and a comparison of the modern observations with the ancient proves incontestibly an *acceleration* in her mean motion. Her mean *tropical* revolution is $27^d\ 7^h\ 43^m\ 4''\cdot7$, and her mean *synodical* revolution $29^d\ 12^h\ 44^m\ 2''\cdot 8$. Her *mean distance* from the earth is 29·982175 times the diameter of the terrestrial equator, or about 237000 miles. The eccentricity of her orbit is ·0548553; the mean distance from the earth being taken equal to unity; but this eccentricity is variable in each revolution. Her *mean longitude*, at the commencement of the present century, was in 3^s 21° 30′ 42.″ 1. Her *velocity* varies in different parts of her orbit; she is swiftest in her perigee (or point nearest the earth,) and slowest when in her apogee (or point farthest from the earth:) her mean diurnal velocity is equal to 13° 10′ 34″ 9, or about thirteen times greater than that of the sun. The greatest *equation* of her *centre* is 6° 17′ 54″·5. The mean longitude of her *perihelion* was, at the commencement of the present century, in 8^s 26° 6′ 5″·1; but the line of the apsides has a motion according to the order of the signs. The period of a sidereal revolution of the apsides, in $3232^d\ 13^h\ 56'\ 16''\cdot8$, or nearly nine years. The period of a tropical revolution of the apsides, is but $3231^d\ 11^h\ 24'\ 8''\cdot6$. But these periods are not uniform; for they have a secular irregularity, and are retarded whilst the motion of the moon itself is accelerated. The period of an *anomalistic* revolution of the moon is $27^d\ 13^h\ 18'\ 37''\ 4$. Her orbit is *inclined* to the plane of the ecliptic in an angle of 5° 9′; but this inclination is variable. The greatest inequality, which sometimes extends to 8′ 47″ 1, is proportional to the cosine of the angle on which the inequality in the motion of the nodes depends. Her orbit, at the commencement of the present century, crossed the ecliptic in 0^s 15° 55′ 26″·3; but the places of this crossing, that is, of her *nodes*, is variable; these having a retrograde motion, and make a sidereal revolution in $6793^d\ 10^h\ 6'\ 30''\ 0$, or in about 18·6 Julian years. This variation, however, is subject to many inequalities; of which, the greatest is proportional to the sine of double the distance of the moon from the sun, and extends to 1° 37′ 45″·0 at its maximum. A synodical revolution of the nodes is performed in $346^d\ 14^h\ 52'\ 43''\cdot6$; but the motion of the nodes is subject also to a secular inequality, dependent on the acceleration of the moon's mean motion. The *rotation* of the moon on her axis is equal and uniform, and is performed in the same time as the tropical revolution in her orbit; whence she always presents nearly the same face to the earth. But as the motion of the moon in her orbit is periodically variable, we sometimes see more of her eastern edge, and sometimes more of her western edge; which appearance is called the *libration* of the moon in *longitude*. The *axis* of the moon is inclined to the plane of the ecliptic in an angle of 88° 29′ 49″; in consequence of which position of the moon, her poles alternately become visible to and obscured from us: and this phenomenon is called her *libration* in *latitude*. There is also another optical deception arising from the moon being seen from the surface of the earth, instead of the centre, which is called her *diurnal libration*. There are also other inequalities in the moon's motion, arising from the action and influence of the sun. The principal of these are as follows:

1. The *evection;* whose constant effect is to diminish the equation of the centre in the syzigies, and to augment it in the quadratures. If this diminution and increase were always the same, the evection would depend only on the angular distance of the moon from the sun; but its absolute value varies also with the distance of the moon from the perigee of its orbit. After a long series of observations, we are enabled to represent this inequality by supposing it equal to the sine of double the distance of the moon from the sun, minus the distance of the moon from its perigee. At its maximum it amounts to 1° 18′ 2″·4. 2. The *variation;* which disappears in the syzigies and quadratures, and is greatest in the octants, being then equal to 31′ 44″·1; whence it is proportional to the sine of double the distance of the moon from the sun. Its duration is half a synodical revolution of the moon. 3. The *annual equation;* which follows exactly the same law as the equation of the centre of the sun, with a contrary sign; for when the earth is in its perihelion, the orbit of the moon is enlarged by the action of the sun; and the moon therefore requires more time to perform her revolution; but as the earth proceeds towards its aphelion, the moon's orbit contracts; whence the period of this inequality is an anomalistic year: and at its maximum, it amounts to 11′ 15″·9; it is, however, subject to a secular inequality. The *figure* of the moon is that of an oblate spheroid, like the earth; her mean *diameter* being in the proportion to that of the earth as 5823 to 21332, or as 1 to 3·665. Whence this will be about 2160 miles. Her *volume*, compared with that of the earth, is $\frac{1}{49}$; but her *mass* is only $\frac{1}{68\cdot3}$. The *apparent diameter* of the moon varies according to her distance from the earth; when nearest to us it is 33′ 31″·1; but at her greatest distance it is 29′ 21″·9; so that her mean apparent diameter is 31′ 26″·5, and her mean *horizontal parallax* is equal to 57′ 34″·2.

The *phases* of the moon are caused by the reflection of the sun's light, and depend on the relative positions of the sun, the earth, and the moon.

An *eclipse* of the moon can take place only at the time of her opposition to the sun; and is caused by her passing through the shadow of the earth; which shadow is three and a half times longer than the distance between the moon and the earth; and its breadth, where it is traversed by the moon, is about two and two-thirds times greater than the diameter of the moon: the breadth of the earth's shadow, where it is traversed by the moon, is equal to the difference between the semi-diameter of the sun, and the sum of the horizontal parallaxes of the sun and moon.

The moon cannot be eclipsed, however, if her distance from the place of her node, at the time of her opposition, exceeds 13° 21′: but if it is within 7° 47′, there will certainly be an eclipse. The duration of the eclipse will depend on the apparent diameter of the moon, and on the breadth of the shadow at the point where she traverses it.

The sun cannot be eclipsed unless the moon be in conjunction; and then only when the centres of the sun and moon are in a line, or nearly in a line with the eye of the spectator on the earth. In the first case, if the apparent diameter of the moon be greater than that of the sun, the eclipse will be *total;* but if it be less, it will be *annular*. In other cases, the eclipse will be *partial*.

The sun cannot be totally obscured for a longer period of time than four minutes; but the moon may be obscured for a much longer period.

Eclipses, of which there cannot be less than two in a year, nor more than seven, generally in the same order and magnitude at the end of 223 lunations; for in that number of synodical revolutions there are $6585^{d}\ 7^{h}\ 42^{m}\ 31''{\cdot}7$, and $6585^{d}\ 18^{h}\ 41^{m}\ 45''{\cdot}6$ there are nineteen mean synodical revolutions of the moon's nodes; therefore at the end of $6585^{d}\ 7^{h}\ 24^{m}\ 31''{\cdot}7$ the moon's mean longitude will be only $28'\ 32''$ behind the mean place of her nodes.

The atmosphere of the moon, if it has any, must be extremely attenuated, more rare than we can produce with our best air-pumps. The light of the moon is only one 300000th part of that of the sun; and the refraction of the rays of light, at the surface of our earth, must be at least 1000 times greater than at the surface of the moon: this, at least, is the opinion of many of our most celebrated astronomers, and it would perhaps be thought presumptive in us to doubt such authority.

When we look at the surface of the moon with a good telescope, we find its surface surprisingly diversified. Besides the large dark spots that are visible to the naked eye, we perceive extensive valleys, and long ridges of high elevated mountains projecting their shadows on the plains below. Single mountains here and there rise to a great height, while hollows, which from accurate measurement are found to be three miles deep, and almost perfectly circular, are excavated in the plains. The margin of these circular cavities is frequently elevated a little above the general level, and a lofty eminence rises in the centre of the cavity.

When the moon approaches to her opposition with the sun, the elevations and depressions on her surface in a great measure disappear, while her disc is marked with a number of brilliant points and permanent radiations.

These various appearances have been accurately represented in maps of the moon's surface. This was first attempted, but in a very rude manner, by Riccioli. Hevelius, in his "Selenographia," afterwards gave more just delineations of the lunar disc, during the whole of her progress round the earth. A map of the full moon was drawn by Cassini, and has been copied, though extremely incorrect, into most of our modern treatises of astronomy. Excellent drawings of the moon were also made by Tobias, Mayer, and by Mr. Russel; but the most accurate and complete are those of the celebrated Schroeter, who has given highly magnificent views of several parts of the moon's surface. The most favourable time for viewing the lunar disc, is when she is about five days old; the irregularities in her surface being then the most conspicuous.

It is difficult to say, with any degree of probability, what the immense cavities, of which we have been speaking, are, or may have been; but we cannot help thinking that our earth would assume nearly the same appearance if all the lakes and seas were removed; and who can say but that this may have been the case in the lunar regions? Astronomers have formerly supposed that the dark part of the moon's surface were large lakes and seas; but it is obvious, on an attentive observation with a good telescope, that there are ridges and unevennesses in those parts, which plainly indicate that they are not fluid but solid, like the other parts of her surface; whence it appears, that there is very little of any fluid matter in this luminary, and hence probably is the reason that her atmosphere is so different from our own; if, indeed, this can be justly inferred from the circumstances usually adduced as arguments in favour of this hypothesis, which we confess appears very doubtful.

We have observed, that mountains may always be seen on the surface of the lunar disc; and even volcanoes have been frequently observed, from which some philosophers have supposed the ærolìths, that at times fall to the earth, to have been projected: which it appears, from computation, would only require a velocity of about 6200 feet per second, to cause them to pass from this body to the earth.

Age of the MOON, is the number of days since the new moon, which is found by the following rule: To the epact add the number and day of the month, which will be the age required, if less than thirty; and if it exceed thirty, subtract this number from it, and the remainder will be the age.

Harvest MOON, is a remarkable phenomenon relating to the rising of this luminary in the harvest season. During the time she is at the full, and for a few days before and after, in all about a week, there is less difference in the time of her rising between any two successive nights at this than at any other time of the year. By this means she affords an immediate supply of light after sun-set, which is very beneficial in gathering in the fruits of the earth; and hence it is, that this lunation has been termed the harvest moon.

In order to conceive this phenomenon, it may first be considered, that the moon is always opposite to the sun when she is full; that she is full in the signs Pisces and Aries in our harvest months, these being the signs opposite to Virgo and Libra, the signs occupied by the sun about the same season; and because those parts of the ecliptic rise in a shorter space of time than others, as may easily be shown and illustrated by the celestial globe; consequently, when the moon is about her full in harvest, she rises with less difference of time, or more immediately after sun-set, than when she is full at other seasons of the year.

MOON *Dial*, is a dial which shows the hours of the night by the light of the moon.

MORTALITY. Bills of mortality are accounts or registers specifying the number of births and burials which happen in any town, city, or parish, and on which is founded the doctrine of *Life Annuities* and *Assurances*.

MOTION, or *Local Motion*, in *Mechanics*, is a continued and successive change of place, or it is that affection of matter by which it passes from one point of space to another. Motion is of various kinds, as follows:

Absolute MOTION, is the absolute change of places in a moving body independent of any other motion whatever; in which general sense, however, it never falls under our observation. All those motions which we consider as absolute, are in fact only relative; being referred to the earth, which is itself in motion. By absolute motion, therefore, we must only understand that which is so with regard to some fixed point upon the earth; this being the sense in which it is delivered by writers on this subject.

Accelerated MOTION, is that which is continually receiving constant accessions of velocity. See ACCELERATED *Motion*.

Angular MOTION, is the motion of a body as referred to a centre, about which it revolves.

Compound MOTION, is that which is produced by two or more powers acting in different directions. See PARALLELOGRAM of *Forces*.

Equable MOTION, or *Uniform Motion*, is when the body moves continually with the same velocity, passing over equal spaces in equal times.

Natural MOTION, is that which is natural to bodies, or that which arises from the action of gravity.

Relative MOTION, is the change of relative place in one or more moving bodies; thus two vessels at sea are in *absolute motion* (according to the qualified signification of this term) to a spectator standing on the shore, but they are only in relative motion with regard to each other.

Retarded MOTION, is that which suffers continual diminution of velocity, the laws of which are the reverse of those for accelerated motion. See **Acceleration** and **Retardation.**

Projectile **Motion**, is that which is not natural, but impressed by some external cause; as when a ball is projected from a piece of ordnance, &c.

Rectilinear **Motion**, that which is performed in right lines.

Rotatory **Motion**. See *Rotation.*

Laws of **Motion**, as delivered by Newton in his "Principia," and on which he has supported the whole system of his philosophy, are the three following:

1. Every body perseveres in its state of rest or uniform motion in a right line, until a change is effected by the agency of some external force.

2. Any change effected in the quiescence or motion of a body is in the direction of the force impressed, and is proportional to it in quantity.

3. Action and reaction are equal and in contrary directions.

When speaking of these axioms, or laws of motion, it ought always to be recollected that they are not the efficient operative causes of any thing. A law presupposes an agent; for it is only the mode, according to which an agent proceeds: it implies a power, for it is the order according to which that power acts. Abstracted from this agent, this power, the law does nothing, is nothing: so that a law of nature or of motion can never be assigned as the adequate cause of phenomena, exclusive of power and agency.

The Newtonian axioms are, in reality, intermediate propositions between geometry and philosophy; through which mechanics becomes a mathematical branch of physics, and its conclusions possessed of such coherence and consistency among themselves, and with matter of fact, as are rarely to be found in other branches, which admit not of so intimate a union with the science of quantity.

The evidences from which our assent to these axioms is derived, are of various kinds. 1. From the constant observation of our senses, which tend to suggest the truth of them in the ordinary motion of bodies, as far as the experience of mankind extends. 2. From experiments, properly so called. 3. From arguments *à posteriori*. One or other of these kinds of evidence generally forms a part of our most valuable treatises on mechanics and physics: but there is a fourth way in which the truth of these axioms may be deduced, which is that in which they are shown to be laws of human thought, intuitive consequences of the relations of those ideas which we have of motion, and of the causes of its production and changes.

Sir R. Phillips, in accordance with his new views on the mechanism of the universe, enumerates the following new laws of motion:

1. Matter never originates its own motion.

2. All motion acquired by one body, is the result of motion transferred from some other moving body or bodies.

3. All motion lost by one body is transferred to some other body or bodies.

4. In the material universe, no motion is created or lost, but is in a continued state of transfer, and thereby produces material changes, or phenomena.

5. *Action* is the transferring of motion from the agent to the patient.

6. *Re-Action* is the reception of motion by the patient from the agent.

7. Action and re-action are, therefore, convertible terms; and necessarily equal.

8. Every body continues in motion till it has parted with its motion to other bodies.

9. Parting with motion is effected by impinging, or by contact.

10. *Resistance* is the parting with motion to the atoms of a fluid, or solid, through which the moving body passes; and it is proportional to the density and the cohesion or viscidity of the patient.

11. *Friction* is the parting with motion by continuous contact; and is proportioned to the number of

atoms of the agent opposed to the atoms of the patient.

12. Quantity of motion, (or *Momentum*) is the velocity multiplied by the number of atoms moved.

13. Impulse of motion is the transfer of motion, either by contact, or the intervention of a lever.

14. All motion received by impulse, is in the direction of the impulse.

Definition.—The matter which lies between a body moving, and one moved, is called a *lever;* and levers may consist of either fixed, or fluid, or gaseous matter.

15. Levers of fixed matter transfer the entire impulse of the agent; and resulting velocity of the patient to that of the agent is as their number of atoms.

16. Either gaseous or fluid levers, in propagating or conveying the motion or impulse of the said agent, radiate or diffuse it; and the momentum is, therefore, at different distances, inversely as the squares of the distances.

17. Through both species of lever, the direction of the motion acquired by the patient, is the direction of the simultaneous motion of the agent.

MOVEMENT, is sometimes used in the same sense as motion, but more commonly it implies the internal parts of machines, particularly of clocks and watches.

MULTANGULAR *Figure,* one that has many angles.

MULTILATERAL *Figure,* a figure of many sides.

MULTINOMIAL, an algebraical quantity consisting of more than three terms; though this last is sometimes called a multinomial, but more commonly a trinomial.

MULTINOMIAL *Theorem,* is a general expression or formula, for determining any power or root of a given quantity consisting of any number of terms.

The general form of this theorem is as follows:

$$(A_0 + A_1x + A_2x^2 + A_3^{x3} \ldots\ldots A_nx^n)^m =$$

$$A_0^m + mA_1B_0 \cdot \frac{x}{A_0} + \left.\begin{matrix} 2mA_2B_0 \\ +(m-1)A_1B_1 \end{matrix}\right\} \frac{x^2}{2A_0}$$

$$\left.\begin{matrix} + & 3mA_3B_0 \\ + & (2m-1)\,A_2B_1 \\ + & (m-2)\,A_1B_2 \end{matrix}\right\} \frac{x2}{3A_0}$$

$$\left.\begin{matrix} + & 4mA_4B_0 \\ + & (3m-1)\,A_3B_1 \\ + & (2m-2)\,A_2B_2 \\ + & (m-3)\,A_1B_3 \end{matrix}\right\} \frac{x^2}{4A_0} + \&c.$$

Where $B_0 = A_0^m$; and B_1, B_2, B_3, &c. are the co-efficients of the terms immediately preceding those in which they first appear. And the manner of applying this theorem to any particular case, is by substituting the numbers or letters in the given example, for A_0, A_1, A_2, &c.; and m, as in the binomial theorem, which see.

Thus, for example, let it be proposed to cube the series

$$1 + x + x^2 + x^3 + x^4 + \&c.$$

Here $A_0 = 1$, $A_1 = 1$, $A_2 = 1$, &c. and $m = 3$.

Whence

$$A_0^m = 1^m = 1 = B_0$$

$$mA_1B_0 = 3 \times 1 \times 1 = 3 = B_1$$

$$\frac{2mA_2B_0 + (m-1)A_1B_1}{2A} = \frac{6+2\cdot3}{2} = 6 = B_2$$

$$\frac{3mA_3B_0 + (2m-1)A_2B_1 + (m-2)A_1B_2}{3A_0} = \frac{9+15+6}{3} = B_3, \&c.\ \&c.$$

Therefore

$$1 + 3x + 6x^2 + 10x^3 + 15x^4 + \&c.$$

is the cube required.

Required the cube root of the series $1 + \frac{1}{2}x + \frac{1}{3}x^2 + \frac{1}{4}x^3 + \&c.$

Here $A = 1$, $A = \frac{1}{2}$, $A = \frac{1}{3}$ &c. and $m = \frac{1}{3}$.

Whence

$$A_0^m = 1^{\frac{1}{3}} = 1 = B_0$$

$= \frac{1}{6} = B_1$

$1)\ AB_{11}$

$= \frac{1}{12} = B_2$

$)\ AB_{21} + (m-2)\ AB_{12}$

$3A_0$

$= \frac{35}{648} = B_3$, &c. &c.

Therefore

$1 + \frac{1}{6}x + \frac{1}{12}x^2 + \frac{35}{648}x^3 +$ &c.

is cube root of the series proposed. See Bonnycastle's Algebra.

MULTIPLE, in *Arithmetic*, is a number which contains another number a certain number of times.

Common MULTIPLE of two or more numbers, is that which contains those numbers a certain number of times.

To find the least common Multiple of several Numbers.—Divide them repeatedly by numbers taken by trial, till no number will divide two of the remainders, then the product of all the divisors and remainders is the least common multiple.

Example.—Required the least common multiple of 7, 35, 42, 30, and 24.

÷ by 7	7, 35, 42, 20, 24
÷ by 3	1, 5, 6, 20, 24
÷ by 5	1, 5, 2, 20, 8
÷ by 2	1, 1, 2, 4, 8
÷ by 2	1, 1, 1, 2, 4
	1, 1, 1, 1, 2

Then 7 × 3 × 5 × 2 × 2 × 2 = 840, the least common multiple.

MULTIPLICAND, in *Arithmetic*, whatever is multiplied.

MULTIPLICATION, is one of the principal operations in arithmetic and algebra; and consists in finding the amount of a given number or quantity, called the multiplicand, when repeated a certain number of times expressed by the multiplier; and this amount is generally termed the product; also the multiplier and multiplicand are commonly called factors. Multiplication is either simple or compound.

MULTIPLICATION is *Simple* when the proposed quantities are integral numbers. In this the multiplicand must be multiplied by each figure of the multiplier, and the products added together, each product must be placed as far to the left as the figures from which it arises, reckoning both in the multiplier and multiplicand.

Proof of MULTIPLICATION. 1. Invert the operation, by making the multiplier and multiplicand change places, and if you thus obtain the same result, it is highly probable the work is right.

2. Cast out all the 9's from the multiplier, multiplicand, and product; and multiply the overplus of the two former together, and cast the 9's out of this product; then if this remainder be the same as that arising from the total product, the operation is probably right; but if not, it is certainly wrong. The error must, when this proof answers, always be 9, or a multiple of 9.

This proof depends upon this property, that any number divided by 9, will leave the same remainder, as the sum of its digits when divided by the same number.

3. Another proof is, the 1st, 3d, 5th, &c. being taken from the sum of the digits in the 2d, 4th, &c. places; the remainder, when divided by 11, will leave the same overplus, as the whole number when divided by 11. If the former sum be greater than the latter, as many times 11 must be added to it, as will render the latter sum the greater of the two. This being observed, the proof by this number will be the same as in the former case.

EXAMPLE.

45684 multiplicand
 4374 multiplier

182736
319788
137052
182736

199821816 product.

The other proof by inverting the operation depends upon this; that the product of two numbers is the same which ever of the two is the multiplier; or generally that a times b, is the same as b times a; which, though generally considered as an axiom, is, in fact, a proposition, and one that is not very easily demonstrated, except by a geometrical diagram.

Compound **Multiplication**, is the method of finding the product arising from a compound and simple quantity; or of two compound quantities of different kinds; or even in some cases of two compound quantities of the same kind. In all these varieties the product is of the same kind with the multiplicand.

Rule. Place the multiplier under the lowest denomination of the multiplicand; and multiply this denomination by the multiplier. Find how many units of the next higher denomination are contained in the product; set down the remainder, and carry the units to the next product, with which proceed as before, and so on through all the denominations to the last; and the result will be the answer required.

If the multiplier exceed 12, the operation will be much simplified, as follows:

1. If the given multiplier be a composite number, multiply successively by each of its factors, instead of the whole number at once.

2. If the given multiplier be not a composite number, take that which is nearest to it, and multiply by its factors as before; then add or subtract as many times the first line, as the number so taken is less or greater than the multiplier.

3 Multiply by 10 till there be as many lines as there are figures in the multiplier. Multiply the first line by the units, the second by the tens, and so on; then add.

Also if there be any fractional part belonging to the multiplier, take such part of the multiplicand, as this fraction is of a unit, and add it to the result before found.

EXAMPLES.

Multiply £7 13*s.* 4½*d.* by 24.

$$\begin{array}{lrrr} & \pounds & s. & d. \\ 24 = 4 \times 6 & 7 & 13 & 4\frac{1}{2} \\ & & & 4 \\ \hline & 30 & 13 & 6 \\ & & & 6 \\ \hline \text{Product } & \pounds 184 & 1 & 0 \\ \hline\hline \end{array}$$

Multiply £3 15*s.* 6½*d.* by 17½.

$$\begin{array}{rrrr} & \pounds & s. & d. \\ 17\frac{1}{2} = 4 \times 4 + 1 + \frac{1}{2} & 3 & 15 & 6\frac{1}{2} \\ & & & 4 \\ \hline 4 = & 15 & 2 & 2 \\ & & & 4 \\ \hline 4 \times 4 = & 60 & 8 & 8 \\ 1 = & 3 & 15 & 6\frac{1}{2} \\ \frac{1}{2} = & 1 & 17 & 9\frac{1}{4} \\ \hline \text{Product } 17\frac{1}{2} = & \pounds 66 & 1 & 11\frac{3}{4} \\ \hline\hline \end{array}$$

Multiply £2 11*s.* 3*d.* by 4256.

$$\begin{array}{rrrlrrr} \pounds. & s. & d. & & \pounds. & s. & d. \\ 2 & 11 & 3 & \times 6 = & 15 & 7 & 6 \\ & & 10 & & & & \\ \hline 25 & 12 & 6 & \times 5 = & 128 & 2 & 6 \\ & & 10 & & & & \\ \hline 256 & 5 & 0 & \times 2 = & 512 & 10 & 0 \\ & & 10 & & & & \\ \hline 2562 & 10 & 0 & \times 4 = & 10250 & 0 & 0 \\ \hline & & & \text{Product} & \pounds 10996 & 0 & 0 \end{array}$$

Multiplication of *Fractions*, is performed by the following

Rule. Reduce all mixed numbers to improper fractions; and then multiply the numerators together for numerator, and the denominators together for denominator, which will be the answer required.

Note. All factors that are common both to the numerators and denominators, may be cancelled or omitted, by which means the result obtained will be in its lowest terms.

EXAMPLES.

1. $\frac{5}{7} \times \frac{4}{9} = \frac{20}{63}$
2. $\frac{5}{12} \times \frac{6}{7} \times \frac{4}{15} = \frac{1}{2} \times \frac{1}{7} \times \frac{4}{3} = \frac{1}{7} \times \frac{2}{3} = \frac{2}{21}$
3. $3\frac{1}{5} \times 4\frac{1}{6} = \frac{16}{5} \times \frac{25}{6} = \frac{8}{1} \times \frac{5}{3} = \frac{40}{3}$

Multiplication of *Decimals,* is performed the same as in the simple rule, except, that in the product as many decimal places must be pointed off from the right hand, as are equal to the number of decimals in both factors; and if there are not so many, the defect must be supplied by prefixing ciphers.

EXAMPLES.

$$\begin{array}{lr} \text{Multiply} & 41{\cdot}364 \\ \text{by} & 7{\cdot}328 \\ \hline & 330\,912 \\ & 827\,28 \\ & 12\,409\,2 \\ & 289\,548 \\ \hline \text{Product} & 303{\cdot}115\,392 \end{array}$$

$$\begin{array}{lr} \text{Multiply} & {\cdot}14\,68 \\ \text{by} & {\cdot}14 \\ \hline & 58\,72 \\ & 146\,8 \\ \hline \text{Product} & {\cdot}0205\,52 \end{array}$$

Multiplication of *Circulating Decimals,* is performed by converting the circulates into their proper fractions, and multiplying them together by the rule for multiplication of fractions; or by finding the product by the interminate decimals, dividing it by as many nines as there are circulating figures, and adding the quotient to the product.

Multiplication in *Algebra,* is the method of finding the product arising from the multiplication of any two or more indeterminate quantities; which may be divided into two cases.

Case 1. When the multiplier and multiplicand are both simple quantities.

Rule. Multiply the co-efficients of the two quantities together, as in arithmetic; and annex to the result all the letters in both factors. But if two or more letters that are alike, as aa, bbb, $xxxx$, &c. enter into the product, these are to be represented by a^2, b^3, x^4, &c. With regard to the sign to be prefixed, if the factors are both affected with the sign +, or both with the sign −; the sign to be prefixed to the product must be +. But if one of the factors be +, and the other −, the sign of the product must be −. This is generally expressed by saying, like signs produce *plus,* and unlike signs *minus.*

EXAMPLES.

1. $4ab \times 6ab = 24aabb = 24a^2b^2$
2. $5xy \times 7x^2y^2 = 35x^3y^3$
3. $3xy \times -4xy^2 = -12x^2y^3$
4. $-4ab \times -3cd = +12abcd$

Case 2. When the multiplier and multiplicand are both compound quantities.

Rule. Beginning at the left-hand, multiply each term in the multiplicand, by each term in the multiplier, by the last rule; and the sum of the several products will be the answer.

Note. It is best to remove the leading term of each product, one place more to the right hand.

EXAMPLES.

$$\begin{array}{llll} a & +b & & \\ a & +b & & \\ \hline a^2 & +ab & & \\ & +ab & +b^2 & \\ \hline a^2 & +2ab & +b^2 & \end{array} \qquad \begin{array}{llll} 7a^2 & +5ab^2 & & \\ 3a^2 & +ab^2 & & \\ \hline 21a^4 & +15a^3b^2 & & \\ & +7a^3b^2 & +5a^2b^4 & \\ \hline 21a^4 & +22a^3b^2 & +5a^2b^4 & \end{array}$$

$$\begin{array}{llll} 7y & +4z & & \\ y & -z & & \\ \hline 7y^2 & +4yz & & \\ & -7yz & -4z^2 & \\ \hline 7y^2 & -3yz & -4z^2 & \end{array} \qquad \begin{array}{llll} 3xy & -2z^2 & & \\ 3xy & -2z^2 & & \\ \hline 9x^2y^2 & -6xyz^2 & & \\ & +6xyz^2 & -4z^2 & \\ \hline 9x^2y & * & -4z^2 & \end{array}$$

Multiplication of *Algebraic Fractions,* is performed by the following

Rule. Reduce all mixed expressions to improper fractions; then multiply all the numerators together for a new numerator; and the denominators together for a new denominator, and the result will be the product required.

Note. All factors that are common to the numerators and denominators, may be cancelled or omitted.

EXAMPLES.

1. $\frac{3ab^2}{cd} \times \frac{4ac^2}{3a^2b} = \frac{b}{d} \times \frac{4c}{1} = \frac{4bc}{d}$
2. $\frac{x^2-y^2}{7z} \times \frac{z^2}{x+y} = \frac{xz-yz}{7}$
3. $\frac{x-y}{3a^2} \times \frac{x+y}{b^2} = \frac{x^2-y^2}{3a^2b^2}$

Multiplication of *Surds*, is the method of finding the product of two or more irrational factors.

Rule. Reduce the given surds to their simplest form; and the radical parts thus arising to like radicals; then multiply the co-efficients together for a new co-efficient, and the surds together for a new surd; which being annexed to the preceding product, will be the answer required.

Note. If the radicals be not the same, but the quantities under them be equal, the multiplication will be performed by the addition of the indices representing those radicals.

EXAMPLES.

1. $\sqrt{7} \times \sqrt{5} \times \sqrt{3} = \sqrt{105}$

2. $\sqrt{3x^3y} \times \sqrt{15x} = x\sqrt{3xy} \times 3\sqrt{2x} = 3x\sqrt{6x^2y} = 3x^2\sqrt{6y}$

3. $\sqrt[3]{7ab} \times \sqrt[3]{4a^2c} = \sqrt[3]{28a^3cb} = a\sqrt[3]{28bc}$

4. $a^{\frac{1}{2}} \times a^{\frac{1}{3}} = a^{\frac{1}{2}+\frac{1}{3}} = a^5 = \sqrt[6]{a^5}$

5. $a^{\frac{n}{m}} \times a^{\frac{p}{q}} = a^{\frac{nq+mp}{mq}} = \sqrt[mq]{a^{nq+mp}}$

6. $a^{\frac{1}{m}} \times b^{\frac{1}{n}} = a^{\frac{n}{mn}} \times b^{\frac{m}{mn}} = \sqrt[mn]{a^n b^m}$

Multiplication by *Logarithms*. See Logarithms.

Multiplication *Table*, a small table containing the products of all the simple digits, at least this is its natural limits, though it is most commonly carried as far as 12 times 12.

MULTIPLIER, or ***Multiplicator***, the number by which another is multiplied.

MURAL *Arch* (from ***murus***, a wall), a wall, or arched wall, placed exactly in the plane of the meridian, for fixing a large quadrant, sextant, or other instrument, to observe the meridian altitude, &c. of the heavenly bodies.

N.

NADIR, that point of the heavens which is diametrically under our feet, or opposite to the zenith which is directly over our heads. The zenith and nadir are the poles of the horizon.

The *Sun's* **Nadir** is sometimes used to denote the vertex, the earth's shadow arising from the interception of the solar rays.

NAPIER'S *Rods* or *Bones*, a method contrived by Lord Napier, the inventor of the logarithms, for the more easy performing of the arithmetical operations of multiplication, division, &c. These rods are five in number, made of bone, ivory, horn, wood, or pasteboard, &c. Their faces are divided into nine little squares, each of which is parted into two triangles by diagonals. In these little squares are written the numbers of the multiplication table; in such manner as that the units, or right-hand figures, are found in the right-hand triangle: and the tens, or the left-hand figures, in the left-hand triangle. To multiply numbers by Napier's bones, dispose the rods in such manner, as that the top figures may exhibit the multiplicand, and to those on the left hand, join the rod of units: in which seek the right-hand figure of the multiplier; and the numbers corresponding to it, in the squares of the other rods, write out, by adding the several numbers occurring in the same rhomb together and their sums. After the same manner write out the numbers corresponding to the other figures of the multiplier, disposing them under one another as in the common multiplication; and lastly, add the several numbers into one sum.

NAVIGATION, the art of sailing, or of conducting a vessel on the ocean, and is usually divided into navigation common, and navigation proper; the former relating to what is otherwise called coasting; or such voyages as are conducted along the coast or shore of a country, and in which the navigator seldom or never loses sight of land. And the other to those voyages made from one country to another, through the trackless paths of the largest seas and oceans.

The Phœnicians, particularly those of Tyre, are generally considered as the first people who made any great advances in this important art. These were afterwards followed up by the Carthagenians, who discovered the Fortunate Islands, or the Canaries; and even, according to some authors, America was visited by this enterprising people, but of this there is not sufficient proof. From Carthage and Tyre commerce and navigation were transferred principally to Alexandria, which latter city, when under the Romans, was only inferior to Rome itself, the latter being supplied with its merchandize wholly from the magazines of the former. Constantinople became afterwards the centre of commerce, and navigation was for a long time pursued with great ardour by the merchants of that city; after this time it began to spread itself, though slowly, amongst the several European cities and nations. Genoa and Venice are particularly distinguished for the active part they took in promoting this important branch of human knowledge. The crusades contributed in a great measure to the revival, or at least to the more rapid progress of commerce and navigation; for the Genoese, the Pisans, Venetians, &c. furnished the transports which carried those vast armies, composed of all the nations of Europe, into Asia upon this wild enterprise, and likewise supplied them with provisions and military stores.

Navigation was not, however, confined only to these religious fanatics; other travellers, beside those whom a mistaken religious zeal sent forth to Asia, ventured into remote countries, from the prospect of commercial advantage, or from motives of mere curiosity. Of these the most eminent was Marco Polo, a Venetian, of a noble family, about the year 1260; and he was succeeded about half a century after by Sir John Maudeville, an Englishman.

But the present art of navigation owed its rise to the invention of the mariner's compass, in the beginning of the fourteenth century; and made considerable progress during the voyages that were began in the year 1418 by Henry Duke of Visco. This learned prince was particularly skilled in cosmography, and employed a person from the island of Majorca to teach navigation, and to make instruments and charts for the sea.

These voyages being greatly extended after the discovery of Porto Santo and Madeira, the art was improved under the succeeding monarchs of Portugal; so much that Roderic and Joseph, physicians to John II., together with one Martin de Bohemia, a Portuguese native of the island of Fayal, a pupil of Regiomontanus, about the year 1485, calculated, for the use of navigators, tables of the sun's declination, and recommended the astrolabe for taking observations at sea. About this time Columbus, a native of Genoa, having thoroughly acquainted himself with the discoveries and observations of the Portuguese, and having also considerable experience in the art of navigation, made a proposal to John II. of exploring a passage to India, by sailing directly towards the west across the Atlantic Ocean.

John rejected this proposal in a very dishonourable manner; and Columbus instantly quitted the kingdom, and landing in Spain towards the close of the year 1484, he resolved to propose his plan to Ferdinand and Isabella, with whom, after repeated application and long delay, he ultimately succeeded. In August, 1492, Columbus, furnished with a small armament of three ships, set sail, and steered directly for the Canary Islands; from thence, holding his course due-west, he stretched away into unfrequented and unknown seas. After many difficulties, he at length arrived at Guanahani, one of the large cluster of islands called the Lucaya, or Bahama Isles. He also discovered Cuba, Hispaniola, and several other small islands; and having left a small colony in a fort at Hispaniola, returned to Spain in March, 1493. In September following he set out on his second voyage, and sailed by the Leeward Islands to Hispaniola. In a third voyage, undertaken in the year 1498, he discovered the continent of America; and in the same year, Vasco de Gama returned to Lisbon, from a voyage to the East Indies by the Cape of Good Hope. Columbus, it is said, before he attempted the discovery of America, consulted Martin de Bohemia, as well as others, and during the course of his voyage instructed the Spaniards in navigation; for the improvement of which art, the emperor Charles V. afterwards founded a lecture at Seville. This art was considerably improved by the discovery of the variation of the compass, and by the use of the cross-staff. At length there were published, in Spanish, two treatises containing a system of the art, which were in great esteem, the first by Pedro de Medina, at Valladolid, in 1545, called "Arte de Navegar;" the other at Seville in 1556, by Martin Cortes, under the title of "Breve Compendio de la Sphera y de la Arte de Navegar, con neuos Instrumentos y Reglas." This is said to have been composed at Cadiz in 1545; and a translation of it was published in London, which passed through several impressions. Besides the improvement which the art of navigation received from the proposals of Werner, and Gemma Frisius, for finding the longitude, it was much promoted by Pedro Nunez, or Nonius, who composed a treatise on this subject so early as 1537, in the Portuguese language, which thirty years after was printed at Basil in Latin, with additions, under the title of "De Arte et Ratione Navigandi." In this work the errors of the plane chart are exposed, and the problem of determining the latitude from two observations of the sun's altitude, and the intermediate azimuth, is resolved. In 1577, Bourne published his treatise, entitled "A Regimen for the Sea," and intended as a supplement to that of Cortes.

The mistakes of Medina were well exposed by Michael Coignet, a native of Antwerp, who, in 1581, published a small treatise, entitled "Instruction Nouvelle des Points plus excellens et necessaires touchant l'Arte de Navigar." In the same year Robert Norman published his discovery of the dipping-needle, in a pamphlet called the "New Attractive," to which is always subjoined William Burrough's "Discourse of the Variation of the Compass." In 1594, Captain John Davis published a small treatise, entitled the "Seamen's Secrets," which was much esteemed in its time. The writers of this period complained much of the errors of the plane chart, which continued still in use, although they were unable to discover a proper remedy, till Gerrard Mercator contrived his universal map, which he published in 1569, without clearly understanding the principles of its construction; these were first discovered by Mr. Edward Wright, who sent an account of the true method of dividing the meridian from Cambridge, where he was a fellow, to Mr. Blundeville, with a short table for that purpose, and a specimen of a chart so divided. These were published by Blundeville in 1594, among his exercises; to the later editions of which was added his "Discourse of Universal Maps," first printed in 1589. However, in 1599, Wright printed his "Correction of certain Errors in Navigation," in which work he shows the reason of this division, the manner of constructing his table, and its uses in navigation. A second edition of this treatise, with farther improvements, was printed in 1610; and a third edition, by Moxon, in 1657. The method of approximation, by what is called the middle latitude, now used by our sailors, occurs in Gunter's works, first printed in 1623. About this time logarithms began to be introduced, which were applied to navigation in a variety of ways by Gunter; though the first author who applied the logarithmic tables to the cases of sailing was Thomas Addison, in his "Arithmetical Navigation," printed in 1625. In 1635, Gellibrand printed "A Discourse Mathematical on the Variation of the Magnetic Needle," containing his discovery of the changes to which the variation is subject. In 1631, Richard Norwood published an excellent "Treatise of Trigonometry," adapted to the invention of logarithms, particularly in applying Neper's generals canons; and for the farther improvement of navigation, he undertook the laborious work of measuring a degree of the meridian for examining the divisions of the log-line. Of this affair he has given a full and clear account in his "Seaman's Practice," first published in 1637; where he also describes his own excellent method of setting down and perfecting a sea-reckoning, &c. This treatise, and that of trigonometry, were continually reprinted, as the principal books for learning scientifically the art of navigation. What he had delivered, especially in the latter of them, concerning this subject, was contracted as a manuel for sailors, in a very small piece, called his "Epitome," which has gone through innumerable editions.

About the year 1645, Mr. Bond published, in Norwood's "Epitome," a very great improvement in Wright's method, by a property in his meridian line, whereby his divisions are more scientifically assigned than the author himself was able to effect; which he deduced from this theorem, that these divisions are analogous to the excesses of the logarithmic tangents of half the respective latitudes, augmented by forty-five degrees above the logarithm of the radius; this he afterwards explained more fully in the third edition of Gunter's works, printed in 1653; and the demonstration of the general theorem was supplied by James Gregory of Aberdeen, in his —"Exercitationes Geometricæ," printed at London 1668; and afterwards by Dr. Halley, in Phil. Trans. No. 219, and by Cotes, Phil. Trans. No. 388. In 1700, Mr. Bond, who imagined that he had discovered the longitude, by having found out the true theory of the magnetic variation, published a general map, on which were delineated curve lines, expressing the paths where the magnetic needle had the same variation: the positions of these curves will, indeed, continually suffer alterations; but they should be corrected from time to time, as they have already been for the years 1744 and 1756, by William Mountaine and James Dobson. The allowances proper to be made for leeway are very particularly set down by John Buckler, and published in a small tract first printed in 1702, entitled "A New Compendium of the whole Art of Navigation," written by William Jones.

As it is now generally agreed that the earth is a spheroid, whose diameter at the poles is shorter than the other, Dr. Murdoch published a tract in 1741, in which he accommodated Wright's sailing to such a figure; and Maclaurin, in the same year, gave a rule in the Phil. Trans. No. 461, for determining the meridional parts of a speroid, which speculation he has farther prosecuted in his "Fluxions," printed at Edinburgh in 1742.

The principal foreign writers on navigation are Bartolomew Crescenti, of Rome, in 1607; father George Fournier, at Paris, in 1633; John Baptist Riccioli, at Bologna, in 1661; father Millet Dechales, in 1674 and 1677; the sieur Blondel St. Austin, in 1671 and 1673; M. Dassier, in 1683; M. Sauveur, in 1692; M. Bouguer, in 1698; father Pazenas, in 1733 and 1741; and M. Peter Bouguer, who, in 1753, published a very elaborate treatise on this subject, entitled "Nouveau Traité de Navigation," in which he gives a variation compass of his own invention, and attempts to reform the log, as he had done in the Memoirs of the Academy of Sciences for 1747. He is also very particular in determining the lunations more accurately than by the common methods, and in describing the correction of the dead reckonings. This book was abridged and improved by M. de Lacaille, in 1760; to these we may add Don George Juan, of Spain, in 1757.

Those who are desirous of perusing a fuller account of the progress of navigation, may consult Dr. James Wilson's preface to Dr. Robertson's "Elements of Navigation."

NAUTICAL, relating to Navigation.

NEAP, or NEEP-*Tides*. *See* TIDES.

NEBULÆ, in *Astronomy*, faint luminous spots in the heavens, some of which consist of clusters of telescopic stars, others appear as luminous spots of different forms. The most considerable is the one in the midway between the two stars on the blade of Orion's sword, marked θ, by Bayer, discovered in the year 1656 by Huygens; it contains only seven stars, and the other part is a bright spot upon a dark ground, and appears like an opening into brighter regions beyond.

Dr. Halley and others have discovered nebulæ in different parts of the heavens. In the Connoissance des Temps, for 1783 and 1784, there is a catalogue of 103 nebulæ, observed by Messier and Mechain. But to Dr. Herschel we are indebted for catalogues of 2000 nebulæ, and clusters of stars which he himself has discovered. Some of them form a round compact system, others are more irregular, of various forms, and some are long and narrow. The globular systems of stars appear thicker in the middle than they would do if the stars were at equal distances from each other; they are therefore condensed towards the centre. That stars should be thus accidentally disposed is too improbable a supposition to be admitted; he supposes, therefore, that they are brought together by their mutual attractions, and that the gradual condensation towards the centre is a proof of a central power of such a kind. He observes, also, that there are some additional circumstances in the appearance of extended clusters and nebulæ, that very much favour the idea of a power lodged in the brightest part; for although the form of them be not globular, it is plain that there is a tendency to sphericity. As the stars in the same nebulæ must be nearly all at the same relative distances from us, and they appear nearly of the same size, their real magnitude must be nearly equal. Granting, therefore, that these nebulæ and clusters of stars are formed by mutual attraction, Dr. Herschel concludes that we may judge of their relative age by the disposition of their component parts, those being the oldest which are most compressed. He supposes, and, indeed, offers arguments to prove, that the milky way is the nebulæ of which our sun is one of its component parts.

NEGATIVE, in *Algebra* and *Arithmetic*, the reverse of positive, as negative *index, exponent, quantity, sign*, &c.

Negative *Index*, of a logarithm, are those which are affected with a negative sign; such are the indices of all numbers less than unity. *See* Logarithms.

Negative *Exponent* or *Power*, is that which is affected with a negative sign, being the form in which the reciprocal of any power is commonly put to prepare it for expansion into a series, and is equally convenient for many other algebraical operations. Thus

$$\frac{1}{x^2} = x^{-2},\ \frac{b}{x^3} = bx^{-3},\ \frac{a-x}{(a+x)^n} =$$

$(a-x)(a+x)^{-n}$, &c.; -2, -3, and $-n$ being negative indices.

That these expressions are equivalent to each other, may be shown as follows:

The known rule for the division of powers, in algebra, is to subtract their indices.

$$\text{Now } \frac{1}{x^2} = \frac{x^1}{x^3} = x^{1-3} = x^{-2}$$

$$\text{In the same way } \frac{b}{x^3} = \frac{bx^1}{x^4} =$$

$bx^{1-4} = bx^{-3}$ and the same for any other similar expressions.

Negative *Quantities*, are those quantities which are preceded or affected with the negative sign. See the following article.

Negative *Sign*, in *Algebra*, is that character, or symbol, which denotes subtraction, being a short line preceding the quantity to be subtracted, and is read *minus*; thus $a-b$ denotes that the quantity b is to be taken from the quantity a, and is read, *a minus b*.

The introduction of this character into algebra has given rise to various controversies, with regard to the legality or illegality of certain conclusions depending upon it; some maintaining, that as a negative quantity is in itself tally imaginary, it ought not to be introduced into a science, the excellency of which depends upon the rigour and certainty of its conclusions; while others, running into the opposite extreme, have endeavoured to illustrate what will not admit of illustration; and thus, like other zealots, have been the greatest enemies of the cause they were so anxious to defend.

It is in vain to attempt to define what can have no possible existence; a quantity less than nothing is totally incomprehensible; and to illustrate it, by reference to a debtor and creditor account, to say the least of it, is highly derogatory to this most extensive and comprehensive science.

The apparent anomalies resulting from the introduction of this character have arisen from giving to this symbol the same generality as belongs to the sign $+$ or addition. When two quantities are to be added together, as a and b, it is perfectly indifferent which of them is placed first, for $a + b$ is, in every respect, the same as $b + a$; but if the difference of them is to be expressed, this is not the case; $a - b$ and $b - a$ being totally different; if a is greater than b, then $a - b$ is a real quantity equal to the difference of a and b; but $b - a$ is an imaginary quantity arising from a suppositious operation, viz. of taking a greater quantity from a less; yet this expression, considered merely as an algebraical symbol, may still enter as such into the steps of any process, and will ultimately produce a legitimate result; but, in order to this, we must first have certain rules laid down for operating on such quantities, accommodated to their particular nature, and which must be such as necessarily arise from principles previously established.

If it be required to add to the quantity c the residual $a - b$, it is obvious that the sum is $c + a - b$. And if, on the other hand, we have to subtract from c the quantity $a - b$, we must first subtract a, which gives $c - a$; but we have now taken away too much by the quantity b, and this, therefore, must be added to the result, making the true remainder $c - a + b$; therefore to subtract a negative quantity is the same as to add a positive one; that is, we must change the sign from $-$ to $+$.

Again, if it were required to multiply $+ a$ by $+ b$, we know the product to be ab; and if $+ a$ be multiplied by a quantity less than b, as $b - c$, the product must necessarily be less than ab; that is, from the product of a multiplied by b, we must take the product of $a \times$ by c; hence $a \times (b - c)$ must be expressed by $ab - ac$; therefore it follows that $+ a \times - c$ gives for a product $- ac$; that is, $+$ *plus* into $-$ *minus*, gives $-$ *minus*.

If now we consider the product arising from the multiplication of the two quantities $(a - b)$ and $(c - d)$, we know that it is less than that of $(a - b) \times c$, or of $ac - bc$, by the product of $(a - b) \times d$: or thus, the product of $(a - b) \times (c - d)$ becomes first $ac - bc - ad$, to which is to be annexed the product of $- b$ by $- d$, and the question is, only, what sign we must employ for this purpose, whether $+$ or $- 1$. Now we have seen that from the product $ac - bc$, we must subtract the product of $(a - b) \times d$, that is, we must subtract a quantity less than ad; we have therefore already subtracted too much by the quantity bd; this product must therefore be added; that is, it must have the sign $+$ prefixed: hence then it appears, that $b \times - d$ give $+ bd$ for a product; or *minus* multiplied by *minus* gives *plus*, or $- \times - = +$.

Whence, generally, like signs produce *plus*, and unlike signs *minus*.

NEWTONIAN *Philosophy*, the doctrine of the universe, and particularly of heavenly bodies, their laws, affections, &c. as delivered by Sir Isaac Newton.

The term Newtonian philosophy is applied very differently; some authors, under this philosophy, include all the corpuscular philosophy, considered as it now stands, corrected and reformed by the discoveries and improvements made in several parts thereof by Sir Isaac Newton. In which sense it is that Gravesande calls his elements of physics, "Introductio Philosophiam Newtonianam." And in the same sense the Newtonian is the same with the new philosophy, and thus stands contradistinguished from the Cartesian, the Peripatetic, and the ancient corpuscular philosophy.

Others, by Newtonian philosophy, mean the method or order which Sir Isaac Newton observes in philosophising, viz. the reasoning and drawing conclusions directly from phenomena exclusive of all previous hypotheses: the beginning from simple principles, deducing the first powers and laws of nature from select phenomena, and then applying those laws, &c. to account for other things. And in this sense the Newtonian philosophy is the same with experimental philosophy, and stands opposed to the ancient corpuscular.

Some, by Newtonian philosophy, mean that wherein physical bodies are considered mathematically, and where geometry and mechanics are applied to the solution of the appearances of nature. In which sense the Newtonian is the same with the mechanical and mathematical philosophy.

Others again, by Newtonian philosophy, understand that part of physical knowledge which Sir Isaac Newton has handled, improved, and demonstrated in his "Principia."

While others mean the new principles which Sir Isaac Newton has brought into philosophy; the new system founded thereon; and the new solutions of phenomena thence deduced; being that which characterises and distinguishes his philosophy from all others.

NIGHT, that part of the natural day during which the sun is below the horizon; though the twilight, both in the morning and evening, is sometimes considered as forming part of the day.

NOCTURNAL, an instrument for taking the altitude of the stars, &c. formerly used at sea, but long since discontinued.

NODE, in the *Doctrine* of *Curves*, is a small oval figure, made by the intersection of one branch of a curve with another. *See* CURVE.

NODE, in *Dialing*, denotes a small hole in the gnomon of a dial, which indicates the hour by its light, as the gnomon itself does by its shadow.

NODES, in *Astronomy*, are the opposite points where the orbit of a planet crosses the ecliptic.

Ascending NODE, is that where the planet ascends from the south to the north side of the ecliptic, which is denoted by the character ☋, and denominated *dragon's head*.

Descending NODE, is that where the planet descends from the north to the south side of the ecliptic, which is denoted by the character ☊, and is called the *dragon's tail*.

The right line joining these two points, is called *the line of the nodes*.

It appears by observation, that in all the planets, the line of the nodes continually changes its place, its motion being in antecedentia, or contrary to the order of the signs, the particular quantity for which, in each planet, will be found under their several names.

NONAGESIMAL *Degree*, called also *Mid-heaven*, is the highest point or 90th degree of the ecliptic, reckoned from its intersection with the horizon at any time.

NONAGON, a figure of nine angles and nine sides. The angle at the centre of a nonagon is 40°, the angle subtended by its sides 140°, and its area when the side is 1 = $\frac{9}{4}$ nat. tang. 70° = 6·1818242.

NONES, were certain days in the Roman calendar, viz. the 5th day of the months January, February, April, June, August, September, November, and December.

NONIUS. *See* VERNIER.

NORMAL, is the same as perpendicular.

NORTH, one of the four cardinal points.

NORTH *Star*. *See* POLE *Star*.

NORTHERN *Signs*, are those that are on the north side of the equator, viz. Aries, Taurus, Gemini, Cancer, Leo, and Virgo.

NORTHING, in *Navigation*, is the difference of latitude which a ship makes in sailing towards the north.

NOTATION, in *Arithmetic*, is the method of expressing, by means of certain symbols or characters, any proposed number or quantity. In the modern analysis, notation implies a method of representing any quantity or operation; and the judicious selection of proper symbols for this purpose, is an important consideration, which every author who undertakes to write on this subject should particularly attend to.

In the common scale of notation every number is expressed by means of the ten characters, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, by giving to each digit a local as well as its proper or natural value, the discovery of which was perhaps one of the most important steps that has ever been made in mathematics, and does as much honour to its inventor as any other in the history of this science.

To whom we are indebted for this improvement is not known, nor the nation whence it dates its origin. Some authors have attributed the honour of it to the Arabs, others to the Greeks, and others again to the Indians; which latter people have certainly a priority of claim to both the other nations; but whether they were the original inventors, is at present, and probably ever will be, undetermined. The simplicity of this method of expressing quantities, and the universal application of it to every species of calculation, render it so familiar to all our ideas of numbers, that we lose sight of the ingenuity of the invention, by that which ought most to distinguish it; that is, the obvious and accurate principles on which it is founded; and instead of considering it as a most ingenious discovery, we are apt to treat it as a necessary consequence following immediately from the nature of the number itself. That this is however a mistaken notion, is evident from a notation of the Greeks and Romans, to whom this method was unknown; in fact, it does not appear to have been introduced into Europe till the latter end of the tenth century, when it was first taught by the Arabs to the Spaniards, with whom they had a communication; and hence it soon after became generally known to all the European nations, and seems to have been brought into England as early as the year 1150. The Arabs, it is evident from several of their manuscripts, derived their knowledge of it, either directly, or through some other nation, from the Indians, it being entitled, in some of their works, *The Indian Arithmetic;* but, as we before observed, whether this latter nation originally made the discovery is unknown.

In the common, or denary scale of notation, the value of every digit increases in a tenfold proportion from the right towards the left; thus, $1111 = 1000 + 100 + 10 + 1 = 10^3 + 10^2 + 10 + 1$; also $3464 = 3 \cdot 10^3 + 4 \cdot 10^2 + 6 \cdot 10 + 4$, and so on of others; the distance of any figure from the right indicating the power of 10, and the digit itself the number of those powers intended to be expressed, on which obvious principle it is evident, that any number whatever may be represented with ease and simplicity; but since also a number may be expressed by assuming any other radix instead of 10, the curious reader will inquire how it happened that the number 10 came to be adopted as the radix of our notation in preference to any other. The fact is, that long before the invention of our present notation, all nations, at least with a very trifling exception, divided their numbers into periods of 10, and this singular coincidence of different people, unknown and unconnected with each other, has been the subject of philosophical investigation and inquiry from the time of Aristotle to the present day, though it is now generally allowed to have had its origin in the formation of man, that is, to his having 10 fingers, by means of which it is highly probable that computation, or at least numbering, was first effected; as we always see children, in making their first efforts in calculation, have recourse to this means for assisting their memory; and hence we may infer, that the present division of numbers into periods of 10, had its origin as soon as numbering was attempted, that is, as soon as man began to associate with each other.

Scales of Notation. In general, any number N may be represented by the formula $N = a.r^{n} + b\,r^{n-1} + c\,r^{n-2} + \&c.\ pr^{2} + qr + w$, where r the radix, may be any number whatever, and a, b, c, &c. integer numbers less than r. And according as r is assumed $= 2, 3, 4$ &c. the different scales or systems receive the following denominations:

If $r = 2$ it is termed the Binary scale
$r = 3$ . . . . Ternary scale
$r = 4$ . . . . Quaternary scale
$r = 5$ . . . . Quinary scale
$r = 6$ . . . . Senary scale
$r = 10$ . . . . Denary scale
$r = 12$ . . . . Duodenary scale.

And since the co-efficients a, b, c, &c. are always less than r, therefore it follows that for any scale we must have as many characters, including the cipher, as is equal to the number expressing the radix of the system; thus, the characters are for the

Binary scale . .	0, 1
Ternary scale .	0, 1, 2
Quaternary scale	0, 1, 2, 3
Quinary scale . .	0, 1, 2, 3, 4
Senary scale . .	0, 1, 2, 3, 4, 5
Denary scale . .	0, 1, 2, 3, 4, 5, 6, 7, 8, 9

And hence it follows, that for the duodenary scale two other characters are necessary for expressing 10 and 11; assuming, therefore, 10 = φ, and 11 π; the digits of this system are

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, φ, π.

To transform a Number from the Denary or Common Scale, to any other Scale.

Rule. Divide the number successively by the radix of the given scale, and the several remainders will be the digits of the number required.

Exam. Transform 1810 into the binary and ternary scales.

2) 1810		3) 1810	
2) 905 rem. =	0	3) 603 rem. =	1
2) 452 ———	1	3) 201 ———	0
2) 226 ———	0	3) 67 ———	0
2) 113 ———	0	3) 22 ———	1
2) 56 ———	1	3) 7 ———	1
2) 28 ———	0	3) 2 ———	1
2) 14 ———	0	0 ———	2
2) 7 ———	0		
2) 3 ———	1		
2) 1 ———	1		
0 ———	1		

Therefore 1810 = 11100010010 in the binary scale.
And 1810 = 2111001 in the ternary scale.

Notation of the *Greeks*. These people had three distinct notations; the most simple of which was, by making the letters of their alphabet the representatives of numbers,

α, 1; β, 2; γ, 3;

and so on. Another method was by means of six capital letters, thus, I [ια for μια] 1; Π [ϖεντε] 5; Δ [δεκα] 10; H [Hεκατον] 100; X [χιλια] 1000; M [μυεια] 10000: and when the letter Π enclosed any of these, except I, it indicated the enclosed letter to be 5 times its proper value, as stated above, thus, $\overline{|\Delta|}$ represented 50; $\overline{|H|}$ 500; $\overline{|X|}$ 5000, and so on. This kind of notation was only used to represent dates and similar cases; but for arithmetical purposes they had a more organised system, in which thirty-six characters were employed, and by means of which any number not exceeding 100000000, might be expressed; though in the first instance it appears that 10000, or a myriad, was the extent of their arithmetic.

Our digits

1, 2, 3, 4, 5, 6, 7, 8, 9,

They expressed by the letters

α, β, γ, δ, ε, ϛ, ζ, η, θ.

For our tens, as

10,20,30,40,50,60,70,80,90,

They employed the letters

ι, κ, λ, μ, ν, ξ, ο, π, ϟ.

The hundreds were expressed by

ρ, σ, τ, υ, φ, χ, ψ, ω, ϡ.

And the thousands by

α̦, β̦, γ̦, δ̦, ε̦, ϛ̦, ζ̦, η̦, θ̦.

That is, they had recourse again to the characters of the simple units, which were distinguished by a small iota or dash placed below them; and with these characters a number under 10000 was readily expressed; and this, as we have observed above, was, for some time, the limit of their arithmetic. Afterwards 10000, or a myriad, was represented by M; and any number of myriads by M placed under the number of them. Thus,

$\overset{\alpha}{M}$, $\overset{\beta}{M}$, $\overset{\gamma}{M}$,

represented 10000, 20000, 130000, &c. and after this character followed the number of units; thus, $\underset{M}{\lambda\gamma}$.θ̦ϡϟθ represented 339999; but this character for myriads being inconvenient, was afterwards changed for Mυ set after the number of myriads; so that the above number was sometimes written λγMυ θ̦ϡϟθ, and this character was latterly entirely omitted, and the myriads were distinguished from the simple units only by a point placed between the two.

By these means the notation of the Greeks was extended to 100000000; and it was afterwards farther improved by Archimedes, Apollonius, &c. The following numbers, when compared with what has been stated above, will serve as an illustration of this method.

991 = ϡϟα

9999 = θ̦ϡϟθ

7382 = ζ̦τπβ

8036 = ηλϛ

6420 = ϛυκ

4001 = δ̦α

9917382 = ϡϟα.ζ̦τπβ

171000 = ιζ.α̦

647499 = ξδ.ζ̦δϟθ

1110000 = ρια.

3000001 = τ.α

99999999 = θϡϟθ.θ̦ϡϟθ

Notation of the *Hebrews*, resembled, in a great measure, that of the Greeks above described; thus, Instead of our units

1, 2, 3, 4, 5, 6, 7, 8, 9,

The Hebrews used their letters

א, ב, ג, ד, ה, ו, ז, ח, ט.

For our tens, as

10,20,30,40,50,60,70,80,90,

They employed

י, כ, ל, מ, נ, ס, ע, פ, צ.

For the hundreds they used

ק, ר, ש, ת, ך, ם, ן, ף, ץ.

And for representing thousands, they had again recourse to their simple units; distinguishing them only by two dots or acute accents.

Notation of the *Romans*. This is still employed by us for dates and other similar purposes; and is too well known to require a very minute description. The Roman numeral characters are seven in number; *viz.* I. one; V. five; X. ten; L. fifty; C. a hundred; D. or IↃ. five hundred; M. one thousand: this last number is also sometimes expressed by Dᗡ. or by CIↃ. And by the various combinations of these characters, any number whatever might be expressed.

NOVEMBER, the 11th month in the Julian year, but the ninth in the year of Romulus, whence its name. In this month, which contains thirty days, the sun enters the sign ♐, usually about the twenty-first day of the month.

NUCLEUS, is used by Hevelius and some other astronomers for the body of a comet, which others call

its head, as distinguished from the tail or beard.

Nucleus is also used by some writers for the central parts of the earth and other planets, which they suppose firmer, and separated from the other parts, as the kernel of a nut is from the shell.

NUMBER, in its most extended signification, has a reference to every abstract quantity that can be made the subject of arithmetical computation; but, in a more limited sense, it signifies only several things of the same kind, and is defined by Euclid to be a multitude of units. Numbers of this latter kind are termed *integral*, to distinguish them from the other kind of numbers, which are of various denominations; as exponential, fractional, logarithmic, surd, &c. See Exponential, Fraction, Surd.

Integral Numbers are distinguished into various classes, as follow: *Absolute, Abstract, Abundant, Amicable, Cardinal, Circular, Composite, Concrete, Figurate, Homogenial, Irrational, Ordinal, Perfect, Polygonal, Prime, Rational,* &c.; for which see the respective adjectives.

Properties of Numbers, are certain theorems relating to the *Divisors, Forms, Powers, Products*, &c. of numbers; many of which are given under the respective articles, and some others of a more general nature are enumerated below.

1. The product of two numbers is the same, whichever of the two is the multiplier.

2. If any number p, be prime to two other numbers, a and b, it is also prime to their product $a\,b$.

3. Every integer number, without exception, is composed of, and may be resolved into, different terms of the geometrical series 1, 2, 4, 8, 16, &c.

4. Every integer number, without exception, may be made up by the addition and subtraction of different terms in the series 1, 3, 9, 27, 81, &c.; and therefore with such a series of weights, any number of pounds may be ascertained.

5. The difference between any cube and its root, is divisible by 6; between any fifth power and its root, is divisible by 10; and, generally, if n be a prime number, the difference $x^n - x$ is divisible by 2^n.

6. Every number whatever is the sum of three, or a less number of triangular numbers; of four, or a less number of squares; of five, or a less number of pentagonals; and so on for hexagonal, heptagonals, &c. &c.

Theory of Numbers, is the investigation and demonstration of certain theorems relating to the properties of numbers. This is a subject with which the ancients were little acquainted; in fact, their method of notation was an insuperable impediment to any investigations of this kind. Diophantus, however, may be said to have treated on the properties of numbers; but it was his commentators, Bachet and Fermat, who first laid the foundation of the present theory: the latter, in particular, discovered many fine theorems, which still retain the name of their inventor; but they were mostly given without demonstration, in his notes on Diophantus, in one of which he distinctly mentions a work that he was preparing on the subject of numbers; but this was unfortunately lost at his death. What Fermat thus left incomplete, was taken up by Euler and Lagrange, who demonstrated several of his theorems, the former in the Acta Petro, and the latter in the Memoirs of Berlin; and, latterly, a very ingenious work, entitled "Essai sur la Théorie des Nombres," in 4to. was published at Paris by Legendre; and another about the same time by M. Gauss, at Strasburg, entitled "Disquisitiones Arithmeticæ," and which has since been translated into French by M. Poullet de Lisle.

Number of *Direction*, in *Chronology*, some one of the 35 numbers between the Easter limits, or between the earliest and latest days on which it can fall; *viz.* between March 22 and April 25, which is 35 days, and is so called because it serves as a direction for finding Easter for any year; being, indeed, the number that expresses how many days after the 21st of March Easter-day falls. Thus Easter-day, falling as below, the number of direction will be as on the lower line.

March.

Easter-day, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31
No. of Direct. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

April.

Easter-day, 1, 2, 3, 4, 5, 6, 7, 8, &c.
No. of Direct 11, 12, 13, 14, 15, 16, 17, 16, &c.

and so on, till the number of direction on the lower line be 35, which will answer to April 25, being the latest that Easter can happen. Therefore add 21 to the number of direction, and the sum will be so many days in March for the Easter day; if the sum exceed 31, the access will be the day in April.

To find the Number of Direction. Enter the following table with the Dominical Letter on the top, and the Golden Number on the right hand; then where the columns meet is the number of direction required for that year.

G. N.	A	B	C	D	E	F	
1	29	27	8	19	30	24	25
2	19	13	14	15	16	17	18
3	5	6	7	8	2	3	4
4	26	27	21	22	23	24	25
5	12	13	14	15	16	10	11
6	33	34	35	29	30	31	32
7	19	20	21	22	23	24	18
8	12	13	7	8	9	10	11
9	26	27	28	29	30	31	32
10	19	20	21	15	16	17	18
11	5	6	7	8	9	10	4
12	26	27	28	29	23	24	25
13	12	13	14	15	16	17	18
14	5	6	7	1	2	3	4
15	26	20	21	22	23	24	25
16	12	13	14	15	9	10	11
17	33	34	28	29	30	31	32
18	19	20	21	22	23	17	16
19	12	6	7	8	9	10	11

Thus, for the year 1790, the Dominical Letter being C, and the Golden Number 5; in the column C and in the line 5 stands 14, the number of direction; to this add 21, which gives 35, subtract 31 and there remains 4; which is the day of [illegible] on which Easter-day fell [illegible] 1790.

NUMERATION, in *Arithmetic*, is the art of reading or estimating the value of any number expressed by the ten characters 1, 2, 3, &c. any how combined or repeated; and is, therefore, the reverse of notation, which is expressing by those characters any number proposed.

In order that the nine significant figures may express not only units, but tens, hundreds, thousands, &c. each character has a local value given to it, as stated under the article NOTATION; so that though when alone they express only units, yet in the second place they denote tens; in the third, hundreds; in the fourth, thousands: thus, 5555 represents five thousands, five hundreds, fifty, and five. Hence then, to express any written number, or to assign the proper value to each character, beginning at the right hand, divide the proposed number into periods of three places each, and consider two of these as forming one period of six places: then each of these greater periods has a name common to all the figures of which it is composed; the first six being *units;* the next six, *millions;* the third, *millions* of *millions*, or *billions;* the fourth, *trillions;* and so on: also every half period of three places is read separately, or by itself, so many hundreds, tens, and units; only after the left-hand half of each period, the word thousands is repeated; and at the end of each complete period, its common name is added.

The French enumerate their numbers in a different manner, calling the first three, hundreds and units; the next three, thousands; the next three, millions; the next three, billions; the next three, trillions, &c. See Billion.

Numerator of a *Fraction*, is that number which stands above the line, and shows how many parts the fraction consists of; as the denominator represents the number of parts into which the unit is supposed to be divided.

NUMERICAL, or Numeral, something that relates to number.

NUMERAL *Algebra*, are those cases in which numbers are employed, in contradistinction to *Literal Algebra*, or that in which the letters of the alphabet are made use of.

NUTATION, in *Astronomy*, a kind of trepidation, or tremulous motion of the axis of the earth, whereby its inclination to the plane of the ecliptic is not always the same, but varies backwards and forwards some seconds; and the period of these variations is nine years. This nutation was discovered by Dr. Bradley, who published an account of his discovery in the year 1737.

O.

OBJECT *Glass*, in *Optical Instruments*, is that which is placed towards the object, the other extreme lens being called the eye-glass, being that to which the eye is directed.

OBLATE, flattened or shortened.

OBLIQUE, aslant, indirect, or deviating either from perpendicularity or parallelism.

OBLONG, is properly a right-angled parallogram, of which the length and breadth are unequal. The same word, however, is commonly employed to denote any figure which is longer than it is broad, or even a solid; thus a prolate sphere is sometimes called an oblong spheroid.

OBSERVATION, in *Astronomy* and *Navigation*, denotes the measuring, with some instrument proper for the purpose, the angular distance, altitude, &c. of the sun, moon, or other celestial body.

OBSERVATORY, a place destined for observing the heavenly bodies.

OBTUSE, literally implies any thing blunt or dull, in contradistinction to acute, sharp, or pointed.

OCCIDENT, in *Astronomy* and *Geography*, is the same as westward, or point of the horizon where the sun sets. A planet is said to be occident when it sets after the sun.

Occident *Equinoctial*, that point of the horizon where the sun sets, when he crosses the equinoctial, or enters the sign Aries or Libra.

OCCULTATION, the obscuration of a planet or star by the interposition of the moon, or other planet, between it and our eye.

OCEAN, that immense collection of water which forms the greater portion of the terrestrial globe, being commonly estimated at double the space occupied by the dry land.

OCTAGON, in *Geometry*, is a figure of eight sides and angles, which, when the sides and angles are all equal, is called a regular octagon, and when they are not both equal, an irregular octagon.

The angle at the centre of an octagon is 45 degrees, and the angle of its sides 135 degrees. The area of a regular octagon whose side is $1 = 2\,(1 + \sqrt{2}) = 4{\cdot}8284271$; and therefore when the side is s, the area $= 4{\cdot}8284271\,s^2$, and the radius of its circumscribing circle

$$= \frac{s}{\sqrt{(2-\sqrt{2})}}$$

To construct a regular octagon on a given line.—From the extremities of the line, draw perpendiculars to that side of it, upon which the octagon is to be constructed, and produce them indefinitely. Produce also the line both ways. Then bisect the angles made by the perpendiculars and line produced, and from the extremities of the line draw lines through the points of section. Make them equal to the given line, and three sides will be constructed. Two sides more are obtained by drawing parallels to the indefinite perpendiculars, and the remaining three, by cutting the perpendiculars from the extremities of them, with a distance equal to the given side.

To inscribe an Octagon in a given Circle.—Inscribe a square in the given circle; then bisect each of the four equal arcs intercepted by the sides of the square, which will be the arcs subtended by the sides of the octagon.

OCTAHEDRON, or **Octaedron**, one of the five regular bodies, contained under eight equal and equilateral triangles. Or an octahedron may be conceived to be made up of eight equal triangular pyramids, whose vertices unite in one common point, which is the centre of the solid, and of its circumscribed spheres.

To find the surface and solidity of an Octahedron, the side of one of its equal faces being given.

Let s represent the given side, then

$$\text{surface} = 2\,s^2\sqrt{3} = 3{\cdot}4641016\ s^2$$
$$\text{solidity} = \tfrac{1}{3}\,s^3\sqrt{2} = {\cdot}4714045\ s^3$$

The radius of the circumscribing and inscribed sphere being given to find its side or lineal edge, its surface and solidity.

Let R and r represent the radii of the circumscribing and inscribed sphere, then

$$\text{side} = R\sqrt{2} = r\sqrt{6}$$
$$\text{surface} = 4\,R^2\sqrt{3} = 12\,r^2\sqrt{3}$$
$$\text{solidity} = \tfrac{4}{3}\,R^3 = 4\,r^3\sqrt{3}$$

Or writing s for the side, we have

radius of circum. sphere $\frac{1}{2}\,s\sqrt{2}$
radius of inscrib. sphere $\frac{1}{6}\,s\sqrt{6}$

See Hutton's Mensuration, p. 251, 2d edition.

OCTANT, the eighth part of a circle.

Octant, or *Octile*, is also an ancient term in *Astronomy*, to denote one of the aspects, *viz.* when two planets are distant from each other 45°.

OCTOBER, the eighth month of the ancient Roman calendar, but the tenth according to the Julian year. This month contains 31 days, on about the 22d of which the sun enters the sign Scorpio.

ODD *Number*, that which can not be divided into two equal integral parts.

ODOMETER, is an instrument for measuring the distance travelled over by a post-chaise or other carriage; it is attached to the wheel, and shows the distance by means of an index and dial-plate.

OFFSETS, in *Surveying*, are those short perpendiculars that are measured on the sides of irregular figures for the more accurate determination of the area. See Surveying.

Offset *Staff*, a staff or rod used in surveying for measuring offsets. It is commonly made of light wood, ten links in length, divided and numbered from one end to the other.

OLBERS, a name sometimes given to the planet Pallas.

OLYMPIAD, in *Chronology*, a period of four years, by which the Greeks reckoned their time; being thus called from the olympic games held every fourth year. The first olympiad began the year 3938 of the Julian period, corresponding to 776 years before Christ. The computation by these games ended with the 404th, being the 440th of the present christian era.

OMBROMETER, a name given by Mr. Pickering to the pluviameter or rain-gauge.

OMPHALOPTER, a name sometimes given to a convex lens.

OPAKE, or **Opaque**, not translucent nor transparent, or that which prevents the free admission of the rays of light.

OPERA *Glass*, in *Optics*, is so called from its use in playhouses, though it is sometimes called a diagonal perspective, from its construction. It consists of a tube about four inches long; in each side of which there is a hole, and exactly against the middle a plane mirror, which reflects the rays falling upon it to a convex-glass; through which they are refracted to the concave eye-glass, whence they emerge parallel to the eye at the end of the tube. The peculiar artifice of this glass is to view a person at a small distance, so that no one shall know who is observed, for the instrument points to a different object from that which is viewed, and as there is a hole on each side, it is impossible to know on which hand the object is situated, which you are viewing. The position of the object will be erect through the concave eye-glass.

OPPOSITION, in *Astronomy*, is that aspect of any two heavenly bodies, when they are diametrically opposite each other. See *Aspect*.

OPTIC, or OPTICAL, any thing relating to the science of optics.

OPTIC *Inequality*, in *Astronomy*, is an apparent irregularity in the motions of the planets and other celestial bodies, being thus called because it does not arise from any real inequality of the moving body, but from the situation of the eye of the observer. See APPARENT *Motion*.

OPTIC *Place* of a planet, is its place as seen by the eye.

OPTIC *Pyramid*, is a pyramid formed by the visual rays proceeding from the eye, and passing through the extremities of any picture, when these rays are continued to terminate in a plane perpendicular to the observer.

OPTICS, the science of *Vision*, including catoptrics and dioptrics, and by some authors perspective is also included under the general term optics; as also chromatics, or the theory of colours, and all the phenomena of visible objects.

The reflection of the rays of light, from a polished surface, is a circumstance so common and obvious, that it could not long remain unnoticed by the most superficial observer; even the equality of the angle of incidence and reflection appears to have been known from earliest periods; but this was not the case with regard to the angle of incidence and refraction. It was, indeed, known at an early period that an oar, or other straight piece of wood, partially immersed in water, no longer appeared straight, yet ages after this elapsed before any attempts were made to ascertain the relation between the angles of incidence and refraction. Empedocles was the first on record that wrote systematically on light; and Euclid composed a treatise on the ancient optics and catoptrics; dioptrics being less known to the ancients, though it was not quite unnoticed by them; for among the phenomena at the beginning of that work, Euclid remarks the effect of bringing an object into view by refraction, in the bottom of a vessel, by pouring water into it, which could not be seen over the edge of the vessel before the water was poured in; and other authors speak of the effects of glass globes, &c. both as burning-glasses, and as to bodies seen through them. Euclid's work, the genuineness of which has been doubted, is chiefly on catoptrics, or reflected rays; in which he shows the chief properties of them in plane, convex, and concave surfaces, in his usual geometrical manner, beginning with that concerning the equality of the angles of incidence and reflection, which he demonstrates; and in the last proposition, shows the effect of a concave speculum, as a burning-glass when exposed to the rays of the sun. The effects of burning-glasses, both by refraction and reflection, are noticed by several others of the ancients, and it has been thought that the Romans had a method of lighting their sacred fire by some such means. Aristophanes, in one of his comedies, introduces a person as making use of a transparent stone to cancel a bond that was against him, by thus melting the wax on which such instruments were then executed. If we give credit to what some ancient historians have written concerning the exploits of Archimedes, we shall be induced to think that he constructed some very powerful burning mirrors. It is even allowed that this eminent geometrician wrote a treatise on the subject of them, though it be not now extant; as also concerning the appearance of a ring or circle under water, and therefore could not have been ignorant of the common phenomena of refraction. We find, indeed, many questions concerning optical appearances in the works of Aristotle. This author was sensible that it is the reflection of light from the atmosphere which prevents total darkness after the sun sets, and in those places where he does not shine in the day-time. He was also of opinion, that rainbows, halos, and mock suns, were all occasioned by the reflection of the sun-beams in different circumstances, by which an imperfect image of his body was produced, the colour only being exhibited, and not his proper figure. The ancients were not only acquainted with the more ordinary appearances of refraction, but knew also the production of colours by refracted light. Seneca says, that when the light of the sun shines through an angular piece of glass, it shows all the colours of the rainbow. These colours, however, he says, are false, such as are seen in a pigeon's neck, when it changes its position; and of the same nature, he says, is a speculum, which, without having any colour of its own, assumes that of any other body.

It appears also that the ancients were not unacquainted with the magnifying power of glass globes filled with water, though they probably knew nothing of the reason of this power; and it is supposed that the ancient engravers made use of a glass globe filled with water to magnify their figures, that they might work to more advantage. Ptolemy, about the middle of the second century, wrote a considerable treatise on optics. This work was supposed, till very lately, to have been lost, but from some late Paris journals it appears to have been recently discovered in a library of that city, and Delambre read an interesting account of it to the academy, from which it appears that Ptolemy was not only acquainted with the refraction of light, but had determined the ratio of the angles of refraction as compared with those of incidence; and it is singular that his results differ very little from those of Newton, not more than might have arisen from his using water and glass of a little different specific gravities to those employed by Newton; he was not, however, aware of the ratio of the sines of those angles. He there also treats of the effects of refractions in astronomical observations, and shows that it decreases from the horizon to the zenith, where it is zero, &c. &c. The more early astronomers were not aware that the intervals between the stars appear less when near the horizon than in the meridian, and on this account they must have been much embarrassed in their observations; but it is evident that Ptolemy was aware of this defect by the caution which he gives to allow something for it, whenever recourse is had to ancient observations. This philosopher also advances a very ingenious hypothesis to account for the remarkably great apparent size of the sun and moon when seen near the horizon. The mind, he says, judges of the size of objects by means of a preconceived idea of their distance from us; and this distance is fancied to be greater when a number of objects are interposed between the eye and the body we are viewing, which is the case when we see the heavenly bodies near the horizon. In his Almagest, however, he ascribes this appearance to a refraction of the rays by vapours, which actually enlarges the angle under which the luminaries appear, just as the angle is enlarged by which an object is seen from under water.

Alhazen, an Arabian writer, was the next author of any importance, who wrote on this subject about the year 1100. Alhazen made many experiments on refraction, at the surface between air and water, air and glass, and water and glass; and hence he deduced several properties of atmospherical refraction, such as, that it increases the altitude of all objects in the heavens; and he first advanced that the stars are sometimes seen above the horizon by means of refraction, when they are really below it; which observation was confirmed by Vitellio, Walther, and especially by the observations of Tycho Brahe. Alhazen observed, that refraction contracts the diameters and distances of the heavenly bodies, and that it is the cause of the twinkling of the stars. This refractive power he ascribed not to the vapours contained in the air, but to its different degrees of transparency. And it was his opinion, that so far from being the cause of the heavenly bodies appearing larger near the horizon, that it would make them appear less; observing that two stars appear nearer together, in the horizon, than near the meridian. This phenomena he ranks among optical deceptions. We judge of distance, he says, by comparing the angle under which objects appear, with their supposed distance; so that if these angles be nearly equal, and the distance of one object be conceived greater than that of the other, this will be imagined to be larger. And he farther observes, that the sky near the horizon is always imagined to be farther from us than any other part of the concave hemisphere.

In the writings of Alhazen, we likewise find the first distinct account of the magnifying power of glasses, and it is not improbable that his writings on this head gave rise to the useful invention of spectacles; for he says, that if an object be applied close to the base of the larger segment of a sphere of glass, it will appear magnified. He also treats of the appearance of an object through a globe, and says that he was the first who observed the refraction of rays in it.

In 1270, Vitellio, a native of Poland, published a treatise on optics, containing all that was valuable in Alhazen, and digested in a better manner. He observes that light is always lost by refraction, which makes objects appear less luminous. He gave a table of the results of his experiments on the refracting power of air, water, and glass, corresponding to different angles of incidence. He ascribes the twinkling of the stars to the motion of the air in which the light is refracted; and he illustrates this hypothesis by observing, that they twinkle still more when viewed in water put in motion. He also shows, that refraction is necessary, as well as reflection, to form the rainbow; because the body which the rays fall upon is a transparent substance, at the surface of which one part of the light is always reflected, and another refracted. And he makes some ingenious attempts to explain refraction, or to ascertain the law of it. He also considers the foci of glass spheres, and the apparent size of objects seen through them, though with but little accuracy. See *Refraction.*

Contemporary with Vitellio was Roger Bacon, a man of very extensive genius, who wrote upon almost every branch of science; though it is thought his improvements in optics were not carried far beyond those of Alhazen and Vitellio; to him, however, has been attributed the invention of the magic lantern. One of the next who distinguished himself in this way was Maurolycus, teacher of mathematics at Messina. In a treatise "De Lumine et Umbra," published in 1575, he demonstrates, that the crystalline humour of the eye is a lens which collects the rays of light issuing from the objects, and throws them upon the retina, where the focus of each pencil is formed. From this principle he discovered the reason why some people are short-sighted, and others long-sighted; also why the former are relieved by concave glasses, and others by convex ones.

Contemporary with Maurolycus was John Baptista Porta of Naples. He discovered the camera obscura, which throws considerable light on the nature of vision. His house was the constant resort of all the ingenious persons at Naples, whom he formed into what he called an academy of secrets, each member being obliged to contribute something that was not generally known, and might be useful. By this means he was furnished with materials for his "Magia Naturalis," which contains his account of the camera obscura, and the first edition of which was published, as he informs us, when he was not quite fifteen years old. He also gave the first published hint of the magic lantern, which Kircher afterwards followed and improved. His experiments with the camera obscura convinced him, that vision is performed by the intermission of something into the eye, and not by visual rays proceeding from it, as had been formerly imagined; and he was the first who fully satisfied himself and others upon this subject. He justly considered the eye as a camera obscura, and the pupil the hole in the window-shutter; but he was mistaken in supposing that the crystalline humoûr corresponds to the wall which receives the images; nor was it discovered till the year 1604, that this office is performed by the retina. He made a variety of just remarks concerning vision, and particularly explained several cases in which we imagine things to be without the eye, when the appearances are occasioned by some affection of the eye itself, or by some motion within the eye. He remarked also, that in certain circumstances, vision will be assisted by convex or concave glasses; and he seems even to have made some small advances towards the discovery of telescopes. Other treatises on optics, with various and gradual improvements, were afterwards successively published by several authors, particularly by Newton, whose genius nowhere displays itself to greater advantages than in the beautiful theory which he has established.

ORB, a spherical shell, or hollow sphere, whence the denomination of orbs to the several supposed moveable spheres, of which the ancients conceived the heavens to consist; and by the introduction of which they endeavoured to account for all the celestial motions and phenomena.

ORBIT, the path of a planet, comet, or other celestial body. The orbits of the several planets were, even after the restoration of the Pythagorean system of astronomy by Copernicus, supposed to be circular, having the sun in their common centre, which, indeed, was such a rational and simple hypothesis, that it is not at all singular both Copernicus and Kepler, as well as the other astronomers of that day, were so unwilling to give up this idea. However, Kepler, after an immense number of observations upon the planet Mars, found that it was impossible to reconcile his observations with that theory, and he therefore abandoned it; but now, though he changed the figure from a circle to an ellipse, he still, by supposing the sun in the centre, found nearly the same difficulty in accounting for some of the observed phenomena. At length, however, it happily occurred to him to place the sun in one of the foci, with which position every observed irregularity perfectly agreed; and by that perseverance, for which he is so eminently distinguished, he came finally to those fundamental laws, which still bear his name, *viz.*

1. The planets all revolve in eliptic orbits situated in planes passing through the centre of the sun, the latter body occupying one of the foci of the ellipse.

2. Equal areas are described in equal times. That is, if a line be supposed to join the central and revolving body, that line passes over or describes equal areas in equal times.

3. The squares of the times of revolution in planetary bodies, are as the cubes of their distances from the sun.

ORDER, of a *Line*, in the *Theory* of *Curves*, is denoted by the dimension of the equation by which the line is defined or expressed; it being said to be of 1st, 2d, 3d, &c. order, according as the equation is of the 1st, 2d, 3d, &c. degree or dimension.

ORDINATE, in the *Theory* of *Curves*, is any right line drawn from a point in the absciss to terminate in the curve; and if it be drawn perpendicular to the absciss, it is called a *right ordinate.* It is a general property of the ordinates of a curve, that when they are perpendicular to the axis, or principal diameter of a line of the 2d order, that is, in the circle and conic sections, the ordinates are all bisected by that axis, making the sum of all the ordinates on one side equal to that of those on the other; and in lines of the 3d order, where a line may cut the curve in three places, the ordinate on the one side is always equal to the sum of the ordinates on the other; and the same for curves of any dimension, *viz.* the sum of the ordinates on one side of the axis, is always equal to the sum of those on the other.

ORGANICAL *Description* of Curves, is the method of describing them on a plane by means of instruments. Of these, the compasses and ruler are the most simple, and are the only instrument admitted into plane or elementary geometry; but other instruments have been invented for the description of various curves.

ORIENT, the east or eastern point of the horizon.

Orient *Equinoctial*, that point of the horizon where the sun rises when in the equinoctial.

Orient *Estival*, is that point of the horizon where the sun rises in the middle of summer.

Orient *Hybernal*, that point of the horizon where the sun rises in the middle of winter.

ORIENTAL, any thing or place situated to the eastward of an observer.

ORION, one of the grandest of the ancient constellations, situated in the southern hemisphere.

ORRERY, an astronomical instrument for exhibiting the several motions of the heavenly bodies. The first machine of this kind was constructed by Mr. Graham, but its name is derived from one made by Mr. Rowley for the Earl of Orrery, which was supposed by Sir R. Steel to be the first ever constructed; and he therefore gave it the above name in honour of the Earl, and attributes the invention to Mr. Rowley; which name it has ever since retained, though the error on which it was adopted has been long corrected.

ORTHOGRAPHIC *Projection* of the *Sphere*, is that projection which is made upon a plane passing through the middle of the sphere, by an eye placed vertically at an infinite distance.

OSCILLATION, in *Mechanics*, vibration, or the reciprocal ascent and descent of a pendulum.

Axis of Oscillation, is a right line passing through the point of suspension parallel to the horizon.

Centre of Oscillation, is that point in a vibrating body into which, if all the matter of the body were collected, the vibrations would be performed in the same time. See Centre of *Oscillation*.

Huygens, in his work "De Horologia Oscillatorio," first gave a general solution of the problem of oscillatory bodies, by reducing all compound bodies or pendulums to simple ones; or, which is the same, by finding their common centre of oscillation. This solution he founded on this principle, that the common centre of gravity of several bodies, connected together, must return precisely to the same height whence it fell; whether those weights return conjointly, or whether after their fall they return separately, each with the velocity it had then required.

This hypothesis gave rise to various discussions, some adopting it and others objecting to it; and others again, who were disposed to think it correct, were not willing to adopt it without demonstration.

After various disputes, James Bernouilli published, in the Memoirs of the Academy of Sciences for 1703, a perfectly satisfactory geometrical demonstration of Huygens's hypothesis, by referring all the different weights to a lever; and, after his death, which happened in 1705, his brother John gave another demonstration, much more simple than that of the former, which he published in the same Memoirs for 1714; and at the same time, Dr. Brook Taylor published his demonstration in his *Methodus Incrementorum*, which gave rise to some dispute between these two authors, each accusing the other of plagiarism; the particulars of which may be seen in the Leipsic Acts for 1716, as also in the works of John Bernouilli, published in 1742.

OSCULATION, in the *Theory* of *Curves*, denotes the contact between any curve and its osculatory circle; or that circle which has the same curvature as the curve at the given point of osculation.

OSTENSIVE *Demonstration*, is a direct geometrical demonstration, in contradistinction to an apogogical one, or that which depends upon a reductio ad absurdum.

OTACOUSTIC, a name sometimes given to a hearing trumpet, and other instruments for improving the sense of hearing.

OVAL, an oblong curvilinear figure, having two unequal diameters, and bounded by a curve line returning into itself. Under this general definition of an oval is included the ellipse, which is a regular oval; and all other figures which resemble the ellipse, though without possessing its properties, are classed under the same general denomination.

P.

PALLAS, one of the new planets discovered in 1802. It is situated between the orbits of Mars and Jupiter, and is nearly of the same magnitude with Ceres, but of a less ruddy colour. It is surrounded with a nobulosity of less extent, and performs its annual revolution in nearly the same period. The planet Pallas, however, is distinguished in a very remarkable manner from Ceres and all other primary planets, by the immense inclination of its orbit. While these bodies are revolving round the sun in almost circular paths, rising only a few degrees above the plane of the ecliptic, Pallas ascends above this plane at an angle of about thirty-five degrees, which is nearly five times greater than the inclination of Mercury. From the eccentricity of Pallas being greater than that of Ceres, or from a difference of position in the line of their apsides, while their mean distances are nearly equal, the orbits of these two planets mutually intersect each other; a phenomenon, which is altogether anomalous in the solar system. The atmosphere of Pallas, according to the observation of Schroeter, is to that of Ceres as 101 to 146, or nearly as 2 to 3. It undergoes similar changes, but the light of the planet exhibits greater variation. On the first of April the atmosphere suddenly cleared up, and the solid nucleus, or disc of the planet, was alone visible. About twenty-four hours afterwards she appeared pale and surrounded with fog, and this appearance continued during the third and fourth of April. Schroeter has shown, that this phenomenon does not arise from the diurnal rotation of the planet.

The diameter of Pallas has not been determined with sufficient accuracy. Dr. Herschel makes it only eighty miles, which is but one-half the diameter of Ceres; while Schroeter makes it no less than 2099 miles, which is considerably larger than the magnitude that he assigned to Ceres. The elements of the orbit of Pallas, and the other particulars which are known respecting this planet, are given in the following table.

Tropical revolution . .	$4^{y}\ 7^{m}\ 11^{d}$
Sidereal revolution, from Maskelyne's Table • • • • • • • •	$1703^{d}\ 16^{h}\ 48'$
Annual motion • • •	$2^{\circ}\ 18'\ 11''$
Mean longitude, Jan. 1, 1804 • • • • • •	$9^{s}\ 29^{\circ}\ 52'\ 58''$
Place of ascending node in 1802, from Maskelyne's Table	$5^{s}\ 22^{\circ}\ 28'\ 57''$
Place of ascending node in 1804, according to Lalande • • •	$5^{s}\ 22^{\circ}\ 28'$
Place of perihelion •	$4^{s}\ 1^{\circ}\ 7'$
Eccentricity, the mean distance being one • • • • • •	0·24630
Inclination of orbit in 1801, according to Maskelyne's Table	$34^{\circ}\ 50'\ 40''$
Inclination of orbit in 1804, according to Lalande • • • • • •	$34^{\circ}\ 39'$
Greatest equation of centre • • • • • • • •	$28^{\circ}\ 25'$
Mean distance from the sun, that of the earth being one • •	2·7910
Mean distance in English miles • • • • •	266000000

Diameter in English miles, according to Herschel	80	app. 0″ 5
Ditto, according to Schroeter	2099	app. 6″·5
Ditto, comprehending the atmosphere of Pallas	3036	
Height of the atmosphere	468	

PARABOLA, is one of the conic sections formed by the intersection of a plane and a cone, when the plane passes parallel to the side of the cone. This figure, like all the other conic sections, may be treated of in three different ways; *viz.* 1. As being produced by the intersection of a plane and cone. 2. According to its description in plano. And, 3. As being generated by the motion of a variable line or ordinate, along another fixed line or directrix; in which case the properties of the curve are deduced from the equation by which it is defined or expressed.

Properties of the Parabola.—1. The abscisses of the parabola are to each other, as the squares of their corresponding ordinates.

2. In the parabola the distance from the vertex to the focus is equal to one-fourth of the parameter, or to half the ordinate at the focus.

3. A line drawn from the focus of a parabola to any point in the curve, is equal to the sum of the focal distance, and the absciss of the ordinate to that point.

4. If through a point in the axis produced, taken so that the distance from the vertex is the same as that of the focus, a line be drawn perpendicular to the axis, that line is called the *Directrix* of the *Parabola*, which has this property; *viz.* that if there be drawn any number of lines parallel to the axis and meeting the curve, they will be equal to the lines from the same points in the curves to the focus.

5. In any parabola the absciss of any ordinate is equal to the distance of the vertex, from the point where the tangent to the parabola at the extremity of that ordinate cuts the axis produced.

6. The same properties as have been stated above, and a variety of others belonging to the axis, abscisses, and ordinates of the axis, are equally true of any other diameters.

The equation of the parabola is $yy = 4\,cx \sin^2 \frac{1}{2}$ B; therefore if we call the constant quantity $4c \sin^2 \frac{1}{2}$ B $= p$, we shall have $yy = px$.

Hence the squares of the ordinates of the parabola are to each other as their abscissæ.

Therefore if we take one abscissa, double another, the squares on the two corresponding ordinates will be in the ratio of 2 to 1.

The indefinite line A L, (plate parabola, fig. 1,) is called the axis of the parabola, the point A is its vertex, A Q is an abscissa, M Q the corresponding ordinate to this abscissa, and the constant quantity p is called the parameter, or *latus rectum*, of the axis.

We can always determine this quantity p by the equation $yy = px$, which gives $x : y = y : p$. For this purpose it is sufficient to take any abscissa and its corresponding ordinate, and a third proportional to these two lines will be the parameter of the parabola passing through their extremities.

If we take the abscissa A F $= \frac{1}{4} p$, the point F is called the focus, and the ordinate D F passing through this point will have for its expression $\sqrt{\frac{1}{4} p^2} = \frac{1}{2} p$. Therefore the double ordinate D d passing through the focus is equal to the parameter.

If on L A continued, we take A G $=$ A F $= \frac{1}{4} p$, and if through the point G we draw the indefinite line E G e parallel to the ordinate M Q, this line E G e parallel to the ordinate M Q, this line E Ge is called the *directrix*.

$$\text{Now FM} = \sqrt{\left\{yy + (x - \tfrac{1}{4} p)^2\right\}}$$

$$= \sqrt{\left\{px + (x - \tfrac{1}{4} p)^2\right\}} = x + \tfrac{1}{4} p$$

$=$ A Q $+$ A G $=$ M H, therefore F M $=$ M H; hence the distance from any point M of the parabola to the directrix, is equal to the distance from this same point to the focus F. This property suggests an easy method of describing the parabola by a continuous movement, as will be shewn at the end of the next section.

Let it now be proposed to draw a tangent M T (fig. 2,) at a given point M of the parabola.

If we suppose the arc M m infinitely small, its continuation M m T will be the tangent required. Now if we draw the perpendiculars M Q, m q on the directrix, and the lines M F, m F to the focus F, and draw m r parallel to Q q, and if from the point F as a centre, and with the radius F m, we suppose an arc described infinitely small m s, which may be considered as a line, we shall have M Q = M F, m q = m F, and therefore M Q − m q, or M r = M F − m F, or M s. The right-angled triangles M m r, M m s are therefore equal and similar, and consequently the angle m M r or T M F = q M m = Q M T = M T F. Hence the triangle M T F is isosceles, and consequently if we take F T = F M, the line M T drawn through the points T and M will be the tangent required.

The angle M T F = L M O = F M T. Therefore all luminous or sonorous rays O M parallel to the axis A P, must on their impinging on the parabola A M be reflected to the focus F, since we know that the angle of reflection is equal to the angle of incidence.

Since F M = $x + \frac{1}{4} p$, we have F T $- \frac{1}{4} p$ = A T = x. Therefore P T, or the subtangent = $2x$. Consequently in the parabola the subtangent* is always double the abscissa.

The tangent M T = $\sqrt{(px + 4xx)}$ = $\sqrt{(4\,MF \times x)}$. If we draw the line M N perpendicular to the parabola, or which is the same thing, to its tangent M T at the point M, we shall have P N = $\frac{PM}{PT} = \frac{px}{2x}$ = $\frac{1}{2}p$. Therefore, in the parabola, the subnormal* is always equal to the semi-parameter. As for the normal M N its expression is $\sqrt{(px + \frac{1}{4}p^2)} = \sqrt{(MF \times p)}$. It is, therefore, a mean proportional between the distance from the point M to the focus, and the parameter.

Any line M O (fig. 3,) parallel to the axis of a parabola, is called in general a diameter. The point M is its vertex; four times the distance from this point to the focus F is its parameter q; its ordinates are right lines N P parallel to the tangent at M, and the abscissae of these ordinates are the lines M P.

To find the equation of the co-ordinates of the diameter M O, call M P, x', P N, y', A Q = A T = a; we shall have M Q = $\sqrt{ap}$, $q = p + 4a$, M T = $\sqrt{aq}$; and if we draw N L perpendicular to the axis, the similar triangles N R L, M T Q, will give

M T : N R = M Q : N L = Q T : R L

or $\sqrt{aq} : y' = \sqrt{aq} = \sqrt{ap} : NL = \frac{y'\sqrt{ap}}{\sqrt{aq}} - \sqrt{ap} = 2a : RL = \frac{2ay'}{\sqrt{aq}} - 2a$

But A R = R T − A T = $x' - a$; therefore A L = $x' + a + \frac{2ay'}{\sqrt{aq}}$, and by the property of the parabola, N L^2 = p × A L, or $\left(\sqrt{ap} + \frac{y'\sqrt{ap}}{\sqrt{aq}}\right)^2 = ap + px' + \frac{2apy'}{\sqrt{aq}}$; from which by reduction we obtain $y'y' = qx'$, an equation similar to that which was before found for the co-ordinates of the axis. Hence we may conclude that any diameter M O bisects all the ordinates N n.

The two following problems afford an easy application of the above principles.

1. Given the axis A L of a parabola and its parameter p, to find a diameter M O, which shall make with its ordinates a given angle M P n = A. *(See preceding figure.)*

This problem merely requires us to find the point Q where the perpendicular M Q meets the axis. For this purpose call A Q = a, the triangle M T Q will give

A T : M Q = radius : tang A (be-

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* In any curve A M, (fig. 4,) if T M is a tangent to the curve at the point M and A P and P M co-ordinates to the same point; then T lled the subtangent, M N r to T M is called the N P, the subnormal.

Parabola.

Fig. 1.

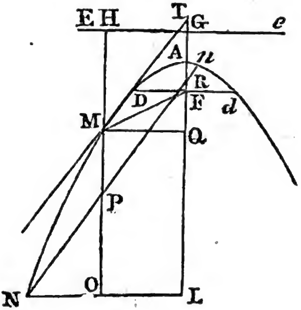


Fig. 2.

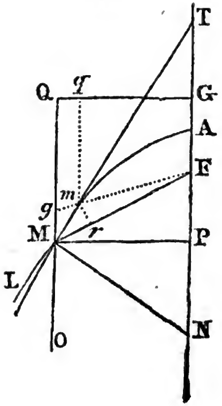
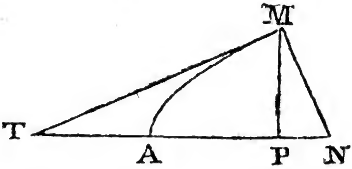


Fig. 3. See Fig. 1.

Fig. 4.



cause $\angle MPn = \angle MTQ$) or in symbols, $2x : \sqrt{px} :: 1 : \text{tang } A$

Hence $x = \frac{p}{4\,\text{tang}^2 A} = \frac{1}{4} p \cot^2 A$, and the parameter of the diameter MO, or $q = p + 4x = p\,(1 + \cot^2 A)$ $= p \operatorname{cosec}^2 A = \frac{p}{\sin^2 A}$

It is easy to see that this problem has two solutions.

2. Given the parameter q of the diameter MO, with the vertex M of that diameter, and the angle A which it makes with its ordinates; to find the axis AL, its vertex A, and its parameter p. *(See the preceding figure.)*

This problem requires us to find the distance MQ from the axis to the diameter, and then the distance AQ, in order to obtain the vertex A and the parameter p. Preserving the same denominations as in the preceding problem, we have $MQ = \sqrt{px}$, $q = p + 4x = \frac{p}{\sin^2 A}$. Hence we obtain $p = q$, $\sin^2 A$, $x = \frac{1}{4} q \cos^2 A$, and $MQ = \pm \frac{1}{2} q \sin A \cos A = + \frac{1}{4} q \sin 2A$.

Biquadratic **Parabola**, a curve of the third order, having two infinite legs, and is generally expressed by one of these three equations; viz.

$a^3 x = y^4$;

$a^3 x = y^4 - a^2 y^2$;

$a^3 x = y^4 - \overline{a + b} \cdot y^3 - a b y^2$.

Cartesian **Parabola**, is a curve of the second order, expressed by the equation $xy = ax^3 + bx^2 + cx + d$, containing four infinite legs; viz. two hyperbolic and two parabolic.

Cubic **Parabola**, is also a curve of the second order, having two infinite legs tending contrary ways. If the absciss touch the curve in a certain point, the relation between the absciss and ordinate is expressed by the equation

$$y = ax^3 + bx^2 + cx + d$$

and when b, c, and d are each zero, the equation then becomes $y = ax^3$ as in the annexed figure.

The area of the cubic parabola is equal to three quarters of its circumscribing cylinder; but it cannot be rectified even by means of the conic sections.

Diverging **Parabola**, is a name given by Newton to a species of five different lines of the third order expressed by the equation

$$y^2 = ax^3 + bx^2 + cx + d$$

The first is a bell-formed parabola, having an oval at its head, which is the case when the equation $0 = ax^3 + bx^2 + cx + d$ has three real and unequal roots. The 2d is also bell-formed, with a *punctum conjugatum;* which happens when the above equation has its two less roots equal. The 3d is when the two greater roots are equal. The fourth is when the same equation has two imaginary roots. The fifth is when the roots of the proposed equation are all equal to each other; the most simple equation being in this case $py^2 = x^3$; which is the semi-cubical, or *Neilian* parabola.

If a solid, generated by the rotation of a semi-cubical parabola about its axis, be cut by a plane, each of these five parabolas will be exhibited by its sections, according to the positions of the cutting plane.

Infinite **Parabolas** or **Paraboloides**, are parabolas of the higher orders, defined by the general equation $a^{m-n} x^n = y^m$; in all of which the proportion of the area to that of the circumscribing parallelogram, is as m to $m + n$.

Parabolic *Arcs*, are the arcs of the curve of a parabola, the length of which may be found by the following formula:

Let p = parameter, x any absciss from the vertex, and y the corresponding right ordinate; also make $\frac{2y}{p} = q$, and $\sqrt{(1 + q^2)} = s$; then

1. Parabolic arc $= \frac{1}{2} p \left\{ qs + \text{hyp. log.}\, (q + s) \right\}$

2. Parabolic arc $= 2y \left\{ 1 + \frac{q^2}{2 \cdot 3} - \frac{q^4}{2 \cdot 4 \cdot 5} + \frac{3 q^6}{2 \cdot 4 \cdot 6 \cdot 7} - \frac{3 \cdot 5 q^8}{2 \cdot 4 \cdot 6 \cdot 8 \cdot 9} + \text{\&c.} \right\}$

3. Parabolic arc $= 2y \left\{ 1 + \frac{q^2}{} \right.$

$$A - \frac{1 . 3 q^2}{4 . 5} B + \frac{3 . 5 q^2}{6 . 7} C - \&c. \Big\}$$

Where A, B, C represent the preceding terms. To which may be added, the following approximations:

4. Parabolic arc $= 2 \sqrt{(y^2 + \frac{4}{3} x^2)}$ nearly.

5. Parabolic arc $= \frac{2}{5} \left\{ \sqrt{(y^2 + \frac{4}{3} x^2)} - \frac{y^2 + \frac{2}{3} x^2}{\frac{1}{4} y} \right\}$

Parabolic *Frustrum* or *Zone*, is the space included between any two parallel double ordinates.

Let D *d* represent the two ends, and *a* the perpendicular distance between them; then

area of zone $= \frac{2}{3} a \times \frac{D^3 - d^3}{D^2 - d^2}$

When $d = o$ the area of the whole parabola becomes

area of parabola $= \frac{2}{3} a D$.

Parabolic *Pyramidoid*, is a solid generated by supposing all the squares of the ordinates applicates to the parabola, so placed that the axis shall pass through all their centres at right angles; in which case the aggregate of the planes will form the solid called the *Parabolic Pyramidoid*; the solidity of which is equal to the product of the base, and half the altitude.

Parabolic *Spiral* or *Helicoid*. See *Spiral*.

Parabolic *Spindle*, is the solid generated by the rotation of a parabola about any double ordinate; the solidity of which is found as follows:

Let *m* denote the middle diameter, and *l* the length of the spindle; then

1. Solidity $= {\cdot}418879\, l\, m^2$

And the solidity of a middle frustrum of such solid, is

Solidity $= {\cdot}05236\, l\, (8 m^2 + 3 d^2 + 4 d m)$

where *d* denotes the diameter of the end of the spindle. See *Hutton's Mensuration*.

PARABOLOID, or **Parabolic** *Conoid*, is the solid generated by the rotation of a parabola about its axis, which remains fixed; the surface and solidity of which may be found by the following formulæ:

Let *y* represent the radius of the circular base, *x* the altitude of the solid, and 3 1416 $= p$; then

1. Surface $= \frac{2 p y}{12 x^2} \left\{ (y^2 + 4x^2)^{\frac{3}{2}} - y^3 \right\}$

2. Solidity $= \frac{1}{2} p y^2 x$.

Frustrum of a **Paraboloid**, is the lower solid formed by a plane passing parallel to the base of a paraboloid.

Let D denote the greater diameter, *d* the less, P the parameter, and *h* the height of the frustrum 3·1416 $= p$; then

1. Surface $= \frac{(P^2 + D^2)^{\frac{3}{2}} - (P^2 + d^2)^{\frac{3}{2}}}{P} \times \frac{1}{6} p$

2. Solidity $= {\cdot}3927\, h\, (D^2 + d^2)$

or when $d = o$

Solidity $= {\cdot}3927\, h\, D^2$ for the whole paraboloid.

These formulæ only obtain when the base of the frustrum is perpendicular to the axis of the solid.

PARACENTRIC *Motion*, is the motion of a planet towards the centre of attraction or the sun. The orbits of the planets being elliptical, they are sometimes nearer and sometimes more remote from the sun; and the difference in this distance is what is called the paracentric motion.

PARALLACTIC *Angle*, is the angle subtended by two lines drawn from the centre of a planet, the one to the centre of the earth, and the other to some point on its surface. See **Parallax**.

PARALLAX, in *Astronomy*, an arc of the heavens intercepted between the true and apparent place of any of the heavenly bodies; that is, between its place as viewed from the centre of the earth, and from some point on its surface.

The parallax of any celestial body is $= o$, when that body is in the zenith, because then, the line passing through the eye of the observer, passes also through the centre of the earth; and it encreases gradually toward the horizon, being the greatest possible when a body is situated there. This is obvious from its cause, that being the angle which the semi-diameter of the earth subtends at the body, which diminishes as the position of the semi-diameter makes a less angle with the line of observation, and that and it vanish together. The parallax makes the body appear lower than it would be, if viewed from the centre.

The *Horizontal* PARALLAX, or that which has place when the star is in the horizon, is the greatest.

Parallax receives particular denominations, according to the circle upon which it is computed.

PARALLAX of *Altitude*, is the difference between the true and apparent altitude of a planet, as above described.

PARALLAX of *Right Ascension* and *Descension*, is an arch of the equinoctial, by which the parallax of altitude increases the ascension and diminishes the descension.

PARALLAX of *Declination*, is an arch of a circle of declination, by which the parallax of altitude increases or diminishes the declination of a star.

PARALLAX of *Latitude*, is an arch of a circle of latitude, by which the parallax of altitude increases or diminishes the latitude.

Menstrual PARALLAX of the *Sun*, is an angle formed by two right lines; one drawn from the earth to the sun, and another drawn from the sun to the moon, at either of their quadratures.

PARALLAX of the *Annual Orbit* of the *Earth*, is the difference between the heliocentric and geocentric place of a planet, or the angle at any planet subtended by the distance between the earth and sun.

PARALLAX of the *Planets*. The exact determination of the parallax of the planets is of the greatest importance in astronomy, as it is from that, or indeed from the parallax of any one of them, we estimate their several distances. For if the parallax, or the angle which is subtended by the terrestrial radius, at any planet be known, its distance from the earth is also known; whence its distance from the sun, as also the distances of all the other planets, will be known also, from the third law of Kepler, *viz.* that the squares of the periodic times are as the cubes of the distances.

There are many different methods of determining the parallax of the planets; but being an extremely delicate operation, on account of the smallness of the angles, we had several different results; till the parallax of Venus was determined by means of her transits over the sun's disc, in 1761 and 1769.

Laplace has determined the parallax of the sun to be 8″¾, whence its distance is found to be about 94 million miles.

And hence the parallaxes of the several planets are determined as below:

Greatest horizontal parallel of the . .	Sun . .	8″.75
	Mercury	14″.58
	Venus .	29″.16
	Mars . .	17″.50
	Jupiter .	2″.08
	Saturn .	1″027
	Uranus .	0″415

PARALLAX of the *Moon*. This is much more considerable than in any other of the heavenly bodies, on account of its proximity to the earth, and is much easier determined than any of the others; one of the most simple and correct methods being as follows:

Observe the moon's meridian altitude with the greatest accuracy, and mark the moment of observation: this time being equated, compute her true longitude and latitude, and from these find her declination, and from her declination, and the elevation of the equator, find her true meridian altitude; if the observed altitude be not meridian, reduce it to the true for the time of observation; take the refraction from the observed altitude, and subtract the remainder from the true altitude; and the remainder is the moon's parallax; by this means the parallax of the moon has been found as follows, *viz.*

Greatest parallax . .	61′	32″
Least parallax	54	4
Mean parallax	57	48
But Laplace makes her mean parallax	56	34″.2

PARALLAX of the *Fixed Stars*. As the distances of the heavenly bodies are determined by means of their parallaxes, every possible method has been attempted to ascertain the parallax of some of the fixed stars, but their distance is so immense, that not only have they no parallax, as compared with the terrestrial radius, but even the whole diameter of the earth's orbit, though near 200 million miles, is not sufficient to render any difference in their apparent places at all evident, which, if it only amounted to one second, would be discoverable by modern instruments. This distance, therefore, great as it is, is not more than a mere point compared with the distance of the nearest of the fixed stars.

Parallel, in *Geometry*, is applied to lines, figures, and bodies, which are every where equidistant from each other, or which, if ever so far produced, would never meet.

Parallel *Right Lines*, are those which, though infinitely produced, would never meet.

To draw a line parallel to a given line through a given point. Draw a line meeting the given line in any point; and make at the given point an angle equal to the alternate angle made at the given line, then the line making this angle is the parallel.

Parallel *Ruler*, an instrument consisting of two wooden, brass, or steel rulers, equally broad throughout, and so joined together by the cross blades, as to open to different intervals, and accede and recede, yet still retain their parallelism. The use of this instrument is obvious; for one of the rulers being applied to a given line, another drawn along the extreme edge of the other will be parallel to it; and thus, having given only one line, and erected a perpendicular upon it, we may draw any number of parallel lines or perpendiculars to them; by only observing to set off the exact distance of every line with the point of the compasses.

But the best parallel rulers are those whose bars cross each other, and turn on a joint at their intersection; one of each bar moving on a centre, and the other ends sliding in grooves as the rulers recede.

PARALLELEPIPED, or **Parallelopiped**, or **Parallelopipedon**, is a solid figure contained under six parallelograms, the opposite of which are equal and parallel; or it is a prism whose base is a parallelogram. See Prism.

PARALLELISM, the quality of being parallel.

Parallelism of the *Earth's Axis*, is used to denote that invariable position of the terrestrial axis, by which it always points to the same point in the heavens, abstracting from the trifling effect of nutation, &c.

PARALLELOGRAM, in *Geometry*, is a quadrilateral right-lined figure, whose opposite sides are parallel.

Parallelograms receive particular denominations according to the equality or inequality of its sides and angles.

Thus a *rectangle, rhombus, rhomboid*, and *square*, are each a particular species of parallelograms, for the properties of which see the several articles.

Other properties common to every parallelogram may be enumerated as follows:

1. A parallelogram has its opposite sides and angles equal to each other, and the diagonal divides it into two equal triangles.

2. The two adjacent angles of any parallelogram are together equal to two right angles.

3. Parallelograms having equal bases and altitude are equal; on equal basis they are to each other as their altitudes; and with equal altitudes they are to each other as their bases; and generally parallelograms are to each other in the compound ratio of their bases and altitudes.

4. The sum of the squares of the diagonals of any parallelogram is equal to the sum of the squares of the four sides.

5. The complements about the diagonals of any parallelogram are equal to each other.

Parallelogram of *Forces*, is a term used to denote the composition of forces, or the finding a single force that shall be equivalent to two or more given forces when acting in given directions, the principles of which may be thus illustrated.

The simultaneous action of two impulsive forces on a body which would impress upon it separately the velocities, in directions making an angle with each other, will cause that body to move uniformly over the diagonal of the parallelogram whose sides are in the directions of those forces.

Let the body be situated on a plane, the containing sides of which make any given angle with each other, and let the plane be moved parallel to one of its containing sides, and with an uniform velocity, so that it may arrive at the other extremity of that side, that is, that the plane may have moved its whole extent in any unit of time; it is obvious that, with regard to the plane, the body has remained at rest, but that with regard to space, it has passed over a line equal to the side of the plane. Suppose, again, that the body is acted upon at the same time by another force, which would carry it parallel to the other containing side of the plane, and make it arrive at the extremity of that side at the same instant that the motion of the plane brings it to the extremity of the other; then it is obvious, that the body will have arrived at the angle of a parallelogram, of which the lines of motion are the containing sides, opposite to that from which the body is supposed to set out; and that its path will have been along the diagonal of that parallelogram. At the half of the supposed unit of time, it will be at the middle of the parallelogram, or at the middle of the diagonal; and its situation at any other period may be easily found. If the angle made by the two forces, or motions which are the measures of those forces, be a right angle, the force produced, or the motion which is the measure of that force, will be equal to the square root of the sum of their squares; if less than a right angle, it will be greater than this; and, if greater than a right angle, it will be less. If the angle vanish by the forces and motion coinciding, the result will be their sum; and if it vanish by their opposing each other, it will be their difference.

It could likewise be shewn, that if a body be acted on by two similar variable forces, whose directions and magnitudes are expressed by the adjacent sides of a parallelogram meeting in the body, it will describe the diagonal of the parallelogram.

Hence it appears, that in order to find a force that shall be equivalent to two forces, whose quantity and direction is given, we have only to find the diagonal of the parallelogram, by the sides of which the given forces are represented, and this will express the quantity and direction of the force which is equivalent to them both.

And in the same manner may be found the equivalent to three or more forces, by first reducing two of them to one single and equivalent force; then this and one of the other given forces; and so on till they are all reduced to one equivalent force.

Suppose, for example, it were required to compound three given forces; or to find the quantity and direction of one single force, which shall be equivalent to three given forces.

First reduce the two to one, by completing the parallelogram. Then reduce the result of these and the third to one, by completing a second parallelogram.

Hence conversely, any single direct force may be resolved into two or more oblique forces; which is done by merely describing any parallelogram, such that the line representing the given single force may be its diagonal; and as these may be an indefinite number of parallelograms having the same diagonal, so may any single force be resolved in an indefinite number of ways into two or more oblique forces, that shall produce the same effect as the single given force.

Parallelogram of the *Hyperbola*, is used to denote the modulus, or any of the equal parallelograms inscribed between the curve and its asymptote.

PARALLELOGRAM *Protractor*, a mathematical instrument, consisting of a semicircle of brass, with four rulers in form of a parallelogram, made to move to any angle: one of these rulers is an index, which shows on the semicircle the quantity of any inward or outward angle.

PARALLELOGRAM, is also the name of an instrument for copying plans, otherwise called a PENTAGRATH.

PARAMETER, a certain and constant right line in each of the three conic sections, and otherwise called the *Latus Rectum*, because it measures the conjugate axis by the same ratio, which has place between the axes themselves, being alway a third proportional to them, viz. a third proportional to the transverse and conjugate axes in the ellipse and hyperbola, and, which is the same thing, a third proportional to any absciss and its corresponding ordinate in the parabola.

PARHELION, or PARHELIUM, denotes a mock sun or meteor, appearing as a very bright light near the sun, being formed by the reflection of his beams, in a cloud properly situated to receive them.

PARTIAL *Differences, Theory of*, is one of the higher branches of the modern analysis, for the developement of which we are principally indebted to d'Alembert, though the subject had been previously investigated by other mathematicians.

Integration of *Equations* of PARTIAL *Differences.* Let u be any function of x, y, and z, and δ, d, D, the characteristics of their differentiation relatively to each of the variables, then an equation compounded in any manner, with any of the quantities,

$$x,\quad y,\quad z,\quad \&c.$$

$$\frac{\delta u}{dx},\quad \frac{du}{dy},\quad \frac{\mathrm{D}u}{dz},\quad \&c.$$

$$\frac{\delta\delta u}{dx^2},\quad \frac{d\delta u}{dxdy},\quad \frac{\mathrm{D}\delta u}{dxdz},\quad \&c.$$

is called an equation of partial differences, and which may be compounded or not, with constant or known quantities. The degree of the equation is estimated by the highest power of any of the differentials that enters into it.

Thus, $\frac{\delta u}{dx} + \frac{du}{dy} - xy = 0$

is a partial differential equation of the first degree, the solution or integration of which depends upon finding such relation between the variables x and y and the function u; that the latter being first differenced with x as variable and y constant, and divided by dx; *plus* the same differenced with y as variable, and x constant, and divided by dy, the sum of the two partial differences $\frac{\delta u}{dx} + \frac{du}{dy}$ may be equal to xy, and the method of performing this is called the *integral calculus of partial differences.*

Again

$$\frac{\delta u}{dx} + \frac{du}{dy} - x^2 - 4xy - y^2 = 0$$

is also an equation of partial differences, the integral of which is $u = x^2y + y^2x$.

For differencing this first relatively to x variable, and dividing by dx, we have

$$\frac{\delta u}{du} = 2xy + y^2.$$

And the same with reference to y gives

$$\frac{du}{dy} = 2x^2 + 2xy;$$

consequently

$$\frac{\delta u}{dx} + \frac{du}{dy} = x^2 + y^2 + 4xy.$$

This integral it obtained merely by the reverse operation by which the function was thrown into partial differences, and is therefore very obvious; but when the equation arises in the solution of a problem or otherwise, providing its origin is not known, its integration is frequently extremely difficult, and requires the most profound and accurate analytical judgment.

Let there be proposed the equation

$$\frac{d\,d\,u}{d\,y^2} = \frac{a^2\,\delta\,\delta\,u}{d\,x^4}$$

Suppose
$du = pdx + qdy$; $dp = rdx + sdy$;
$dq = sdx + tdy$, then $t = a^2 r$
therefore
$dq = sdx + a^2 rdy$, and
$adp = ardx + asdy$.

Whence $adp + dq = (s + ar)\ (dx + ady)$ which is one complete differential, and therefore its equal $(s + ar)\ (dx + ady)$ must be one likewise; and consequently $s + ar$ is some function of $x + ay$, and $ap + q$ another function also of $x + ay$.

Let therefore $ap + q = f(x + ay)$
and $ap - q = f'\,(x - ay)$
the latter being obtained in the same manner as the former.

Whence $2adu =$
$(dx + ady).f\,(x + ay) +$
$(dx - ady).f_{\prime}(x - ay)$
and integrating, $2an = \phi(x + ay) + \phi'\ (x - ay)$ an integral equation, whatever may be the functions ϕ and ϕ', which functions are to be determined from the nature of the problem. See Mem. of Berlin for 1747, 1748, 1750, and 1763; vols. i. and ii. of the Memoires of Turin; and the Opuscules of d'Alembert, and the other papers referred to in the preceding part of this article.

PARTICLE, the minute part of a body, or an assemblage of several atoms of which natural bodies are composed. This term is frequently used as synonymous with atom, corpuscule, and molecule, but sometimes it is distinguished from them.

PARTICULAR *Integral*, in the *Integral Calculus*, is that which arises in the integration of any differential equation, by giving a particular value to the arbitrary quantity or quantities that enter into the general integral.

Thus, for example, in the equation

$$y = \frac{2\,xdy}{dx} - \frac{ydy^2}{ax^4}$$

the general integral is

$$y = 4\,m\,(x - m)$$

and by making $m = 1$, this becomes $y = 4\ (x - 1,)$ which is a *particular integral.*

We call also particular integrals those which are obtained by means of an equation of inferior dimensions; thus, in the above, if we make $x = y$, we shall have

$$y = \frac{2\,y\,d\,y}{d\,y} - \frac{y\,d\,y^2}{d\,y} = 2\,y - y$$

which obviously answer the conditions of the equation.

So also in the equation
$xdx + ydy = dy\,\sqrt{}\,(x^2 + y^2 - a^2)$
it is easy to see, that $x^2 + y^2 = a^2)$ answers the condition of the equation, and is therefore called by some a particular integral. But neither this, nor that found above, viz. $x = y$, are deducible from the general integration of the equation whence they are derived, and they thus differ from the particular integral above defined; so that this term ought not to be employed to denote the two distinct cases; and accordingly some authors distinguish the integrals arising from such equations into three kinds, viz. *complete integral*, which is that drawn from the general integration; an *incomplete integral*, which is that found as above, by means of an equation of inferior dimensions; and, lastly, a *particular integral*, which is that arising from giving to the general indeterminate quantities in the complete integral some particular value. Others, again, as Lagrange in his "Leçons des Functions," instead of incomplete integrals calls them *valeures singulaire.*

If, in the case of two variables, we consider these as the rectangular co-ordinates of a curve, it is obvious that we may imagine as many different curves to be described, as there may be particular values given to the general indeterminate quantity; and if it happens that these curves have one constant quantity common to them all, then the same tangent touching all the curves, must involve in its equation the element of one of those curves, yet this tangent is not one of the curves, and therefore its equation is not obtainable from the complete integration.

PEDOMETER, or **PODOMETER**,

a mechanical instrument in form of a watch; consisting of various wheels, with teeth catching in one another, all disposed in the same plane; which, by means of a chain or string, fastened to a man's foot, or to the wheel of a chariot, advance a notch each step, or each revolution of the wheel; so that the number being marked on the edge of each wheel, one may number the paces, or measure exactly the distance from one place to another.

PELECOID, or **PELECOIDES**, *hatchet form*, in *Geometry*, contained under the two inverted quadrantal arcs, and a semicircle.

The area of the pelecoid is demonstrated to be equal to the square of the quadrantal chord.

PENCIL of *Rays*, in *Optics*, denotes a number of rays diverging from some luminous point, which, after falling upon and passing through a lens, converge again on entering the eye.

PENDULUM, in *Mechanics*, denotes any heavy body B, suspended upon, and moving about some fixed point at A, as a centre.

A is called the *centre of motion*, or *of suspension*, and the line passing through this point, parallel to the horizon, the *axis of oscillation*, and that point in the body B, into which, if all the matter of the body was condensed, so as still to perform its oscillations in the same time, is called the centre of oscillation; and the distance of this point from the point of suspension, is accounted the *length* of the pendulum.

Pendulums receive particular denominations, according to the materials of which they are composed, or the purposes they are intended to answer; for an account of which see the subsequent part of this article; we shall here confine our remarks to the theory of the simple pendulum.

Simple **PENDULUM**, theoretically considered, is a single weight attached to a string supposed to be devoid of gravity, and oscillating freely without resistance, either from friction or the air. Its properties are these:

I. In small circular arcs. 1. The vibrations of the same pendulum, or of different pendulums of the same length, and in the same place, in very small arcs of circles, are always performed in the same time. 2. The velocity of the bob, in the lowest point of the arc, will be very nearly as the length of the chord of the arc which it describes in its descent. 3. The times of vibration of different pendulums are, to each other, as the square roots of their lengths. Or their lengths are as the squares of the number of vibrations performed in the same time. 4. On the supposition above made, viz. that there is neither friction nor resistance opposed to the motion of the pendulum, that motion would be perpetual, that is, the force which it acquires in its descent would carry it up to the same height on the opposite side, which would be again repeated in its descent, and thus it would continue to oscillate constantly through the same arc, and consequently its vibrations uniform and perpetual. But as it is impossible to divest it of these two retarding forces, it is obvious, as well from theory as practice, that without the application of some external force, the vibrations will be shortened in every ascent, and the motion of the pendulum ultimately cease. 5. The time of vibration in a circular arc of any sensible magnitude, is expressed by the following formulæ:

$$\text{time} = p\sqrt{\frac{d}{4g}} \times \left\{1 + \frac{1^2 a}{2^2 d} + \frac{1^2.3^2.a^2}{2^2.4^2.d^2} + \frac{1^2.3^2.5^2 a^3}{2^2.4^2.6^2 d^3} + \&c.\right\}$$

Where $d =$ or twice the length of the pendulum, $a =$ the versed sine of half the arc of vibration, $g = 16\frac{1}{12}$ half the force of gravity, and $p = 3{\cdot}1416$ the circumference of a circle whose diameter is 1. 6. When a is very small, as we have supposed in the preceding cases, then writing $2\,l$ instead of d, and omitting all the terms in the above series beyond the second as being inconsiderable, we shall have

$$\text{time} = p\sqrt{\frac{l}{2g}} \times \left(1 + \frac{a}{g\,l}\right) =$$

$\frac{p}{2g}\sqrt{\frac{l}{2g}} \times (8\,l + a)$ 7. Or if instead of using the versed sine of the half arc, we introduce the degrees in that arc, the expression becomes

$$\text{time} = p\sqrt{\frac{l}{2g}} \times \left(1 + \frac{d^2}{5234}\right).$$

II. In cycloidal Arcs.—It is a known property of the cycloid that its evolute is a cycloid, similar and equal to the former. If therefore a pendulum be suspended from the point of concurrence of two equal inverted semicycloidal cheeks, and be made to vibrate between them, the bob will itself describe a cycloidal arc. The properties of a pendulum thus suspended are as follow: 1. The time of oscillation in all arcs, whether great or small, are performed in the same time. 2. The time of vibration in any arc, is to the time in which a heavy body would fall by the force of gravity through half the length of the pendulum; as the circumference of a circle is to its diameter. Now by the laws of falling bodies $\sqrt{\frac{l}{2g}}$ = the time of a body falling through ½ l, or ½ the length; therefore, putting $p = 3{\cdot}1416$, the circumference of a circle whose diameter is 1, we have as $1'' : p = : \sqrt{\frac{l}{2g}} : p\sqrt{\frac{l}{2g}}$ the time of vibration of a pendulum whose length is l. 3. It is obvious in this formula, as well as in those given above, that the time of vibration depends not only on the length of the pendulum, but also on the value of g, which is different for different latitudes; and, consequently, the lengths of the pendulum designed to vibrate any proposed number of times in a second, in any place, must be regulated by the value of g, or by the force of gravity at that place. 4. The value of g in the latitude of London is $16\frac{1}{12}$ feet, or 193 inches, which being substituted for g in the expression $p\sqrt{\frac{l}{2g}} = 1''$, gives $l = 39{\cdot}11$, or $39\frac{1}{9}$ inches for the length of the second's pendulum, which is extremely near the true length, as determined from accurate experiments, which show it to be about $39\frac{1}{8}$ inches. 5. From the above formula may also be found the force of gravity in any particular place, the length of the second's pendulum being previously found from experiment; for, squaring the above, we have $\frac{p^2 l}{2g} = 1$, or $g = \frac{1}{2} p^2 l$, the descent of a heavy body in one second, in terms of the length of the second's pendulum; and this is a much more accurate method of finding the force of gravity, than any other which depends simply on experiments. 6. In speaking of the simple pendulum as vibrating in very small arcs, we found that the time of one vibration was very nearly $= p\sqrt{\frac{l}{2g}} \times \left(1 + \frac{a}{8\,l}\right)$, where a is the versed sine of half the arc of vibration, whereas in a cycloid the time is simply $p = \sqrt{\frac{l}{2g}}$, and therefore the time lost each vibration in the former case is $\frac{a\,p}{8\,l} \times \sqrt{\frac{l}{2g}}$, or estimating by the degrees of the arc it is $\frac{D^2 p}{52524} \times \sqrt{\frac{l}{2g}}$; and hence we may compute how much the pendulum must be shortened in the former case to make its vibrations correspond with one vibrating as in the latter case. 7. We have seen above, that the length of the pendulum varies in different latitudes in consequence of the variation in the force of gravity; and, on this principle, is computed the following table, which shows the length of the second's pendulum for every 5 degrees of latitude from the equator to the pole.

Deg. of Lat.	Length of Pendulum.	Deg. of Lat.	Length of Pendulum.
0	39·027	50	39·126
5	39·029	55	39·142
10	39·032	60	39·158
15	39·036	65	39·168
20	39·044	70	39·177
25	39·057	75	39·187
30	39·070	80	39·191
35	39·084	85	39·195
40	39·097	90	39·197
45	39·111		

The foregoing laws, &c. of the motion of pendulums, cannot strictly hold good, unless the thread that sustains the ball be void of weight, and the gravity of the whole ball be collected into a point. In practice, therefore, a very fine thread, and a small ball, but of a very heavy matter, are to be used. But a thick thread and a bulky ball disturb the motion very much; for, in that case, the simple pendulum becomes a compound one; it being much the same thing, as if several weights were applied to the same inflexible rod in several places.

M. Kraft, in the new Petersburg Memoirs, vols. vi. and vii. has given the result of many experiments upon pendulums, made in different parts of Russia, with deductions from them; from thence he derives this theorem: if x be the length of a pendulum that swings seconds in any given latitude l, and in a temperature of 10 degrees of Reaumur's thermometer, then will the length of that pendulum for that latitude be thus expressed, *viz*.

$x = (439.178 + 2.321 \times \text{sin. } 21)$ lines of a French foot.

We have seen above that pendulums in different latitudes require to be of different lengths, in order that they may perform their vibrations in the same time; but, besides this, there is another irregularity in the motion of a pendulum in the same place, arising from the different degrees of temperature. Heat expanding, and cold contracting, the rod of the pendulum, a certain small variation must necessarily follow in the time of its vibration, to remedy which defect various methods have been invented, for constructing what are commonly called *compensation pendulums*, or such as shall always preserve the same distance between the centre of oscillation and the point of suspension; but of these we have only room to describe one or two of the most approved construction.

Compound or *Compensation* PENDULUMS, have received different denomination from their form and materials, as the *gridiron pendulum, mercurial pendulum, lever pendulum*, &c.

Gridiron PENDULUM. This consists of five rods of steel, and four of brass, placed in an alternate order, the middle rod being of steel, by which the pendulum ball is suspended; these rods of brass and steel are placed in an alternate order, and so connected with each other at their ends, that while the expansion of the steel rods has a tendency to lengthen the pendulum, the expansion of the brass rods acting upwards tends to shorten it. And thus, when the lengths of the brass and steel rods are duly proportioned, their expansions and contractions will exactly balance and correct each other, and so preserve the pendulum invariably of the same length.

The *Mercurial* PENDULUM was the invention of the ingenious Mr. Graham, in consequence of several experiments relating to the materials of which a pendulum might be formed, in 1715. Its rod is made of brass, and branched towards its lower end, so as to embrace a cylindrical glass vessel 13 or 14 inches long, and about 2 inches in diameter; which being filled about 12 inches deep with mercury, forms the weight or ball of the pendulum. If upon trial the expansion of the rod be found too great for that of the mercury, more mercury must be poured into the vessel; if the expansion of the mercury exceeds that of the rod, so as to occasion the clock to gain with an increase of temperature, some mercury must be taken out of the vessel, in order to shorten the column. And thus may the expansion and contraction of the quicksilver in the glass be made exactly to balance the expansion and contraction of the pendulum rod, and thus preserve the distance between the centre of oscillation and the point of suspension invariably the same.

PENETRABILITY, the capability of being penetrated.

PENETRATION, is used principally to denote the forcible entry of one solid body within another by means of a projectile motion, communicated to the former, which enables it to displace those parts of the latter with which it comes in contact. Or, the penetration may be otherwise produced, by the action of some percussive force acting upon one of the bodies when in contact with the other; these two cases, however, differ rather in circumstances than in principle, and therefore in the slight sketch we shall give of this subject, we shall consider the penetrating body to be projected with a certain velocity, and impinging upon the fixed body in a direction perpendicular to its surface. This is a subject of considerable importance in military and naval gunnery, and has been accordingly treated of by different writers on these subjects; Dr. Hutton, in particular, has made several experiments on different substances, and with different charges of powder, and different weight of shot, in order to procure data from which the penetration in other cases may be determined. The mean results of the most accurate of his experiments, as given in vol. iii. of his Tracts, are stated below.

Velocity in feet.	Substance.	Diam. of iron shot in inches.	Penetration.
1600	Elm	1·96	20 inch.
1200	Ditto	1·96	15
1500	Ditto	2·78	30
1060	Ditto	2·78	16
1200	Oak	5·04	34
1300	Earth	5·55	15 feet

In these experiments, the gun was placed so near the object, that the initial velocity of the ball was not changed, and the penetration was made lengthways of the timber, which, in the first four cases, was sound elm, cut off near the root of the tree; the other two experiments were made by Robins, and given by that author, with some others, in his Tract on Gunnery. The first two experiments, with the velocity 1600 and 1200 feet, give 20 and 15 inches for the penetration, being precisely in the ratio of the two velocities; but, this ratio does not obtain in the next two experiments; in fact, it is pretty obvious that this cannot be the case, unless the resisting force of the wood was uniformly the same throughout, which cannot be; its density about the penetrating body increasing every instant, in consequence of the parts forced in by the ball, which accounts for the results, as deduced by Dr. Hutton, on this hypothesis not agreeing with each other.

Supposing the resisting force of the wood to be uniform, and representing it by f, the velocity by v, and the space described by s, also $g = 16\frac{1}{12}$ feet, the space described by a body by gravity in one second, we shall have on the principle of forces $f = \frac{v^2}{4g\,s}$, and applying this formula to each of the above cases for the same substance, each of them will be found to give different results, which, if the resistance was uniform, ought all to agree with each other, or nearly so; and the same objection must necessarily have place with regard to the times of penetration. It is probable, however, that this hypothesis, of uniform resistance, may not much affect the results in drawing a comparison between the penetration of bodies of different densities, magnitudes, &c. because here we do not require any absolute quantities but their ratios; with regard to each other, under this point of view, if d is made to represent the diameter of the ball, n its density, that of water being one, f the height of a column of water equal to the resisting force of the wood or body penetrated, $a = \cdot 7854$, $g = 16$ feet, then it may readily be shown from the doctrine of forces that $f = \frac{n d v^2}{6 s g}$, and $s = \frac{n d v^2}{6 f g}$; from which formulæ, and the preceding experiments, all cases of penetration on the substances there given may be determined.

From these formulæ it appears, that the depth penetrated, is as the density and diameter of the ball, and the square of the velocity, divided by the strength, or resisting force, of the matter or obstacle; so that if equal balls be discharged against the obstacle, the depths will be as the squares of the velocities.

PENNYWEIGHT, the 20th part of an ounce Troy. This weight derived its name from being exactly the weight of an ancient English silver penny.

PENTAGON, in *Geometry*, is a figure of five angles; and, consequently, also of five sides, and when these are both equal it is called a regular pentagon; but, otherwise, it is irregular.

The angle at the centre of a pentagon is 72°, and the angle of its sides 144°. The area of a pentagon, whose side is one, is 1·7204774; consequently, when the side is s, the area $= s^2 \times 1{\cdot}7204774$.

PENTAGRAPH, an instrument with which designs of any kind may be copied in any proportion at pleasure, without being skilled in drawing.

PENUMBRA, in *Astronomy*, a faint or partial shade observed between the perfect shadow and the fall of light in an eclipse.

This arises from the magnitude of the sun, for were he only a luminous point, the shadow would be everywhere perfect; but, in consequence of his diameter, it happens, that a place, which though not illuminated by the whole body of the sun, may, notwithstanding, receive a part of his rays.

Penumbras must be constant attendants of all eclipses, whether of the sun, moon, or planets, primary or secondary; but, with us, they are most obvious in eclipses of the sun, which is the case above alluded to.

PERAMBULATOR, an instrument for measuring distances, called also a pedometer, waywiser, and surveying wheel. This wheel is so constructed as to measure out a pole, or 16½ feet in two revolutions, and consequently its circumference is 8¼ feet, and its diameter 2·626 feet nearly. To the wheel is attached a sort of double pole, where the machinery is contained, which works by means of the revolution of the wheel; and at the other end is a dial, which exhibits the number of miles, furlongs, &c. passed over; the whole being driven forward by a person on foot, or drawn along by a carriage to which it is sometimes attached. This machine is extremely convenient and expeditious, but it is not always quite accurate, in consequence of the unevenness of the road.

PERCH, in *Land Measure*, the 40th part of a square rood, containing 30¼ square yards.

PERCH, is also sometimes used as a measure of length, being equal to 5½ yards, or 16½ feet, and is otherwise called a rod or pole.

PERCUSSION, in *Mechanics*, the striking of one body against another, or the shock arising from the collision of two bodies. This is either *direct* or *oblique*.

Direct PERCUSSION, is when the impulse takes place in a line perpendicular to the plane of impact.

Oblique PERCUSSION, is that which takes place in any direction not perpendicular to the plane of impact.

The *Theory* of PERCUSSION, is a subject which has much engaged the attention of philosophers, particularly with regard to the comparison of percussion and pressure, one party maintaining a perfect congruity between these two forces, while others assert their total incomparability, observing that the least quantity of percussion is greater than any pressure, however great; for, say they, the momentum of a body is measured by its mass into its velocity, if therefore the body A moves with a velocity v, while the body B is at rest, or has no velocity; the momentum of the former is $A \times v$, and of the latter $B \times 0$, and consequently the former is infinitely greater than the latter.

But however plausible this reasoning may appear at first sight, it is evidently erroneous as to the fact. Daily experience will convince us, that though the advantages gained by bodies, moving swiftly, are very great over those which oppose merely a resistance of pressure, yet that they are by no means infinite. Numerous circumstances will suggest themselves to the mind, which prove, that, physically speaking, we may balance any percussive force by an equivalent one of mere pressure, or even we may make the latter greater, so as to overcome the former.

The pile-engine offers a remarkable confirmation of this equality, or even preponderance on the side of pressure. It has, for instance, been found, that in driving piles in a uniform sandy soil of the same density to 47 feet, the piles could not be driven more than 15 feet by any percussive blow that could be communicated by the engine; that is, the friction and resistance of the soil, which may be considered as a pressive force, was greater than any percussive force that could be employed by the pile-engine, although the rammers made use of were extremely great.

And hence, when we are computing the effect of a pile-engine, it will be necessary to estimate first the quantity of percussion that is equivalent to the resistance and friction opposed to the pile; as no momentum short of this, or even just equal to it, will produce any effect, and when the momentum is greater than this, it is only the difference between the two that is effective in producing motion in the pile. And to this circumstance must be attributed the many erroneous solutions that appeared a few years back to the question. "What must be the height of a pile-engine to produce the greatest effect in a given time?" This question, at first sight, appears to be the same, with asking how high must the pile-engine be to produce the greatest momentum in a given time; but by using this principle the solution always gave the height $= 0$; that is, the greatest effect will be produced when the rammer is left at rest on the top of the pile.

But if, instead of proceeding thus, we first estimate, or find from experiment, the height to which the rammer must be drawn, in order that its momentum may be equivalent to the resistance of the pile, and then considering the difference between this and any greater momentum to be only the effective part, a very rational solution will be obtained.

But, before entering upon the solution of this problem, it will be proper to offer a few farther remarks with regard to the comparability of percussion and pressure, because the solution ultimately depends upon a proper comparison of those quantities, and a want of due attention to which seems to have been the cause of the erroneous results generally deduced in the solution of this problem.

Without, indeed, entering into a discussion concerning the congruity or incongruity of these forces, it is obvious, that they may be so employed as to produce the same or equal results. A nail, for example, may be driven to a certain depth into a block of wood by the blow of a hammer, or it may be sunk to the same depth by the pressure of a heavy body; whence, and from numerous other instances, it is obvious, that pressure and percussion, whether congruous or incongruous in their nature, are at least comparable in their effects.

With regard to the above problem, the resistence and friction of the soil against the pile may, as above observed, be considered as a pressure, and the object of our inquiry is to establish a comparison between this resistance or pressure of the soil, and the momentum of the ram, or what part of the whole generated momentum of the latter is employed in overcoming the resistance of the former, in order to determine the effective part of the stroke, which ought alone to be considered in estimating the maximum effect; because any single momentum, less than that which is equivalent to the resistance, would produce no effect whatever.

Now it being admitted that pressure and momentum are at least comparable in their effects, it must also be granted that there is some determinate momentum of the ram, that is equivalent to the resistance of the pile, and the height necessary for producing this momentum must be the first object of our research, which it is obvious, from the various circumstances that may arise in the application of the pile-engine, can only be determined by experiment.

Suppose, for example, that it is found by experiment, that a momentum m drives the pile a inches, and a momentum M. drives it b inches, and let it be required thence to determine the momentum y, which is equal to the resistance of the pile.

The first effective momentum is $m-y$, and the second $M-y$; and now admitting each effect proportional to the cause which produces it, we have

$$m-y : M-y = a : b$$

$$\text{or } bm-by = Ma-ay$$

$$\text{or } y = \frac{Ma-bm}{a-b}$$

which is the momentum equal to the resistance of the pile. Find the height due this momentum, and call it h; then the solution of the problem will stand as follows:

Let x be the required height to produce the max.
a the velocity per second in drawing up,
t the time in falling,
$g = 16\frac{1}{12}$ feet.

Then supposing the momentum to vary as the velocity, and calling the weight of the ram w, we have $2gt = v\,vw = m$, the generated momentum,

$\frac{x}{a}$ = time in drawing up,

$\frac{\sqrt{x}}{\sqrt{g}}$ = time in falling $= t$,

$\frac{x\sqrt{g}+a\sqrt{x}}{a\sqrt{g}}$ the whole time,

$2\sqrt{gx} = v$, or $2w\sqrt{gx} = m$, by substituting $\frac{\sqrt{x}}{\sqrt{g}}$ instead of t.

Also h being the height due to the momentum above determined, this will be expressed by $2w\sqrt{gh}$, and consequently $2w\sqrt{gx} - 2w\sqrt{gh}$ is the effective momentum, which divided by the time necessary for the production of it must be a maximum; that is,

$$\frac{2w\sqrt{gx}-2w\sqrt{gh}}{x\sqrt{g}+a\sqrt{x}} \times a\sqrt{g} = \text{a max.}$$

$$\text{or } \frac{\sqrt{x}-\sqrt{h}}{x\sqrt{g}+a\sqrt{x}} = \text{a max.}$$

This, thrown into fluxions, gives

$$\dot{x}\left\{\frac{1}{2x}(x\sqrt{g}+a\sqrt{x}) - \sqrt{gx} + \sqrt{gh} - \frac{x}{2\sqrt{x}}(\sqrt{x}-\sqrt{h})\right\} = 0$$

which reduced gives

$$\tfrac{1}{2}\sqrt{gx} = \tfrac{1}{2}a\sqrt{\frac{h}{x}} + \sqrt{gh},$$

$$\text{or } x\sqrt{g} = a\sqrt{h} + 2\sqrt{ghx}, \text{ or}$$

$$x - x2\sqrt{hx} = a\frac{\sqrt{h}}{\sqrt{g}}$$

Whence

$$\sqrt{x} = \sqrt{h} \pm \sqrt{\left(h + a\frac{\sqrt{h}}{\sqrt{g}}\right)}$$

$$\text{or } x = \left\{\sqrt{h} \pm \sqrt{\left(h + a\frac{\sqrt{h}}{\sqrt{g}}\right)}\right\}^2$$

which is the height required.

In the above solution we have only determined the height when any single momentum, divided by the time necessary for the production of it, is a maximum; whereas the problem requires, that the sum of all the momenta divided by the whole time should be a maximum. In order to determine this, we must assume a certain law of resistance, to which, apply the preceding principle of solution, a general result will be obtained. But as this assumed law of resistance must be totally suppositious, and probably very different from that which arises in practice, the preceding determination (supposing the value of h to have been found by experiment towards the conclusion of the operation of the engine) is probably more accurate than that which would result from the above supposition of a variable law of resistance.

For it must be observed, that x, as found above, is always the same function of h; that h increases as the blows are repeated, and consequently x also increases each blow; but the distance between the top of the pile and the top of the engine also practically increases each stroke, by the depth the pile has penetrated.

PERFECT *Number*, is one that is equal to the sum of all its divisors, or aliquot parts.

To find a Perfect Number.

Find $2^n - 1$ a prime number, then will $2^{n-1} \times (2^n - 1)$ be a perfect number.

For $2^n - 1 = 1 + 2 + 2^2 + 2^3 + 2^4$, &c. 2^{n-1}; and it is demonstrated by Euclid, in the last prop. of book ix. that if the above series be continued till its sum be a prime, then that sum multiplied by the last term of the series will be a perfect number; which, therefore, is identically the same as the above formula, and the truth of the theorem, under its analytical form, may be demonstrated as follows: Since $2^n - 1$ is a prime, put it $= b$, and $2^{n-1} = 2^m = a^m$; then the above reduces to $a^m b$. Now the sum of the divisors of $a^m b$

$$= \frac{a^{m+1} - 1}{a - 1} \times \frac{b^{1+1} - 1}{b - 1}.$$

But in this formula, the number itself is considered as a divisor, which is excluded in perfect number; therefore the sum of the divisors, exclusive of the number itself, is

$$\frac{a^{m+1} - 1}{a - 1} \times \frac{b^{1+1} - 1}{b - 1} - a^m b.$$

PERIGEE, or PERIGÆUM, in the *Ancient Astronomy*, signified the nearest approach of the sun, or any of the planets, to the earth; or rather that point of their orbit when at their least distance: a term which the moderns have changed to *perihelion*, because it is the earth that is in motion and not the sun, as was supposed by the ancients. The term perigee is, however, still properly used, as applied to the moon, comets, &c. to denote their nearest approach to our planet.

PERIHELION, or PERIHELIUM, that point in the orbit of a planet or comet which is nearest to the sun; being the extremity of their transverse axis nearest to that focus in which the sun is placed; being thus opposed to the aphelion, which is the opposite extremity of the same axis.

The perihelion distances of the several planets, the mean distance of the earth from the sun being taken as unity, are as follows:

Perihelion Distances of the Planets, the Mean Distance of the Earth being Unity.

Mercury	·1815831
Venus	·7164793
Earth	·9831468
Mars	1·4305595
Vesta	2·2707800
Juno	2·4122190
Ceres	2·6800660
Pallas	2·5222080
Jupiter	5·1546127
Saturn	9·4820022
Uranus	19·1366347

PERIMETER, in *Geometry*, the boundary of any figure; being the sum of all the sides in right-lined figures, and means the same as circumference, or periphery, in circular ones.

PERIOD, in *Astronomy*, is the time in which a planet or satellite makes one entire revolution in its orbit, or return again to the same point in the heavens.

PERIPHERY, the same as perimeter or circumference: perimeter, however, is more commonly used in speaking of right-lined figures; and periphery, of curvilinear ones.

PERMUTATIONS, the changes in the position of things; differing from combinations in this, that the latter has no reference to the order in which the quantities are combined; whereas, in the former, this order is considered, and consequently the number of permutations always exceeds the number of combinations. Permutation also differs from what is simple, termed changes in this; that, by changes, is commonly meant only the different order in which a number of things may be arranged, taking all together; whereas permutations implies a combination of a number of things into different sets, and the changes which may then have place amongst them.

In general, the number of permutations may be found by first finding the number of combinations, and then the number of permutations in each; and the product of these is the whole number of permutations.

PERPENDICULAR, in *Geometry*, is formed by one line meeting another so as to make the angles on each side of it equal to each other.

Perpendicular to a *Curve*, is a line perpendicular to the tangent of the curve at that point.

Perpendicular *Action* of *Gravity*, is the direction which it gives to a body falling freely, which is always in a line perpendicular to a tangent to the earth's surface at that point, and not necessarily in a line directed to the centre of the earth.

PERPETUAL *Motion*, is that which possesses within itself the principle of motion; and, consequently, since everybody in nature, when in motion, would continue in that state, every motion once begun would be perpetual but for the operation of some external causes; such are those of friction, resistance, &c.: and since it is also a known principle in mechanics, that no absolute power can be gained by any combination of machinery, except there being at the same time an equal gain in an opposite direction; but that, on the contrary, there must necessarily be some lost from the above causes, it follows that a perpetual motion can never take place from any pure mechanical combination.

PERPETUITY, in the *Doctrine* of *Annuities*, is the number of years in which the simple interest of any principal sum will amount to the same as the principal itself; or it is the quantity arising by dividing 100, or any other principal, by its interest for one year. Thus the perpetuity, at the rate of 5 per cent, is $\frac{100}{5} = 20$ years; and at 4 per cent. $\frac{100}{4} = 25$ years.

PERSPECTIVE, the art of delineating visible objects on a plane surface, such as they appear at a given distance or height upon a transparent plane, supposed to be placed commonly perpendicular to the horizon, between the eye and the object. Perspective is divided into *Aerial* and *Lineal*, the former having principally a reference to the colouring and shading of distant objects, and the latter as above defined, which relates to the position, magnitude, form, &c. of the several lines or contours of objects, &c. This part, in fact, is the only branch of perspective that properly falls within the design of the present work, and to which we shall confine our observations.

PHANTASMAGORIA, denotes a remarkable optical illusion, arising from a particular application of the magic lantern. In the exhibition of this spectacle, the audience are placed in a dark room, having a transparent screen between them and the lantern, which screen ought to be let down after the lights are withdrawn, and unknown to the spectators.

The lantern being then properly adjusted on the opposite side, the figure intended to be exhibited is thrown upon the screen, which will appear to the observers as if placed in free space, and by altering the distance of the lantern the figure may be made to appear of any size; which changes in its dimensions are attributed by the observers to the distance or proximity of the image, so that at one time it appears to be at an immense distance, and at another to be exceedingly near, and over the heads of some part of the audience.

PHASES, in *Astronomy*, denote the various appearance of the moon at different ages, being at one time a crescent, then a semicircle, then gibbous, and lastly full; after which the same phases return again in the same order. Venus and Mercury have the same phases as the moon, and Mars partakes of them, in some measure, being at times gibbous; the same must also have place in a less degree with the other superior planets. The same term is also applied to denote the appearance of the moon or sun when eclipsed.

PHENOMENON, is strictly an appearance, but more commonly confined to those only of an extraordinary nature, particularly as relating to the heavens, or heavenly bodies; as comets, meteors, shooting stars, &c. We also speak of the phenomena of the magnet, of electricity, &c.

PHYSICAL, any thing relating to physics.

PHYSICO-*Mathematics*, is the same as mixed mathematics, being those branches of this science which investigates the laws and actions of bodies, and their combinations, by means of certain data drawn from observation and experiment.

PHYSICS, is a term denoting the same as experimental or natural philosophy; being the doctrine of natural bodies, their phenomena, causes, and effects, with their various affections, motions, and operations.

Experimental Physics, is that which enquires into the nature and reason of things by experiments, as in hydrostatics, pneumatics, optics chemistry, &c.

Mechanical Physics, explains the appearances of nature from the matter, motion, structure, and figures of bodies, and their several parts, according to the established laws of nature.

PIERS, in the theory of *Bridges*, are the walls built to support the arches, and from which as bases they spring.

PILES, in *Building*, are large stakes or beams sharpened at the end, and shod with iron, to be driven into the ground for a foundation to build upon in marshy places.

Pile-*Engine*, is an engine used for the purpose of driving piles. Of the principles of the operation of this engine, see Percussion.

Pile, in *Artillery*, denotes a collection or heap of balls or shells, piled up in a pyramidal form, the base being some regular figure, as an equilateral triangle, square, or rectangle, and the whole pile a series of such figures, the side of each successive row diminishing by one from the bottom upwards.

Therefore the whole number of balls is equal to the sum of a series of triangular numbers, squares, or rectangles, according to the figure of the pile: and may be expressed by the following formulæ,

$$\text{Triangular pile} = \frac{n(n+1)(n+2)}{6}$$

$$\text{Square pile} = \frac{n(n+1)(2n+1)}{6}$$

$$\text{Rectangular pile} = \frac{m(m+1)(3n-m+1)}{6}$$

Where n in the two first formulæ denotes the number of balls in the side of the base; and in the latter, n is the number of balls in the length of the base, and m the number of those in the breadth.

These formulæ are difficult to remember upon an emergency; and therefore the following general rule, for this purpose, which is not commonly known, is deserving the notice of artillery officers.

Rule. In every pile there may be found three parallel lines, the sum of which, multiplied by the number of balls in the triangular face of the pile, and divided by 3, is the number of balls.

In the rectangular pile the three parallel lines, are the two bottom rows in length, and the upper ridge of the pile; and the face the triangular end.

In the square pile any two opposite sides of the square base, and the upper ball, are three parallel lines. And in the triangle pile one side of the bottom row, the opposite extreme ball, and the upper ball, are the three parallel sides; the face in both these cases being any of the equal slant sides of the pile.

PINION, in *Practical Mechanics*, is any small wheel working in the teeth of a larger wheel.

PISTON, in *Mechanics*, denotes a short cylinder working within another hollow cylinder, as water and air-pumps, and some other machines

PLANE, or **Plain**, in *Geometry*, denote a surface or superficial extension, lying evenly between its bounding lines; being such, that if a right line touch it in two points, it will touch through its whole extent.

The same term is also frequently used in astronomy, for an ideal plane passing through certain parts or points of the heaven, as the plane of the *horizon*, of the *ecliptic*, *equator*, &c. by which is to be understood certain ideal planes passing through those circles of the sphere, or on which they are supposed to be described. And on similar principles we say a *vertical* plane, *horizontal* plane, to denote planes passing in those directions.

Plane *Problem*. See Locus and Problem.

Plane *Table*, in *Surveying*, a very simple instrument, whereby the draught of a field is taken on the spot, without any future protraction. It is generally of an oblong rectangular figure, and supported by a fulcrum, so as to turn every way by means of a ball and socket. It has a moveable frame, which serves to hold fast a clean paper; and the sides of the frame, facing the paper, are divided into equal parts every way. It has also a box with a magnetic needle, and a large index with two sights; and lastly, on the edge of the frame, are marked degrees and minutes. To use this instrument, take a sheet of paper which will cover it, and wet it to make it expand; then spread it flat on the table, pressing down the frame on the edges to stretch it, and keep it fixed there; and when the paper is become dry, it will by contracting again, stretch itself smooth and flat from any cramp and unevenness. On this paper is to be drawn the plan or form of the thing measured.

Thus, begin at any proper part of the ground, and make a point on a convenient part of the paper or table, to represent that place on the ground; then fix in that point one leg of the compasses, or of a fine steel pin, and apply to it the graduated edge of the index, moving it round till, through the sights, you perceive some remarkable object, as the corner of a field, &c.; and from the station point draw a line with the point of the compasses along the graduated edge of the index, which is called setting or taking the object; then set another object or corner, and draw its line; do the same by another, and so on, till as many objects are taken as may be thought fit. Then measure from the station towards as many of the objects as may be necessary, but not more; taking the requisite offsets to corners or crooks in the hedges, laying the measures down on their respective lines on the table. Then, at any convenient place measured to, fix the table in the same position, and set the objects which appear from that place, and so on as before. And thus continue till the work is finished, measuring such lines only as are necessary, and determining as many as may be, by intersecting lines of direction, drawn from different stations.

PLANET, a wandering star, as distinguished from the fixed stars, which always preserve the same relative position with respect to each other. Hence it follows that comets and satellites are included, according to the original signification of this term, under the same general denomination; in fact, the early astronomers had no idea of comets being permanent bodies, and as they were also unacquainted with any satellite but the moon, which, with the sun, was supposed to revolve about the earth, it was natural for them to class both under the same general appellation.

But modern astronomers, in order to make a distinction between these, define a planet to be a celestial body revolving about the sun, as a centre, with a moderate degree of eccentricity; thus, excluding comets, the eccentricity of whose orbits is very considerable; and the satellites which revolve about their primaries as the primaries do about the sun. These last are, however, sometimes called *secondary planets*.

The planets belonging to our

system, as included under this definition, are therefore Mercury, Venus, the Earth, Mars, Juno, Pallas, Ceres, Vesta, Jupiter, Saturn, and Uranus, or the Georgium Sidus.

Of these, Mercury, Venus, Mars, Jupiter, and Saturn, have been known from the highest antiquity; but the other five have been discovered within a very few years; *viz.* Uranus, by Dr. Herschel, March 13, 1781; Ceres, by M. Piazzi, January 1, 1801; Pallas, by Dr. Olbers, March 28, 1802; Juno, by Mr. Harding, September 1, 1804; and lastly, Vesta, by Dr. Olbers, March 29, 1807, being the second planet discovered by this celebrated observer.

Astronomers use certain characters to designate the different planets, which are as follow: *viz.* Mercury ☿; Venus ♀; Earth ⊕; Mars ♂; Jupiter ♃; Saturn ♄; Ceres ⚳; Pallas ⚴; and Uranus ♅; but the other new planets have not at present, we believe, received any such distinguishing character.

All the planets perform their revolutions in elliptical orbits about the sun, which is situated in one of the foci of the ellipse; that the orbit of each planet lies in a plane, which passes through the centre of the sun; that those which are nearest the centre move with a greater velocity than those that are more remote; that the same planet is also quicker or slower according as it is in that part of its orbit which is nearer or more remote from the central body; and that all their motions are performed in the same order from west to east, or according to the order of the signs. And farther, the motions and distances of the several planets are related to each other by certain and invariable laws, viz. that the cubes of their periodic times of revolution, are as the squares of their distances from the sun; and that equal areas are described by the same planet in equal times; that is, if we suppose a line drawn from the sun to a planet, and to move about it as a centre, that line will pass over or describe equal areas in equal times: and therefore since the periods of the planetary revolution are accurately known from observation, we may hence find the exact proportional distance of all these bodies. It has also been discovered by means of certain spots, observable on the discs of the planets, that they have each a rotatory motion about their own axes, the time of which, however, seems to follow no particular law, either with regard to their magnitude or distance from the sun. Hence then it appears that these bodies have each a diversity of seasons, their spring, summer, autumn, and winter, resembling those in our planets; that they have likewise the same alternations of day and night; in short, that they are equally fitted to the accommodation of inhabitants, and that in all probability millions of beings are placed upon them. The excessive heat and cold experienced in those regions, in consequence of their proximity or remoteness from the sun, as have been supposed and even computed by some authors, appear to be wholly imaginary. We have no reason, for example, to suppose that even Uranus, the most distant planet in our system, has a temperature at all different from us, or even from Mercury, who revolves much nearer to the sun than ourselves. The light and heat received and experienced on our globe, seems to depend much more upon the constitution of the atmosphere than upon any other cause; or why have we mountains covered with perpetual snow; and whence that diminution of light and heat experienced by aerial voyagers; but in consequence of the extreme rarity of the air in the upper regions? And if this be granted, it follows immediately that a simple modification in the atmospheres of the several planets, would render the temperature of each supportable even by terrestrial inhabitants. The idea of the temperature of the several planets depending upon their distances from the sun, arises from considering that body, not simply as the cause of heat, but as an immense mass of fire, possessing in itself, independent of any other agent, the power of heat; whereas there is every reason to conclude that it is only by a combination of the solar rays with certain parts of our atmosphere that the effect is produced. Water poured upon unslaked lime generates heat in the combined mass, and if we could imagine a being existing in such a mass, we should have no difficulty in conceiving, that he would attribute that quality to the water which we attribute to the sun, although in this case the contrary is evident.

Elements of the PLANETS, are certain quantities which are necessary to be known in order to determine the theory of their elliptic motion.

Astronomers reckon seven of those quantities, of which five relate to the elliptic motion, viz. 1. The duration of the sidereal revolution. 2. The mean distance, or semi-axis major. 3. The eccentricity, from which is derived the greatest equation of the centre. 4. The mean longitude of the planet at any given epoch. 5. The longitude of the perihelion at the same epoch.

The two other elements relate to the position of the orbit, and are, 1. The longitude at a given epoch of the nodes of the orbit with the ecliptic. 2. The inclination of the orbit to this plane. These several elements, as given by Laplace, "Systeme du Monde," 3d edition, are contained in the following table, which presents also a general view of the planetary system.

Elements and General View of the Planetary System.

Names of the Planets.	Duration of a Sidereal Revolution.	Mean Distance from the Sun.	Ratio of the Eccentricity to half the Major Axis.	Mean Longitude, Jan. 1, 1801.	Mean Longitude of the Perihelion	Longitude of the ascending Node.	Inclination of the Orbit to the Ecliptic
	Days						*
Mercury	87·96925804	·3870981	·20551494	182°15047	82°6256	51°0651	7°78058
Venus.	224 70082390	·7233323	·00685298	11°93672	142°9077	83°1972	3°76936
Earth . .	365·25638350	1 0000000	·01685318	111°28179	110°5571	0°0000	0°00000
Mars . . .	686 97961860	1·5236935	·09313400	71°24145	369°3407	53°3605	2°05663
Vesta . . .	1335·20500000	2·3730000	·09322000	297°12990	277°4630	114°4630	7°94010
Juno . . .	1590·99800000	2·6671630	·25494400	322°79380	59°2349	190°1228	....
Ceres . .	1681·53900000	2·7674060	·07834860	294°16820	162°9565	89°9083	11°80680
Pallas . .	1681·70800000	2·7675920	·24538400	280°68580	134°7040	191°7148	38°46540
Jupiter .	4332·59630780	5 2027911	·04817840	124°67781	12°3812	109°3624	1°46034
Saturn .	10758 96984000	9 5387705	05616830	150°38010	99°0549	124°36662	2°77102
Uranus .	30688·71268720	19 1833050	04667030	197°54244	185°9574	80°9488	0°85990

* These angles, as also those in the preceding columns, answer to the French division of the circle: the same according to the usual division are given in table III. as copied from the Edinburgh Encyclopedia.

Elements and General View of the Planetary System.

Names of the Planets.	Mean Diam. in English miles.	Mean Distance from the Sun in millions of Miles.	Mean apparent Diam. as seen from the Earth.	Mean Diam. as seen from the Sun.	Densities, that of Water being one.	Proportional Quantities of Matter.	Diurnal Rotation on their own Axis.
							d h m s
Sun	883246	....	32′ 1″·5	....	$1\frac{2}{15}$	333928	25 14 8 0
Mercury	3224	37,000000	10	16″	$9\frac{1}{3}$	0·1654	14 24 5 28
Venus ..	7687	68,000000	58	30	$5\frac{11}{13}$	0 8899	0 23 20 54
Earth ...	7911	95,000000	....	17·2	$4\frac{19}{20}$	1	1 0 0 0
Moon ...	2180	95,000000	31′ 8	4·6	$3\frac{7}{17}$	0·025	29 17 44 3
Mars ...	4189	144,000000	27	10·	$3\frac{2}{7}$	0·0875	0 24 39 22
Ceres ...	163 1024	263,000000	1 6·4	....	....	....	....
Pallas...	80 2099	265,000000	0·5 0·5	....	....	....	....
Juno ...	1425	252,000000	3	....	....	....	....
Vesta ...	238	225,000000	0·5	....	....	....	....
Jupiter .	89170	490,000000	39	37	$1\frac{1}{27}$	312·1	0 9 55 37
Saturn .	79042	900,000000	18	16	$0\frac{13}{32}$	97·76	0 10 16 2
Uranus .	35112	1,800,000000	3·54	4	$1\frac{1}{100}$	16·84	....

Names of the Planets.	Inclination of Axes to Orbits.	Inclination of Orbit to the Ecliptic.	Place of Aphelion in Jan. 1800.	Motion of Aphelion in 100 yrs.	Motion of the Nodes in 100 yrs	Eccentricities mean Dist. being 100,000.	Greatest Equation of the Centre.
Sun ---	82° 44′ 0″	------	---------	------	------	------	-------
Mercury	--------	7° 0′ 0″	8s 14° 20′50″	1°33′ 45″	1°12′ 10″	7955·4	23°40′ 0″
Venus --	--------	3 23 25	10 7 59 1	1 21 0	0 51 40	498	0 47 20
Earth --	66 32	0 0 0	9 8 40 12	0 19 35	------	1681·395	1 55 30·9
Moon --	88 17	5 9 3	---------	------	------	------	-------
Mars --	59 22	1 51 0	5 2 24 4	1 51 40	0 46 40	14183·7	10 40 40
Ceres --	--------	10 37 0	4 25 57 15	------	------	8141	9 20 8
Pallas --	--------	34 50 40	10 1 7 0	------	------	24630	28 25 0
Juno ---	--------	21 0	7 29 49 33	------	------	25096	-------
Vesta --	--------	7 8 46	2 9 42 53	------	------	9322	-------
Jupiter -	90 nearly	1 8 56	6 11 8 20	1 34 33	0 59 30	25013·3	5 30 38
Saturn -	60 probably	2 29 50	8 29 4 11	1 50 7	0 55 30	53640·42	6 26 42
Uranus	--------	0 46 20	11 16 30 31	1 29 2	44 35	90804	5 27 16

PLANETARIUM, an astronomical machine, contrived to represent the motions, orbit, &c. of the planets, as they really are in nature, according to the Copernican system. The larger sort of them are called orreries.

PLANETARY, something that relates to the planets.

Planetary *System*, is the system or assemblage of the planets, primary and secondary, moving in their respective orbits, round their common centre the sun.

PLANIMETRY, that part of geometry which considers lines, and plane figures, without any regard to heights or depths. Planimetry is particularly restricted to the mensuration of planes and other surfaces; as contradistinguished from stereometry, or the mensuration of solids, or capacities of length, breadth, and depth.

Planimetry, is performed by means of the squares of long measure, as square inches, square feet, square yards, &c.; that is, by squares whose side is an inch, a foot, a yard, &c. So that the area, or content, of any surface, is said to be found, when it is known how many such square inches, feet, yards, &c. it contains.

PLANISPHERE, a projection of the sphere, and its various circles, on a plane; as upon paper or the like. In this sense, maps of the heavens and the earth, exhibiting the meridians and other circles of the sphere, may be called planispheres.

Planisphere is sometimes also considered as an astronomical instrument, used in observing the motions of the heavenly bodies; being a projection of the celestial sphere upon a plane representing the stars, constellations, &c. in their proper situations, distances, &c. As the astrolabe, which is a common name for all such projections. In all planispheres, the eye is supposed to be in a point, viewing all the circles of the sphere, and referring them to a plane beyond them, against which the sphere is as it were flattened; and this plane is called the plane of projection, which is always some one of the circles of the sphere itself, or parallel to some one of them.

PLANO, *Concave* and *Convex Lens;* a lens which is flat on the one side and concave or convex on the other.

PLATONIC *Bodies*, the same as *Regular Bodies*, which see.

Platonic *Year*, or the *great year*, is the period of time determined by the revolution of the equinoxes, upon a supposition of the precession going on uniformly till they have made one complete revolution.

PLEIADES, an assemblage of seven stars in the neck of the constellation of Taurus: of which, however, there are but six now visible to the naked eye.

PLENIUM, in *Philosophy*, is that state of things in which every part of space, or extension, is supposed to be full of matter; in opposition to a vacuum, which is a space devoid of all matter.

PLOTTING, in *Surveying*, is the art of laying down on paper, &c. the several angles and lines of a tract of ground surveyed by a theodolite, &c. and chain. In surveying with the plain table, the plotting is saved; the several angles and distances being laid down on the spot as fast as they are taken. But, in working with the theodolite, semicircle, or circumferentor, the angles are taken in degrees; and the distances in chains and links, so that there remains an after operation to reduce these numbers into lines, and so to form a draught, plan, or map; this operation is called plotting. Plotting then is performed by two instruments, the protractor and plotting-scale. By the first, the several angles observed in the field with a theodolite, or the like, and entered down in degrees in the field-book, are protracted on paper in their just quantity. By the latter the several distances measured with the chain, and entered down in the like manner in the field-book, are laid down in their just proportion.

Plotting *Scale*, a mathematical instrument, usually of wood, sometimes of brass, or other matter; and either a foot or half a foot long. On one side of the instrument are several scales or lines, divided into equal parts. The first division of the first scale is subdivided into ten equal parts, to which is prefixed the number of 10, signifying that ten of these subdivisions make an inch; or that the divisions of that scale are decimals of inches. The first division of the second scale is likewise subdivided into 10, to which is prefixed the number 16, denoting that sixteen of these subdivisions make an inch. The first division of the third scale is subdivided in like manner into 20, to which is prefixed the number 20; to that of the fourth scale is prefixed the number 24; to that of the fifth 32; that of the sixth 40; that of the seventh 48; denoting the number of subdivisions equal to an inch in each respectively. The two last scales are broken off to make room for two lines of chords. There are also on the back side of the instrument a diagonal scale.

As to the use of the plotting-scale, if we were required to lay down any distance upon paper, suppose 6 chains 50 links: draw an indefinite line; then sitting one foot of the compasses at figure 6 on the scale, *e. gr.* the scale of 20 in an inch, extend the other to 5 of the subdivisions for the 50 links; this distance being transferred to the line, will exhibit the 6 chains 50 links required.

If it be desired to have 6 chains 50 links, take up more or less space, take them off from a greater or lesser scale, *i. e.* from a scale that has more or fewer divisions in an inch.

To find the chains and links contained in a right line, or one that is just drawn, according to any scale, *e. gr.* that of 20 in an inch. Take the length of the line in the compasses, and applying it to the given scale, you will find it extend from the number 6 of the great divisions to 5 of the small ones: hence the given line contains 6 chains, 50 links.

PLUMBLINE, a line having a plummet or weight attached to it, in order to find a perpendicular.

PLUNGER, the solid brass cylinder used as a forcer in forcing pumps.

PLUS, in *Algebra*, the affirmative or positive sign +, which signifies addition. The ancient algebraists used the word plus at full length, after which the initial was employed, and finally the sign +, which was first introduced by Stefelius in his arithmetic.

PLUVIAMETER. See RAIN *Gage*.

PNEUMATICS, is that branch of natural philosophy which treats of the weight, pressure, elasticity, &c. of elastic fluids, but more particularly of the air, the history and principles of which will be found under the articles *Air*, *Atmosphere*, *Barometer*, &c.

POINT, in *Geometry*, according to Euclid's definition, is that which has no parts or dimensions, neither length, breadth, nor depth; and therefore marks position only.

Point receives also various denominations in the doctrine of curves, optics, perspective, &c. which are all defined under the respective adjectives.

Physical POINT, is the smallest or least sensible object of sight, and is thus distinguished from a geometrical point, which has only position, being of no magnitude or dimension.

POINTS of the *Compass*, are the 32 principal points, into which the compass card is divided, or the points of the horizon towards which these are directed. See *Compass*.

POLAR, something relating to or situated near the poles.

POLAR *Circles*. See *Circle*.

POLARITY, the quality of a thing having poles, or a tendency to turn itself into a certain position, but more particularly used with reference to the magnet.

POLE, in *Astronomy*, one of the extremities of the imaginary axis on which the sphere is supposed to revolve. These two points are each 90 degrees from the equator, that towards the north being called the north pole, and the other the south pole.

POLE, in *Geography*, one of the points on which the terraqueous globe turns, each of them being 90 degrees distant from the equator, and are denominated the north or south pole, according as they point towards the north or south points of the heavens.

In consequence of the inclination of the terrestrial axis to the plane of the ecliptic, and its parallelism during its annual motion in its orbit, these parts of the world have only one day and one night throughout the year, each continuing for about six months.

It is singular that though the poles have a longer duration of light than any other parts of the globe, yet the name by which they are denoted in most languages, both ancient and modern, is derived from terms signifying darkness and obscurity; but though they really enjoy longer light upon the whole than any other parts, yet, in consequence of the obliquity with which the rays of the sun fall upon them, and the great length of winter night, the cold is so intense, that those parts of the globe that lie near the poles have never been fully explored, though the attempt has been repeatedly made by the most celebrated navigators.

Elevation of the POLE, is an angle subtended between the horizon of any place and a line drawn from thence to the pole, which is always equal to the latitude of the place.

POLE, in *Spherics*, a point equally distant from every part of the circumference of a great circle of the sphere; or it is a point 90° distant from the plane of a circle, and in a line, called the axis, passing perpendicularly through the centre. The zenith and nadir are the poles of the horizon; and the poles of the equator are the same with those of the sphere.

POLES of the *Ecliptic*, are two points on the surface of the sphere, 23° 30′ distant from the poles of the world, and 90 distant from every part of the ecliptic.

POLES, in *Magnetism*, are two points of a loadstone, corresponding to the poles of the world; the one pointing to the north, the other to the south.

POLE, in *Land Measure*, is a linear measure equal to 5½ yards; or a square measure of 30¼ square yards.

POLE, or *Polar Star*, is a star of the second magnitude, the last in the tail of Ursa minor. Its longitude Mr. Flamsteed makes 24° 14′ 41″; its latitude, 66° 4′ 11″. The nearness of this star to the pole, whence it happens that it never sets, renders it of vast service in navigation, &c. for determining the meridian line, the latitude, &c.

POLEMOSCOPE, in optics, a kind of reflecting perspective-glass invented by Hevelius, who commends it as useful in sieges, &c. for discovering what the enemy is doing, while the spectator lies hid behind an obstacle.

POLITICAL *Arithmetic*, is the art of reasoning by figures upon matters relating to government, such as the revenues, number of people, extent and value of land, taxes, trade, &c. in any nation. These calculations are generally made with a view to ascertain the comparative strength, prosperity, &c. of any two or more nations, and is otherwise called *Statistics*.

POLLUX, in *Astronomy*, one of the Twins in the constellation Gemini, also a fixed star of the second magnitude in that constellation.

POLYACOUSTIC, any thing that multiplies sound.

POLYGON, in *Geometry*, a multilateral figure, or a figure whose perimeter consists of more than four sides, and consequently having more than four angles. If the angles be all equal among themselves, the polygon is said to be a regular one; otherwise it is irregular. Polygons also take particular names according to the number of their sides; thus a polygon of

3 sides is called a trigon,
4 sides is called a tetragon,
5 sides is called a pentagon,
6 sides is called a hexagon, &c.

and a circle may be considered as a polygon of an infinite number of small sides, or as the limit of the polygons.

Polygons have various properties, as below:

1. Every polygon may be divid-

ed into as many triangles as it has sides.

2. The angles of any polygon taken together, make twice as many right-angles, wanting 4, as the figure hath sides; which property, as well as the former, belongs to both regular and irregular polygons.

3. Every regular polygon may be either inscribed in a circle, or described about it; which is not necessarily the case if the polygons be irregular.

But an equilateral figure inscribed in a circle is always equiangular; though an equiangular figure inscribed in a circle is not always equilateral, but only when the number of sides is odd. For if the sides be of an even number, then they may either be all equal, or else half of them may be equal, and the other half equal to each other, but different from the former half, the equals being placed alternately.

4. Every polygon, circumscribed about a circle, is equal to a right-angled triangle, of which one leg is the radius of the circle, and the other the perimeter or sum of all the sides of the polygon. Or the polygon is equal to half the rectangle under its perimeter and the radius of its inscribed circle, or the perpendicular from its centre upon one side of the polygon.

Hence, the area of a circle being less than that of its circumscribing polygon, and greater than that of its inscribed polygon, the circle is the limit of the inscribed and circumscribed polygons: in like manner, the circumference of the circle is the limit between the perimeters of the said polygons. See *Circle*.

5. The following table exhibits the angles and areas of all the polygons, up to the dodecagon, viz. the angle at the centre, the angle of the polygon, and the area of the polygon when each side is 1.

No. of sides.	Name of Polygon	Ang. F at cent.	Ang. C of Polygon	Area
3	Trigon	120°	60°	0·4330127
4	Tetragon	90	90	1·0000000
5	Pentagon	72	108	1·7204774
6	Hexagon	60	120	2·5980762
7	Heptagon	$51\frac{3}{7}$	$128\frac{4}{7}$	3·6339124
8	Octagon	45	135	4·8284271
9	Nonagon	40	140	6·1818242
10	Decagon	36	144	7·6942088
11	Undecagon	$32\frac{8}{11}$	$147\frac{3}{11}$	9.3656399
12	Dodecagon	30	150	11·1961524

Therefore, to find the area of any regular polygon, not exceeding 12 sides, square the side, and multiply that square by the corresponding tabular number in the preceding table. Or, generally, if s represent the length of one of the equal sides, and n the number of them; then $s^2 \times \frac{n}{4}$ tang. $\left(\frac{90n-180}{n}\right)$ = area of the polygon.

To inscribe a Polygon within, or to circumscribe a Polygon about, a given Circle.

Bisect two of the angles of the given polygon, and by the right lines; and from the point where they meet, with the radius equal to either of them, describe a circle which will circumscribe the polygon. Next to circumscribe a polygon, divide 360 by the number of sides required, which will give the angle at the centre; draw a radii, including this angle; they will cut off one side, and this applied round the circle will give the polygon.

2. On a given line to describe any given regular polygon. Find the angle of the polygon in the table, and at each extremity of the given line make an angle equal to half that angle, produce the lines till they meet at the centre, then describe the circle, and the construction becomes the same as before.

Otherwise. To inscribe a Polygon in a Circle.—Draw a diameter, and divide it into as many equal parts as the figure has sides. From the extremities of the diameter as centres, with the radius = the diameter, describe arcs crossing each other. From the point of section, through the second division of the diameter, draw a line. Join the points, and the distance between the point where it cuts the circle, and the nearest extremity of the diameter will give the side of the polygon.

Another method, something more accurate, is by erecting a perpendicular from the centre, of such a length that the part without the circle shall be equal to $\frac{3}{4}$ of that within, and drawing a line from its extremity through the second division as before.

In the preceding part of the article it is observed, that any regular polygon may be inscribed in, or circumscribed about a circle; but this must be understood under certain modifications; all that is meant is, that there is nothing in the nature of the problem to render it impossible; and not that any polygon may be geometrically inscribed.

In fact, the number of polygons that admit of a geometrical construction is very limited, viz. the equilateral triangle, the square and pentagon, and those figures whose number of sides are some multiples of these; to which Gauss has lately added the 17 sided polygon, and its multiples, and some others, viz. all those polygons whose number of sides is a prime, and of the form $2^m + 1$.

It is obvious that the side of a polygon, inscribed in a circle, corresponds to the chord of the angle of that polygon at the centre, or to twice the sine of half that angle, and that the perpendicular falling from the centre to that side also answers to the cosine of the same half angle; if, therefore, the sine or cosine of the angle can be found by a quadratic equation, the polygon itself may be constructed geometrically and not otherwise.

It is also known, from the Cotesian Theorem, that if a circle be divided into n equal parts, the double cosines of each of the angles thus formed are the sums of the pairs of imaginary roots being to the equation $x^n - 1 = 0$. That is, all the imaginary roots of the equations $x^n - 1 = 0$, are comprised in the general

$$x^2 - 2 \cos. \frac{2k\pi}{n} x + 1 = 0$$

k being any integer not divisible by n, and π representing the semicircumference.

And since the coefficient of the second term of any equation is equal to the sum of all its roots with their signs changed, it follows that $2 \cos. \frac{2k\pi}{n}$ is equal to the sum of two of the imaginary roots of the equation $x^n - 1 = 0$; and, consequently, the possibility of exhibiting the cosines of these angles analytically, depends upon the solution of the equation $x^n - 1 = 0$.

If, therefore, it were required to find the cosine of $\frac{360°}{3}$, we should have $n = 3$, or $x^3 - 1 = 0$. Here the real root being 1, we have

$$\frac{x^3 - 1}{x - 1} = x^2 + x + 1 = 0;$$

the imaginary roots of which are

$$\begin{cases} x = -\frac{1}{2} + \frac{1}{2}\sqrt{-3} \\ x = -\frac{1}{2} - \frac{1}{2}\sqrt{-3} \end{cases}$$

and their sum $= -1$, that is, twice the cosine of $120° = -1$, or the cosine $= -\frac{1}{2}$.

And, therefore, since every equation $x^n - 1 = 0$ may always be reduced to half its original degree if even, or to half the original degree *minus* 1 if odd, (see Reciprocal Equations) it follows, that there is no difficulty in the solution of $x^5 - 1 = 0$, or in the geometrical construction of a pentagon. But if $n = 7$, or when the figure is a heptagon, the equation only reduces to a cubic; and as we cannot give a geometrical construction of a cubic equation, so we cannot give a geometrical construction of a heptagon.

Gauss, however, in his "Disquisitiones Arithmeticæ," published in 1800, has shown a method of solving any equation $x^n - 1 = 0$, when n is a prime, by reducing it to other simpler equations; thus, if

n is a prime, and $n-1$ be resolved into its factors $n-1=a^{\alpha}$, b^{β}, c^{γ}, &c. then the solution will be effected by α equations of the degree a; β of the degree b; γ of the degree c, &c.

Thus, if $n=7$, then since $n-1=6=2^1 . 3^1$, the solution will be obtained by one quadratic, and one cubic equation; if $n=13$, then $n-1=12=2^2\, 3^1$, the solution is obtained by means of two quadratics and one cubic; but if $n=17$, then since $17-1=16=2^4$, the solution will be obtained by means of four quadratic equations; and, consequently, such a polygon may be inscribed geometrically in a circle; and the same has place when n is any prime number of the form 2^m+1.

By this means Gauss has found the cosine of $\frac{360^\circ}{17}=\frac{1}{2}$

$$\left\{\begin{array}{l} \frac{1}{2}\quad\left\{\frac{1}{2}(-\frac{1}{2}+\frac{1}{2}\sqrt{17})+\frac{1}{2}\sqrt{\frac{1}{2}(17-\sqrt{17})}\right\}+\\ \frac{1}{2}\sqrt{\left\{\frac{1}{2}(-\frac{1}{2}+\frac{1}{2}\sqrt{17})+\frac{1}{2}\sqrt{\frac{1}{2}(17-\sqrt{17})}\right\}}-\\ 4\quad\left\{\frac{1}{2}(-\frac{1}{2}+\frac{1}{2}\sqrt{17})+\frac{1}{2}\sqrt{\frac{1}{2}(17-\sqrt{17})}\right\}\end{array}\right.$$

Now 5, 17, 257, 65537, are prime numbers of this form, and therefore each of these admits of a geometrical construction. We know also, from other principles, that if any two polygons of an unequal number of sides, prime to each other, can be inscribed geometrically in a circle, that the polygon, the number of the sides of which is equal to the product of these two, can also be inscribed geometrically; also all polygons of which the number of sides is any power of 2, may be inscribed by continual bisections; and again, all those whose number of sides is equal to any inscribable polygon into any power of 2.

Hence we have the following series of polygon, each of which admits of a geometrical construction.

Polygons of less than 100 sides, admitting of geometrical construction.

No. of Sides.	No. of Sides.
$3=3$	$30=2.15$
$4=2^2$	$32=2^5$
$5=2^2+1$	$34=2.17$
$6=2\cdot3$	$40=2^3\,5$
$8=2^3$	$48=2^4.3$
$10=2.5$	$51=3.17$
$12=2^2.15$	$60=2^2.15$
$15=3.5$	$64=2^6$
$16=2^4$	$68=2^2.17$
$17=2^4+1$	$80=2^4\,5$
$20=2^2.5$	$85=5.17$
$24=2^3.3$	$96=2^5.3$

And to the above we may add the three consecutive polygons 255, 256, 257; each of which is inscribable in a circle for $255=3.5.17$; $256=2^8$ and $257=2^8+1$.

The next three consecutive polygons that admit of a geometrical construction are:

$65535=255.257$
$65536=2^{16}$
$65537=2^{16}+1$.

For more on this subject, see Gauss's "Disquisitiones Arithmeticæ," or the French translation of the same, entitled "Recherches Arithmetique;" Le Gendre's "Essai sur la Theorie des Nombres;" and Barlow's "Elementary Investigation of the Theory of Numbers."

POLYGONAL *Numbers*, are those that are formed of the sums of different and independent arithmetical series, and are termed *Natural*, *Triangular*, *Quadrangular*, *Pentagonal*, *Hexagonal*, &c. *Numbers*; according to the series from which they are generated.

Lineal, or *Natural Numbers*, are formed from the successive sums of a series of units; thus

Units · · · · 1, 1, 1, 1, 1, 1, &c.
Nat. num. · 1, 2, 3, 4, 5, 6, &c.

Triangular Numbers, are the successive sums of an arithmetical series, beginning with unity, the common difference of which is 1; thus

Arith. series 1, 2, 3, 4, 5, 6, 7, &c.
Trian. num. 1, 3, 6, 10, 15, 21, 28, &c.

Quadrangular, or *Square Numbers*, are the successive sums of an arithmetical progression, beginning with unity, the common difference of which is 2: thus

Arith. ser. 1, 3, 5, 7, 9, 11, 13, &c.
Quad. or sq 1, 4, 9, 16, 25, 36, 49, &c.

Pentagonal Numbers, are the sums of an arithmetical series, the common difference of which is 3; thus

Arith. ser. 1, 4, 7, 10, 13, 16, 19, &c.
Pentagon. 1, 5, 12, 22, 35, 51, 70, &c.

And, universally, the *m, gonal Series of Numbers*, is formed from the successive sums of an arithmetical progression, beginning with unity, the common difference of which is $m-2$.

The general form, which includes every order of *Polygonal Numbers*, is

$$\frac{(m-2)\,n^2-(m-4)\,n}{2}$$

where m is the denomination of the order; therefore, making successively $m=3, 4, 5,$ &c. we have the following results as to the forms of polygonals.

$m=3$ triangular num. $=\dfrac{n^2+n}{2}$

$m=4$ square $=\dfrac{2n^2-0n}{2}$ $=n^2$

$m=5$ pentagolans $=\dfrac{3n^2-n}{2}$

$m=6$ hexagonals $=\dfrac{4n^2-2n}{2}$

By means of the above general form, any polygonal number, of which the root n is given, may be readily ascertained. Thus, by making $m=3$, $m=4$, $m=5$, &c. and in each series $n=1, 2, 3, 4, 5,$ &c. we shall obtain the same numbers as given above, under each respective denomination.

Also any polygonal number and its denomination being given, the root of the polygon is readily obtained. For let

$$p=\frac{(m-2)\,n^2-(m-4)\,n}{2}$$

represent any given polygonal, of which the denomination m is known; then

$$(m-2)\,n^2-(m-4)\,n=2p$$

or $n^2-\left(\dfrac{m-4}{m-2}\right)n=\dfrac{2p}{m-2}$

whence $n=$

$$\frac{m-4\pm\sqrt{(2p\,(m-2)+(m-4)^2)}}{2m-4}$$

which is a general form for the root of every polygonal number.

Fermat, at page 15, in one of his notes to prop. 9 of Diophantus on Multangular Numbers, has given particular rules for finding the roots of given polygonal numbers, without the extraction of the square root; but they are of little or no use, and, therefore, we shall not enumerate them.

We may also find the sum of any series of polygonals by means of the foregoing general formula, for representing still the denomination of any order of polygonals by m, and for abridging make $m-2=d$ the common difference of the series from which they are generated; and let n be the number of terms in the series whose sum is required; then we shall have

$$\left(\frac{n^2-1}{6}\,d+\frac{n+1}{2}\right)n=\left(\frac{n^2-1}{6}(m-2)+\frac{n+1}{2}\right)n$$ for the sum of the n terms sought.

Hence, substituting successively the numbers 3, 4, 5, &c. for m, there is obtained the following particular cases or formulæ; viz.

Triangulars . . $\dfrac{n^2+3n+2}{6}\,n$

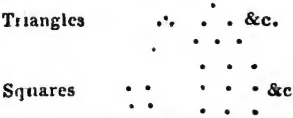
Squares . . . $\dfrac{2n^2+3n+1}{6}\,n$

Pentagonals . $\dfrac{3n^2+3n+0}{6}\,n$

Hexagonals . . $\dfrac{4n^2+3n-1}{6}\,n$

&c. &c.

The denomination of polygonals seems to have been given to these class of numbers from the circumstance, that they may be represented by the particular figures, the name of which they bare; and the *side of the figure* is the same as what we call the *root of the polygon*. Thus



See some particular rules for arranging the points, in a note subjoined to the English edition of Euler's 'Algebra."

POLYGONOMETRY, is an extension of the science of trigonometry, having the same reference to polygons in general, as trigonometry has to triangles in particular.

1. In any polygon, any one side is equal to the sum of all the rectangles of each of the other sides, drawn into the cosines of the angles made by those sides and the proposed side.

2. The perpendicular let fall from the highest point or summit of a polygon, upon the opposite side or base, is equal to the sum of the products of each of the sides on either side comprised between that summit and the base, drawn into the sines of their respective inclinations to the base.

3. The square of any side of a polygon is equal to the sum of the squares of all the other sides, minus twice the sum of the products of all the other sides, multipled two and two by the co-sines of the angles included beween them.

4. Twice the area of any polygon is equal to the sum of the rectangles of its sides, except one, taken two and two, by the sines of the sum of the exterior angles contained by those sides; that is, the angles formed by producing those sides.

POLYHEDRON, a body or solid contained by many rectilinear planes or sides. When the sides of the polyhedron are regular polygons, all similar and equal, then the polyhedron becomes a regular body, and may be inscribed in a sphere; that is, a sphere may be described about it, so that its surface shall touch all the angles or corners of the solid. There are but five of these regular bodies, *viz.* the *tetrahedron*, the *hexahedron* or cube, the *octahedron*, the *dodecahedron*, and the *icosahedron.*

POLYHEDRON, *Gnomonical*, is a stone with several faces, on which are projected various kinds of dials.

POLYSCOPE or **POLYHEDRON**, in *Optics*, a multiplying-glass, or one which represents a single object to the eye, as if there were many. It consists of several plane surfaces, disposed under a convex form, through each of which the object is seen.

PORES, are the small interstices between the solid particles of bodies.

PORISM, in *Geometry*, is a peculiar sort of proposition, which has been differently defined by different writers; some considering it as a general theorem or canon, deduced from a geometrical locus, and serving for the solution of other general and difficult problems; while others make it a lemma, or proposition almost self-evident.

Dr. Simson defines it a proposition, either in the form of a problem or a theorem, in which it is proposed either to investigate or demonstrate.

Euclid wrote three books of porisms, being a curious collection of various things relating to the analysis of the more difficult and general problems. Those books however are lost; and nothing remains in the works of the ancient geometricians concerning this subject, besides what Pappus has preserved, in a very imperfect and obscure state, in his " Mathematical Collections," *viz.* in the introduction to the seventh book.

Several attempts have been made to restore these writings. Fermat has given a few propositions, which are to be found in the collection of his works, in folio, 1679, p. 116. The like was done by Bullialdus, in his " Exercitationes Geometricæ," 4to. 1657. Dr. Robert Simson gave also a specimen, in two propositions, in the Phil. Trans. vol. xxxii. p. 330; and besides left behind him a considerable treatise on the subject of porisms, which has been printed in an edition of his works, at the expense of the late Earl of Stanhope, in 4to. 1776.

The whole three books of Euclid were also restored by Albert Girard, as appears by two notices that he gave, first in his " Trigonometry," printed in French, at the Hague, in 1629, and also in his edition of the works of Stevinus, printed at Leyden in 1634, p. 459; but whether his intention of publishing them was ever carried into execution we have not been able to learn.

A learned paper on the subject

of porisms, by the late very ingenious Professor Playfair, has also been inserted in the third volume of the Transactions of the Royal Society of Edinburgh.

"The restoration of the ancient books of geometry (says this learned professor) would have been impossible without the coincidence of two circumstances, of which, though the one is purely accidental, the other is essentially connected with the nature of the mathematical sciences. The first of these circumstances is the preservation of a short abstract of those books drawn up by Pappus Alexandrinus, together with a series of such lemmata as he judged useful to facilitate the study of them. The second is the necessary connection that takes place among the objects of every mathematical work, which, by excluding whatever is arbitrary, makes it possible to determine the whole course of an investigation when only a few points in it are known. From the union of these circumstances, mathematics has enjoyed an advantage of which no other branch of knowledge can partake; and while the critic or the historian has only been able to lament the fate of those books of Livy and Tacitus which are lost, the geometer has had the high satisfaction to behold the works of Euclid and Apollonius reviving under his hands.

"The first restorers of the ancient books were not, however, aware of the full extent of the work which they had undertaken. They thought it sufficient to demonstrate the propositions which they knew from Pappus to have been contained in those books; but they did not follow the ancient method of investigation, and few of them appear to have had any idea of the elegant and simple analysis by which these propositions were originally discovered, and by which the Greek geometry was peculiarly distinguished.

"Among these few, Fermat and Halley are to be particularly remarked. The former, one of the greatest mathematicians of the last age, and a man in all respects of superior abilities, had very just notions of the geometrical analysis, and appears often abundantly skilful in the use of it; yet, in his restoration of the Loci Plani, it is remarkable, that in the most difficult propositions he lays aside the analytical method, and contents himself with giving the synthetical demonstration. The latter, among the great number and variety of his literary occupations, found time for a most attentive study of the ancient mathematicians, and was an instance of what experience shows to be much rarer than might be expected, a man equally well acquainted with the ancient and the modern geometry, and equally disposed to do justice to the merit of both. He restored the books of Apollonius, on the problem "De Sectione Spatii," according to the true principles of the ancient analysis.

"These books, however, are but short, so that the first restoration of considerable extent that can be reckoned complete, is that of the Loci Plani by Dr. Simson, published in 1749, which, if it differs at all from the work it is intended to replace, seems to do so only by its greater excellence. Thus much at least is certain, that the method of the ancient geometers does not appear to greater advantage in the most entire of their writings than in the restoration above mentioned; and that Dr. Simson has often sacrificed the elegance to which his own analysis would have led, in order to tread more exactly in what the lemmata of Pappus pointed out to him as the track which Appollonius had pursued.

"There was another subject, that of porisms, the most intricate and enigmatical of any thing in the ancient geometry, which was still reserved to exercise the genius of Dr. Simson, and to call forth that enthusiastic admiration of antiquity, and that unwearied perseverance in research, for which he was so peculiarly distinguished. A treatise in three books, which Euclid had composed on porisms, was lost, and all that remained concerning them was an abstract of that treatise, inserted by Pappus Alexandrinus in his "Mathematical Collections;" in which, had it been entire, the geometers of later times would doubtless have found wherewithal to console themselves for the loss of the original work. But unfortunately, it has suffered so much from the injuries of time, that all that we can immediately learn from it is, that the ancients put a high value on the propositions which they called porisms, and regarded them as a very important part of their analysis. The porisms of Euclid are said to be 'Collectio artificiosissima multarum rerum quæ spectant ad analysin difficiliorum et generalium problematum.' The curiosity, however, which is excited by this encomium is quickly disappointed; for when Pappus proceeds to explain what a porism is, he lays down two definitions of it, one of which is rejected by him as imperfect; while the other, which is stated as correct, is too vague and indefinite to convey any useful information.

"These defects might nevertheless have been supplied, if the enumeration which he next gives of Euclid's propositions had been entire; but on account of the extreme brevity of his enunciations, and their reference to a diagram which is lost, and for the constructing of which no directions are given, they are all, except one, perfectly unintelligible. For these reasons, the fragment in question is so obscure, that even to the learning and penetration of Dr. Halley, it seemed impossible that it could ever be explained; and he therefore concluded, after giving the Greek text with all possible correctness, and adding the Latin translation, 'Hactenus porismatum descriptio nec mihi intellecta, nec lectori profutura. Neque aliter fieri potuit, tam ob defectum schematis cujus sit mentio, quam ob omissa quædam et transposita, vel aliter vitiata in propositionis generalis expositione, unde quid sibi velit Pappus haud mihi datum est conjicere. His adde dictionis modum nimis contractum, ac in re difficili, qualis hæc est minime usurpandum.'

"It is true, however, that before this time Fermat had attempted to explain the nature of porisms, and not altogether without success. Guiding his conjectures by the definition which Pappus censures as imperfect, because it defined porisms only 'ab accidente,' *viz.* 'porisma est quod deficit hypothesi a Theoremate Locali,' he formed to himself a tolerably just notion of these propositions, and illustrated his general description by examples that are in effect porisms. But he was able to proceed no farther; and he neither proved that his notion of a porism was the same with Euclid's, nor attempted to restore or explain any one of Euclid's propositions; much less did he suppose that they were to be investigated by an analysis peculiar to themselves. And so imperfect, indeed, was this attempt, that the complete restoration of the porisms was necessary to prove, that Fermat had even approximated to the truth.

"All this did not, however, deter Dr. Simson from turning his thoughts to the same subject, which he appears to have done very early, and long before the publication of the Loci Plani in 1749.

"The subject of porisms is not, however, exhausted, nor is it yet placed in so clear a light as to need no farther illustration. It yet remains to inquire into the probable origin of these propositions; that is to say, into the steps by which the ancient geometers appear to have been led to the discovery of them.

"It remains also to point out the relations in which they stand to the other classes of geometrical truths; to consider the species of analysis, whether geometrical or algebraical, that belongs to them; and, if possible, to assign the reason why they have so long escaped the notice of modern mathematicians. It is to these points that the following observations are chiefly directed.

"I begin with describing the steps that appear to have led the ancient geometers to the discovery of porisms; and must here supply the want of express testimony by probable reasonings, such as are necessary, whenever we would trace remote discoveries to their sources, and which have more weight in mathematics than in any other of the sciences.

"It cannot be doubted that it has been the solution of problems, which, in all states of the mathematical sciences, has led to the discovery of most geometrical truths. The first mathematical inquiries, in particular, must have occurred in the form of questions, where something was given, and something required to be done; and by the reasonings necessary to answer these questions, or to discover the relation between the things that were given and those that were to be found, many truths were suggested which came afterwards to be subjects of separate demonstration. The number of these was the greater, that the ancient geometers always undertook the solution of problems with a scrupulous and minute attention, which would scarcely suffer any of the collateral truths to escape their observation. We know, from the examples which they have left us, that they never considered a problem as resolved till they had distinguished all its varieties, and evolved separately every different case that could occur, carefully remarking whatever change might arise in the construction, from any change that was supposed to take place among the magnitudes which were given.

"Now, as this cautious method of proceeding was not better calculated to avoid error, than to lay hold of every truth that was connected with the main object of inquiry, these geometers soon observed that there were many problems which, in certain circumstances, would admit of no solution whatever, and that the general construction by which they were resolved would fail, in consequence of a particular relation being supposed among the quantities which were given.

"Such problems were then said to become impossible, and it was readily perceived that this always happened when one of the conditions prescribed was inconsistent with the rest, so that the supposition of their being united in the same subject, involved a contradiction. Thus, when it was required to divide a given line, so that the rectangle under its segment should be equal to a given space, it was evident that if this space was greater than the square of half the given line, the thing required could not possibly be done; the two conditions, the one defining the magnitude of the line, and the other that of the rectangle under its segments, being then inconsistent with one another. Hence an infinity of beautiful propositions concerning the maxima and the minima of quantities, or the limits of the possible relations which quantities may stand in to one another.

"Such cases as these would occur even in the solution of the simplest problems; but when geometers proceeded to the analysis of such as were more complicated, they must have remarked that their constructions would sometimes fail, for a reason directly contrary to that which has now been assigned. Instances would be found where the lines that, by their intersection, were to determine the thing sought, instead of intersecting one another, as they did in general, or of not meeting at all, as in the above-mentioned case of impossibility, would coincide with one another entirely, and leave the question of consequence unresolved. But though this circumstance must have created considerable embarrassment to the geometers who first observed it, as being perhaps the only instance in which the language of their own science had yet appeared to them ambiguous or obscure, it would not probably be long till they found out the true interpretation to be put on it. After a little reflection they would conclude that since, in the general problem, the magnitude required was determined by the intersection of the two lines above-mentioned, that is to say, by the points common to them both; so, in the case of their coincidence, as all their points were in common, every one of these points must afford a solution, which solutions therefore must be infinite in number; and also, though infinite in number, they must all be related to one another, and to the things given by certain laws, which the position of the two coinciding lines must necessarily determine.

"On inquiring farther into the peculiarity in the state of the data which had produced this unexpected result, it might likewise be remarked, that the whole proceeded from one of the conditions of the problem involving another, or necessarily including it; so that they both together made, in fact, but one, and did not leave a sufficient number of independent conditions to confine the problem to a single solution, or to any determinate number of solutions. It was not difficult afterwards to perceive that these cases of problems formed very curious propositions, of an intermediate nature between problems and theorems, and that they admitted of being enunciated separately in a manner peculiarly elegant and concise. It was to such propositions, so enunciated, that the ancient geometers gave the name of porisms.

PORISTIC *Method*, in *Mathematics*, is that which determines when, by what means, and how many different ways a problem may be resolved.

POSITION, in *Astronomy*, relates to the sphere. The position of the sphere is either right, parallel, or oblique; whence arise the inequality of days, the difference of seasons, &c.

Circles of POSITION, are circles passing through the common intersections of the horizon and meridian, and through any degree of the ecliptic, or the centre of any star, or other point in the heavens, used for finding out the position or situation of any star. These are usually counted six in number, cutting the equator into twelve equal parts, which the astrologers call the celestial houses.

POSITION, in *Arithmetic*, called also false position, or supposition, or rule of false, is a rule so called, because it consists in calculating by false numbers supposed or taken at random, according to the process described in any question or problem proposed, as if they were the true members; and then, from the results compared with that given in the question, the true numbers are found. It is sometimes also called trial and error, because it proceeds by trials of false numbers, and thence finds out the true ones by a comparison of the errors.

POSITION is either *single* or *double*.

Single POSITION, is that which has place when the results are proportional to their suppositions, and therefore only one supposition is necessary.

Rule. Assume any number for that required, and perform the operations described in the question with it. Then say, as the result obtained is to the number assumed, so is the result in the question to the answer.

EXAM. A person after spending $\frac{1}{3}$ and $\frac{1}{4}$ of his money, has yet remaining 60*l.*; what had he at first?

Suppose he had at first 120*l.*

Now $\frac{1}{3}$ of 120 is 40
$\frac{1}{4}$ of it is 30

their sum is 70
which taken from 120

leaves 50

Then $50 : 120 = 60 : 144$, the answer.

Proof.

$\frac{1}{3}$ of 144 is 48
$\frac{1}{4}$ of 144 is 36

their sum 84
taken from 144

leaves 60
as per question.

Double POSITION, is a method of answering questions similar to those in single position, but which have not their results proportional to their positions. The rule for this purpose given by Mr. Bonnycastle in his Arithmetic, 8vo. edition, is as follows:

Take any two convenient numbers, and proceed with them separately, according to the conditions of the question, noting the results obtained from it.

Then, as the difference of these results is to the difference of the supposed numbers, so is the difference between the true result and either of the former, to the correction of the number belonging to the result used; which correction, being added to that number, when it is too little, or subtracted from it when it is too great, will give the answer required.

Exam. What number is that which, being multiplied by 6, the product increased by 18, and the sum divided by 9, the quotient shall be 20.

Let the two supposed numbers be 18 and 30.

$$\begin{array}{lrr} \text{Then} & 18 & 30 \\ & 6 & 6 \\ \hline & 108 & 180 \\ & 18 & 18 \\ & 9)126 & 9)198 \\ \hline & 14 \left.\begin{matrix}\\ \\ \end{matrix}\right\} \begin{matrix}\text{the 1st}\\ \text{result}\end{matrix} & 22 \left.\begin{matrix}\\ \\ \end{matrix}\right\} \begin{matrix}\text{the 2d}\\ \text{result}\end{matrix} \\ \hline\hline \end{array}$$

$$\begin{array}{lrcl} \text{Then} & 22 - 14 : 30 - 18 & = & 20 - 14 \\ \text{Or} & 8 : 12 & = & 6 : 9 \end{array}$$

the correction to 1st supposition; therefore $9 + 18 = 27$, the number sought.

An expert arithmetician will generally be able to solve questions of this nature without any supposition. Thus, in the first of the preceding examples, there was spent $\frac{1}{3}$ and $\frac{1}{4}$, the sum of which is $\frac{7}{12}$, hence there was $\frac{5}{12}$ remaining, $=$ 60*l.*, which, multiplied by 12 and divided by 5, produces 144*l.* the answer.

In questions like the second, one has only to retrace the steps of the operation: Thus,

$20 \times 9 = 180 - 18$, $162 \div 6 = 27$, the answer.

Given in **Position**, in *Geometry*, is an expression made use of to denote that the position or direction of a line is given or known.

Geometry of **Position**, is a species of geometry first treated of by Carnot, the object of which is to investigate and determine the relation that has place between the position of the different parts of a geometrical figure with regard to each other, or with regard to some determinate line or figure first fixed upon as a term of comparison, and which is called the *primitive system*, while that compared with it is denominated the *transformed system;* and as long as the different parts of the transformed system have the same directions or positions with regard to each other, their relation is said to be *direct;* but when they are different, *inverse*.

These positions, in the analytical representations of geometrical figures, are commonly indicated by the signs prefixed to the letters or symbols representing these lines or quantities; and the geometry of position, at the same time that it gives very great facility to the investigation of many very interesting researches, sets in a clear and indisputable light all those apparent mysteries and anomalies connected with the introduction of the negative sign into analytical investigations.

POSITIVE *Electricity*. In the Franklinian system, all bodies supposed to contain more than their natural quantity of electric matter are said to be positively electrified; and those from whom some part of their electricity is supposed to be taken away, are said to be electrified negatively. These two electricities being first produced, one from glass, the other from amber or rosin, the former was called vitreous, the other resinous, electricity.

Positive *Quantities*, in *Algebra*, are those which are affected with the sign +, being affirmative or additive, in contradistinction to negative quantities, which are to be subtracted.

POSTULATE, in *Geometry*, a demand or petition, or a supposition so easy and self-evidently true, as needs no explanation or illustration; differing from an axiom only in the manner in which it is put, *viz.* as a request instead of an assertion.

POUND, an English weight of different denominations, as Avoirdupoise, Troy, Apothecaries, &c. The pound avoirdupoise is sixteen ounces of the same weight, but the other pounds are each equal to twelve ounces. The pound avoirdupose is to the pound troy as 5760 to 6999½, or nearly as 576 to 700.

Pound is also the highest denomination used by the English in their money accounts, being equal to 20 shillings.

POWER, in *Arithmetic* in *Algebra*, that which arises by the successive multiplication of any number or quantity into itself, the degree of the power being always denominated by the number of equal factors that are employed, and is thus always one more than the number of multiplications; thus

$2 = \dots 2^1$, ... 1st power of 2.
$2 \times 2 = 2^2$, ... 2d power or square.
$2 \times 2 \times 2 =$ . . 2^3, 3d power or cube.
$2 \times 2 \times 2 \times 2$. 2^4, 4th power,
&c. &c.

So also
$x =$ . . . . x^1, 1st power.
$x \times x =$ . . . . . x^2, 2d power.
$x \times x \times x =$ . . . x^3, 3d power.
$x \times x \times x \times x$ . . x^4, 4th power
&c. &c.

Hence it appears that the idex which denotes the degree of any power is always equal to the number of equal factors from which that power arises; or one more than the number of operations.—See **Exponent** and **Involution**.

The early writers on arithmetic and algebra, gave names to the several powers, which we now denote by their indices.

Thus the 2d was called the *square*, and the 3d the *cube*, as we do at present; but they carried this much farther, calling the 4th power the *quadrato-quadratum;* the 5th, *quadrato-cubus;* 6th, *cubo-cubus;* 7th, *quadrato-quadrato cubus*, &c. but these denominations are now rejected for the more simple and obvious denominations by means of the exponent.

The powers of numbers have several curious properties, as to their forms, divisors, &c. of which the following are the most remarkable:

1. The difference of any two equal powers of different numbers, is divisible by the difference of their roots, that is

$x^n - y^n$ is divisible by $x - y$

This is true, whether n be even or odd.

2. $x^n - y^n$ is divisible, both by $x - y$ and $x + y$, if n be even.

3. $x^n + y^n$ is divisible by $x + y$ if n be odd.

Thus also

4. $x^m - x^n$ is divisible by $x - 1$, and by $x + 1$ if $m - n$ be even.

5. $x^m - x^n$ is divisible by $x - 1$ if $m - n$ be odd.

6. $x^m + x^n$ is divisible by $x + 1$ if $m - n$ be odd.

7. Neither the sum nor difference of any two equal integral powers above the second, can be equal to a complete power of the same denomination.

8. If m be a prime number, and x any number not divisible by m, then will the remainder arising from the division of x by m, be the same as that from the division of x^m by m; and consequently $x^m - 1 - 1$, is always divisible by m.

9. Hence is obtained the forms of several powers, that is, the remainders that they will leave when divided by given numbers; thus x^2, x^3, x^4, &c. denoting the powers of any numbers whatever we have.

Forms.
x^2 . . $3n$, or $3n +$
x^3 . . . .
x^4 . . . $5n$, or $5n + 1$
x^5 . . . .
x^6 . . . $7n$, or $7n + 1$

Forms.
$5n$, or $5n \pm 1$.
$7n$, or $7n \pm 1$.
. . . .
$11n$, or $11n \pm 1$.
$13n$, or $13n \pm 1$.

Forms.
x^8 . . . .
x^9 . . . .
x^{10} . $11n$, or $11n + 1$
x^{11} . . . .
x^{12} . $13n$, or $13n + 1$

Forms.
$17n$, or $17n \pm 1$.
$19n$, or $19n \pm 1$.
. . . .
$23n$, or $23n \pm 1$.

And generally if x^m be any power, then if $m+1$ be a prime number

x^m is of the form $(m+1)n$, or $(m+1)n+1$ and if $2m+1$ be a prime, then

x^m is of the form $(2m+1)n$ or $(2m+1)n \pm 1$.

That is, when divided by $2m+1$, the remainder will either be 0, or ±1.

10. The *n*th differences of any number of consecutive *n*th powers is constant, and equal to 1 . 2 . 3 . 4. &c. *n*.

Thus let 1 4 9 16 $\Big\{$ be any consecutive square.
1st diff. 3 5 7
2d diff. 2 2 &c. = 1 . 2.

Again 1 8 27 64 125 $\Big\{$ consecutive cubes.
1st diff. 7 19 37 61
2d diff. 12 18 24
3d diff. 6 6 &c. = 1 . 2 . 3.

And in the same manner the 4th difference of 4th powers
= 1.2.3.4 = 24
of 5th powers = 1 . 2 . 3 . 4 . 5 = 120
and so on for any higher powers. See the demonstration of these and several other curious properties of powers in chap. vi. Barlow's Theory of Numbers.

Power of the *Hyperbola*, is the 4th power of its conjugate axis.

Power, in *Mechanics*, denotes some force which, being applied to a machine, tends to produce motion; whether it does actually produce it or not. In the former case, it is called a moving power; in the latter, a sustaining power.

Power is also used in *Mechanics*, for any of the six simple machines, *viz.* the lever, the balance, the screw, the wheel and axle, the wedge, and the pulley.

Power of a *Glass*, in *Optics*, is by some used for the distance between the convexity and the solar focus.

PRACTICE, is an arithmetical rule, principally employed in those questions in which the amount of a certain number of things is required, the price of each being given; being a more ready and expeditious method than *Compound Multiplication*, by which rule the same questions may also be resolved.

PRECESSION of the *Equinoxes*, in *Astronomy*, is a slow retrograde motion of the equinoctial points, *viz.* from east to west, or contrary to the order of the signs. The sun every year crosses the equator in two points, which are called the equinoctial points; and as it is natural to compute the course of the year from the moment when the sun is in one of those points which gives equal days and nights to all parts of the globe, it became very early an important problem in practical astronomy to determine them with some degree of accuracy; which was by no means difficult, as it did not require that accurate kind of observation that is so indispensable in many other cases. It is only necessary for this purpose, to observe the sun's declination on the noon of two or three days before and after the time of the equinoxes. On two consecutive days of this number, his declination must have changed from north to south, or from south to north. If his declination on one day was observed to be 21′ north, and on the next 5′ south, it follows that his declination was nothing, or that he was in the equinoctial point about twenty-three minutes after seven in the morning of the second day. Knowing then the precise moments, and the rate of the sun's motion in the ecliptic, it is easy to ascertain the precise point of the ecliptic in which the equator intersected it.

The motion of the stars *in antecedentia* was long considered as an inexplicable mystery in the science of astronomy; but it is now as clearly understood and accounted for, as any of the other celestial phenomena.

It is well known, that while the earth revolves round the sun from west to east, in the plane of the ecliptic, in the course of a year, it turns round its own axis from west to east in 23h 56′ 4″, which axis is inclined to this plane in an angle of nearly 23° 28′: and that this axis turns round a line perpendicular to the ecliptic in 25745 years from east to west, keeping nearly the same inclination to the ecliptic.—By this means, its pole in the sphere of the starry heavens describes a circle round the pole of the ecliptic at the distance of 23° 28′ nearly. The consequence of this must be, that the terrestrial equator, when produced to the sphere of the starry heavens, will cut the ecliptic in two opposite points, through which the sun must pass when he makes the day and night equal; and that these points must shift to the westward, at the rate of $50\frac{1}{3}$ seconds annually, which is the precession of the equinoxes.

To find the Precession in right Ascension and Declination.

Put $d =$ the declination of a star,
and $a =$ its right ascension;
then their annual variations of pressions will be nearly as follow: *viz.*
$20''\cdot084 \times \cos.\, a =$ the annual precession in declination,
and $46''\cdot0619 + 20''\cdot084 \times \sin.\, a \times \text{tang.}\, d =$ that of right ascension.

PRESS, in *Practical Mechanics*, a machine made of iron or wood, serving to compress bodies close, and therefore into a less compass. The common presses consists of six parts or members, *viz.* two flat smooth planks or boards, between which the compression takes place, two screws or worms fastened to the lower plank, and passing through two holes in the upper, and two nuts, serving to drive the upper plank, being moveable, against the lower one, which is fixed.

Hydraulic or Hydrostatic PRESS. This, otherwise called Bramah's press, is a machine which exerts a very powerful effect, and the principle of it is at the same time simple. It consists of a strong frame, with a moveable bottom plate, which gives the pressure. Attached to the under side of this plate there is a piston, which works in a strong cylinder. Water is forced into this cylinder, through a tube of small aperture, by a common forcing pump, and there are valves, generally conical ones, which prevent the water from returning. A safety-valve is also fitted to the cylinder, the aperture of which is very small and covered by a lever, to which is hung a weight. When water is forced into the cylinder it drives up the piston and exerts the pressure, and the safety-valve prevents the apparatus from bursting. In a well-constructed press one man may by pumping in a pailfull of water, exert a pressure of many tons.

PRESSURE, in *Physics*, is properly the action of a body which makes a continual effort or endeavour to move another body on which it rests; such as the action of a heavy body supported by a horizontal table; and is thus distinguished from percussion or momentary force of action. Since action and re-action are equal and contrary, it is obvious that pressure equally relates to both bodies, *viz.* the one which presses and that which receives the pressure. See a few remarks on the difference between percussion and pressure, under the article PERCUSSION.

PRESSURE of *Fluids*, is of two kinds, *viz.* of elastic and non-elastic fluids.

PRESSURE of *Non-elastic Fluids.* The upper surface of a homogeneous heavy fluid in any vessel, or any system of communicating vessels, is horizontal.

This is usually explained by saying, that since the parts of a fluid are easily moveable in any direction, the higher particles will descend by reason of their superior gravity, and raise the lower parts till the whole comes to rest in a horizontal plane. Now, what is called the horizontal plane is, in fact, a portion of a spherical surface, whose centre is the centre of the earth: hence it will follow, that if a fluid gravitate towards any centre, it will dispose itself into a spherical figure, the centre of which is the centre of force.

If a fluid, considered without weight, is contained in any vessel whatever, and an orifice being made in the vessel, any pressure whatever be applied thereto, that pressure will be distributed equally in all directions. Hence:—

1. Not only is the pressure transmitted equally in all directions, but it acts perpendicularly upon every point of the surface of the vessel which contains the fluid.

For, if the pressure which acts upon the surface were not exerted perpendicularly, it is easy to see that it could not be entirely annihilated by the re-action of that surface; the surplus of force would, therefore, occasion fresh action upon the particles of the fluid, which must of consequence be transmitted in all directions, and thus necessarily occasion a motion in the fluid; that is, the fluid could not be at rest in the vessel, which is contrary to experience.

2. If the parts of a fluid contained in any vessel, open towards any part, are solicited by any forces whatever, and remain notwithstanding in equilibrio, these forces must be perpendicular to the open surface. For the equilibrium would obtain, in like manner, if a cover or a piston of the same figure as the open surface were applied to it; and it is manifest that, in this latter case, the forces which act at the surface, or their resultant, must be perpendicular to that surface.

3. If, therefore, the forces which act upon the particles of the fluid are those of gravity, we shall see that the direction of gravity is necessarily perpendicular to the surface of a tranquil fluid; consequently, the surface of a heavy fluid must be horizontal to be in equilibrio, whatever may be the figure of the vessel in which it is contained.

4. If a vessel, closed throughout except a small orifice, is full of a fluid without weight; then, if any pressure be applied at that orifice, the resulting pressure on the plane surface, or bottom, will neither depend upon the quantity of fluid in the vessel, nor on its shape; but, since the pressure applied at the orifice, is transmitted equally in all directions, the actual pressure upon the bottom will be to the pressure at the orifice, as the area of the bottom is to that of the orifice.

5. In the same manner will the pressure applied at the orifice, be exerted in raising the top of the vessel; so that if the top be a plane, of which the orifice forms a part, the vertical pressure tending to force the top upwards will be to the force applied at the surface, as the surface of the top to the area of the orifice. The hydraulic press is founded upon this principle.

The pressure of a fluid on the horizontal base of a vessel in which it is contained, is as the base and perpendicular altitude, whatever be the figure of the vessel that contains it; the upper surface of the fluid being supposed horizontal.

PRIMARY *Planets*, are such as revolve about the sun as a centre; such are Mercury, Venus, Terra the Earth, Mars, Vesta, Juno, Pallas, Ceres, Jupiter, Saturn, and Uranus, or the Georgium Sidus; being thus called in contradistinction to the secondary planets or satellites, which revolve about their respective primaries.

PRIME *Numbers*, are those which have no divisors, or which cannot be divided into any number of equal integral parts, less than the number of units of which they are composed; such as 2, 3, 5, 7, 11, 13, 17, &c. These numbers have formed a subject of investigation and inquiry, from the earliest date down to the present day; and a rule for finding them is still amongst the desiderata of mathematicians. Eratosthenes invented what he called a sieve for this purpose, because by this he sifted away, or separated, those numbers that were not primes from those that were, and by this means ascertained the latter; which is an indirect method, and cannot be applied to a particular case, without a general calculation.

The principle of this method, which is the same that has since been employed by modern writers, for ascertaining those numbers, is as follows:

Having written down, in their proper order, all the odd numbers, from 1 to any extent required; as,

$$\begin{array}{cccccccccc}
1 & 3 & 5 & 7 & \dot{9} & 11 & 13 & \ddot{15} & 17 & 19 \\
\ddot{21} & 23 & \dot{25} & \dot{27} & 29 & 31 & \dot{33} & \ddot{35} & 37 & \dot{39} \\
41 & 43 & \ddot{45} & 47 & \dot{49} & \dot{51} & 53 & \dot{55} & \dot{57} & 59 \\
61 & \ddot{63} & \dot{65} & 67 & \dot{69} & 71 & 73 & \ddot{75} & \dot{77} & 79 \\
\dot{81} & 83 & \dot{85} & \dot{87} & 89 & \dot{91} & \dot{93} & \dot{95} & 97 & \dot{99}
\end{array}$$

We begin with the first prime

number 3, and over every third number, from that place, we put a point, because all those numbers are divisible by 3; as 9, 15, 21, &c.

Then from 5, a point is placed over every fifth number, all these being divisible by 5, such are 15, 25, 35, &c.

Again, from 7, every 7th number is pointed in the same manner, such as 21, 35, 49, &c.

And having done this, all the numbers that now remain without points are prime numbers; for there is no prime number between 7 and $\sqrt{100}$; because if a number cannot be divided by a prime number less than the square root of itself, it is itself a prime number; adding therefore to the above, the prime number 2, which is the only even prime we have,

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97,

which are all the prime numbers under 100.

The method of finding a prime number beyond a certain limit, by a direct process, is considered one of the most difficult problems in the theory of numbers; which, like the quadrature of the circle, the trisection of an angle, and the duplication of the cube, have engaged the attention of many able mathematicians, but without arriving at any satisfactory result. It was generally supposed, that this might be done by some formula, as $42x^2 + 17$, $3x^2 + 5x + 19$, &c.; in which, by assuming any number whatever for x, the whole formula would be a prime; but it may be demonstrated, that no such formula can exist; still, however, though no rule for finding these numbers has yet been discovered, yet many very interesting properties of them have been demonstrated, the principal of which are as follow:

1. If a number cannot be divided by another number less than the square root of itself, that number is a prime.

2. All prime numbers are of one of the forms $4n + 1$, or $4n - 1$; that is, if a prime number be divided by 4, the remainder will be ± 1. So, also, all prime numbers are of one of the forms $6n \pm 1$; and various other forms may be given, but the converse, as observed above, has not place; namely, that every number in these forms is a prime.

3. The number of prime numbers is infinite.

4. There cannot be three prime numbers in arithmetical progression, unless their common difference be divisible by $1 \times 2 \times 3$; or unless the first of these primes be the number 3, in which case there may be three prime numbers in such an arithmetical progression; but there cannot be more than 3.

5. If n be a prime number, then will

$$1 . 2 . 3 . 4 . 5, \&c.\ n, + 1$$

be divisible by n.

6. Every prime number of the form $4n + 1$, is the sum of two squares; as $5 = 2^2 + 1^2$, $13 = 3^2 + 2^2$, $17 = 4^2 + 1^2$, &c.; but a prime number, not of this form, cannot be resolved into two squares.

7. Every prime number of the form $8n + 1$, is of the three forms $y^2 + z^2$, $y^2 + 2z^2$, $y^2 - 8z^2$.

Thus $41 = 5^2 + 4^2 = 3^2 + 2.4^2 = 7^2 - 2.2^2$.

And $73 = 8^2 + 3^2 = 1^2 + 2.6^2 = 9^3 - 2.2^2$.

8. Every prime number $8n + 7$, is of the form $y^2 - 2z^2$.

Thus $31 = 7^2 - 2.3^2$; and $47 = 7^2 - 2.1^2$.

9. Every prime number $8n + 3$, is of the form $y^2 + 2z^2$.

For example, 11, 19, and 43, are primes of this form; and $11 = 3^2 + 2.1^2$, $19 = 1^2 + 2.3^2$, and $43 = 5^2 + 2 . 3^2$.

10. If n be a prime number, and r any number whatever not divisible by n; then will r^n, when divided by n, leave the same remainder as r divided by n.

11. Supposing still n and r, as in the preceding case, $r^{n-1} - 1$ will be divisible by n.

12. The square of every prime number of the form $4n + 1$, is of the form $y^2 + 25z^2$.

For the demonstration of these, and various other properties of prime numbers, see Barlow's "Elementary Investigations," &c.

Prime *Vertical*, is that vertical circle, or azimuth, which is perpendicular to the meridian, and passes through the east and west points of the horizon.

PRIME *Vertical Dials*, are those that are projected on the plane of the prime vertical circle, or on a plane parallel to it. These are otherwise called direct, erect, north, or south dials.

PRIME of the *Moon*, is the new moon at her first appearance, for about three days after her change. It means also the GOLDEN *Number*.

PRIMUM *Mobile*, in the *Ptolemaic Astronomy*, the ninth or highest sphere of the heavens, whose centre is that of the world, and in comparison of which the earth is but a point. This the ancients supposed to contain all other spheres within it, and to give motion to them, turning itself, and all of them, quite round in twenty-four hours.

PRINCIPAL, in *Arithmetic* or in *Commerce*, is the sum lent upon interest, either simple or compound.

PRISM, in *Geometry*, is a body, or solid, whose two ends are any plain figures which are parallel, equal, and similar; and its sides connecting those ends are parallelograms. Hence, every section parallel to the base, is equal and similar to the base; and the prism may be considered as generated by the parallel motion of this plane figure.

Prisms receive particular names, according to the figure of their bases; as a *triangular* prism, a *square* prism, a *pentagonal* prism, a *hexagonal* prism, and so on. And hence the denomination prism comprises also the cube and parallelopipedon, the former being a square prism, and the latter a rectangular one. And even a cylinder may be considered as a round prism, or one that has an infinite number of sides. Also a prism is said to be regular or irregular, according as the figure of its end is a regular or an irregular polygon.

The axis of a prism, is the line conceived to be drawn lengthways through the middle of it, connecting the centre of one end with that of the other end.

Prisms, again, are either right or oblique.

A right prism is that whose sides and its axis are perpendicular to its ends, like an upright tower. And

An oblique prism, is when the axis and sides are oblique to the ends; so that, when set upon one end, it inclines on one hand more than on the other.

The principal properties of prisms are,

1. That all prisms are to one another in the ratio compounded of their bases and heights.

2. Similar prisms are to one another in the triplicate ratio of their like sides.

3. A prism is triple of a pyramid of equal base and height; and the solid content of a prism is found by multiplying the base by the perpendicular height.

4 The upright surface of a right prism is equal to a rectangle of the same height, and its breadth equal to the perimeter of the base, or end. And, therefore, such upright surface of a right prism, is found by multiplying the perimeter of the base by the perpendicular height. Also the upright surface of an oblique prism is found by computing those of all its parallelogram sides separately, and adding them together.

And if to the upright surface be added the areas of the two ends, the sum will be the whole surface of the prism.

PRISM, in *Optics*, is an instrument employed for showing the properties of solar light, and consists merely of a triangular prism of glass, which separates the rays of light in their passage through it, in consequence of the different degrees of refrangibility that has place in the component part of the same ray.

It is, for instance, by means of this instrument that the origin of colours is shown to be owing to the composition which takes place in the rays of light, each heterogeneous ray consisting of innumerable rays of different colours. Thus, a ray being let into a darkened room, through a small round aperture, and falling on a triangular glass prism, is by the refraction of the prism considerably dilated, and will exhibit on the opposite wall an oblong image, called a spectrum, variously coloured, the extremities of which are bounded by semicircles, and the sides rectilinear. The colours are commonly divided into seven, which, however, have various shades, gradually intermixing at their juncture. Their order, beginning from the side of the refracting angle of the prism, is red, orange, yellow, green, blue, purple, and violet. The obvious conclusion from this experiment is, that the several component parts of solar light have different degrees of refrangibility, and that each subsequent ray in the order above mentioned, is more refrangible than the preceding.

As a circular image would be depicted by the solar ray unrefracted by the prism, so each ray that suffers no dilation by the prism, would mark out a circular image in the original direction of the ray. Hence, it appears that the spectrum is composed of innumerable circles of different colours. The mixture, therefore, is proportionable to the number of circles mixed together; but all such circles lie between those of two contingent circles, consequently the mixture is proportionable to the interval of those centres; *viz.* to the breadth of the spectrum. Consequently, if the breadth can be diminished, retaining the length of the rectilinear sides, the mixture will be lessened proportionably; and this is done by the following process:—At a considerable distance from the hole, place a double convex lens, whose focal length is equal to half that distance, and place the prism behind the lens; then, at a distance behind the lens, equal to the distance from the hole, will be formed a spectrum, the length of whose rectilinear sides is the same as before, but its breadth much less; for the undiminished breadth was equal to a line subtending, at the distance of the spectrum from the hole, an angle equal to the apparent diameter of the sun, together with a line equal to the diameter of the hole; but the reduced breadth is equal to the diameter of the hole only: the image of the hole formed by the lens, at the distance of double its focal length, is equal to the hole; therefore its several images, in the different kinds of rays, are equal to the same; *viz.* the breadth of the reduced spectrum is equal to the diameter of the hole.

It is also known from experiment that a prism placed in an horizontal position will project the ray into an oblong form, but if another horizontal prism be applied, similar to the former, to receive the refracted light emerging from the first, and having its refracting angle turned the contrary way from that of the former, the light, after passing through both prisms, will assume a circular form, as if it had not been at all refracted. But if the light, after emerging from the first prism, be received on another prism, perpendicular to the former; it will be refracted by this into a position inclined to the former; but its breadth will remain the same.

In order now to show that the different colours suffer no manner of change from any number of refractions, let there be placed close to the prism a perforated board, and let the refracted light transmitted through the hole be received on another board parallel to the former, and likewise perforated with a small hole; and behind this hole place another prism, with its refracting angle downwards, and turn the first prism slowly about its axis, and the light will then move up and down the second board; let the different colours be turned successively, and mark the place of the different coloured rays on the wall after their refraction at the second prism; it will then be found that the red is seen the lowest, and the violet the highest, and the rest in the intermediate space in their order.

From these experiments, aided by some others which our limits will not admit of detailing, the following conclusions have been drawn, *viz.* The solar rays may be resolved into different coloured rays; these coloured rays are immutable, either by reflection or refraction. That from the mixture of these coloured rays in due proportion solar light may be produced; and consequently, that the differently coloured rays exist in solar light, though, when blended together in their natural proportions, it exhibits no traces of colour.

PRISMOID, a figure resembling a prism.

PROBABILITY of an *Event*, in the doctrine of *Chances*, is the ratio of the number of chances by which the event may happen, to the number by which it may both happen and fail. So that, if there be constituted a fraction, of which the numerator is the number of chances for the events happening, and the denominator the number for both happening and failing, that fraction will properly express the value of the probability of the event's happening.

PROBLEM, in *Geometry*, is a proposition wherein some operation or construction is required; as to divide a line or angle, erect or let fall perpendiculars.

Problem, in *Algebra*, is a question or proposition which requires some unknown truth to be investigated, and the truth of the discovery proved.

Problem of the *Three Bodies*, is the term by which is denoted the celebrated problem of finding the inequalities of the lunar orbit. This problem, in all its generality, is as follows: three bodies of given magnitudes, as the sun, the earth, and moon, being projected into space with given velocities, and in given directions, and attracting each other according to a given law (the inverse ratio of the squares of their distances from each other, and directly as their masses): it is required to determine the nature of the curve that one of them, as the moon, describes about one of the others, as the earth. Such is the general state of the problem, but in the case in question, there are certain conditions which render it less difficult, *viz.* 1. That one of the bodies, the sun, is very great in comparison with the other two, and nearly at rest. 2. Its distance from the other two is so great that it may be considered the same for both. 3. The orbit of the moon about the earth is nearly an ellipse, and the aberrations from which is, therefore, all that is required. The problem, even with these limitations, is sufficiently difficult, and has engaged the attention of the most celebrated analysts of modern times.

PRODUCT, in *Arithmetic* and *Algebra*, is the quantity arising from the multiplication of two or more factors.

Some of the most remarkable properties of products, as to their forms, with reference to the factors whence they were produced, may be stated as follows:

1. The product of the sum of two numbers by their difference is equal to the difference of their squares; that is,

$$(x+y)(x-y)=x^2-y^2.$$

2. The double of the sum of two squares is also the sum of two squares; that is,

$$(x^2+y^2)\times 2=(x+y)^2+(x-y)^2.$$

Consequently, the sum of two squares, multiplied by any power of 2, is the sum of two squares; thus for example:

$5=2^2+1^2$; $5\times 2=10=3^2+1^2$ $10\times 2=20=4^2+2^2$; $40=6^2+2^2$, &c.

3. The product of the sum of two squares, by the sum of two squares, is itself the sum of two squares; for

$$(x^2+y^2)(x'^2+y'^2)=\begin{cases}(xx'+yy')^2+(xy'-x'y)^2 \text{ or}\\ (xx'-yy')^2+(xy'+x'y)^2\end{cases}$$

$$\begin{array}{lr} \text{Thus} & 5=2^2+1^2 \\ & \underline{13=3^2+2^2} \\ \text{product} & \underline{\underline{65=8^2+1^2}} \text{ or } 7^2+4^2 \end{array}$$

4. The product of the sum of four squares, by the sum of four squares, is also the sum of four squares; for $(w^2+x^2+y^2+z^2)\times(w'^2+x'^2+y'^2+z'^2)=(ww'+xx'+yy'+zz')^2+(wx'-xw'+yz'-y'z)^2+(uy'-xz'-yw'+zx')^2+(wz'+xy'-yx'-zu')^2$

5. The product of two numbers, or formulæ of the form x^2+ay^2, is also the same form; for

$$(x^2+ay^2)(x'^2+ay'^2)=\begin{cases}(xx'+ayy')^2+a(xy'-yx')^2\\ (xx'-ayy')^2+a(xy'+yx')^2\end{cases}$$

6. The two formulæ $x^2+y^2+z^2$,

and $x'^2 + y'^2 + 2z'^2$ are such, that the double of either gives the other thus:

$$(x^2 + y^2 + z^2) \times 2$$
$$= \begin{cases} 2x^2 + 2y^2 + 2z^2 = \\ (x+y)^2 + (x-y)^2 + 2z^2 \end{cases}$$

Again,

$$(x'^2 + y'^2 + 2z'^2) \times 2$$
$$= \begin{cases} 2x'^2 + 2y'^2 + 4z'^2 = \\ (x'+y')^2 (+ x' - y'^2 + (2z')^2 \end{cases}$$

$$\text{Thus} \quad 14 = 3^2 + 2^2 + 1^2$$
$$\text{mult. by} \quad \underline{2}$$
$$\text{the prod. } 28 = \begin{cases} (3 +{}^{2}2)^2 + (3-2)^2 \\ \quad + 2\ 1^2 \\ 5^2 + 1^2 + 2^2 \end{cases}$$
$$\text{Again,} \quad 15 = 3^2 + 2^2 + 2 . 1^2$$
$$\text{mult. by} \quad \underline{2}$$
$$30 = \begin{cases} (3+2)^2 + (3-2)^2 \\ \quad + 2^2 = \\ 5^2 + 1^2 + 2^2 \end{cases}$$

See Barlow's Theory of Numbers, p. 180.

PROGRESSION, in *Arithmetic* and *Algebra*, a series of numbers advancing or proceeding in the same manner, or according to a certain law, &c.

Progression is either arithmetical, or geometrical.

Arithmetical PROGRESSION, is a series of three or more quantities that have all the same common difference; as 3, 5, 7, &c. which have the common difference 2; and a, $a + d$, $a + 2d$, &c. which have all the same difference d.

In an arithmetical progression, the chief properties are these: 1st, The sum of any two terms, is equal to the sum of every other two that are taken at equal distances from the two former, and equal to double the middle term when there is one equally distant between those two: so, in the series 0, 1, 2, 3, 4, 5, 6 &c. $0 + 6 = 1 + 5 = 2 + 4 =$ twice 3 or 6. 2d. The sum of all the terms of any arithmetical progression, is equal to the sum of as many terms of which each is the arithmetical mean between the extremes; or equal to half the sum of the extremes multiplied by the number of terms: so, the sum of these ten terms 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, is $\frac{0+9}{2} \times 10$, or 9×5, which is 45; and the reason of this will appear by inverting the terms, setting them under the former terms, and adding each two together, which will make double the same series;

$$\begin{array}{rl} \text{thus} & 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \\ \text{inverted} & \underline{9, 8. 7. 6, 5. 4. 3, 2. 1. 0,} \\ \text{sums} & 9, 9, 9, 9, 9, 9, 9, 9, 9, 9. \end{array}$$

where the double series being the same number of nines, or sum of the extremes, the single series must be the half of that sum. 3d. The last, or any term, of such a series, is equal to the first term, with the product added of the common difference multiplied by 1 less than the number of terms, when the series ascends or increases; or the same product subtracted when the series descends or decreases; so, of the series 1, 2, 3, 4, &c. whose common difference is 1, the 50th term is $1 + (1 \times 49)$, or $1 + 49$, that is 50; and of the series 50, 49, 48, &c. the 50th term is $50 - 1 \times 49$, or $50 - 49$, which is 1. Also, if we denote by a the least term,

z the greatest term,
d the common difference,
n the number of the terms,
and s the sum of the series;

then the principal properties are expressed by these equations, *viz.*

$$\left.\begin{array}{l} z = a + d . (n-1) \\ a = z - d . (n-1) \end{array}\right\}$$
$$s = (a + z)\tfrac{1}{2}n,$$
$$s = (z - \tfrac{1}{2}d . \overline{n-1})\, n$$
$$s = (a + \tfrac{1}{2}d . \overline{n-1})\, n$$

Moreover, when the first term a, is 0 or nothing; the theorems become $z = d\,(n-1)$ and $s = \frac{1}{2}zn$. See Bonnycastle's Arithmetic and Algebra.

Geometrical PROGRESSION, a progression in which the terms have all successively the same ratio; as 1, 2, 4, 8, 16, &c. where the common ratio is 2.

The general and common property of a geometrical progression is, that the product of any two terms, or the square of any one single term, is equal to the product of every other two terms that are taken at an equal distance on both sides from the former. So of these terms,

1, 2, 4, 8, 16, 32, 64, &c.

$1 \times 64 = 2 \times 32 = 4 \times 16 = 8 \times 8 = 64.$

In any geometrical progression, if
a denote the least term
z the greatest term,
r the common ratio,
n the number of the terms,
s the sum of the series, or all the terms;

then any of these quantities may be found from the others, by means of these general values or equations, *viz.*

$$r = \left(\frac{z}{a}\right)^{\frac{1}{n-1}}$$

$$z = a \times r^{n-1},$$

$$a = \frac{z}{r^{n-1}},$$

$$n = \frac{\log. \frac{rz}{a}}{\log. r} = \frac{\log. r + \log. z - \log. a}{\log. r}$$

$$s = \frac{r^{n}-1}{r-1} \times a = \frac{r^{n}-1}{r-1} \times \frac{z}{r^{n-1}} = \frac{rz-a}{r-1}$$

When the series is infinite, then the least term a is nothing, and the sum $s = \frac{rz}{r-1}$

In any increasing geometrical progression, or series beginning with 1, the 3d, 5th, 7th, &c. terms will be squares; the 4th, 7th, 10th, &c. cubes; and the 7th will be both a square and a cube. Thus in the series 1, r, r^2, r^3, r^4, r^5, r^6, r^7, r^8, r^9, &c. r^2, r^4, r^6, r^8, are squares; r^3, r^6, r^9, cubes; and r^6 both a square and a cube.

PROJECTILES, is that branch of mechanics, which relates to the motion, velocity, range, &c. of a heavy body projected into void space by any external force, and then left to the free action of gravity, by which it descends to the earth.

Till the time of Galileo the true path of a projectile was unknown; it was supposed that its first motion was in a right line in the direction of the impelling force, and that it afterwards described a curve; but the nature of that curve was wholly unknown. Tartaglia, however, pointed out the error of supposing any part of the path to be a right line; but it was Galileo who first demonstrated that, on the supposition of gravity acting in parallel lines, the body would describe the curve of a parabola. This, it is true, is not theoretically correct, for the action of gravity, it is well known, is not made in lines parallel to each other, but in lines perpendicular to the earth's surface: the error, however, is so trifling as to be of no importance, and this supposition of its parallelism is therefore universally adopted.

There is no branch of mixed mathematics that presents a more elegant and interesting theory than projectiles; but it unfortunately happens, also, that there is none in which theory and practice are more at variance, a circumstance which is principally to be attributed to the resistance of the air, which is very considerable, whereas in theory the motion is supposed to take place in void space, or, as it is commonly said, in a non-resisting medium. The following propositions comprise the theory of projectiles in non-resisting mediums:

1. If a heavy body be projected into a non-resisting medium, either parallel or in any way inclined to the horizon, it will by this motion, combined with that resulting from the action of gravity, describe the curve of a parabola.

Cor. 1. The velocity in the line of direction being uniform, the horizontal velocity is also uniform, and it is to the former as radius to the cosine of the angle of elevation.

Cor. 2. The velocity in the direction of gravity at any point, is to the first projectile velocity, as twice the ordinate, or descent from the line of direction, is to the distance measured upon that line.

Cor. 3. The squares of the distances in the line of direction is equal to four times the rectangle of the ordinate and projectile force or impetus.

2. Let the impetus $= m$; the velocity of projection $= v = 2\sqrt{g\,m}$, the time of flight $= t$; the descent of gravity in one second $16\frac{1}{12} = g$. And call the angle of elevation $=$ A, the angle of inclination of the plane $=$ B, and the sum of these two $=$ C; then since the velocity is uniform we have in the first

place, the range in the line of direction $= t\,v$.

Again, as $(= \cos. B) : \sin. C = tv : H$, the final ordinate

whence $H = \frac{\sin.\ C}{\cos.\ B}\ t\,v = g\,t^2 = 4\,h$,

h being the greatest height of the projectile,

whence $v = \frac{\cos.\ B}{\sin.\ C}\,g\,t$ and $t = \frac{\sin.\ C}{\cos.\ B} \cdot \frac{v}{g}$

Again, As $\sin. A\ (= \cos. B) : = \cos. A = t\,v$: or, the range $r = \frac{\cos.\ A}{\cos.\ B}\,t\,v$

Now substituting in this last expression, for the range, the values of v, t, &c. as found above, we have

$$r = \begin{cases} \frac{\cos.\ A}{\cos.\ B}\ t\,v = \frac{\cos.\ A}{\sin.\ C}\ g\,t^2 = \\ \frac{\cos.\ A.\ \sin.\ C}{\cos.\ ^2B} \cdot \frac{v^2}{g} = \\ \frac{\cos.\ A.\ \sin\ C}{\cos.\ ^2B}\,4m = \frac{\cos.\ A}{\sin.\ C}\,4h. \end{cases}$$

And from these we readily draw the following values of v, t, and p.

$$v = \begin{cases} \frac{\cos. B\ r}{\cos. A\ t} = \cos. B \sqrt{\frac{g\,r}{\cos. A. \sin. C}} \\ = \frac{\cos.\ B}{\sin.\ C}\,gt = \frac{2 \cos\ B}{\sin. C}\sqrt{gh} \\ = \sqrt{4mg} \end{cases}$$

$$t = \begin{cases} \frac{2 \sin.\ C}{\cos.\ B}\sqrt{\frac{m}{g}} = \frac{\sin.\ C}{\cos.\ B}\,\frac{v}{g} = \surd \\ \frac{\sin.\ C}{\cos.\ A}\,\frac{r}{g} = 2\sqrt{\frac{h}{g}} = \frac{\cos. B}{\cos. A}\,\frac{r}{v} \end{cases}$$

$$h = \begin{cases} \frac{\sin.\ ^2C}{\cos.\ B}\,m = \frac{\sin.\ ^2C}{\cos.\ ^2B}\,\frac{v^2}{4g} = \\ \frac{\sin.\ C}{\cos.\ A}\,\frac{r}{4} = \frac{gt^2}{4} \end{cases}$$

And from these formulæ it is easy to find expressions for the sine or cosine of the angle of elevation, when these are to be determined.

When the plane is horizontal these expressions become more simple.

$$r = \begin{cases} \cos.\ A\,.\,tv = \frac{\cos.\ A}{\sin.\ A}\,g\,t^2 = \\ \cos.\ A.\ \sin.\ A.\ \frac{v^2}{g} = \cos.\ A. \\ \sin.\ A.\ 4m = \frac{\cos.\ A}{\sin.\ A} \cdot 4\,h \end{cases}$$

$$\begin{cases} \frac{r}{\cos.\ A\,.\,t} = \sqrt{\frac{g\,r}{\cos.\ A.\ \sin.\ A}} \\ = \frac{g\,t}{\sin. A} = \frac{2\sqrt{g\,h}}{\sin.\ A} = \sqrt{m\,g} \end{cases}$$

$$t = \begin{cases} 2 \sin.\ A.\ \sqrt{\frac{m}{g}} = \frac{\sin.\ A\,.\,v}{g} = \surd \\ \frac{\sin.\ A.r}{\cos. A.g} = 2\sqrt{\frac{h}{g}} = \frac{r}{\cos. A. v} \end{cases}$$

$$h = \begin{cases} \sin.\ ^2A\ m = \frac{\sin\ ^2A\ v^2}{4g} = \\ \frac{\sin.\ A.\ r}{4 \cos.\ A} = \frac{gt^2}{4} \end{cases}$$

And these again, by substituting $\sin.\ A.\ \cos.\ A = \frac{1}{2}\ \sin.\ 2\ A$ and $\frac{\sin.\ A}{\cos.\ A} = \tan.\ A$, are rendered still more simple, and particularly those which relate to the angle of elevation. We have thus,

$$r = \frac{gt^2}{\tan.\ A} = \sin.\ 2\ A\ \frac{v^2}{2g} = \sin.\ 2\ A\ .\ 2m$$

$$\sin.\ a = \frac{gt}{v} = \frac{2}{v}\sqrt{gm} = \frac{tv}{4h}$$

$$\tan.\ a = \frac{g\,t^2}{r} = \frac{4\,m}{r} = \frac{g\,t^2}{r}$$

$$\sin.\ 2a = \frac{2r\,g}{v^2} = \frac{r}{2\,h} = \frac{2g\,r}{v^2}$$

From which last value of r it is obvious that the range varies as the sine of the double angle of elevation, and therefore that it is the greatest when this angle is 45°, or when the angle subtended by the plane and impetus is bisected, and the same is also true in the latter sense, for the oblique plane.

2. And in both cases all angles, equally above and below, that which gives the maximum range, will give equal ranges, as may be readily deduced from the preceding formulæ.

3. All things being the same, the range varies, as v^2, or as the square of the velocity; as does also the greatest height of the projectile.

4. The time of flight varies as the velocity, or as the square root of the impetus.

PROJECTION, in *Mechanics*, the art of giving a body its projectile motion.

PROJECTION of the *Sphere in Plano*, is a representation of the several points or places of the surface of the sphere, and of the circles described upon it, upon a transparent plane placed between the eye and the sphere, or such as they appear to the eye placed at a given distance.

The principal use of the projection of the sphere is in the construction of planispheres, maps, and charts, which are said to be of this or that projection, according to the several situations of the eye, and the perspective plane, with regard to the meridians, parallels, and other points or places so represented.

The most usual projection of maps of the world, is that on the plane of the meridian, which exhibits a right sphere, the first meridian being the horizon. The next is that on the plane of the equator, which has the pole in the centre, and the meridians the radii of a circle, &c.

The projection of the sphere is usually divided into orthographic and stereographic; to which may be added gnomonic.

Orthographic PROJECTION, is that in which the surface of the sphere is drawn upon a plane cutting it in the middle; the eye being placed at an infinite distance vertically to one of the hemispheres.

Stereographic PROJECTION of the *Sphere*, is that in which the surface and circles of the sphere are drawn upon the plane of a great circle, the eye being in the pole of that circle.

Gnomonical PROJECTION of the *Sphere*, is that in which the surface of the sphere is drawn upon an external plane, commonly touching it, the eye being at the centre of the sphere.

Stereographic Projection.

Prop. 1. Any circle passing through the projecting point, is projected into a right line. For, all lines drawn from the projecting point to this circle, pass through its intersection with the plane of projection, which is a right line. Hence, 1. A great circle passing through the poles of the primitive circle, is projected into a right line passing through the centre. 2. Any circle passing through the projecting point, is projected into a right line perpendicular to the line of measures, and distant from the centre, the semi-tangent of its nearest distance from the pole opposite to the projecting point.

Prop. 2. Every circle that does not pass through the projecting point, is projected into a circle.

Case 1. (Plate, projection of the sphere, fig. 1.) Let the circle E F be parallel to the primitive B D, lines drawn to all points of it, from the projecting point A, will form a conic surface, which, being cut parallel to the base, by the plane B D, the section G H, into which E F is projected, will be a circle; by the conic sections.

Case 2. (fig. 2.) Let BH be the line of measures to the circle E F; draw F K parallel to B D, then will the arc A K equal A F, and therefore the angle A F K or A H G, is equal to A E F; wherefore, in the triangles A E F and A G H, the angles at E and H are equal, and the angle A common; therefore, the angles at F and G are equal. Whence the cone of rays A E F, the base of which E F is a circle, is cut by a sub-contrary section, by the plane of projection B D; and, therefore, by the conic sections, the section G H, which is the projection of the circle E F, will also be a circle. Hence

When A F is equal to A G, the circle E F is projected into a circle equal to itself. For then the similar triangles A H G and A E F will also be equal, and G H equal to E F.

Prop. 3. Any point on the surface of the sphere is projected into a point, distant from the centre, the semi-tangent of its distance from the pole opposite to the projecting point. Thus the point E, (fig. 2.) is projected into G, and F into H; and C G is the semi-tangent of E M, and C H of M F. Hence,

1. A great circular perpendicular to the primitive, is projected into a line of semi-tangents passing through the centre, and produced *ad infinitum.*—For M F is projected into C H its semi-tangent, and F M into the semi-tangent C G. 2. Any arc E M of a great circle, perpendicular to the primitive, is projected into its semi-tangent. Thus, E M is projected into G C.

3. Any arc EMF of a great circle is projected into the sum of its semi-tangents, of its greatest and least distances from the opposite pole M, if it lie on both sides of M; or the difference of the semi-tangents when all on one side.

Prop. 4. The angle made by two circles on the surface of the sphere, is equal to that made by their representations upon the plane of projection.

Let the angle BPK, (fig. 3.) be projected; through the angular point P and the centre C draw the plane of a great circle PED, perpendicular to the plane of projection EFG. Let a plane PHG touch the sphere in P; then, since the circle EPD is perpendicular both to this plane and to the plane of projection, it is perpendicular to their intersection GH. But the angles made by circles are the same as those made by their tangents; therefore, in the plane PGH, draw the tangents PH, PF, PG, to the arcs PB, PD, PK, and they will be projected into the lines pH, pF, pK; then the $< CpA$ will $= < HpG$. For the angle CPF is a right angle $= CpA + CAP$; therefore, taking away the equal angles CPA and CAP, and $< pPE = CpA$ or PpF; consequently, $pF = PF$. Therefore, in the right-angled triangles PFG and pFG there are two sides equal, and the included angle is a right angle; therefore, the hypothenuse $PG = pG$; and, for the same reason, in the right-angled triangles PFH and pFH, $PH = pH$. Lastly, in the triangles PHG and pHG, all the sides are respectively equal; and, therefore $< P = < p$. Hence,

The angle made by two circles on the sphere, is equal to the angle made by the radii of their projection at the point of intersection. For the angle made by two circles on a plane, is the same with that made by their radii drawn to the point of intersection.

Prop. 5. The centre of a lesser circle projected, perpendicular to the primitive, is in the line of measures, distant from the centre of the primitive, the secant of the lesser circle's distance from its own pole, and its radius is the tangent of that distance.

Let A, (fig. 4.) be the projecting point, EF the circle to be projected, GH the projected diameter. From the centre C and D draw CF, DF; and the triangles CFI, DFI, are right-angled at I; then $< IFC = < FCA = 2 < FEA$, or $2 < FEG = 2 < FHG = < FDG$. Therefore, $IFC + IFD = FDG + IFD$ equal a right angle; that is, CFD is a right-angle, and the line CD is the secant of BF, and the radius FD is the tangent of it. Hence, if these circles be actually described, it is plain the radius FD is a tangent to the primitive at F, where the lesser circle cuts it.

Prop. 6. The centre of projection of a great circle is in the line of measures, distant from the centre of the primitive the tangent of its inclination to the primitive; and its radius is the secant of its inclination.

Let A (fig. 5.) be the projecting point, EF the great circle, GH the projected diameter, D the centre, draw DA. The angle EAF being in a semi-circle is a right-angle. In the right-angled triangle GAH, AC is perpendicular to GH; therefore, $< GAC = AHC$ and their double, $ECB = ADC$, and their complements $ECI = CAD$. Therefore, CD is the tangent of ECI, and the radius AD is its secant.

Hence, 1. If the great oblique circle AGBH, (fig. 6.) be actually described upon the primitive AB, all great circles passing through G, will have the centre of their projection in the line RS, drawn through the centre D perpendicular to the line of measures IH. For, since all great circles cut one another at the distance of a semicircle, all circles passing through G must cut at the opposite point H; and therefore their centres must be in the line RDS. 2. Hence, also, if any oblique circle GLH be required to make any given angle with another circle BGAH, it will be projected the same way, with regard to GAH considered as a primitive, and RS its line of measures, as the circle BGA is on the primitive BIA, and line of measures ID; and, therefore, the tangent of the angle AGL, to the radius GD, set off from D to N, gives the centre of GL. For the < NGD will then be equal to AGL, by *Cor.* 2, *Prop.* 4; and, therefore, GLH is truly projected. N. B. Of all great circles in the projection, the primitive is the least; for the radius of any oblique great circle being the secant of the inclination, is greater than the radius of the primitive; as the secant is always greater than the primitive.

Prop. 7. The projected extremities of the diameter of any circle, (fig. 5.) are in the line of measures, distant from the centre of the primitive circle the semi-tangent of its nearest and greatest distances from the pole of projection opposite to the projecting point.

For the diameter of the circle EF, (fig. 7.) is projected into GH from the projecting point A; but GC is the semi-tangent of BE, and CH the semi-tangent of BF. Hence, the points where an inclined great circle cuts the line of measures, within and without the primitive, is distant from the centre of the primitive the tangent and co-tangent of half the complement of the inclination of the circle to the primitive. For CG is equal to the tangent of half EB, or of half the complement of IE the inclination, and because the < EAF is a right angle, CH is the co-tangent of GAC, or half EB.

Prop. 8. The projected poles of any circle are in the line of measures within and without the primitive, and distant from its centre the tangent and co-tangent of half the inclination to the primitive.

For the poles, P*p* (fig. 8.) of the circle EF, are projected into D and *d*, and CD is the tangent of CAD, or half BCP, that is, of half GCI, the inclination of the circle ICK parallel to EF. Likewise, C*d* is the tangent of CA*d*, or the co-tangent of CAD. Hence, 1. The pole of the primitive is its centre, and the pole of a right circle is in the primitive. 2. The projected centre of any circle is always between the projected pole nearest to it on the sphere and the centre of the primitive; and the projected centres of all circles lie between the projected poles. For the middle point of EF, or its centre, is projected into S; and all the points in P*p*, in which are all the centres, are projected into D*d*. 3. If P, (fig. 9.) be the projected centre of any circle EFG, any right line EG, FH passing through P will intercept equal arcs EF, GH. For, in any circle of the sphere, any two lines passing through the centre intercept equal arcs, and these are projected into right lines passing through the projected centre P, and therefore EF, GH, represent equal arcs.

Prop. 9. If EFGH, *efgh* (fig. 9.) represent two equal circles, whereof EFG is as far distant from its pole P, as *efg* is from the projecting point, any two right lines, *e*EP and *f*FP being drawn through P, will intercept equal arcs in the representations of these circles; if P falls within the circles on the same side; but on the contrary side if without; that is, EF = *ef*, and GH = *gh*. For, by the nature of the section of a sphere, any two circles passing through two given points, or poles, on the surface of the sphere, will intercept equal arcs of two other circles equi-distant from these poles. Therefore, the circles EFG and *efg* on the sphere, are equally cut by the planes of any two circles passing through the projecting point and the pole P on the sphere. But these circles, by *Prop.* 1, are projected into the right lines P*e* and P*f* passing through P; and the intercepted arcs representing equal arcs on the sphere, are therefore equal; that is, EF = *ef*, and GH = *gh*. Hence,

1. If a circle is projected into a right line EF (fig. 10.) perpendicular to the line of measures EG, and if from the centre C a circle *ef*P be described passing through its pole P, and P*f* be drawn, then the arc *ef* = EF; and, if any other circle be described, the vertex of which is P, the arc *ef* will always be equal to EF. 2. Hence, also, if from the pole of a great circle there be drawn two right lines, the intercepted arc of the projected great circle will be

equal to the intercepted arch of the primitive. 3. After the same manner, if there be two equal circles E F, *ef*, (fig. 11.) whereof one is as far from the pole P, as the other is from the pole of projection *e*, opposite to the projecting point. Then any circle drawn through the points P and C will intercept the equal arc EF $=$ *ef*, and G H $=$ *gh*, between it and the line of measures P C G. For this is true on the sphere, and their projections are the same. 4. If, from an angular point be drawn two right lines through the poles of its sides, the intercepted arc of the primitive will be equal to that angle.

Prop. 10. If Q H, (fig. 12.) N K, be two equal circles, whereof N K is as far from the projecting point, as Q H from its pole P, and if they be projected into the circles, of which the radii are M C or C L, and D F or F G, F being the centre of D G, and E the projected pole, then the pole E will be distant from their centres in proportion to the radii of the circles, that is, $CE : EF = CL : DF$ or FG.

For, since N K and M L (fig. 13) are parallel, and the arc $NI = PH$, therefore, $< FLI = NKI$, or $nKI = GIP$; therefore, the triangles I E L and I E G are similar; whence $EL : EI = EI : EG$. Again, the angle $EMI = KNI = PIQ$; and, therefore, the triangles I E M and I E D are similar, whence $EM : EI = EI : ED$. Therefore, $EI^2 = EL \times EG = EM \times ED$; consequently, $EM : EL = EG : ED$; and, by composition

$$\frac{EM + EL}{2} \cdot \frac{EM - EL}{2} = \frac{EG + ED}{2} \cdot \frac{EG - ED}{2};$$

that is, $CM : EC = FG : EF$. Hence, 1. If the circle K N be as far from the projecting point, as Q H is from either of its poles, and if E and O be its projected poles, then will $EL : EM = ED : EG = OD : OG$. 2. If F be the centre, and F D the radius of any circle Q H, and E and O the projected poles, then as $EF : DF = DF : FO$.

The above propositions embrace the principal parts of the theory, and from them the solution of the problems is easily deduced.

PROLATE, in *Geometry*, a term applied to a spheroid produced by the revolution of a semi-ellipsis about its transverse diameter; and is thus distinguished from an oblate sphere, which is produced by the revolution of the ellipse about its conjugate diameter.

PROPORTION, in *Mathematics*, is an equality or similitude of ratio; thus if the ratio a to b is the same as that of c to d; that is, if $\frac{a}{b} = \frac{c}{d}$, then a, b, c, d, are in proportion, which is denoted by placing the quantities thus; $a : b = c : d$, and is read as *a is to be, so is c to d.*

Euclid, in the 5th definition of his 5th book, gives a general definition of four proportionals, or when, of four terms, the first has the same ratio to the 2d, as the 3d has to the 4th, viz. When any equimultiples whatever of the first and third being taken, and any equimultiples whatever of the 2d and 4th; if the multiple of the first be less than that of the 2d, the multiple of the 3d is also less than that of the 4th; or if the multiple of the 1st be equal to that of the 2d, the multiple of the 3d is also equal to that of the 4th; or if the multiple of the first be greater than that of the 2d, the multiple of the 3d is also greater than that of the 4th. And this definition is general for all kinds of magnitudes or quantities whatever.

Also, in the 7th book, Euclid gives another definition of proportionals, viz. when the first is the same equimultiple of the 2d, as the 3d is of the 4th, or the same part or parts of it. But this definition appertains only to numbers and commensurable quantities. See *Ratio.*

Proportion, though sometimes confounded with ratio, differs from it in this, that ratio has only a relation to two quantities of the same kind, whereas proportion relates to the comparison of two such ratios.

Proportion differs also from progression, in this, that in the former it is only required that there should

[illegible]

[illegible] &c.

[illegible]

direct	[illegible]
inversion	[illegible]
alternate	[illegible]
composition	[illegible]
conversion	[illegible]
division	[illegible]

Proportion is also distinguished into arithmetical, geometrical, and harmonical.

Arithmetical Proportion, is the equality of two arithmetical ratios or differences. As in the numbers [illegible], where the difference between [illegible] is the same as the difference between [illegible]. And here the sum of the extreme terms is equal to the sum of the means, or to double the single mean, when there is but one. As [illegible].

Geometrical Proportion, is the equality between two geometrical ratios, or between the quotients of the terms. See the preceding article.

Harmonical Proportion, is when the first term is to the third as the difference between the 1st and 2d is to the difference between the 2d and 3d; or in four terms, when the 1st is to the 4th as the difference between the 1st and 2d is to the difference between the 3d and 4th; or the reciprocals of an arithmetical proportion are in harmonical proportion. As 6, 4, 3; because $6 : 3 = 6 - 4 = 2 : 4 - 3 = 1$; or because $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$, are in arithmetical proportion, making $\frac{1}{6} + \frac{1}{3} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$. Also the four 24, 16, 12, 9, are in harmonical proportion, be $24 : 9 = 8 : 3$.

of Proportion, a name the French, and some ors, call the sector.

PROPORTIONAL, relating to proportion, as proportional *compasses*, *parts*, *scales*, *spiral*, &c. for which see the respective terms.

Proportional also denotes one of the terms of a proportion, which receives particular denominations according to the place it holds in the proportion, as a *mean* proportional, a *third*, *fourth*, &c. proportional.

Mean Proportional, is the middle term of three continued geometrical proportionals. See Mean *Proportional*.

Fourth Proportional, is the fourth term of a geometrical proportion, which is found arithmetically by dividing the product of the second and third terms by the first.

Third Proportional, is the third of three terms in continued proportion, and is found arithmetically by dividing the square of the second term by the first.

To cut a Line in extreme and mean Proportion; that is, so that the whole line may be to the greater part, as the greater part is to the less.

Algebraically,—Put the line $= a$, and the segments $= x$ and $a - x$.

$$\text{Then } \frac{x^2}{a-x} = a, \text{ or}$$

$$x^2 = a^2 - ax, \text{ or}$$

$$x^2 + ax = a^2$$

$$\text{Comp. sq. } x^2 + ax + \frac{a^2}{4} = \frac{5a^2}{4}$$

$$\therefore x + \frac{a}{2} = \sqrt{\frac{5a^2}{4}} = \sqrt{5}\,\frac{a}{2}$$

$$x = \sqrt{5}\,\frac{a}{2} - \frac{a}{2}$$

But $\sqrt{5} = 2.236$ nearly, therefore the mean segment $= \frac{a}{2} \times 1.236 = a \times .618$ nearly.

PROPOSITION, in *Mathematics*, is either some truth advanced, which is to be demonstrated, or some operation proposed which is to be performed and shown to be that which was required. Being, in the former case, called a theorem, and in the latter a problem.

PROTRACTOR, is the name of an instrument used for protracting or laying down on paper the angles

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be an equality between the ratio of the 1st and 2d term, and that of the 3d and 4th, whereas to constitute a progression there must be the same ratio between each two adjacent terms; these two cases, however, are sometimes distinguished by the terms *discrete* and *continued* proportion.

PROPORTION, is also *direct* and *inverse*, or *reciprocal*, *alternate*, &c.

Thus if the ratio of a to b is equal to the ratio c to d, then,

direct	$a : b = c : d$
inversion	$b : a = d : c$
alternate	$a : c = b : d$
composition	$a + b : b = c + d : d$
conversion	$a + b : a = c + d : c$
division	$\begin{cases} a - b : a = c - d : c \\ a - b : b = c - d : d \end{cases}$

PROPORTION, is again distinguished into arithmetical, geometrical, and harmonical.

Arithmetical PROPORTION, is the equality of two arithmetical ratios or differences. As in the numbers 12, 9, 6; where the difference between 12 and 9 is the same as the difference between 9 and 6, viz. 3.

And here the sum of the extreme terms is equal to the sum of the means, or to double the single mean when there is but one. As $12 + 6 = 9 + 9 = 18$.

Geometrical PROPORTION, is the equality between two geometrical ratios, or between the quotients of the terms. See the preceding article.

Harmonical PROPORTION, is when the first term is to the third as the difference between the 1st and 2d is to the difference between the 2d and 3d; or in four terms, when the 1st is to the 4th as the difference between the 1st and 2d is to the difference between the 3d and 4th; or the reciprocals of an arithmetical proportion are in harmonical proportion. As 6, 4, 3; because $6 : 3 = 6 - 4 = 2 : 4 - 3 = 1$; or because $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$, are in arithmetical proportion, making $\frac{1}{6} + \frac{1}{3} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$. Also the four 24, 16, 12, 9, are in harmonical proportion, because $24 : 9 = 8 : 3$.

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Algebraically,—Put the line $= a$, and the segments $= x$ and $a - x$.

Then $\frac{x^2}{a - x} = a$, or

$x^2 = a^2 - a x$, or

$x^2 + a x = a^2$

Comp. sq. $x^2 + a x + \frac{a^2}{4} = \frac{5 a^2}{4}$

$$\sqrt{\cdot}\ x + \frac{a}{2} = \sqrt{\frac{5 a^2}{4}} = \sqrt{5}\,\frac{a}{2}$$

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PROTRACTOR, is the name of an instrument used for protracting or laying down on paper the angles

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P

P

of a field or other figure. The protractor is a small semicircle of brass, or other solid matter, the limb or circumference of which is nicely divided into one hundred and eighty degrees; it serves not only to draw angles on paper, or any plane, but also to examine the extent of those already laid down. For this last purpose, let the small point in the centre of the protractor be placed above the angular point, and make the side coincide with one of the sides that contain the angle proposed; then the number of degrees cut off by the other side, computing on the protractor, will show the quantity of the angle that was to be measured.

PULLEY, in *Mechanics*, one of the mechanical powers, consisting of a small wheel having a groove around it, and turning on an axis; and hence by means of a rope it is employed to raise weights, or to draw them in any direction. The following are the principal properties of the pulley.

1. In the single fixed pulley, there is an equilibrium when the power and weight are equal.

Let a power and weight P, W, (Plate pulley, fig. 1.) equal to each other, act by means of a perfectly flexible rope P D W, which passes over the fixed pulley A D B; then, whatever force is exerted at D in the direction D A P, by the power, an equal force is exerted by the weight in the direction D B W; these forces will therefore keep each other at rest. Hence, 1. Conversely, when there is an equilibrium, the power and weight are equal. 2. The proposition is true in whatever direction the power is applied; the only alteration made, by changing its direction, is in the pressure upon the centre of motion.

2. In the single moveable pulley, whose strings are parallel, the power is to the weight as 1 to 2.

A string fixed at E. (fig. 23) passes under the moveable pulley A, and over the fixed pulley B; the weight is annexed to the centre of the pulley A, and the power is applied at P. Then, since the strings E A, B A are in the direction in which the weight acts, they exactly sustain it; and they are equally stretched in every point, therefore they sustain it equally between them; or each sustains half the weight. Also, whatever weight A B sustains, P sustains; therefore P : W = 1 : 2.

3. In general, in the single moveable pulley, the power is to the weight as radius to twice the cosine of the angle, which either string makes with the direction in which the weight acts. Let A W, (fig. 3) be the direction in which the weight acts; produce B D till it meets A W in C, from A draw A D at right angles to A C, meeting B C in D; then, if C D be taken to represent the power at P, or the power which acts in the direction D B, C A will represent that part of it which is effective in sustaining the weight, and A D will be counteracted by an equal and opposite force, arising from the tension of the string C E; also, the two strings are equally effective in sustaining the weight; therefore, 2 A C will represent the whole weight sustained; consequently, P : W = C D : 2 A C = rad. : 2 cos. D C A. Hence, 1. If the figure be inverted, and E and B be considered as a power and weight which sustain each other upon the fixed pulley A, W is the pressure upon the centre of motion; consequently, the power : the pressure = radius : 2 cos. D C A. 2. When the strings are parallel, the angle D C A vanishes, and its co-sine becomes the radius; in this case, the power : the pressure = 1 : 2.

4. In a system where the same string passes round any number of pulleys, and the parts of it between the pulleys are parallel, P : W = 1 : the number of strings at the lower block. Figures 4 and 5.

Since the parallel parts, or strings at the lower block, are in the direction in which the weight acts, they exactly support the whole weight; also, the tension in every point of these strings is the same, otherwise the system would not be at rest, and consequently each of them sustains an equal weight; whence it follows, that, if there be n strings, each sustains $\frac{1}{n}$th part of the weight; therefore, P sustains $\frac{1}{n}$th part of the weight, or $P : W = \frac{1}{n} = 1 = 1 : n$. Hence, If two systems of this kind be combined, in which there are m and n strings, respectively, at the lower blocks, $P : W = 1 : mn$.

5. In a system where each pulley hangs by a separate string, and the strings are parallel, P : W = 1 : that power of 2 whose index is the number of moveable pulleys. In this system, a string passes over the fixed pulley A, (fig. 6) and under the moveable pulley B, and is fixed at E; another string is fixed at B, passes under the moveable pulley C, and is fixed at F; &c. in such a manner that the strings are parallel.

Then, when there is an equilibrium, P :
the weight at B = 1 : 2
the weight at B :
the weight at C = 1 : 2
the weight at C :
the weight at D = 1 : 2
&c.

Comp. $P : W = 1 : 2 \times 2 \times 2 \times$, &c. continued to as many factors as there are moveable pulleys; that is, when there are n such pulleys, $P : W = 1 : 2^n$. Hence, 1. The power and weight are wholly sustained at A, E, F, G, &c. which points sustain respectively, 2 P, P, 2 P, 4 P, &c. 2. When the strings are not parallel, P : W = rad. : 2 cos. of the angle which the string makes with the direction in which the weight acts, in each case.

6. In a system of n pulleys, each hanging by a separate string, where the strings are attached to the weight, as is represented in (fig. 7) $P : W = 1 : 2^n - 1$.

A string, fixed to the weight at F, passes over the pulley C, and is again fixed to the pulley B; another string, fixed at E, passes over the pulley B, and is fixed to the pulley A; &c. in such a manner that the strings are parallel.

Then, if P be the power, the weight sustained by the string D A is P; also the pressure downwards upon A, or the weight which the string A B sustains, is 2 P; therefore the string E B sustains 2 P; &c. and the whole weight sustained is P + 2 P + 4 P +, &c. Hence $P : W = 1 : 1 + 2 + 4 +$, &c. to n terms $= 1 : 2^n - 1$. Hence, 1. Both the power and the weight are sustained at H. 2. When the strings are not parallel, the power in each case is to the corresponding pressure upon the centre of the pulley = rad. : 2 cos. of the angle made by the string with the direction in which the weight acts (*art.* 189). Also, by the resolution of forces, the power in each case, or pressure upon the former pulley, is to the weight it sustains = rad. : cos. of the angle made by the string with the direction in which the weight acts.

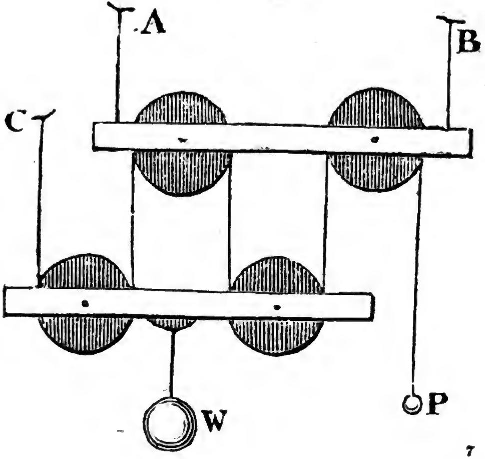
PUMP, an hydraulic engine used for raising water, sometimes by means of the pressure of the atmosphere, sometimes by forcing, and at others by a combination of both.

Machines for raising water, it is pretty certain, must have been employed from a very early date; but the pump, as now constructed by the moderns, was not known to the ancient Greeks or Romans, nor has any thing approximating towards it been discovered in any of the remote parts which the enterprising spirit of Europeans has of late years explored on the continents of Asia, America, &c. Vitruvius ascribes the invention of the pump to Ctesibius of Athens, or as some say of Alexandria, about 120 years before Christ; but the pump of this Grecian is still very different from those now in common use, which may be divided into three kinds, *viz.* the *forcing pump*, the *lifting pump*, and the *sucking pump*, though the two latter are not indeed essentially different from each other.

Forcing Pump. This consists of a hollow cylinder, called the working barrel, open at both ends, and having a valve at the bottom, opening upwards. This cylinder is filled by a solid piston, covered externally with leather or tow, by which means it fits the box of the cylinder exactly, and allows no water to escape by its sides. There is a pipe, which communicates laterally with this cylinder, and

Pulley.

5



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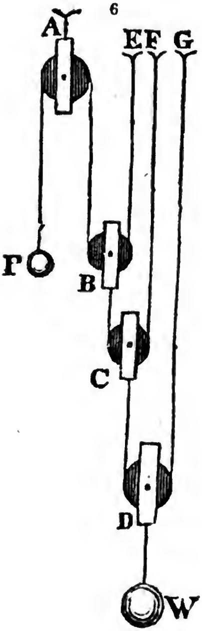

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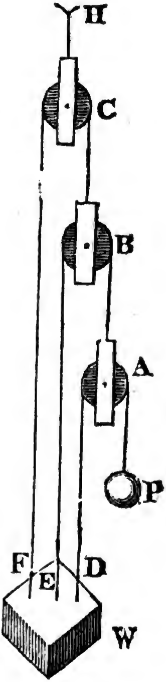


has a valve at some convenient place, as near as possible to its junction with the cylinder. This valve also opens upwards. This pipe, usually called the rising pipe or main, terminates at the place where the water must be delivered.

Now suppose this apparatus set into the water, so that the upper end of the cylinder may be under or even with the surface of the water; the water will open the valve, and after filling the barrel and lateral pipe, will also open the valve in the lateral pipe, and at last stand at an equal height within and without. Now let the piston be put in at the top of the working barrel, and thrust down. It will push the water before it. This will shut the valve at the bottom, and the water will make its way through the other valve, and fill a part of the rising pipe, equal to the internal capacity of the working barrel. When this downward motion of the piston ceases, the side valve will fall down by its own weight and shut this passage. Now let the piston be drawn up again: the side valve hinders the water in the rising pipe from returning into the working barrel. But now the bottom valve is opened by the pressure of the external water, and the water enters and fills the cylinder as the piston rises. When the piston has got to the top, let it be thrust down again: the bottom valve will again be shut, and the water will be forced through the passage at the side valve, and rise along the main, pushing before it the water already there. Repeating this operation, the water must at last arrive at the top, however remote.

Lifting Pump. The simplest form and situation of the lifting pump has one valve in the plunger, and another fixed in the working barrel; and both of these open upwards. The pump is immersed in the cistern till both the valve and piston are under the surface of the surrounding water. By this means the water enters the pump, opening both valves, and finally stands on a level within and without.

Now draw up the piston to the surface. It must lift up the water which is above it, because the valve in the piston remains shut by its own weight. In the mean time, the pressure of the surrounding water forces it into the working barrel, through the fixed valve; and the barrel is now filled with water. Now, let the piston be pushed down again; the fixed valve immediately shuts by its own weight, and in opposition to the endeavours which the water in the barrel makes to escape this way. This attempt to compress the water in the barrel causes it to open the valve in the piston: or rather, this valve yields to our endeavour to push the piston down through the water in the working barrel. By this means we get the piston to the bottom of the barrel; and it has now above it the whole pillar of water reaching in the pump. Drawing up the piston to the surface a second time, must lift this double column along with it, and its surface now will be higher. By this means the water brought up by the successive strokes of the piston rises to such a height in this cistern, as to produce an efflux by the spout nearly equable.

Sucking Pump. This does not differ essentially from the lifting pump above described, except that in the former the piston is supposed to work in the water of the reservoir, which is in many cases very inconvenient on account of the length of the rod necessary for such a construction. In the sucking pump therefore the piston is situated considerably above the surface, and in most cases in the very body of the pump, if this is not more than 32 or 33 feet above the surface of the water in the well.

In this construction, suppose the piston to be forced down; then the elasticity of air will shut the fixed valve and open that of the piston, and a certain quantity of air will escape. Now, let the piston be drawn up, and the valve in it will be shut, and the air in the barrel will expand itself so as to occupy the whole space; but its elasticity being thus diminished, the pressure of the external atmosphere on the surface of the water in the well will open the fixed valve, and force into the barrel a quantity of water sufficient to establish an equilibrium, between the internal and external air. Let us suppose the water thus admitted rises to any height, then repeating the stroke of the piston, another portion of air will be excluded, and on raising it again, another portion of water will be admitted into the barrel; and thus, by repeated strokes, it will finally arrive at the piston, if this do not exceed 32 or 33 feet, after which the operation will be precisely the same as in the former case. As to the limit of 33 feet, it follows necessarily, from what has been observed under the articles of AIR and ATMOSPHERE, where it is shown that a column of water of 33 feet is about equal to a column of atmosphere of equal base; and hence it follows, that this construction can only be made use of when the distance between the surface of the water and the sucker is within the above limit.

PUNCHEON, a measure for liquors containing 84 gallons.

PURSUIT, *Curve* of, is one generated by the motion of a point which is always directed towards another point, also in motion along a right line, the velocity of the two points bearing any determinate ratio to each other.

Thus let A and B be two bodies, the one A moving along the line with any given velocity v; and the other B, moving with a velocity V, and in such a manner as to be always directed towards the body A, then is the curve thus described by B, the curve of chase or the curve of pursuit, the equations and properties of which are as follow;

Let the velocity of B be to the velocity of A as $n : 1$. Call also the perpendicular $BA = a$, $AD = x$, $B'D = y$, then the equation of the curve is

$$2x = -\frac{a^n y^{1-n}}{1-n} + \frac{a^{-n} y^{n+1}}{1+n} + \frac{2na}{1-n^2}$$

The line B A, or B′ A′, joining any two contemporaneous positions of A and B, is a tangent to the curve at that point.

If the velocity of B be greater than that of A, or n be less than unity, the two bodies will meet when the latter has moved over a distance expressed by $\frac{na}{1-n^2}$, because when y becomes $1 = 0$, $2x = \frac{2na}{1-n^2}$

But if the velocity of B be less than that of A, it is obvious that the former can never come up with the latter; but its nearest approach will be when $y = \left(\frac{n-1}{n+1}\right)^{\frac{1}{2n}} \times a$.

PYRAMID, in *Geometry*, is a solid having any plane figure for its base, and triangles for its sides, all terminating in one common point or vortex. If the base of the pyramid is a regular figure, the solid is called a regular pyramid, which then takes particular names according to the number of its sides, as triangular, square, pentagonal, &c. the same as the prism.

If the perpendicular demitted from its vertex falls on the centre of the base, the solid is called a *right* pyramid; but if not, it is *oblique*.

The principal properties of the pyramid may be stated as follows:

1. Every pyramid is one-third of a prism of equal base and altitude.
2. Pyramids of equal bases and altitudes are equal to each other, whether the figure of their bases be similar or dissimilar.
3. Any section of a pyramid parallel to its base will be similar to the base, and these areas will be to each other as the squares of their distances from the vertex.
4. Pyramids, when their bases are equal, are to each other as their altitudes, and when their altitudes are equal they are to each other as their bases; and when neither are equal, they are to each other in the compound ratio of their bases and altitudes.

To find the Solidity of a Pyramid. Multiply the area of the base by its perpendicular altitude, and one-third of the product will be the solidity.

To find the Surface of a Pyramid.

Multiply the perimeter of the base by the slant altitude of one of its faces, and half the product will be the surface. Or, find the area of one of its triangular faces, and multiply by the number of them, which is the same thing.

Frustrum of a Pyramid. Is tne solid formed by cutting off the upper part of a pyramid by a section parallel to its base.

To find the Solidity and Surface of a Frustrum of a Pyramid. Let A represent the area of the greater end, *a* that of the less, and *h* its height or altitude; also let S and *s* represent the corresponding sides of the two ends, and *p* the tabular number, answering to the particular figure of the base; then,

1. Solidity $= (A + \sqrt{Aa} + a \times \frac{1}{3} h$

2. Solidity $= (S^2 + Ss + s^2) + \frac{1}{3} p h$

3. Surface $= (S + s) \times nb$

where *n* is the number of sides, and *b* the slant height of the frustrum.

PYRAMIDAL *Numbers*, are the same as are otherwise called figurate numbers.

PYRAMIDOID, a solid resembling the pyramid.

PYROMETER, an instrument for measuring the expansion of bodies by heat. It is a chemical, not a mechanical instrument.

Q.

QUADRANGLE, a figure having four angles, and consequently four sides; it is otherwise called a quadrilateral.

QUADRANT, in *Geometry*, is the 4th part of a circle, being bounded by two radii perpendicular to each other, and a quarter of the circumference, or 90°.

QUADRANT, is also a name common to several mathematical instruments, used for measuring altitudes and angular distances, which are commonly distinguished from each other, either by the names of their authors, or the purposes they are intended to answer. Thus we have *Adams's, Cole's, Davis's, Gunter's, Hadley's*, &c. QUADRANTS; as also the *Astronomical, Surveying*, &c. QUADRANTS.

This is the most simple form of the quadrant, under which it is necessarily subjected to very considerable inaccuracies, to remedy which a variety of forms have been given to it by different authors; but that of Hadley's is so decidedly superior to any other of a portable kind, that we shall limit our remarks principally to the description and use of this instrument; and a slight account of the astronomical and Gunter quadrants.

The *Astronomical* QUADRANT, is a large one, usually made of brass or iron bars; having its limb nicely divided, either diagonally or otherwise, into degrees, minutes, and seconds, if room will permit, and furnished either with two pair of plain sights or two telescopes, one on the side of the quadrant, and the other moveable about the centre by means of the screw. Dented wheels serve to direct the instrument to any object. The application of this useful instrument, in taking observations of the sun, planets, and fixed stars, is obvious; for being turned horizontally upon its axis, by means of the telescope, till the object is seen through the moveable telescope, then the degrees, &c. cut by the index, give the altitude, &c. required.

Gunter's QUADRANT, so called from its inventor, Edmund Gunter, beside the apparatus of other quadrants, has a stereographic projection of the sphere on the plane of the equinoctial; and also a calendar of the months, next to the divisions of the limb; by which, beside the common purposes of other quadrants, several useful questions in astronomy, &c. are easily reduced.

Hadley's QUADRANT, is thus named after its inventor, John Hadley, Esq. and is now universally used as by far the best of any for nautical and other observations, where a portable instrument is required.

It seems the first idea of this excellent instrument was suggested by Dr. Hook; for Dr. Sprat, in his History of the Royal Society, p. 246, mentions the invention of a new instrument for taking angles by reflection, by which means the eye at once sees the two objects both as touching the same point, though distant almost a semi-circle; which is of great use for making exact observations at sea.

This instrument is described and illustrated by a figure in Hook's Posthumous Works, p. 502. But, as it admitted of only one reflection, it would not answer the purpose. The matter, however, was at last effected by Sir Isaac Newton, who communicated to Dr. Halley a paper of his own writing, containing a description of an instrument with two reflections, which soon after the doctor's death was found among his papers by Mr. Jones, by whom it was communicated to the Royal Society, and it was published in the Phil. Trans. for the year 1742. See also Abridg. vol. viii. p. 129. How it happened that Dr. Halley never mentioned this in his lifetime is difficult to say; more especially as Mr. Hadley had described his instrument in the Trans. for 1731, which is constructed on the same principles. See also Abr. vol. vi. p. 139. Mr. Hadley, who was well acquainted with Sir Isaac Newton, might have heard him say, that Dr. Hook's proposal could be effected by means of a double reflection; and perhaps in consequence of this hint, he might apply himself, without any previous knowledge of what Newton had actually done, to the construction of this instrument. Mr. Godfrey too, of Pennsylvania, had recourse to a similar expedient; for which reason some gentlemen of that colony have ascribed the invention of this excellent instrument to him. The truth may probably be, that each of these gentlemen discovered the method independent of one another. See Abr. Phil. Trans. vol. viii. p. 366; also Trans. of the American Society, vol. i. p. 21; Appendix, Hutton's Dictionary.

This instrument consists of the following particulars;—1. An octant, or the eighth part of a circle. 2. An index. 3. The speculum. 4. Two horizontal glasses. 5. Two screens, and 6. Two sight-vanes.

The octant consists of two radii, strengthened by the braces, and the arch; which, though containing only 45°, is nevertheless divided into 90 primary divisions, each of which stands for a degree, and are numbered 0, 10, 20, 30, &c. to 90; beginning at each end of the arch for the convenience of numbering both ways, either for altitudes or zenith distances: also each degree is subdivided into minutes, by means of a vernier. But the number of these divisions varies with the size of the instrument.

The index is a flat bar, moveable about the centre of the instrument; and that part of it which slides over the graduated arch is open in the middle, with a Vernier scale on the lower part of it, under which is a screw serving to fasten it, perpendicular to the plane of the instrument, the middle part of the former coinciding with the centre of the latter; and because the speculum is fixed to the index, the position of it will be altered by moving the index along the arch. The image of an observed object is received on the speculum, and from thence reflected on one of the horizon-glasses; which are two small pieces of looking-glass placed on one of the limbs, their faces being turned obliquely to the speculum, from which they receive the reflected images of objects. One of these glasses has only its lower part silvered, and set in brass-work; the upper part being left transparent to view the horizon. The other glass has in its middle a transparent slit, through which the horizon is to be seen. And because the warping of the materials, and other accidents, may distend them from their true situation, there are three screws passing through their feet, by which they may be easily replaced.

The screens are two pieces of coloured glass, set in two square brass frames, and which serve to take off the glare of the sun's rays, which would otherwise be too strong for the eye: the one is tinged much deeper than the other; and as they both move on the same centre; both or either of them may be used.

The sight-vanes are two pins, standing perpendicularly to the plane of the instrument. One has one hole in it, opposite to the transparent slit in the horizon-glass; the other has two holes in it, the one opposite to the middle of the transparent part of the other horizon-glass, and the other rather lower than the quicksilvered part: this vane has a piece of brass on the back of it, which moves round a centre, and serves to cover either of the holes.

To rectify the Quadrant. For the Fore Observation.—Bring the index close to the bottom, so that the middle of the Vernier's scale stand against 0 degrees. Hold the plane of the instrument vertical with the arch downwards; look through the right-hand hole in the vane, and direct the sight through the transparent part of the horizon-glass, to observe the horizon. If the horizon seen both at the quicksilvered part, and through the transparent part coincide, or make one straight line, then the glass is adjusted; but if one of the horizon-lines should stand above the other, slacken the screw in the middle of the lever, and move the lever backwards or forwards, as there may be occasion, until the lines coincide; then fasten the screw, and all is ready for use.

To take the Sun's Altitude.—Fix the screens above the horizon-glass, using either or both of them, according to the strength of the sun's rays, by turning one or both the frames of those glasses close against the plane or face of the instrument; then your face being turned towards the sun, hold the quadrant by the braces, or by either radius, as is found most convenient, so as to be in a vertical position, with the arch downwards. Put the eye close to the right-hand hole in the vane, look at the horizon through the transparent part of the horizon-glass, at the same time sliding the index with the left hand, until the image of the sun, seen in the quicksilvered part, falls in with the edge of the horizon, taking either the upper or the under edge of the solar image. Swing the instrument gently from side to side; and when the edge of the sun is observed not to cut but to touch the horizon line, like a tangent, the observation is made. Then will the degrees on the arch, reckoning from the end next your body, give the altitude of that edge of the sun which was brought to the horizon. If the lower edge was observed, then sixteen minutes, added to the said degrees, gives the altitude of the sun's centre; but, if the upper edge was used, the sixteen minutes must be subtracted.

To take the Altitude of a Star.—Look directly up at the star, through the vane and transparent part of the glass, the index being close to the button; then will the image of the star, by reflection, be seen in the silvered part, right against the star seen through the other part. Move the index forward, and as the image descends, let the quadrant descend also, to keep it in the silvered part, till it comes down in a line with the horizon, seen through the transparent part, and the observation is made.

To rectify for the Back Observation. Slacken the screw in the middle of the handle, behind that glass which has the half transparent; turn the button on one side, and bring the index as many degrees before 0 as is equal to double the dip of the horizon at your height above the water; hold the instrument vertical, with the arch downwards; look through the hole of the single-hole vane; and if the horizon, seen through the transparent slit in the glass, coincide with the image of the horizon seen in the silvered part of the same glass, then the glass is in its proper position; but if not, set it by the handle, and fasten the screw as before.

To take the Sun's Altitude by the Back Observation.—Put the stem of the screens into the hole at the single hole vane, and in proportion to the strength or faintness of the sun's rays, let either one, or both, or neither of the frames of those glasses, be turned close to the face of the limb; hold the instrument in a vertical position with the arch downward, by the braces, with the left hand; turn your back to the sun, and put one eye close to the hole in the vane, observing the horizon through the transparent slit in the horizon-glass; with the right hand move the index, till the reflected image of the sun be seen in the silvered part of the glass, and in a right line with the horizon; swing the instrument to and fro, and if the observation be well made, the sun's image will be observed to brush the horizon, and the degrees reckoned from that part of the arch farthest from your body, will give the sun's altitude at the time of observation; observing to add 16′ for the sun's semidiameter, if the sun's upper edge be used, and subtract the same for the lower edge.

Two corrections are necessary to be made before the altitude can be accurately determined, viz. one on account of the observer's eye being raised above the level of the sea, and the other on account of the refraction of the atmosphere, especially in small altitudes.

The following tables, therefore, show the corrections to be made on both these accounts; the first referring to the dip of the horizon, and the other to the refraction in altitude.

Dip of the Horizon.

Height of the Eye in feet.	Dip.	Height of the Eye in feet.	Dip.	Height of the Eye in feet.	Dip.
1	0′ 57″	12	3′ 18″	35	5′ 29″
2	1 21	14	3 24	40	6 2
3	1 39	16	3 47	45	6 24
4	1 55	18	4 3	50	6 41
5	2 8	20	4 16	60	7 23
6	2 20	22	4 28	70	7 29
7	2 31	24	4 40	80	8 32
8	2 42	26	4 52	90	9 3
9	2 52	28	5 3	100	9 33
10	3 1	30	5 14		

Refraction in Altitude.

App. Alt. in deg.	Refraction.	App. Alt. in deg.	Refraction.	App. Alt. in deg.	Refraction.
0	33′ 0″	8	6′ 29″	40	1′ 8″
¼	30 35	9	5 48	45	0 57
½	28 22	10	5 15	50	0 48
1	24 29	11	4 47	55	0 40
2	18 35	12	4 23	60	0 33
3	14 36	15	3 30	65	0 26
4	11 51	20	2 35	70	0 21
5	9 54	25	2 2	75	0 15
6	8 29	30	1 33	80	0 10
7	7 20	35	1 21	85	0 5

1. In the fore observations, add he sum of both corrections to the bserved zenith distance, for the rue zenith distance; or subtract the said sum from the observed altitude, for the true one. 2. In the back observation, add the dip and subtract the refraction, for alti-

tudes; and for zenith distances, the contrary, *viz.* subtract the dip, and add the refraction.

Exam. By a back observation, the altitude of the sun's lower edge was found by Hadley's quadrant to be 25° 12′; the eye being 30 feet above the horizon. By the tables, the dip on 30 feet is 5′ 14″, and the refraction on 25° 12′ is 2′ 1″. Hence

Apparent alt. lower limb	25°	12′	0″
Sun's semi-diameter, sub.	0	16	0
Appar. alt. of centre	24°	56	0
Dip of horizon, add	0	5	14
	25	1	14
Refraction subtract	0	2	1
True alt. of centre	24	59	13

In case of the moon, besides the two corrections above, another is to be made for her parallax. But for all these particulars there are requisite tables in the Nautical Almanack; and also in the common books on Navigation.

Quadrant of *Altitude*, is a thin piece of metal, in general applied to a globe, and marked with the degrees, from 0 to 90: when fixed on the zenith, it shows either the altitude or zenith distance of any point on the globe. It is also used as a measure of distances generally.

QUADRANTAL *Triangle*, a spherical triangle, having a quadrant or an arc of 90° for one of its sides.

QUADRATIC *Equation*, in *Algebra*, is an equation, in which the highest power of the unknown quantity is of the second degree. If this power enters alone it is called a *simple quadratic*, and when the second power and simple quantity both occur, it is termed an *adfected quadratic equation*.

The solution of a simple quadratic, is performed by the same rules as are given under simple equations. And the solution of the other class of quadratics depends upon the following principles.

Every quadratic equation when properly reduced, falls under one or other of the following forms, *viz.*:

1st $x^2 + a x - b = 0$
2d $x^2 - a x - b = 0$
3d $x^2 - a x + b = 0$

And the roots of these will be expressed by the following formulæ:

$$\text{1st } x = -\frac{a}{2} \pm \sqrt{\left(\frac{a^2}{4} + b\right)}$$

$$\text{2d } x = \frac{a}{2} \pm \sqrt{\left(\frac{a^2}{4} + b\right)}$$

$$\text{3d } x = \frac{a}{2} \pm \sqrt{\left(\frac{a^2}{4} - b\right)}$$

In each of which formulæ x has two values, arising out of the ambiguous sign ±, which is agreeable to what is stated under equations; *viz.* that every equation has as many roots as there are units in its highest power, which in the present case is two.

These formulæ may be all included in one rule, as follows; *viz.* having reduced an equation to either of the above forms, transpose the absolute quantity b, to the right-hand side of the equation, then the values of the unknown quantity are equal to half the co-efficient of the second term with its sign changed; plus and minus; the square root of the square of that half co-efficient, prefixed to b, with its proper sign whether plus or minus.

It is evident in form. 3, that when b is greater than $\frac{a^2}{4}$ both the values of x are imaginary, or impossible.

The principles on which the preceding formulæ are obtained are these, that $(x \pm \frac{1}{2} a)^2 = x^2 \pm a x + \frac{1}{4} a^2$; therefore, if an equation be proposed under the form $x^2 \pm a x = \pm b$; by adding $\frac{1}{4} a^2$, to both sides, we have

$$x^2 \pm a x + \tfrac{1}{4} a^2 = \tfrac{1}{4} a^2 \pm b$$

By extracting the roots

$$x \pm \tfrac{1}{2} a \pm \sqrt{(\tfrac{1}{4} a^2 \pm b)}$$

By transposition

$$x = \pm \tfrac{1}{2} a \pm \sqrt{(\tfrac{1}{4} a \pm b}$$

which includes all the foregoing forms.

Quadratic *Equations*, may also be solved by means of a table of sines and tangents.

QUADRATRIX, or *Quadratix*,

is a name given to different mechanical or transcendental curves, of these the most remarkable is the quadratrix of Dinostrates. It is generated as follows:

Suppose that a straight line A G (Plate *Quadrature*, fig. 1.) a tangent at A, moves uniformly parallel to itself along the diameter A a, and that at the same instant of its setting out from the point A, the radius A C turns uniformly about the centre C towards the point E, so that it coincides with C E at the moment in which the right line A G also coincides with it; by the continual intersection of these two lines we shall have a curve A M D, called the *quadratrix*.

From this description it follows that any space A P passed over by the right line A G is to the circular arc A B described in the same time by the extremity of the radius, as any other space A C passed over by this right line is to the corresponding arc A B E, described by the radius. Call A P = x, P M = y, A P = u, A C = a, A B E = 90° = c, we shall have 1°. $x : a = u : c$ = angle A C B : angle A C E, therefore $u = \frac{cx}{a}$. 2dly. C P : P M = C A : A G, or $a \quad x : y = a :$ tang. u, therefore $y = \frac{a - x}{a}$ tang. $\frac{cx}{a}$; and this will be the equation of the co-ordinates of the quadratrix when the point A is the origin of the abcissæ.

But if we place their origin at the centre C, making C P = x, we shall have $y = \frac{x}{a}$ tang. $(c - \frac{cx}{a} =$

$$\frac{x}{a} \cot \frac{cx}{a} = \cfrac{x \left(a - \frac{c^2 x^2}{2a^3} + \frac{c^4 x^4}{2.3.4.\,a^7} - \&c.\right.}{\cfrac{a - \frac{c^2 x^2}{2a^3} + \frac{c^4 x^4}{2.3.4\,a^7} - \&c.}{\cfrac{\frac{cx}{a} - \frac{c^3 x^3}{2.3.\,a^5} + \frac{c^5 x^5}{2.3.4\,5.a^9} - \&c.}{\frac{c}{a} - \frac{c^2 x^2}{2.3.\,a^5} + \frac{c^5 x^4}{2.3.4.5.\,a^9} - \&c.}}}$$

therefore when $x = o$, y which will become the *base* C D, will have for its expression $\frac{a^2}{c}$; consequently, if the base of the quadratrix were known, we should immediately have the quadrature of the circle; it is this circumstance which gave the name of quadratrix to this curve.

If from the centre C and with the radius C D we describe the quadrant D L K, its length will be equal to the radius C A for $\frac{aa}{c}$: D L K = $a : c$, therefore D L K = a. We shall also have P C = the arc L D; for $\frac{aa}{c}$: K L = $a : u$; therefore K L = x = A P, and P C = L D.

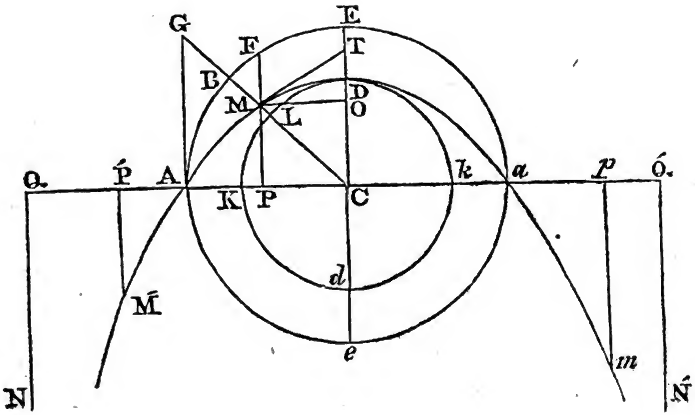
Let us now take negative abscissæ A P′ and substitute their value in the first equation. It will become $y = -\frac{(a + x)}{a}$ tang $\frac{cx}{a}$ which gives the negative ordinates P′ M′. Therefore the curve has a branch A M′, to which we shall find that a right line Q N drawn at the distance A Q = a is the asymptote, by supposing y infinite; for then we shall find that tang $\frac{cx}{a} = \infty$, and consequently that $x = a$.

If, after having coincided with C E, the right line A G, and the radius C A continue to move, the one descending towards a, the other revolving in the same direction; it is visible that their intersection will describe the part Da of the quadratrix.

It is likewise evident, that if this curve could be described geometrically we should immediately be able to assign an angle of any number of degrees, for example of $\frac{l}{m}$ 90°. For this purpose, we need only divide A C at the point P, so that A P should be to A C = 1 : m; for then drawing the ordinate P M, and the radius C B, the angle A C B would = $\frac{1}{m}$ 90° since $x : a = u : c = 1 : m$.

QUADRATURE, in *Astronomy*, that aspect of the moon when she

Quadratrix.



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ASTOR, LENOX AND
TILDEN FOUNDATIONS.

is a quadrant, or 90° distant from the sun.

Quadrature, in *Geometry*, is the finding a square equal in area to another figure, or in other words, finding the areas of plane surfaces.

This, so far as it relates to rectilinear figures, is extremely simple, as these may all be reduced to triangles; the sum of the area of which will be the whole area required.

But the quadrature of curves is a subject of much more profound investigation, of which the ancients had very little knowledge; this being a branch of geometry which we owe almost entirely to the moderns, and particularly to the invention of the fluxional or differential calculus.

The first curvilinear space, whose quadrature was correctly ascertained, was the lune of Hippocrates, after which we hear of no other till Archimedes found the area of the parabola; which is the only curve whose quadrature he was able to find, though he showed the relation of different curves surfaces to each other, particularly of his spirals to their circumscribed circles, as he did the ratio of the surface of a sphere to the area of a great circle of the same, &c. He also, by the method of exhaustions, gave some neat approximations, both for the circle and some other figures, but the general quadrature of curves required a much more profound analysis than any that was known to the ancients.

The quadrature of the circle, in particular, is a problem which has engaged the attention of mathematicians from the earliest period to the present time, though it is now generally considered to be impossible, and is, perhaps, but seldom attempted, except by novices who are not sufficiently acquainted with the difficulty and the almost infinite number of ways in which it has been attempted, of which an entertaining and instructive account is given by Montucla in vol. iv. of his "Histoire des Mathematiques," and in his edition of the "Mathematical Recreations," by Ozanam, of which an English translation has been given by Dr. Hutton; to which work the reader is referred, as our limits will not admit of any detail of them in this place.

It does not appear that the quadrature of any curve, beside those mentioned above, had been found so late as the beginning of the 17th century, but soon after we have accounts of the partial quadrature of several, viz. first by Lord Brouncker and Sir Christopher Wren, and afterwards by a few other mathematicians. In 1668, Newton discovered a method which would apply generally to all quadrable curves, though he did not then publish it, and the first that appeared in any analytical form was given by Mercator in 1688, which was a demonstration of Brouncker's quadrature of the hyperbola by Dr. Wallis's method of converting algebraical fractions into infinite series.

Huygens and Wren both discovered the quadrature of certain cycloidal spaces, and had some controversy concerning the priority of the method. Leibnitz afterwards succeeded in finding the quadrature of another curve, and J. Bernouilli, in 1699, discovered the quadrature of an infinity of cycloidal spaces, both circles and sectors. But all these partial methods were now superseded by the important discovery of the method of fluxions, which rendered every problem of this kind extremely simple and complete, at least in those cases where any finite quadrature was obtainable; and in others they were expressed with nearly the same ease, though not so simply, by means of infinite series.

Quadrature by *Fluxions.* Let AM (fig. 2,) represent any curve, and let its axis be AP; and suppose PM an ordinate to the point M; to find the quadrature of the space AMP, draw another ordinate mp, and the line Mr parallel to Pp; we shall then have the surface of the space $MmpP = MP \times Pp + Mmr$. Conceive now that the point m approaches towards the point M, it is obvious that the triangle Mmr will diminish more and more, but it will not become zero till the point m falls upon the point M; then M m p P will become the fluxion of the space A M P; P p will be $\dot{x}$, and we shall have *flux* (A M P) $= y\dot{x}$, and consequently AMP $= \int y\dot{x} + C$, which by Art. 74

$$= C + xy - \frac{xx\dot{y}}{2\dot{x}} \quad \frac{x^3\ddot{y}}{2.3.\dot{x}^2}$$

$$\frac{x^4\dddot{y}}{2.3.4\dot{x}^3} + \&c.$$

Therefore, also the space A Q M

$$= \int x\dot{y} = C + xy - \frac{y^2\dot{x}}{2\dot{y}} +$$

$$\frac{y^3\ddot{x}}{2.3.y^2} - \&c.$$

Ex. 1. Let the circular arc QMB, (fig. 3,) be a quadrant, described from the centre A, and with the radius a; we shall have $y = \sqrt{aa - xx}$, and the space A Q M P $= \int \dot{x}\sqrt{aa - xx} + C = C + ax -$

$$\frac{x^3}{2.3a} - \frac{1x^5}{2.4.5^3} - \frac{1.3x^7}{2.4.6.7a^5} \cdots \frac{1.3.5}{2.4.6.8} \cdot \frac{x^9}{9a^7} - \frac{1.3.5.7}{2.4.6.8.10} \cdot$$

$$\frac{x^{11}}{11a^9} - \&c.$$

Make $x = o$, and we shall have A Q M P $= o$, and consequently $C = o$.

Therefore A Q M P $= ax - \frac{1}{2} \cdot$

$$\frac{x^3}{3a} - \frac{1}{2.4}\frac{x^5}{5a^3}\frac{1.3}{2.4.6}\frac{x^7}{7a^5} - \&c.$$

Ex. 2. In the ellipse $y = \frac{b}{a}\sqrt{(aa - xx)}$. Therefore $\int y\dot{x} = \frac{b}{a}$

$$\left(ax - \frac{1}{2} \cdot \frac{x^3}{3a} - \frac{1}{2.4} \cdot \frac{x^5}{5a^3} - \&c.\right)$$

Ex. 3. In the parabola $y\dot{x} = p^{\frac{1}{2}} x^{\frac{1}{2}} \dot{x}$, and $\int y\dot{x} = \frac{2}{3} p^{\frac{1}{2}} x^{\frac{3}{2}} = \frac{2}{3} xy$, or the space A P M (fig. 4,) is two-thirds of the circumscribed rectangle. The equation which comprehends parabolas of all degrees is $y^m = x^n a^{m-n}$; therefore $m\,l\,y = n\,l\,x + (m-n)\,l\,a$, consequently

$$\frac{m\dot{y}}{y} = \frac{n\dot{x}}{x},$$

or $m : n = y\dot{x} : x\dot{y} = \int y\dot{x} : \int \dot{x}y =$ A M P : A M Q.

From which it follows that the space A M P is to the circumscribed rectangle A P M Q $= m : m + n$.

Ex. 4. In the equilateral hyperbola (fig. 5,) $xy = aa$, and $y\dot{x} = \frac{aa\dot{x}}{x}$. Consequently $\int y\dot{x} = aa\,l\,x + C$. If we wish to compute the spaces from the vertex A, when $x = o$, the space $= o$. Therefore C $= -aa\,l\,o$, and the space Q′APMN $= aa\,l\,x - aa\,l\,x - aa\,l\,o = \infty$.

If $x =$ A D, then Q′ A D B N $= aa\,l\,a - aa\,l\,o$; consequently B D P M $= aa\,l\,x - aa\,l\,a = aa\,l\,\frac{x}{a}$

The following is the doctrine of Quadrature according to the differential method. Let s be the area of a curved plane A P M B (plate vi. fig. 1); of the abscissa A P $= x$ and A P $= x + h$, the area s is required.

The area A B M′ P $= s + \frac{ds}{dx}h + \frac{d^2s\,h^2}{dx^2\,2} + \&c.$; and hence we shall have the mixtilinear area P M M′P′ $= \frac{ds}{dx}h + \frac{d^2s\,h^2}{dx^2\,2} + \&c.$; that area lies between the two rectangles P M′ and P′ M, hence it is easy to have the analytical expressions,

P M′ = P′ M′ × P P′ $= f(x+h)h$; and

P′ M = P M × P P′ $= fx.h$.

the ratio of these rectangles is $\frac{f(x+h)\,h}{fx.h} = \frac{f(x+h)}{fx}$; or in the case of the limit, the ratio is reduced to $\frac{fx}{fx} = 1$.

But the mixtilinear surface P M M′ P lies between the two rectangles, differing less from the rectangle P′ M than from P′ M′; hence, if in the case of the limit $\frac{\text{P M}'}{\text{P}'\text{ M}} = 1$, much more will 1 be the limit of the ratio,

$$\frac{\text{area}\quad P\,M\,M'\,P'}{\text{rectangle}\quad P'\,M}$$

substituting for these terms their analytical expressions, we shall have

$$\frac{\frac{ds}{dx}h+\frac{d^2s\,h^2}{dx^2\,2}+x}{fx\,.\,h}=\frac{\frac{ds}{dx}+\frac{d^2s\,h}{dx^2\,2}}{fx}$$

Passing to the limit and making $h=o$, we shall find $\frac{ds}{dx\,.fx}=1$, or $ds=fx\,.\,dx$; and by putting for fx its value, we shall have $ds=y\,dx$.

We can also determine the differential of the area of a curve, by the method of infinitesimals in the following manner (fig. 2).

The trapezium $P\,M\,M'\,P'=\frac{P\,M+P\,M'}{2}\times P\,P=\frac{y+(y+dy)}{2}\times dx=y\,dx+\frac{dx\,dy}{2}$; rejecting $dx\,dy$ as an infinitesimal of the second degree, there remains $y\,dx$ for the differential.

For the first application, required the area of a portion B M P (fig. 3) of a parabola. Let $y^2=mx$ be the equation of that parabola, of which the origin is B; we find, by differentiating that $2\,y\,dy=m\,dx$; hence $dx=\frac{2\,y}{m}\,dy$; and consequently $y\,dx=\frac{2\,y^2}{m}\,dy$. Integrating we have $\int\frac{2\,y^2}{m}\,dy=\frac{2\,y^3}{3\,m}+C$.

For determining the constant quantity C, observe that when $y=o$, the integral which expresses the surface sought is o at the same time. That hypothesis reduces the equation to $o=o+C$; hence $\int y\,dx=\frac{2\,y^3}{3\,m}=\frac{2\,y}{3\,m}\times y^2\,\frac{2\,y}{3\,m}\times mx=\frac{2}{3}\,x\,y$.

We have now some important observations to make on the determining of the constant quantity; for which we shall resolve the same problem, and take the parabola whose equation is $y^2=m+nx$.

The origin of the abscissæ here is not at the vertex of the curve; for on making $y=o$, the equation gives $x=-\frac{m}{n}$; and as that abscissa terminates at the point B (fig. 4), or $y=o$, we carry $\frac{m}{n}$ from B to A, and the point A is the origin. That being done, and proceeding as above, we find $2\,y\,dy=n\,dx$; hence $y\,dx=\frac{2\,y^2}{n}\,dy$ and $\int y\,dx=\frac{2\,y^3}{3\,n}+C$.

For determining the constant quantity, observe that the surface A D M P, which here represents the integral, becomes o when the ordinate M P coincides with A D; now, A D being the ordinate which passes through the origin where the abscissa $x=o$, the equation we have given, upon that hypothesis y or $D\,A=\sqrt{m}$. Then making $s\,y\,d\,x=o$, and $y=\sqrt{m}$, these values reduce the equation to $o=\frac{2\,m^{\frac{3}{2}}}{3\,n}+C$; from which we find $C=-\frac{2\,m^{\frac{3}{2}}}{3\,n}$; and consequently the integral sought is $\int y\,dx=\frac{2y^3}{3\,n}-\frac{2\,m^{\frac{3}{2}}}{3\,2}=$ the area A D M P.

In the above, we have drawn into the equation of the curve the value of dx, by substituting in the formula $y\,dx$ and integrating afterwards. We shall now proceed differently, and place in that equation the value of y rather than that of dx; because, for obtaining the integral, it is sufficient that the proposed differential does not contain more than one variable quantity; therefore, we may adopt the substitution which requires the least calculation.

An integral, such as $\int fx\,.\,dx$ can always represent the area of a curve of which the equation shall be $y=fx$. In fact, that equation being given, if we substitute the value of y in the formula $\int y\,dx$, we shall have $\int fx\,.\,dx$ for the surface of that curve. This is, therefore, a problem which we can always reduce to the integration of one function of a single variable quantity; and we may say that quadratures are reduced to this problem.

Suppose X to be a function of x, and that upon integrating $X\,dx$ we have obtained $\int X\,dx = Fx + C$. That integral, in which the constant quantity C is not yet determined, is a general indefinite integral, or more simply an indefinite integral, and it is completed when we have determined the constant quantity.

If, by a hypothesis, we determine that constant quantity; if we suppose, for instance, that $\int X\,dx$ vanishes when $x = a$, the equation gives, in that case, $o = Fa + C$; hence, $C = -Fa$, and the equation becomes $\int X\,dx = Fx - Fa$. That integral $Fx - Fa$ is then a particular integral; and in that the number of particular integrals of a differential expression is unlimited, seeing that we can establish an infinite number of hypotheses for the constant quantity.

When we make the supposition that the integral is o and $x = a$; that is, when we take (fig. 5,) an abscissa $AB = a$, the surface is comprised between the limit BD and the indefinite limit MP, which corresponds to $AP = x$, then the operation by which we determine a particular integral is the same as that which fixes the position of the limit BD, since we reckon the integral from that. The second limit shall be invariable, if we give to x a determinate value b; then the particular integral $\int y\,dx = Fx - Fa$, whence

$$\int y\,dx = Fb - Fa,$$

and the surface BDMP shall be no more arbitrary. In this case the integral gets the name of a definite integral, and we say it is taken from $x = a$ to $x = b$, seeking now the definite integral of $x^m\,dx$, we are then understood to know two values, a and b, which satisfy the indefinite integral $\frac{x^{m+1}}{m+1} + C$. Suppose that the first corresponds to $\int y\,dx = o$, we shall have $\frac{a^{m+1}}{m+1} + C = o$, and the particular integral will be $\int x^m\,dx = \frac{x^{m+1}}{m+1} - \frac{a^{m+1}}{m+1}$. We afterwards take $x = b$, and we shall have for the particular integral $\int x^m\,dx = \frac{b^{m+1}}{m+1} - \frac{a^{m+1}}{m+1}$. We arrive also at that integral, in making successively $x = a$ and $x = b$ in the indefinite integral, and we have $\frac{a^{m+1}}{m+1} + C$ and $\frac{b^{m+1}}{m+1} + C$; we afterwards diminish the first result by the second; but in taking that difference, we should always take care that the part subtracted is the value of the function of x at the origin of the integral.

For a third application, let us determine the area of a right-angled triangle ABC; the equation of the side AC being $y = ax$, and putting that value of y in the formula $y\,dx$, we obtain $ax\,dx$, hence $\int y\,dx = \int ax\,dx = \frac{ax^2}{2} + C$. The surface being nothing when $x = o$ the constant quantity $C = o$, hence the area $ABC = \frac{ax^2}{2} = \frac{x^2}{2} \times ax = \frac{xy}{2}$. If in the formula $y\,dx$ we put the value of y drawn from the equation of the circle, we find $\int dx\sqrt{a^2 - x^2}$ for the expression of the area of the circle. That integral has for its value,

$$\frac{x}{2}\sqrt{a^2 - x^2} + \tfrac{1}{2}a^2 \operatorname{arc}\left(\sin = \frac{x}{a}\right) + C$$

The part $\frac{1}{2}a^2 \operatorname{arc}\left(\sin = \frac{x}{a}\right)$ cannot be determined unless we suppose the ratio of the diameter and circumference to be known; and thus we see that the integration of $dx\sqrt{a^2 - x^2}$ does not lead to the solution of the problem of the quadrature of the circle. It is the same in the quadrature of the ellipse, which depends upon

$$\frac{b}{a}\int dx\sqrt{a^2 - x^2}.$$

From a comparison of the two

expressions we deduce this proportion,

area of the ellipse : area of the circle $= \frac{b}{a}\int dx\sqrt{a^2-x^2} : \int dx\sqrt{a^2-x^2}$; or ellipse : circle $= \frac{b}{a} : 1$, whence area of the ellipse $= \frac{b}{a} \times$ area of the circle $= \frac{b}{a}\pi a^2 = \pi a b$.

QUADRILATERAL, a figure of four sides, and therefore of four angles, under which general term is included the RECTANGLE, SQUARE, RHOMBUS, RHOMBOID, PARALLELOGRAM TRAPEZIUM, and TRAPEZOID.

All quadrilaterals have the following properties. The sum of their four angles is equal to two right angles; and if the sum of each pair of opposite angles be equal to two right angles, the figure may be inscribed in a circle, otherwise it cannot; and in all such quadrilaterals the sum of the rectangles of the opposite sides is equal to the rectangle of the two diagonals.

QUADRILLION, according to English arithmeticians, is the 4th power of a million; but according to the French only the square of a million, or the fourth power of 1000. See BILLION.

QUALITY, is defined by Mr. Locke, to be the power in a subject of producing any idea in the mind; thus a snow-ball having the power to produce in us the ideas of white, cold, and round, these powers, as they are in the snow-ball, he calls qualities; and as they are sensations, or perceptions in our understanding, he calls them ideas. It has been demonstrated that every quality that is propagated from a centre, such as light, heat, cold, odour, &c. has its intensity either increased or decreased, in the duplicate ratio of the distances from the centre inversely. So at the double distance from the earth's centre, or from a luminous or a hot body, the weight, or light, or heat, is but a fourth part, and, at three times the distance, is but a ninth.

QUANTITY, any thing capable of estimation or mensuration; or which, being compared with another thing of the same kind, may be said to be greater or less than it, equal or unequal to it. Mathematics is the science or doctrine of quantity, which being made up of parts is capable of being made greater or less. It is increased by addition, and diminished by subtraction; which are therefore the two primary operations that relate to quantity. Hence it is that any quantity may be supposed to enter into algebraic computations two different ways, which have contrary effects, *viz.* either as an increment or decrement.

A quantity which is to be added is called a positive quantity, and a quantity to be subtracted is said to be negative. Quantities are said to be like or similar, that are of the same denomination; they are represented by the same letter or letters equally repeated; but quantities of different denominations, or represented by a different letter or letters, are said to be unlike or dissimilar. A quantity consisting of more than one term is called a compound quantity; whereas that consisting of one term only is denominated a simple quantity.

The quantity of matter in any body is the product of its density into its bulk; or a quantity arising from the joint consideration of its magnitude and density; thus, if a body be twice as dense, and takes up twice as much space as another, it will be four times as great. This quantity of matter is best discoverable by the absolute weight of bodies.

The quantity of motion in any body is the factum of the velocity into the mass, or it is a measure arising from the joint consideration of the quantity of matter, and the velocity or the motion of the body, the motion of any whole being the sum or aggregate of the motion in all its several parts. Hence in a body twice as great as another moved with an equal velocity, the quantity of motion is double; if the velocity be double also, the quantity of motion will be quadruple. Hence the quantity of motion is the same with what we call momentum or impetus of a moving body.

QUART, an English measure of capacity, being the fourth part of a gallon.

QUARTER, the fourth part of a thing.

QUARTER, in *Weight*, is equal to 28lbs. or the fourth part of a hundred weight.

QUARTER, in *Measure*, is 8 bushels corn measure, and in coal measure it is the fourth of a chaldron.

QUARTER, in speaking of the *Moon's Age*, is a fourth part of one lunation.

QUARTER *Point*, in *Navigation*, is the fourth part of the measure of one of the principal points, or of the arc intercepted between two adjacent points.

QUOTIENT, the quantity which arises by dividing one number or quantity by another. See DIVISION.

R.

RADIANT *Point*, any point from which rays proceed.

RADICAL *Sign*, in *Algebra*, is the character by which the root of a quantity is expressed, and is formed thus $\surd$, while the particular root is indicated by a figure on the left of the sign: thus $\sqrt[2]{a}$, $\sqrt[3]{a}$, $\sqrt[4]{a}$, &c. denote the square root, cube root, and biquadratic root of the quantity a, or of any other quantity placed under the like signs. When it is a compound quantity whose root is to be expressed, it is put in a parenthesis, and the sign prefixed; thus, $\sqrt[3]{}\,(a^2 + b^2)$, means the cube root of the sum of a^2 *plus* b^2, or it is otherwise indicated by a line thus, $\sqrt[3]{a^2 + b^2}$; the characteristic ^{2} is generally omitted in the square root, so that instead of writing $\sqrt[2]{a}$ for the square root of a, we merely write $\sqrt{a}$, by which the square root is always to be understood.

RADII, the plural of RADIUS.

RADIUS, in *Geometry*, the semi-diameter of a circle, or a right line drawn from the centre to the circumference. It is implied in the definition of a circle, and it is apparent from its construction, that all the radii of the same circle are equal. The radius is sometimes called, in trigonometry, the sinus totus, or whole sine. The length of the radius of any circle is equal to that of an arc of 57·2957795 degrees of the same circle.

RADIUS, in the *Higher Geometry*. *Radius* of the *Evoluta*, *Radius Osculi*, called also the *Radius* of *Concavity*, and the *Radius* of *Curvature*, is the radius of a circle having the same curvature, in a given point of the curve, with that of the curve in that point.

RADIUS *Vector*, is used for a right line drawn from the centre of force in any curve in which a body is supposed to move by a centripetal force, to that point of the curve where the body is supposed to be. In the elliptical orbit of a planet let $a =$ semi-axis major, $a\,e =$ distance from the centre to the focus, or $e =$ eccentricity to semi-axis major $= 1$, $v =$ true anomaly, and $u =$ eccentric anomaly, then the radius vector r is expressed by either of the following formulæ $r = a\ (1 + e \cos.\ u)$, or $r = \dfrac{a\,(1 - e^2)}{1 - e \cos.\ v}$.

RADIX, the same as root, but used in a different sense by different authors; thus we say *radix* of a system of logarithms, a system of notation, &c. meaning the fundamental quantity on which the system is constructed, or by which all the others are compared.

RADIX of a *System* of *Logarithms*, is that number which, involved to the power, denoted by the logarithm of a number, is equal to that number. Thus, under the article logarithm, we have shown, if $r^x = a$, that x is the logarithm of a, and r is called the radix of the system. This radix in the Common or Briggs's Logarithms is 10, and in the Naperian or Hyperbolic Logarithms, it is 2.71828182, &c. and generally the radix in any system of logarithms, is that number whose logarithm in that system is unity.

Radix of a *System* of *Notation*, is that number which indicates the local value of the figures, and is in all systems represented by a unit and cipher (10), which is *ten* in the common system, *two* in the binary, *three* in the ternary, &c.

Radix is also used as a term of comparison between any finite function and its expansion or developement; thus we know that

$$\frac{1}{r+r}=1r+r^2-r^3+r^4-\&c.$$

in which case $\frac{1}{1+r}$ is sometimes called the radix of the series

$$1-r+r^2-r^3+r^4-\&c.$$

RAIN, water that descends from the atmosphere in the form of drops of greater or less magnitude.

The quantity of rain which have fallen in different places have been accurately observed, and from which it appears that much depends upon local situation. The quantity of rain which fell at Paris in the course of a year, taken at a medium of six years, was 20·19 inches; and in London the medium quantity per annum, for the same number of years, was 23·001. Much, however, depends upon the height of the rain-gage from the surface of the earth, more than upon the comparative altitudes of it with reference to the surface of the sea, or any fixed point; the rain-gage on the top of a mountain giving nearly as much as that in the plain beneath; whereas, one gage placed on the top of a house or church, and another below, give very different quantities.

RAINBOW. The rainbow is a circular image of the sun, variously coloured. It is thus produced: the solar rays, entering the drops of falling rain, are refracted to their farther surfaces, and thence, by one or more reflections, transmitted to the eye. At their emergence from the drop, as well as at their entrance, they suffer a refraction, by which the rays are eparated into their different colours, and thus, therefore, are exhibited to an eye properly placed to receive them. That this is the true account of the formation of the rainbow appears from the following considerations. 1. That a bow is never seen but when rain is falling, and the sun shining at the same time, and that the sun and bow are always in opposite quarters of the heavens; this every one's experience can testity. 2. That the same appearance can be artificially represented by means of water thrown into the air, when the spectator is placed in a proper position with his back turned to the sun.

Lunar **Rainbow.** The moon sometimes also exhibits the phenomenon of an iris by the refraction of her rays, in drops of rain in the night-time.

Marine **Rainbow**, the sea-bow, is a phenomenon sometimes observed in a much agitated sea, when wind, sweeping part of the tops of the waves, carry them aloft, so that the rays of the sun are refracted, &c, as in a common shower.

RANGE, in *Projectiles*, is the distance to which a body is projected.

RARE, in *Physics*, a relative term, the reverse of dense, being used to denote a considerable porocity, or vacuity, between the particles of a body; as the word dense implies a contiguity or closeness of the particles.

RAREFACTION, in *Physics*, is the making a body to expand or occupy more room or space without the accession of new matter. It is by rarefaction that gunpowder takes effect; and to the same principle also we owe oclipiles, thermometers, &c. The degree to which air is rarefiable exceeds all imagination; perhaps, indeed, its degree of expansion is absolutely beyond all limits. Upon the rarefaction of the air is founded the method of measuring altitudes by the barometer; in all cases of which the rarity of the air is found to be inversely as the force that compresses it, or inversely as the weight of all the air above it at any place.

The open air, in which we breathe, says Sir Isaac Newton, is 8 or 900 times lighter than water, and by consequence 8 or 900 times rarer. And since the air is compressed by the weight of the incumbent atmosphere, and the density of the air is proportionable to the compressing force, it follows, by computation, that at the height of about seven English miles from the earth, the air is four times rarer than at the surface of the earth; and at the height of 14 miles, it is 16 times rarer than at the surface of the earth; and at the height of 21, 28, or 35 miles, it is respectively 64, 256, or 1024 times rarer, or thereabouts; and at the height of 70, or 140, and 210 miles, it is about 1,000,000, 1,000,000,000,000, or 1,000,000,000,000,000,000, &c.

Mr. Cotes has found, from experiments made with a thermometer, that linseed oil is rarefied in the proportion of 40 to 39 with the heat of the human body; in that of to 15 to 14, with that degree of heat wherein water is made to boil; in the proportion of 15 to 13 in that degree of heat wherein melted tin begins to harden; and, finally, in the proportion of 23 to 20, in that degree wherein melted tin arrives at a perfect solidity. The same author discovered that the rarefaction of the air, in the same degree of heat, is ten times greater than that of linseed oil; and the rarefaction of the oil about fifteen times greater than that of the spirit of wine.

RARITY, lightness, thinness, the reverse of density.

RATIO, is the relation of two quantities of the same kind with respect to quantity, and is by some authors divided into arithmetical and geometrical ratio: *viz.* arithmetical when the term is used with respect to the difference of the two quantities, and geometrical when it relates to the number of times in which the one of those quantities is contained in the other; thus the ratio of 6 to 3 is $\frac{6}{3} = 2$. The leading term of the ratio being called the *antecedent*, and the latter the *consequent;* also the quotient or division of the former by the latter, is called the index or *exponent* of the ratio. The equality of ratios constitute proportion. See PROPORTION.

RATIO is also distinguished by some authors, (principally of the old school), into a variety of denominations, many of which are totally useless; but which cannot notwithstanding be passed over in the present article.

Irrational RATIO, is when one of the terms of the ratio, at least, is an irrational quantity, such is the ratio of $\sqrt{3}$ to 1.

Rational RATIO, is when there is no irrational quantity enters, or when the same irrational quantity enters into both terms, thus $\sqrt{6}$ to $\sqrt{24} = \sqrt{6}$ to $2\sqrt{6} = 1$ to 2.

RATIO of *Equality*, is when the terms expressing the ratio are equal, and therefore the exponent $= 1$.

RATIO of *Greater Inequality*, is when the antecedent exceeds the consequent; and *ratio of less inequality*, is when the latter exceeds the former.

For several other distinctions, as *Compound Duplicate, Subduplicate, Triplicate, Subtriplicate,* &c. see the several articles.

Reduction of RATIOS, is the reducing them to less terms, the ratio of 36 to 24 = 6 to 4 = 3 to 2. Sometimes, however, when the terms of the ratio are very large it is difficult to reduce them in this manner, and then recourse must be had to the method of finding the greatest common measure or common divisor, which also fails if the two terms are prime to each other.

In this case, though the exact ratio cannot be found in less terms, it is frequently desirable to find an approximate ratio expressed in less terms, and recourse must then be had to continued fractions; *viz.* convert the given ratio into a continued fraction, and thence find the series of converging fractions, each of which will be an approximate ratio of the proposed one, and so much the more accurate as it is farther advanced in the series. See CONTINUED *Fractions.*

Let 1103 to 887 be a ratio, to

which it is required to find an approximate ratio in less terms.

887) 1103(1

216)887(4

23)216(9

9)23(2

5)9(1

4)5(1

1)4(4

Hence by the rule in continued fractions,

quoto. 1 4 9 2 1 1 4

app. ratios $\frac{1}{1}, \frac{5}{4}, \frac{46}{37}, \frac{97}{78}, \frac{143}{115}, \frac{240}{193}, \frac{1103}{887}$

Each of which fractions approximates nearer to the true ratio than any of those which precede it, and each nearer than any ratio that can be expressed in less terms.

RATIONAL *Fractions*, is the term commonly used to express those fractions which may be decomposed into other fractions, the sum of which is equal to the given fraction; this cannot in all cases be effected, but when it can such fractions are called rational fractions.

In simple numerical fractions, the decomposition, when it can be effected, is drawn from the indeterminate analysis. Thus, let it be proposed to resolve the fraction $\frac{19}{35}$ into two other fractions. Since $35 = 7 . 5$, let the two required fractions be $\frac{x}{7}$ and $\frac{y}{5}$; then their sum $\frac{5x + 7y}{35} = \frac{19}{35}$; whence $5x + 7y = 19$, which is an indeterminate equation of the first degree, and the solution of it gives $x = 1$ and $y = 2$; therefore $\frac{19}{35} = \frac{1}{7} + \frac{2}{5}$

If the denominator is a prime number, the decomposition is impossible, as it is also in some other cases; but if x and y are prime factors of the denominator, and the numerator is greater than $xy - x - y$, then the decomposition is always possible.

If the denominator consists of three or more prime factors, then making it equal to $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} +$ &c. the decomposition may still be effected by means of an indeterminate equation of the first degree.

But the principal use of this decomposition is in the inverse method of fluxions, or the integral calculus, for which purpose they were first investigated by Leibnitz, and have been since much extended by the researches of Cotes, Euler, Simpson, Lagrange, &c. the former in particular has treated the subject at length, in his "Harmonia Mensurarum," and Euler has a very elegant chapter on this subject in his Analysis "Infinitorum," vol. i. to which works the reader is referred, as we can only give a slight sketch of this theory in the present articles.

Let

$$\frac{(x^m + ax^{m-1} + bx^{m-2} + \&c.)\,\dot{x}}{x^n + px^{n-1} + qx^{n-2} + \&c.}$$

be any proposed fluxion of which the fluent is required. This place is the same as

$$\frac{x^m\dot{x}}{x^n + px^{n-1} + qx^{n-2} + \&c.} + \frac{ax^{m-1}\dot{x}}{x^n + px^{n-1} + qx^{n-2} + \&c.} + \frac{bx^{m-2}\dot{x}}{x^n + px^{n-1} + qx^{n-2} + \&c.} +$$

And the whole fluent will be equal to the several fluents of these parts, each of which is included in the general form

$$\frac{1}{x^n + px^{n-1} + qx^{n-2} + \&c.} + \phi.x\mu.\dot{x}$$

and our object is now to decompose this first factor into its simple fractions.

Now the denominator of this fraction, from the known theory of equations, may be considered as made up of the factors $(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$, &c. $\alpha, \beta, \gamma, \delta$, being the roots of the equation arising from making the denominator equal to zero, and which thus become known.

Let us therefore suppose the above fraction to be equal to

$$\frac{A}{x-\alpha} + \frac{B}{x-\beta} + \frac{C}{x-\gamma} + \frac{D}{x-\delta} + \&c.$$

the reduction of which to a common denominator, and comparing the numerators, gives

$$\left.\begin{array}{l} A\,(x-\beta)\,(x-\gamma)\,(x-\delta),\&c. \\ +\,B\,(x-\alpha)\,(x-\gamma)\,(x-\delta),\&c. \\ +\,C\,(x-\alpha)\,(x-\beta)\,(x-\delta),\&c. \end{array}\right\}$$
$= 1.$

Or making successively $x=\alpha$, β, γ, δ, &c. (which will not alter the value, these being the roots of the equation) gives

$$A\,(\alpha-\beta)\,(\alpha-\gamma)\,(\alpha-\delta)\ \&c. = 1$$
$$B\,(\beta-\alpha)\,(\beta-\gamma)\,(\beta-\delta)\ \&c. = 1$$
$$C\,(\gamma-\alpha)\,(\gamma-\beta)\,(\gamma-\delta)\ \&c. = 1$$
&c. &c.

Or,

$$A = \frac{1}{(\alpha-\beta)\ (\alpha-\gamma)\ (\alpha-\delta)\ \&c.}$$
$$B = \frac{1}{(\beta-\alpha)\ (\beta-\gamma)\ (\beta-\delta)\ \&c.}$$
$$C = \frac{1}{(\gamma-\alpha)\ (\gamma-\beta)\ (\gamma-\delta)\ \&c.}$$

whence the values of A, B, C, &c. become known, and therefore the general fluxion

$$\frac{1}{x^n + px^{n-1} + qx^{n-2} + rx^{n-3} +}$$
$$\times \phi x \mu \dot{x}$$

is decomposed into the following particular ones

$$\frac{\phi x \mu \dot{x}}{x-\alpha} + \frac{\phi x \mu \dot{x}}{x-\beta} + \frac{\phi x \mu \dot{x}}{x-\gamma} + \&c.$$

the fluents of which are easily obtained. For

$$\frac{\phi x \mu \dot{x}}{x-\alpha} = \phi x \mu^{-1}\,\dot{x} + \phi \alpha x \mu^{-2}$$
$$\dot{x} + \&c.\ \frac{\phi \alpha \mu^{-1}\,\dot{x}}{x-\alpha}$$

the fluents of which become

$$\frac{\phi x \mu}{\mu-1} + \frac{\phi \alpha x \mu^{-1}}{\mu-2} + \&c. + \phi \alpha \mu_{-1}$$
hyp. log. $(x-\alpha)$

This method admits of considerable abridgment, but the limits of our article will not admit of farther explanation; and we must therefore refer the reader, who is desirous of information on this subject, to the "Analysis Infinitorum" of Euler, as above mentioned; and to Simpson's "Fluxions," vol. i.; and shall in this place merely give one example by way of illustration.

Let it be proposed, for example, to find the fluent of

$$\frac{\dot{x}}{x^3-6x^2+11\,x-6}$$

Here the equation $x^3-6x^2+11x-6=0$, gives $x=1, 2$, and 3; that is, $\alpha=1, \beta=2, \gamma=3$, and the fraction is therefore decomposed into

$$\frac{A\,\dot{x}}{x-\alpha} + \frac{B\,\dot{x}}{x-\beta} + \frac{C\,\dot{x}}{x-\gamma},\ \text{or}\ \frac{A\,\dot{x}}{x-1}$$
$$+\ \frac{B\,\dot{x}}{x-2} + \frac{C\,\dot{x}}{x-3}$$

where $A = \frac{1}{(1-2)\,(1-3)} = \frac{1}{2}$;

$$B = \frac{1}{(2-1)\,(2-3)} = -1;$$

$C = \frac{1}{(3-1)\,(3-2)} = \frac{1}{2}$; therefore

$$\frac{\dot{x}}{x^3-6x^2+11x-6} = \frac{\dot{x}}{2\,(x-1)}\ \frac{\dot{x}}{x-2}$$
$$+\ \frac{\dot{x}}{2\,(x-3)}$$

the fluents of which are $\frac{1}{2}$ hyp. log. $(x-1)$ $-$ hyp. log. $(x-2)$ $+\ \frac{1}{2}$ hyp. log. (-3), or fluent of

$$\frac{\dot{x}}{x^3-6x^2+11-6} = \text{hyp.}$$
$$\text{log.}\ \ \frac{\sqrt{(x^2-4x+3)}}{x-3}$$

RATIONAL *Quantities*, in *Algebra*, are those which are expressed without any radical signs, being equivalent to integers, or fractions, in arithmetic, which are called rational numbers, or quantities, in contradistinction to irrational or surd quantities. SEE SURDS.

RAY, in *Optics*, a beam of light propagated from a radiant point.

If the ray comes direct from the radiant point to the eye it is said to be *direct;* if it first strike upon any body, and is hence transmitted to the eye, it is said to be *reflected*, and the ray itself is called a *reflected ray;* and if the ray in its passage to the eye be bent or turned out of its direct course by passing through any medium, it is said to be *refracted*, and is thence called a *refracted ray*. When two or more rays proceed in directions parallel to each other, they are called *parallel rays*. If they converge towards each other, they are called *converging rays*, and if they diverge, *diverging rays;* and those which pass directly to the eye in any case are called *visual rays*.

Among other qualities of rays it has been found by experiment that there is a very great difference in the heating power of the different rays from the sun.

It appears from the experiments of Dr. Herschel, that the heating power increases from the middle of the spectrum to the red rays, and is greatest beyond it where the rays are invisible. Hence it is inferred that the rays of light and caloric nearly accompany each other, and that the latter are in different proportions in the different coloured rays; these are easily separated from each other, as when the sun's rays are transmitted through a transparent body, the rays of light pass on seemingly undiminished, but the rays of caloric are intercepted. When the sun's rays are directed to an opaque body, the rays of light are reflected, and the rays of caloric are absorbed and retained. This is the case with the light of the moon, which, however much it be concentrated, gives no indication of being accompanied with heat. It has also been shown that the different rays of light produce different chemical effects on the metallic salts and oxyds. These effects increase on the opposite direction of the spectrum, from the heating power of the rays. From the middle of the spectrum, towards the violet end, they become more powerful, and produce the greatest effect beyond the visible rays. From these discoveries it appears that the solar rays are of three kinds: 1. Rays which produce heat. 2. Rays which produce colour; and 3. Rays which deprive metallic substances of their oxygen. The first set of rays is in greatest abundance, or are most powerful towards the red end of the spectrum, and are least refracted. The second set, or those which illuminate objects, are most powerful in the middle of the spectrum. And the third set produce the greatest effect towards the violet end, where the rays are most refracted. The solar rays pass through transparent bodies without increasing their temperature. The atmosphere, for instance, receives no increase of temperature, by transmitting the sun's rays, till these rays are reflected from other bodies, or are communicated to it by bodies which have absorbed them. This is also proved by the sun's rays being transmitted through convex lenses, producing a high degree of temperature when they are concentrated, but giving no increase of temperature to the glass itself. By this method the heat which proceeds from the sun can be greatly increased. Indeed, the intensity of temperature produced in this way is equal to that of the hottest furnace. This is done either by reflecting the sun's rays from a concave polished mirror, or by concentrating or collecting them by the refractive power of convex lenses, and directing the rays thus concentrated on the combustible body.

RECIPROCAL, in *Arithmetic* and *Algebra*, is the quotient arising from dividing unity by any quantity; thus $1 \div \frac{x}{y} = \frac{y}{x}$ is the reciprocal of the fraction $\frac{x}{y}$.

Reciprocal *Equations*, are those which contain several pairs of roots, which are the reciprocal of each other. Thus an equation whose roots are $a, \frac{1}{a}$; $b, \frac{1}{b}$; $c, \frac{1}{c}$, &c. is called a reciprocal equation; the solution of which always depends upon the solution of an equation of half its dimension if it be even, or upon half the dimension *minus* 1, if it be odd.

Thus far, in fact, it differs in no respect from any other equation in which a similar relation is known to have place between its roots; but what is the most characteristic of these equations is, that they are known to be reciprocal as soon as they appear by the order and signs of their co-efficients; that is, the terms equally distant from either extreme have the same co-efficient, thus,

$$x^5 + 5x^4 + 7x^3 + 7x^2 + 5x + 1 = 0$$

is a reciprocal equation.

And therefore when an equation appears under this form, it is immediately known to be a reciprocal one, and may thence be depressed to an equation of half its dimension.

Let $x^{2n} + px^{2n-1} + qx^{2n-2} +$ &c. $qx^2 + px + 1 = 0$ be any reciprocal equation, whose roots are $a, \frac{1}{a}, b, \frac{1}{b}, c, \frac{1}{c}$, &c. Then from the theory of equations, we may consider this to be made up of the factors

$$(x-a)\left(x-\frac{1}{a}\right)(x-b)\left(x-\frac{1}{b}\right)$$

$$(x-c)\left(x-\frac{1}{c}\right) \text{ \&c.}$$

Or putting $a+\frac{1}{a}=m$, $b+\frac{1}{b}=n$, $c+\frac{1}{c}=r$, &c. these become

$(x^2+mx+1)\ (x^2+nx+1)\ (x^2+rx+1)$ &c.

If therefore we really perform this multiplication, and equate the co-efficients, it is obvious, since the multiplication is reduced to half the number of factors, the equation by which the values of m, n, r, &c. are obtained will be of only half the dimension of the original equation; and having found these, since

$$x^2 + mx + 1 = 0$$
$$x^2 + nx + 1 = 0$$
$$x^2 + rx + 1 = 0 \text{ \&c.}$$

we shall have

$$x = -\frac{m}{2} \pm \sqrt{\left(\frac{m^2}{4} - 1\right)}$$

$$x = -\frac{n}{2} \pm \sqrt{\left(\frac{n^2}{4} - 1\right)}$$

$$x = -\frac{r}{2} + \sqrt{\left(\frac{r^2}{4} - 1\right)}$$

&c. &c.

Thus, for example, let there be proposed the equation

$$x^4 + 5x^3 + 7x^2 + 5x + 1 = 0$$

multiply together

$$\begin{array}{l} \quad x^2 + mx + 1 \\ \text{and } \underline{x^2 + nx + 1} \end{array}$$

$$x^4 \left.\begin{array}{l} +m \\ +n \end{array}\right\} x^3 \left.\begin{array}{l} +mn \\ +\ 2 \end{array}\right\} x^2 \left.\begin{array}{l} +m \\ \ \ n \end{array}\right\} x+1$$

Comparing the co-efficients, we have $m+n=5$, and $mn+2=7$.

Hence

$$m = \frac{5+\sqrt{5}}{2}, \text{ and } n = \frac{5-\sqrt{5}}{2}$$

therefore

$$x = \frac{5+\sqrt{5}}{4} \pm \sqrt{\left(\frac{30-10\sqrt{5}}{16} - 1\right)}$$

and

$$x = \frac{5-\sqrt{5}}{4} \pm \sqrt{\left(\frac{30+10\sqrt{5}}{16} - 1\right)}$$

which are the four roots of the proposed equation.

If the equation be of odd dimensions, then either +1 or −1, is one of the roots which will depend upon the signs of the co-efficients, and may therefore be determined immediately from inspection, and the whole equation thus reduced to another one degree less, but still reciprocal; and which may then be reduced to half its new dimension according to the preceding rule. On this subject, see Bonnycastle's Algebra, vol. ii.

RECIPROCAL *Figures*, in *Geometry*, are such as have the antecedent and consequents of the same ratio in both figures.

RECIPROCAL *Proportion*, is when the reciprocal of the two last terms have the same ratio as the quantities of the first terms, or when the antecedents are compared with the reciprocals of the consequents, thus,

$$5 : 8 = 24 : 15$$

is a reciprocal proportion, because

$$5 : 8 = \tfrac{1}{24} : \tfrac{1}{15}, \text{ or}$$
$$5 : \tfrac{1}{24} = 8 : \tfrac{1}{15}.$$

RECIPROCAL *Ratio*, is the ratio of the reciprocals of two quantities.

RECIPROCALLY, the property of being reciprocal; thus we say, that in bodies of the same weight, the density is reciprocally as the magnitude; viz. the greater the magnitude the less is the density; and the less the magnitude, the greater the density. So again, the space being given, the velocity is reciprocally as the time.

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TILDEN FOUNDATIONS.

Fig. 1.

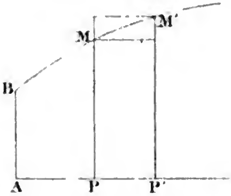


Fig. 2.

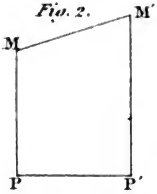


Fig. 3.

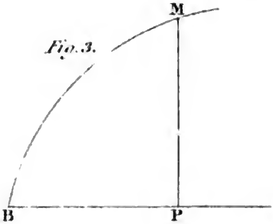


Fig. 4.

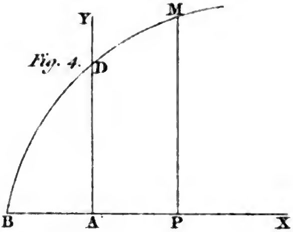


Fig. 5.

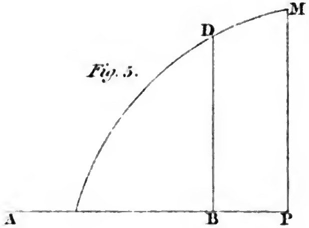


Fig. 6.

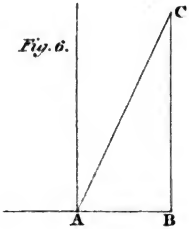


Fig. 7.

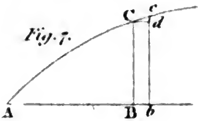


Fig. 9.

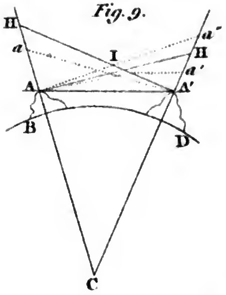
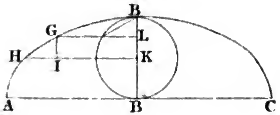


Fig. 8.



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RECIPROCITY. The law of reciprocity is a term employed by Legendre in his "Théorie des Nombres," to denote a reciprocal law that has place between prime numbers of different forms, which is this, that m and n being prime odd numbers,

the remainder of $m^{\frac{n-1}{2}}$ divided by $n =$
the remainder of $n^{\frac{m-1}{2}}$ divided by m.
if m and n are not both of the form $4x-1$.
and if they are both of this form, then
the remainder of $m^{\frac{n-1}{2}}$ divided by $n = -$
the remainder of $n^{\frac{m-1}{2}}$ divided by m
but with a contrary sign. See "Essai sur la Théorie des Nombres, part 3.

RECKONING, in *Navigation*, is that account whereby, at any time, the latitude and longitude of a ship becomes known, and hence the course he ought to steer to gain the desired port. This is sometimes made from observations and sometimes from the logboard, in which latter case it is called the dead reckoning.

RECOIL, or **REBOUND,** of a *Piece of Ordnance*, is that flying backwards which is always observed on the discharge taking place, and which arises from the exploded powder acting equally upon the ball and the gun, so that the momentum of the gun, with its carriage, is equal to the momentum of the ball, or rather to the momentum of the ball and half, and powder.

RECTANGLE, in *Geometry*, is a figure having all its angles right angles, being a particular species of parallelogram, and consequently possessing all the properties belonging to the latter figure; besides which, it has the following ones peculiar to itself, viz. If from any point in the plane of a rectangle, lines be drawn from any point either within or without, or in any of its sides to the four angles of the figure, the sum of the squares of two of those lines going to the opposite angles of the figure, is equal to the sum of the squares of the lines joining the other opposite angles.

To find the area of a rectangle, multiply its length by its breadth, and the product will be the area.

RECTANGULAR *Figures* and *Solids*, are those which have one or more right-angles. With regard to solids they are commonly said to be rectangular when their axis are perpendicular to the planes of their bases.

RECTANGULAR *Section* of a *Cone*, was a term used by the ancients, before Apollonius, for the parabolic section.

RECTIFICATION, in *Geometry*, is the finding of a right-line equal to a proposed curve; a problem that even in the present state of analysis is, in many cases, attended with some difficulty, and was in all totally beyond the reach of the ancient geometers, who were not able to assign the length of any curve line whatever, though they could, in a few cases, assign the area of a curvilinear space.

The first rectification of a curve was effected by Mr. Neal, as we are informed by Dr. Wallis, at the conclusion of his Treatise on the Cissoid. This was the semi-cubical parabola, and Neal's rectification of it was published in July or August, 1657, and two years after the same was done by Van Haureat, in Holland, as may be seen in Schooten's Commentary on Des Cartes's "Geometry."

It is, however, to the doctrine of fluxions that we owe the complete rectification of curve lines, in finite terms, in all cases where they admit of it, and in others by means of infinite series, circular arcs, logarithms, &c. of which method we shall give a general view in the present article.

Let A C c (*Plate* 6, fig. 7) be any curve line, A B an absciss, and B C a perpendicular ordinate; also $b\,c$ another ordinate indefinitely near to B C; and C d drawn parallel to the absciss A B. Put the absciss A B $= x$, the ordinate B C $= y$, and the curve A C $= z$; then C $d =$ B b

$= \dot{x}$, the fluxion of the absciss A B; and $cd = \dot{y}$, the fluxion of the ordinate B C, and C$c = \dot{z}$, the fluxion of the curve A C.

Now the triangles Cdc being right-angled at d, we have C$c = \sqrt{(\text{C}\,d^2 + cd)^2}$, or $\dot{z} = \sqrt{(\dot{x}^2 + \dot{y})^2}$; that is, the fluxion of the curve is equal to the square root of the sum of the squares of the fluxions of the absciss and ordinate; and, therefore, substituting the value of one of these in terms of the other, drawn from the equation of the curve; the fluxion of the curve will be given in terms of one variable quantity, the fluent of which will be the length of the curve in terms of the same quantity, and which will be general for all values of that variable quantity. Taking, then, any known value of this quantity, the length of the curve will also become known.

Exam. To find the length of the semi-cubical parabola, of which the equation is

$$a x^2 = y^3, \text{ or } x = \frac{y^{\frac{3}{2}}}{a^{\frac{1}{2}}}, \text{ and}$$

consequently $\dot{x} = \dfrac{3y^{\frac{1}{2}}\dot{y}}{2a^{\frac{1}{2}}}$ or $\dot{x}^2 = \dfrac{9y\,\dot{y}^2}{4a}$

substituting, therefore, this value of $\dot{x}^2$ in the general expression

$\dot{z} = \sqrt{(\dot{x}^2 + \dot{y}^2)}$ we have

$$\dot{z} = \sqrt{\left(\frac{9y\dot{y}^2}{4a} + \dot{y}^2\right)} = \frac{1}{2a^{\frac{1}{2}}}\,\dot{y}$$

$\sqrt{(9y + 4a)}$ the fluent of which is

$$z = \frac{1}{27\,a^{\frac{1}{2}}} \times (9y + 4\,a)^{\frac{3}{2}} + \text{corr.}$$

Now when the arc $= 0$, then $y = 0$; $\therefore 0 = \dfrac{4a^{\frac{3}{2}}}{27\,a^{\frac{1}{2}}} + \text{C}$

$$\text{or C} = \frac{-4a^{\frac{3}{2}}}{27\;a^{\frac{1}{2}}} = \frac{-8\,a^{\frac{3}{2}}}{27}.$$

Whence the complete fluent is

$$z = \frac{9y + 4a)^{\frac{3}{2}}}{27a^{\frac{1}{2}}} - \frac{8a^{\frac{3}{2}}}{27}$$

Again, in the common parabola, Here the equation is $ax = y^2$, or $2\,bx = y^2$, putting $a = 2b$, whence $x = \dfrac{y^2}{2\,b}$, or $\dot{x} = \dfrac{y\,\dot{y}}{b}$ and $\dot{x}^2 = \dfrac{y^2\dot{y}^2}{b^2}$. Substitute this value of $\dot{x}^2$ in the general expression $\dot{z} = \sqrt{(\dot{x}^2 + \dot{y}^2)}$, and we have

$$\dot{z} = \sqrt{\left(\frac{y^2\dot{y}^2}{b^2} + \dot{y}^2\right)} = \frac{\dot{y}}{b}\,\sqrt{(y^2 + b^2)}$$

the fluent of which is

$$z = \frac{y}{2\,b}\,(y^3 + b^2)^{\frac{1}{2}} + \tfrac{1}{2}b \times \text{hyp. log.}\,(y + \sqrt{y^2 + b^2}) + \text{corr.}$$

Now when $z = 0$, $y = 0$; and we have therefore
$0 = \frac{1}{2}b \times$ hyp. log. b + corr. or corr. $= -\frac{1}{2}b \times$ hyp. log. b

Whence

$$z = \frac{y}{2\,b}\,(y^3 + b^2)^{\frac{1}{2}} + \tfrac{1}{2}b \text{ hyp. log. } \frac{y + \sqrt{(y^2 + b^2)}}{b}$$

Instead of reasoning in all cases from the expression $\dot{z} = \sqrt{(\dot{x}^2 + \dot{y}^2)}$ it is sometimes useful to adopt other methods according to the nature of the curve, as in the following example.

To find the length of the Cycloidal Arc, A B C. (*Plate* 6, fig. 8.)

Take B D $= a$, B E $= x$, B G $= z$, G H in the direction of a tangent at G $= \dot{z}$; draw H I K parallel to G E, and let E K $= \dot{x}$, then by similar triangles B E F and G I H.

B E : B F = G I : G H

or since B F $= \sqrt{(\text{D B . B F})} = \sqrt{a\,x}$, we have

$$x : a^{\frac{1}{2}}\,x^{\frac{1}{2}} = \dot{x} : \dot{z} = \frac{a^{\frac{1}{2}}\dot{x}}{x^{\frac{1}{2}}} = a^{\frac{1}{2}}\,x^{-\frac{1}{2}}$$

$\dot{x}$ the fluent of which is $z = 2a^{\frac{1}{2}}\,x^{\frac{1}{2}} = 2\sqrt{a\,x}$, which needs no correction, for when $z = 0$, $x = 0$, and the fluent itself becomes zero, as it ought to do.

That is, the cycloidal arc B C = 2 B F, and therefore B A = 2 B D, or the whole arc = four times the diameter of the generating circle.

4. *To find the Length of a Circular Arc.*

This may be expressed either in terms of the sine, cosine, versed sine, or any other trigonometrical line, as follows:

First let the versed sine $= x$, the sine $= y$, radius $= r$, and arc $= z$, then by the property of the circle

$$y^2 = 2rx - x^2$$

or putting tangent $= t$; $y^2 = \frac{r^2 t^2}{r^2 + t^2}$

and secant $= s$ gives, $y^2 = \frac{s^2 - r^2}{s^2} r^2$

as are readily deduced from the known properties of the circle.

Now by means of these values of y^2, or of $2rx - x^2$ and the general equation $\dot{z} = \sqrt{(\dot{x}^2 + \dot{y}^2)}$ we readily draw the following values of $\dot{z}$, *viz.*

$$\dot{z} = \left\{ \begin{array}{l} \frac{r\dot{x}}{\sqrt{(2rx - x^2)}} = \frac{r\dot{y}}{\sqrt{(r^2 - y^2}} = \\ \frac{r^2 \dot{t}}{r^2 + t^2} \quad = \frac{r^2 \dot{s}}{\sqrt{(s^2 - r^2)}} \end{array} \right.$$

the fluents of which can only be found in series, which are as follows; making radius $r = 1$, *viz.*

$$z = \left(1 + \frac{x}{2.3} + \frac{3x^2}{2.4.5} + \frac{3.5x^3}{2.4.6.7} + \&c.\right) \times \sqrt{x}$$

$$z = \left(1 + \frac{y^2}{2.3} + \frac{3y^4}{2.4.5} + \frac{3.5y^6}{2.4.6.7} + \&c.\right) \times y$$

$$z = \left(1 - \frac{t^2}{3} + \frac{t^4}{5} - \frac{t^6}{7} + \frac{t^8}{9} + \&c.\right) \times t$$

$$z = \left(\frac{s - r}{s} + \frac{s^3 - r^3}{2.3s^3} + \frac{3(s^5 - r^3}{2.4.5s^5} + \&c.\right) \times r$$

where r is retained in the latter for sake of analogy. It is obvious therefore that the arc may be computed by any of these in terms, either of the sine or versed sine, tangent or secant, and consequently also in terms of the cosine, cotangent, cosecant, &c.

Thus in the first taking $x = \frac{1}{2}$, which is the versed sine of 60°, we have

$$\text{arc } 60^\circ = \left(1 + \frac{1}{2^2.3} + \frac{3}{2^3.4.5} + \frac{3.5}{2^5.4.6.7}\right) \sqrt{\tfrac{1}{2}}$$

In the second, assuming $y = \frac{1}{2} =$ sin. 30°, we have

$$\text{arc } 30^\circ = \left(1 + \frac{1}{2^3.3} + \frac{3}{2^5.4.5} + \frac{3.5}{2^7.4.6.7}\right) \times \tfrac{1}{2}.$$

In the third, assuming $t = 1 =$ tang. 45°, we obtain

$$\text{arc } 45^\circ = (1 - \tfrac{1}{3} + \tfrac{1}{5} - \tfrac{1}{7} + \tfrac{1}{9} - \tfrac{1}{11} + \&c.)$$

In the fourth, assuming $s = 2 =$ sec. 60°, gives

$$\text{arc } 60^\circ = \left(\frac{2 - 1}{2} + \frac{2^3 - 1^3}{2^4.3} + \frac{3(2^5 - 1^5)}{2^6.4.5} + \&c.\right.$$

Then multiplying the numbers obtained from these series by the number of times that the arc is contained in the whole circumference, will give the circumference required.

But no one of these series are sufficiently convergent for ascertaining the circumference of the circle to a great degree of accuracy, and therefore other methods have been contrived, in order to produce series better calculated for this purpose, of which that of Machin has been the most popular; though it does not appear that he employed it in his celebrated quadrature or rectification, in which he found the circumference to 100 places of figures.

In order to render these series more converging, it is obvious that less arcs must be assumed, and the difficulty consists only in finding the tangents for example, (using that series) of a small arc, which may be expressed in numbers that are tolerably manageable in the general series.

For this purpose, Machin knowing the tangent of 45° to be 1, and that the tangent of an arc being known, any multiple of it is readily found, considered, that if there were assumed some small simple number for the tangent of an arc, and then the tangent of the double arc were continually taken, until a tangent was found nearly equal to 1, the tangent of 45°; by taking the tangent of this small difference between 45° and the multiple arc, there would be had two very small tangents, the one of the first arc, and the other of this difference. Then computing the arc to these tangents, whether the measure of them in degree, &c. were known or not; the whole arc of 45° would become known, *viz.* by multiplying the first by the assumed multiple, and adding the last arc to the product; if the tangent of the multiple arc were less than 1, or the arc itself less than 45°; but subtracting it if greater.

Having thus laid down his plan of operations, by a few trials he hit upon a number well suited to his purpose, *viz.* knowing the tangent $\frac{1}{4}$ of 45°, or of 11° 15′, to be very nearly equal to $\frac{1}{5}$, he assumed for his first arc, that whose tangent is $\frac{1}{5}$; then since $\tan. 2a = \frac{2 \tan. a}{1 - \tan.^2 a}$ radius being 1, he had $\frac{5}{12}$ for the tangent of his double arc; and again $\frac{120}{119}$ for the double of this, or of quadruple the first, and this being a little greater than the tangent of 45° was well adapted to his views; for by a known trigonometrical property $\tan. (a - 45) = \frac{\tan. a - 1}{\tan. a + 1}$ that is, the tangent of the small arc, which is equal to the excess of his multiple above 45°, was $\frac{\frac{120}{119} - 1}{\frac{120}{119} + 1} = \frac{1}{239}$. He had therefore two arcs to compute, the one whose tangent was $\frac{1}{5}$, and the other having for its tangent $\frac{1}{239}$; and then 4 times the first arc minus the latter, would evidently give the exact arc of 45°, and both these numbers being such as converge very well in the general series, the difficulty attending the usual approximations was avoided, but others still more converging than these have since been discovered.

RECTIFIER, in *Navigation*, an instrument consisting of two parts, which are two circles, either laid the one upon the other, or the former let into the latter, and so fastened together at their centres, that they represent two compasses, one fixed the other moveable, each of them being divided into 32 points corresponding with the points of the compass, and also into 360°, and numbered both ways from north to south, terminating in the east and west in 90°. The use of this instrument is to rectify the ship's course, by determining the variation of the compass.

RECTIFYING the *Globe*, is a previous adjustment of it to prepare it for the solution of any proposed problem.

RECTILINEAL, or *Rectilinear*, consisting of, or being bounded by, right lines.

RECURRING *Series*, is a series so constituted, that each succeeding term is connected with a certain number of the terms immediately preceding it by a constant and invariable law; as the sums or differences of some multiples of those terms, &c.

Thus the series

$$\begin{array}{ccccccc} \alpha, & \beta, & \gamma. & \delta, & \varepsilon, & \theta, & \&c. \\ 1, & 3x, & 5x^2, & 7x^3, & 9x^4, & 11x^5, & \&c. \end{array}$$

is a recurring series, for these terms being respectively represented by $\alpha, \beta, \gamma, \delta$, &c. we have

$$\begin{aligned} \gamma &= 2x\beta - x^2\alpha \\ \delta &= 2x\gamma - x^2\beta \\ \varepsilon &= 2x\delta - x^2\gamma \\ \&c.\ & \quad \&c. \end{aligned}$$

that is, each term is equal to $2x$ times that which precedes it, *minus* x^2 times the one preceding the last.

Or generally let

$$\left\{ \begin{array}{ccccccc} \alpha, & \beta, & \gamma, & \delta, & \varepsilon, & \theta, & \&c. \\ ax, & bx^2, & cx^3, & dx^4, & ex^5, & fx^6, & \&c. \end{array} \right.$$

be any series whose terms are denoted as above by $\alpha, \beta, \gamma, \delta$, &c. and let μ, ν, ρ, &c. represent the successive multiples of the preceding terms, then if we have

$$\begin{aligned} \alpha &= \alpha \\ \beta &= \beta \\ \gamma &= \mu\, x . \beta + \nu\, x^2 . \alpha \\ \delta &= \mu\, x . \gamma + \nu\, x^2 . \beta \\ \varepsilon &= \mu\, x . \delta + \nu\, x^2 . \gamma \end{aligned}$$

then this series is recurring, and $\mu + \nu$ is called the *scale of relation*, which is here only of *two terms*, and this subtracted from unity is called the *differential scale*.

But if

$$\delta = \mu x . \gamma + \nu x^2 . \beta + \rho x^3 . a$$
$$\varepsilon = \mu x . \delta + \nu x^2 . \gamma + \rho x^3 . \beta$$
$$\theta = \mu x . \varepsilon + \nu x^2 . \delta + \rho x^3 . \gamma$$

&c. &c.

then $\mu + \nu + \rho$ is the scale of relation, which is of three terms, and so on of any other.

Now referring to the first of the preceding set of formulæ as the simplest, and calling the infinite sum of the terms $a + \beta + \gamma + \delta$, &c. $= s$, it is obvious that we shall have

$$s = a = \beta + \mu x (s - a) + \nu^2 . s, \text{ or}$$
$$s - \pi x . s - \nu x^2 . s = a + \beta - a\mu x, \text{ or}$$
$$s = \frac{a + \beta - a\mu x}{1 - \mu x - \nu x^2}$$ the sum of the series.

And in a similar way we may find the sum when the scale of relation is of three, four, &c. terms, but the result must be necessarily a little more complicated.

Let us illustrate this by a few examples.

1. Required the sum of n terms of the series,

$$1 + 2x + 3x^2 + 4x^3 + \text{\&c. } nx^{n-1}.$$

First, it may be observed that the formula above given extends to the infinite sum, whereas it is here only required to find the sum of the first n terms, we must therefore first find the infinite sum of the whole series; and then the infinite sum of all the terms beyond nx^{n-1}, and the difference of these will evidently be the sum required.

Now, in the first place, it is obvious that each term is equal to $2x$ times the preceding term *minus* x^2 times that preceding the latter; that is $\mu = 2$, and $\nu = -1$; therefore

$$s = \frac{a + \beta - a\mu x}{1 - \mu x - \nu x^2} = \frac{1 + 2x - 2x}{1 - 2x + x^2} = \frac{1}{(1 - x)^2}$$

In the second case, *viz.* of the terms beyond nx^n, the series is $(n + 1)x^n + (n + 2)x^{n+1} + (n + 3)x^{n+2}$, &c. and in the formula $s = \frac{a + \beta - a\mu x}{1 - \mu x - \nu x^2}$ we have only to substitute $a = (n + 1)x^n$, and $\beta = (n + 2)x^{n+1}$ instead of $a = 1$, and $\beta = 2x$, as in the former. Hence we have $s' =$

$$\frac{(n+1)x^n + (n+2)x^{n+1} - 2(n+1)x^{n+1}}{1 - 2x + x^2},$$

$$\text{or } s' = \frac{(n + 1)x^n + nx^{n+1}}{(1 - x)^2}.$$

And therefore $s - s'$, or the sum of the first n times, is equal to

$$\frac{1 - (n + 1)x^n + nx^{n+1}}{1 - x^2}.$$

2. Required the sum of n terms in the series

$$1 + 3x + 5x^2 + 7x^3 + \text{\&c.}$$

Here by trial we find the scale of relation to be $\mu = 2$, and $\nu = -1$, as before; therefore the infinite sum is

$$s = \frac{a + \beta - a\mu x}{1 - \mu x - \nu x^2} = \frac{1 + 3x - 2x}{1 - 2x + x}$$
$$= \frac{1 + x}{(1 - x)^2}$$

After n terms the series becomes $(2n + 1)x^n + (2n + 3)x^{n+1} + (2n + 5)x^{n+2} +$, &c. which therefore arises from the fraction

$$\frac{(2n+1)x^n + (2n+3)x^{n+1} + 2(2n+1)x^{n+})}{1 - 2x + x^2}$$
$$= \frac{(2n + 1)x^n - (2n - 1)x^{n+1}}{(1 - x)^2}.$$

Whence the sum of n terms is

$$\frac{1 + x - (2n + 1)x^n + (2n - 1)x^{n+1}}{(1 - x)^2}$$

RECURRING *Decimals*, those which are continually repeated in the same order, at certain intervals, as $\frac{2}{3} = \cdot 6066$, and $\frac{3}{11} = \cdot 272727$, &c. See *Circulating Decimals*, and *Repetend.*

It is a singular property of all fractions having a prime denominator (n), that the number of places in the repetend is always equal to $(n - 1)$, or $\frac{1}{2}(n - 1)$, or $\frac{1}{4}(n - 1)$, &c.; and when it is equal to $n - 1$, the same repetend will always recur in the same order, whatever may be the numerator of the fraction, but the period of commencement will be different in all; thus, in the number 7,

$\frac{1}{7}$ = ·1428·57·142857
$\frac{2}{7}$ = ·285714·285714
$\frac{3}{7}$ = ·42857·142571
$\frac{4}{7}$ = ·57142·8571428
$\frac{5}{7}$ = ·714285·714285
$\frac{6}{7}$ = ·857142·857142

RED, in *Physics*, or *Optics*, one of the simple or primary colours of natural bodies, or rather of the rays of light. The red rays are the least refrangible of all the rays of light.

REDUCING *Scale*, or *Surveying Scale*, is a broad thin slip of box, or ivory, having several lines and scales of equal parts upon it; used by surveyors for turning chains and links into roods and acres by inspection. It is used also to reduce maps and draughts from one dimension to another.

REDUCTION, in general, is the converting, or changing, a quantity from one denomination or state to another, without altering its absolute value.

Reduction, in *Arithmetic*, is, by some authors on this subject, distinguished into reduction *ascending*, and reduction *descending*. The former relating to the conversion of a quantity from a lower denomination to a higher; and the latter, when the quantity is to be reduced from a higher denomination to a lower. These cases may, however, be both included under the following rule. Consider how many of the less denominations make one of the greater, then multiply the higher denomination by this number, if the reduction be to a less name, or divide the lower denomination by it, if the reduction be to a higher name.

Reduction of *Fractions*, may be divided into the following cases.

I. To reduce a fraction to its simplest terms.

Rule. Divide both the numerator and denominator of the given fraction, by any numbers that will divide them both without a remainder, so will the fraction be reduced to its simplest form. If the common divisors of the numerator and denominator do not appear from inspection, the greatest common divisor of both must be found by the proper rule for that purpose. See *Divisor.*

Ex. 1. Reduce $\frac{144}{240}$ to its lowest terms.

Here $\frac{144}{240} = \frac{72}{120} = \frac{18}{30} = \frac{3}{5}$ answer.

2. Again $\frac{5184}{6012} = \frac{1296}{1503} = \frac{144}{167}$ answer.

It may be observed, that this method of reduction is frequently much facilitated by the properties of divisors.

II. To reduce a mixed number to an improper fraction; and the converse. Multiply the integral part by the denominator, and to the product add the numerator, which sum placed over the denominator will be the fraction required. And conversely, divide the numerator by the denominator in the second case.

Ex. 1. $4\frac{5}{7} = \frac{4 \times 7 + 5}{7} = \frac{33}{7}$.

2. $5\frac{6}{11} = \frac{5 \times 11 + 6}{11} = \frac{61}{11}$.

3. Conversely $\frac{74}{9} = 8\frac{2}{9}$; $\frac{111}{7} = 15\frac{6}{7}$, &c.

III. To reduce a compound fraction to a simple one. Multiply all the numerators together for a new numerator, and the denominators together for a denominator, and it will be the fraction sought. If any of the proposed quantities be integral, or mixed numbers, they must be first reduced to improper fractions. And if there are any factors, common in the numerators and denominators, they may be omitted.

Ex. 1. Thus $\frac{2}{3}$ of $\frac{3}{4} = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$ answer.

2. Again $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{5}{8} = \frac{2}{3} \times \frac{3}{5} \times \frac{5}{8} = \frac{1}{4}$ answer.

3. So also $\frac{3}{100}$ of $12\frac{6}{7} = \frac{3}{100} \times \frac{90}{7} = \frac{27}{70}$ as required.

4. To reduce fractions, having different denominators, to a common denominator. Multiply each numerator by all the denominators, except its own, for a new numerator; and all the denominators together for the common denominator; or find the least com-

mon multiple of all the denominators, and multiply the numerators accordingly.

Ex. 1. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{8}$, to a common denominator.

Here the common multiple being 24, we have

$$\tfrac{12}{24},\ \tfrac{16}{24},\ \tfrac{20}{24},\ \tfrac{21}{24},$$

for the fractions required.

2. Reduce $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{1}{7}$, to a common denominator.

Here by the first rule

$$\left.\begin{array}{r} 3 \times 5 \times 7 = 105 \\ 4 \times 4 \times 7 = 112 \\ 1 \times 4 \times 5 = 20 \end{array}\right\}\text{numerators.}$$

$$4 \times 5 \times 7 = 140 \left\}\begin{array}{l}\text{common} \\ \text{denominator.}\end{array}\right.$$

Whence $\frac{105}{140}$, $\frac{112}{140}$, $\frac{20}{140}$, are the fractions sought.

V. To reduce a fraction from one denomination to another. Consider how many of the lower denomination make one of the higher; then multiply the numerator of the fraction by this number, if the reduction be to a lower name; but the denominator if it be to a higher.

Ex. 1. Reduce $\frac{5}{61}$ of a pound to the fraction of a farthing.

$\frac{5}{61} \times \frac{20}{1} \times \frac{12}{1} \times \frac{4}{1} = \frac{4800}{61}$ of a farthing.

2. Reduce $\frac{5}{11}$ of a guinea to the fraction of a pound.

$\frac{5}{11} \times \frac{21}{1} \times \frac{1}{20} = \frac{101}{220}$ of a pound.

VI. To reduce a fraction to its proper integral value. Multiply the parts in the next inferior denomination by the numerator of the fraction, and divide by the denominator, as in compound multiplication and division.

Ex. 1. Find the value of $\frac{7}{8}$ of half a guinea.

$$\begin{array}{rrr} & s. & d. \\ & 10 & 6 \\ & & 7 \\ \hline 8)3 & 13 & 6 \\ \hline & 9 & 2\frac{1}{4}\ \textit{Ans.} \end{array}$$

The reduction of complex fractions to simple ones, is no more than dividing one fraction by another.

REDUCTION of *Decimals*, may be divided into the following cases.

I. To reduce a vulgar fraction to its equivalent decimal.—Annex to the numerator as many decimals as may be thought necessary; then divide by the denominator, and point off as many decimal places in the quotient, as there are ciphers annexed. If there be not so many figures in the quotient as are requisite, the defect must be supplied by prefixing ciphers.

Ex. 1. Thus $\frac{5}{24} = {\cdot}208333$, &c.

2. Also $\frac{13}{1728} = {\cdot}0075231$.

II. To reduce quantities of different denominations to their equivalent decimal values. Reduce the compound quantity to its lowest denomination, and the whole integer to the same denomination; then considering the first of these as the numerator, and the latter as the denominator of a fraction, the reduction to decimals may be performed as in the preceding case. Or the same may be otherwise done, as follows: Write the given numbers perpendicularly under each other for dividends, proceeding in order from the least to the greatest. Then opposite each dividend, on the left, place such a number for a divisor, as will bring it to the next superior denomination, and draw a line between them. This done, begin with the uppermost number, and divide it, as is usual, pointing of the decimals, then divide by the second number, and so on to the last, which will be the decimal sought.

EXAMPLES.

Reduce 15*s*. 9$\frac{3}{4}$*d*.; and 12*s*. 10$\frac{1}{2}$*d*. to decimals.

$$\begin{array}{r|l} 4 & 3 \\ \hline 12 & 9{\cdot}75 \\ \hline 20 & 15{\cdot}8125 \\ \hline & {\cdot}790625 \end{array} \text{ decimals } \begin{array}{r|l} 4 & 2 \\ \hline 12 & 10{\cdot}5 \\ \hline 20 & 12{\cdot}875 \\ \hline & {\cdot}64375 \end{array}$$

III. To reduce a decimal to its equivalent integral value. Multiply the given decimal by the number of parts in the next less denomination, and point off as many places to the right-hand for decimals, as there are in the origin number. Then multiply the mals, thus cut off, by the of parts in the next less denomination, reserving again the same number of places for decimals; multiply these again in the same manner, and so on; then the figures separated on the left-hand will express the value of the decimal required.

Ex. Find the value of ·78546 of a pound sterling.

$$\begin{array}{r}
\cdot 78546 \\
20 \\ \hline
15\cdot 70920 \\
12 \\ \hline
9\cdot 51040 \\
4 \\ \hline
2\cdot\ 4160
\end{array}
\quad \textit{Ans. } 15s.\ 9\tfrac{1}{2}d.$$

Reduction of *Algebraic Fractions*, is performed in exactly the same manner as the reduction of common fractions; it will, therefore, be useless to repeat the rules; we shall, however, for the sake of illustration, give an example or two in each case.

1. To reduce an algebraic fraction to its simplest form.

1. Reduce $\frac{x^4 - b^4}{x^5 - b^2 x^3}$ to its lowest terms.

$$\frac{x^4 - b^4}{x^5 - b^2 x^3} = \frac{(x^2 + b^2)(x^2 - b^2)}{(x^2 - b^2)\ x^3} = \frac{x^2 + b^2}{x^3}$$ answer.

2. Reduce $\frac{x^2 - y^2}{x^4 - y^4}$ to its simplest form.

$$\frac{x^2 - y^2}{x^4 - y^4} = \frac{x^2 - y^2}{(x^2 + y^2)(x^2 - y^2)} = \frac{1}{x^2 + y^2}$$ as required.

In these examples, the factors of the terms are found by inspection, and the reduction is therefore very simple; but if this be not the case, the greatest common divisor must be found by the proper rule, and both terms of the fraction divided by it. See **Divisor**.

2. To reduce a mixed algebraical expression to an improper fraction; and the converse.

1. Reduce $x + \frac{a^2 + x^2}{y}$ to an improper fraction.

$$x + \frac{a^2 + x^2}{y} = \frac{x y + a^2 + x^2}{y}$$ as required.

2. Again $1 + 2x - \frac{x - 3}{5x} = \frac{(1 + 2x)\, 5x - (x - 3)}{5x} = \frac{10x^2 + 4x + 3}{5x}$ as required.

3. Thus also conversely

$$\frac{a y + 2y}{a + y} = (a y + 2 y^2) \div (a + y) = y + \frac{y^2}{a + y}.$$

3. To reduce fractions to a common denominator. See case 4. Common fractions.

1. Reduce $\frac{b}{2a^2}, \frac{c}{2a}$, and $\frac{x}{y}$ to a common denominator.

Here the least common multiple of the denominators being $2a^2 y$, we have

$$\frac{b y}{2 a^2 y}, \frac{c a y}{2 a^2 y}, \text{ and } \frac{2 a^2 x}{2 a^2 y}$$

for the fractions required.

2. Reduce $\frac{1}{2}, \frac{a^2}{3}$, and $\frac{a^2 + x^2}{a + x}$, to a common denominator.

Here the least common multiple is $6a + 6x$, therefore the fractions when reduced are

$$\frac{3a + 3x}{6a + 6x}, \frac{2a^3 + 2a^2 x}{6a + 6x}, \text{ and } \frac{6a^2 + 6x^2}{6a + 6x}$$

Reduction of *Surds*, is the method of reducing a surd, or irrational quantity, to its most simple form; which may be divided into the following cases.

1. To reduce quantities having different indices or radicals, to one common index. Let the proposed radicals be represented by their proper fractional indices, then reduce these fractions to a common denominator, which will be the common index required, the numerators of the fractions being made the powers of the given quantities.

EXAMPLES.

1. Reduce $\sqrt[2]{a}$, and $\sqrt[3]{a}$, to a common radical.

Here the proper fractional indices, being $\frac{1}{2}$ and $\frac{1}{3}$, they become, when reduced to a common denominator, $\frac{3}{6}$ and $\frac{2}{6}$, whence $(a^3)^{\frac{1}{6}}$, and $(a^2)^{\frac{1}{6}}$; or $\sqrt[6]{a^3}$, and $\sqrt[6]{a^2}$, are the common radicals required.

2. Again $a^{\frac{1}{n}}$, and $b^{\frac{1}{m}}$, become $(a^m)^{\frac{1}{mn}}$, and $(b^n)^{\frac{1}{mn}}$; or $\sqrt[mn]{a^m}$, and $\sqrt[mn]{b^n}$; and so of others.

2. To reduce surds to their simplest terms.

Find the greatest power contained in the given surds, by dividing it successively by all the powers between itself and unity; and set the root of this power before the quotient, with the proper radical sign between them; or if the surd has any co-efficient, it must be multiplied by the root of the power before found, and the product placed as above. If the given quantity under the radical be a fraction, it must be rendered integral, by multiplying both numerator and denominator by such a number as will make the denominator a complete power; and then the root of this denominator must be taken as a fractional co-efficient to the radical thus obtained.

EXAMPLES.

1. Thus $\sqrt{125} = \sqrt{5^2 \times 5} = 5\sqrt{5}$.
2. Again $\sqrt[3]{243} = \sqrt[3]{3^3 \times 9} = 3\sqrt[3]{9}$.
3. Also $\sqrt{98a^2x} = \sqrt{(7a)^2 \times 2x} = 7a\sqrt{2x}$.
4. Thus also $\sqrt{(x^3 - a^2x^2)} = x\sqrt{(x - a^2.)}$
5. Again $\sqrt{\frac{50}{147}} = \sqrt{\frac{150}{441}} = \frac{1}{21}\sqrt{150} = \frac{1}{21}\sqrt{5^2 \times 6} = \frac{5}{21}\sqrt{6}$.
6. And $\sqrt[3]{\frac{61}{81}} = \sqrt[3]{\frac{549}{.9^3}} = \frac{1}{9}\sqrt[3]{549}$.

In compound surds of the form

$$\frac{\sqrt{a} \pm \sqrt{b}}{\sqrt{c} \pm \sqrt{d}}$$

the rule is, to multiply both numerator and denominator by the terms of the denominator, but connected with a different sign, which renders the denominator rational, and reduces the whole expression: thus

$$\frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} - \sqrt{5}} = \frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} - \sqrt{5}} \times \frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} + \sqrt{5}} = \frac{7 + 2\sqrt{35} + 5}{7 - 5} = \frac{12 + 2\sqrt{35}}{2} = 6 + \sqrt{35}.$$

Again,

$$\frac{\sqrt{18} + \sqrt{8}}{\sqrt{3} - \sqrt{2}} = \frac{\sqrt{18} + \sqrt{8}}{\sqrt{3} - \sqrt{2}} \times \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}} \frac{\sqrt{54} + \sqrt{24} + 6 + 4}{3 - 2} = \sqrt{6} + 10.$$

Reduction of a *Figure, Design* or *Draught*, is properly the reducing of it from a larger to a smaller scale: though the term is sometimes used indifferently, whether the copy be made larger or smaller. In this reduction, the exact form and proportion of the figure must be observed, for which purposes surveyors and architects make use of an instrument called a **Pentagraph**; *Proportional* **Compasses** are also employed for this purpose; see the method of using these instruments under the respective articles.

Measure all the sides and diagonals of the figure by a scale; and lay down the same measures respectively from another scale, in the proportion required.

To reduce a map, figure, or plan, by squares.

Divide the original into a number of small squares, and divide also a fair sheet of paper, of the dimensions required, into the same number of squares, which will be equally greater, or less, according as the new figure is to be equal, greater, or less. This done, in every square of the second figure, draw what is found in the corresponding square in the first, or original plan; so will the figure be reduced as required.

The cross lines forming these squares may be drawn with a pencil, and rubbed out again after the work is finished. But a more ready and convenient way, p cularly when such reductions are often wanted, is to keep always ready frames of squares of several sizes; for then, by barely placing them upon the papers, the corresponding parts may be readily copied. These squares may be made with four inflexible bars, strung across with horse-hair, or the like.

Reduction to the *Ecliptic* in *Astronomy*, is the difference between the argument of latitude, and an arc of the ecliptic intercepted between the place of a planet and the node. To find this reduction, or difference, there are given in a right-angled spherical triangle, the angle of inclination, and the argument of latitude; to find the arc of the ecliptic, then the difference between that and the argument of latitude is the reduction sought.

REFLECTION, or *Reflexion*, in *Mechanics*, is the return or regressive motion of a moveable body, arising from the re-action of some other body on which it impinges.

The reflection of bodies after impact, is attributable to their elasticity, and the more perfectly they possess this property the greater will be their reflection, all other things being the same. In case of perfect elasticity they would be reflected back again with the same velocity, and at an equal angle with which they met the plane; that is, the angle of incidence would be equal to the angle of reflection, and the velocity both before and after impact would be the same, at equal distance from the body on which they impinge. See Incidence and Percussion.

Reflection of the *Rays of Light*, like that of other material particles, is their motion after being reflected from the surfaces of other bodies; and by which means those bodies become visible. For the laws of reflection, as it is connected with the doctrine of optics, see Mirror.

Reflection of *Heat*. In the same manner as we find the rays of light are reflected by polished surfaces, so it is found that the rays of caloric have precisely the same property. The Swedish chemist Scheele discovered, that the angle of reflection of the rays of caloric is equal to the angle of incidence, a fact which has been more fully established by Dr. Herschell. Some very interesting experiments were made by Professor Pictet of Geneva, which proved the same thing. These experiments were conducted in the following manner: two concave mirrors of tin, of nine inches focus, were placed at the distance of twelve feet two inches from each other; in the focus of the one was placed the bulb of a thermometer, and in that of the other a ball of iron two inches in diameter, which was just heated, so as not to be visible in the dark. In the space of six minutes the thermometer rose 22°. A similar effect was produced by substituting a lighted candle in place of the ball of iron.

Supposing that both the light and heat might act in the last experiment, he interposed between the two mirrors a plate of glass, with the view of separating the rays of light from those of caloric. The rays of caloric were thus interrupted by the plate of glass, but the rays of light were not perceptibly diminished. In nine minutes the thermometer sunk 14°; and in seven minutes after the glass was removed, it rose about 12°. He therefore justly concluded, that the caloric reflected by the mirror, was the cause of the rise of the thermometer. He made another experiment, substituting boiling-water in a glass-vessel in place of the iron ball; and when the apparatus was adjusted, and a screen of silk which had been placed between the two mirrors removed, the thermometer rose 3°; namely, from 47° to 50°. The experiments were varied by removing the tin mirrors to the distance of 90 inches from each other. The glass vessel, with boiling water, was placed in one focus, and a sensible thermometer in the other. In the middle space between the mirrors, there was suspended a common glass mirror, so that either side could be turned towards the glass vessel. When the polished side of this mirror was turned towards the glass vessel, the thermometer rose only five-tenths of a degree; but when the other side, which was darkened, was turned towards the glass vessel, the thermometer rose 3° 5′. And in another experiment, performed in the same way, the thermometer rose 3° when the polished side of the mirror was turned to the glass vessel, and 9° when the other side was turned, which experiments show clearly, that the rays of caloric are reflected from polished surfaces, as well as the rays of light. Transparent bodies have the power of refracting the rays of caloric, as well as those of light. They differ also in their refrangibility. So far as experiment goes, the most of the rays of caloric are less refrangible than the red rays of light. The experiments of Dr. Herschel show that the rays of caloric, from hot or burning bodies, as hot iron, hot water, fires, and candles, are refrangible, as well as the rays of caloric, which are emitted by the sun. Whether all transparent bodies have their power of transmitting these rays, or what is the difference in the refractive power of these bodies, is not yet known.

The light which proceeds from the sun seems to be composed of three distinct substances. Scheele discovered that a glass mirror held before the fire, reflected the rays of light, but not the rays of caloric; but when a metallic mirror was placed in the same situation, both heat and light were reflected. The mirror of glass became hot in a short time, but no change of temperature took place on the metallic mirror. This experiment shows that the glass mirror absorbed the rays of caloric, and reflected those of light; while the metallic mirror, suffering no change of temperature, reflected both. And if a plate of glass be held before a burning body, the rays of light are not sensibly interrupted, but the rays of caloric are intercepted; for no sensible heat is observed on the opposite side of the glass; but when the glass has reached a proper degree of temperature, the rays of caloric are transmitted with the same facility as those of light. And thus the rays of light and caloric may be separated; and the curious experiments of Dr. Herschel have clearly proved, that the invisible rays which are emitted by the sun have the greatest heating power. In these experiments the different coloured rays were thrown on the bulb of a very delicate thermometer, and their heating power was observed. That of the violet, green, and red rays were found to be to each other as the following numbers:

Violet	16	0
Green	22	4
Red	55	0

The heating power of the most refrangible rays was least, and this increases as the refrangibility diminishes. The red ray, therefore, has the greatest heating power, and the violet, which is the most refrangible, the least. The illuminating power, it has been already observed, is greatest in the middle of the spectrum, and diminishes towards both extremities; but the heating power, which is least at the violet end, increases from that to the red extremity, and when the thermometer was placed beyond the limit of the red ray, it rose still higher than in the red ray, which has the greatest heating power in the spectrum. The heating power of these invisible rays was greatest at the distance of half an inch beyond the red ray, but it was sensible at the distance of one inch and a half.

REFLECTOIRE *Curve*, is a term given by Mairan to the curvilinear appearance of the plane surface of a bason containing water to an eye placed perpendicularly over it. In this position the bottom of the bason will appear to rise upwards from the centre outwards, but the curviture will be less and less, and at last the surface of the water will be an asymptote to it.

REFRACTED *Angle*, or *Angle of Refraction*, is the angle which a refracted ray makes with the surface of the refracting body. The complement of this angle is, however, sometimes called the refracted angle.

REFRACTION, in *Mechanics*, is the deviation of a body in motion from its direct course, in consequence of the variable density of the mediums in which it moves. This, however, except in speaking of the rays of light, is more commonly called *deflection*.

REFRACTION of *Light*. If the rays of light, after passing through medium, enter another of a different density perpendicular to its surface, they proceed through this medium in the same direction as before. But if they enter obliquely to the surface of a medium, either denser or rarer than what they moved in before, they change their direction in passing through that medium. If the medium which they enter be denser, they move through it in a direction nearer to the perpendicular drawn to its surface. On the contrary, when light passes out of a denser into a rarer medium, it moves in a direction farther from the perpendicular. This refraction is greater or less, that is, the rays are more or less bent or turned aside from their course, as the second medium through which they pass is more or less dense than the first. Thus, for instance, light is more refracted in passing from air into glass, than from air into water; glass being denser than water. And in general, in any two given media, the sine of the angle of incidence has a constant ratio to the sine of the corresponding angle of refraction. Hence, when the angle of incidence is increased, the corresponding angle of refraction is also increased; and if two angles of incidence be equal, the angles of refraction will be equal. The angle of deviation must also vary with the angle of incidence. If a ray of light pass obliquely out of air into glass, the sine of the angle of incidence, is to the sine of the angle of refraction, nearly as 3 to 2; therefore, supposing the sines proportional to the angles, the sine of the angle of deviation, is as the difference between the sine of the angle of incidence and the sine of the angle of refraction, that is, as 3—2, or 1, whence the sine of incidence is to the sine of the angle of deviation as 3 to 1. In like manner it may be shown, that when the ray passes obliquely out of glass into air, the sine of the angle of incidence will be to that of deviation, as 2 to 1. In passing from air into water, the sine of the angle of incidence is to that of refraction, as 4 to 3, and to that of deviation, as 4 to 4—3, or 1; and in passing out of water into air, the sine of the angle of incidence is to that of refraction, as 3 to 4, and to that of deviation as 3 to 1. Hence a ray of light cannot pass out of water into air at a greater angle of incidence than 48° 36′, the sine of which is to radius as 3 to 4. Out of glass into air the angle must not exceed 40° 11′, because the sine of 40° 11′ is to radius as 2 to 3 nearly; consequently, when the sine has a greater proportion to the radius than that above stated, the ray will not be refracted. And it may be observed, that when the angle is within the limit for light to be refracted, some of the rays will be reflected. For the surfaces of all bodies are for the most part uneven, which occasions the dissipation of much light by the most transparent bodies; some being reflected, and some refracted, by the inequalities on the surfaces. Hence a person can see through water, and his image reflected by it, at the same time. Hence also, in the dusk, the furniture in a room may be seen by the reflection of a window, while objects that are without are seen through it.

To prove the refraction of light, take an upright empty vessel into a dark room; make a small hole in the window-shutter, so that a beam of light may fall upon the bottom, where you may make a mark. Then fill the bason with water, without moving it out of its place, and you will see that the ray, instead of falling upon the mark, will fall nearer the centre of the basin. If a piece of looking-glass be laid in the bottom of the vessel, the light will be reflected upon it, and will be observed to suffer the same refraction as in coming in; only in a contrary direction. If the water be made a little muddy, by putting into it a few drops of milk, and if the room be filled with dust, the rays will be rendered much more visible. The same may be proved by another experiment. Put a piece of money into the bason when empty, and walk back till you have just lost sight of the money, which will be hidden by the edge of the bason. Then pour water into the bason, and you will see the money distinctly, though you look at it from the same spot as before; and hence it is that a straight oar, when partly immersed in water, will appear bent.

If the rays of light fall upon a piece of flat glass, they are refracted into a direction nearer to the perpendicular, as described above, while they pass through the glass; but after coming again into air, they are refracted as much in the contrary direction; so that they move exactly parallel to what they did before entering the glass. But on account of the thinness of the glass, this deviation is generally overlooked, and it is considered as passing directly through the glass.

Atmospherical Refraction. It is evident from the nature and progression of light, that rays, in passing from any object through the atmosphere, or part of it, to the eye, do not proceed in a right line: but the atmosphere being composed of an infinitude of strata, (if we may so call them) whose density increases as they are posited nearer the earth, the luminous rays which pass through it are acted on as if they passed successively through media of increasing density, and are therefore inflected more and more towards the earth as the density augments. In consequence of this it is, that rays from objects, whether celestial or terrestrial, proceed in curves which are concave towards the earth; and thus it happens, since the eye always refers the place of objects to the direction in which the rays reach the eye, that is, to the direction of the tangent to the curve at that point, that the apparent or observed elevations of objects are always greater than the true ones. The difference of these elevations, which is in fact, the effect of refraction, is, for the sake of brevity, called refraction; and it is distinguished into two kinds, horizontal, or terrestrial refraction, being that which effects the altitude of hills, towers, and other objects on the earth's surface: and astronomical refraction, or that which is observed with regard to the altitudes of heavenly bodies. Refraction is found to vary with the state of the atmosphere, in regard to heat or cold, and humidity, so that determinations obtained for one state of the atmosphere will not answer correctly for another without modification. Tables commonly exhibit the refraction at different latitudes, for some assumed mean state.

2. With regard to the horizontal refraction, the following method of determining it has been successively practised in the English Trigonometrical Survey.

Let (*Plate* VI. fig. 9) A, A′, be two elevated stations on the surface of the earth, B D the intercepted arc of the earth's surface, C the earth's centre, A H′ A′ H, the horizontal lines at A, A′, produced to meet the opposite vertical lines C H′, C H. Let *a*, *a*′, represent the apparent places of the objects A, A′, then is *a*′ A A′ the refraction observed at A, and *a* A′ A the refraction observed at A′; and half the sum of those angles will be the horizontal refraction, if we assume it equal at each station.

Now an instrument being placed at each station A, A′, the reciprocal observations are made at the same instant of time, which is determined by means of signals, or watches previously regulated for that purpose; that is, the observer at A takes the apparent depression of A′, at the same moment that the other observer takes the apparent depression of A.

In the quadrilateral A C A′ I, the two angles A A′, are right angles, and therefore the angles I and C are together equal to two right angles: but the three angles of the triangle I A A′ are together equal to two right angles; and consequently the angles A and A′ are together equal to the angle C which is measured by the arc B D

If therefore the sum of the two depressions H A′ a, H′ A A′, or, which is equivalent, from the angle C (which is known, because its measure B D is known); the remainder is the sum of the two depressions from the measure of the intercepted terrestrial arc, half the remainder is the refraction.

3. If by reason of the minuteness of the contained arc B D, one of the objects, instead of being depressed, appears elevated, as suppose A′ to a''; then the sum of the angle a'' A A′, and a A′ A will be greater than the sum I A A′ + I A′A, or than C, by the angle of elevation a'' A A′; but if from the former sum there be taken the depression H A′ A, there will remain the sum of the two refractions. So that in this case the rule becomes as follows: take the depression from the sum of the contained arc and elevation, half the remainder is the refraction.

4. The quantity of this terrestrial refraction is estimated by Dr. Maskelyne at one tenth of the distance of the object observed, expressed in degrees of a great circle. So if the distance be 10000 fathoms, its tenth part, 1000 fathoms, is the sixtieth part of a degree of a great circle on the earth, or 1′, which therefore is the refraction in the altitude of the object at that distance.

But M. Legendre is induced, he says, by several experiments, to allow only $\frac{1}{14}$th part of the distance for the refraction in altitude. So that on the distance of 10000 fathoms, the fourteenth part of which is 714 fathoms, he allows only 44″ of terrestrial refraction, so many being contained in the 714 fathoms. See his Memoir concerning the Trigonometrical Operations, &c.

Again, M. Delambre, an able French astronomer, makes the quantity of the terrestrial refraction to be the eleventh part of the arc of distance. But the English measurers, especially Col. Mudge, from a multitude of exact observations, determine the quantity of the medium refraction to be the twelfth part of the said distance.

The quantity of this refraction, however, is found to vary considerably with the different states of the weather and atmosphere, from the $\frac{1}{7}$th to the $\frac{1}{18}$th of the contained arc. Trigonometrical Survey, vol. i. p. 160, 355.

Having thus given the mean results of observation on the terrestrial refraction, it may not be amiss, though we cannot enter at large into the investigation, to present here a correct table of mean astronomical refractions. The table, which has been most commonly given in books of astronomy, is Dr. Bradley's, computed from the formula $r = 57'' \times \text{cot.} (a + 3r)$ where a is the altitude, r the refraction, and $r = 2' 35''$, when $a = 20°$.

But it has been found that the refractions thus computed are rather too small. Laplace, in his "Mecanique Celeste," tom. iv. p. 27, deduces a formula which is strictly similar to Bradley's; for it is $r = m \times \text{tan.} (z - nr)$ where z is the zenith distance, &c. m and n are two constant quantities to be determined from observation. The only advantage of the latter formula over the one given by the English astronomer is, that Laplace and his colleagues have found more correct co-efficients than Bradley had

Now, if $R = 57°.2957795$, the arc equal to the radius, if we make $m = \frac{kR}{n}$ (where k is a constant co-efficient, which, as well as n, is an abstract number) the preceding equation will become

$$\frac{nr}{R} = k \times \text{tan.} (z - nr).$$

Here the refraction r is always very small, as well as the correction nr, the trigonometrical tangent of the arc nr may be substituted for $\frac{nr}{R}$; thus we shall have $\text{tan.}\, nr = k \times \text{tan.} (z - nr)$.

But $nr = \frac{1}{2}z - (\frac{1}{2}z - nr)$, and $z - nr = \frac{1}{2}z + (\frac{1}{2}z - nr)$; consequently

$$\frac{\text{tan.}\, nr}{\text{tan.} (z - nr)} = \frac{\text{tan.}\left(\frac{z}{2} - \frac{z - 2nr}{2}\right)}{\text{tan.}\left(\frac{z}{2} + \frac{z - 2nr}{2}\right)}$$

$= k$, or

$$\frac{\text{sin. } z - \text{sin. } (z - nr)}{\text{sin. } z + \text{sin. } (z + nr)} = k.$$

Hence $\text{sin. } (z - 2\,nr) = \frac{1-k}{1+k}$ sin. z.

This formula is very convenient when the co-efficients n and $\frac{k-1}{k+1}$ are known; and it has been ascertained, by a mean of many observations, that these are, 4 and ·99765175 respectively. Thus Laplace's equation becomes
$\text{sin. } (z - 8r) = {}\cdot 99765175 + \text{sin. } z.$

With respect to the refraction under different temperatures and at different states of the barometer, the following theorems have been given, the first by Dr. Maskelyne, and the second by Dr. Brinkley, viz.

$$\text{1st. Refract.} = \frac{a}{29{\cdot}6} \times \text{tan.} (z - 3r) \times 57'' \times \frac{400}{350} + h$$

$$\text{2d. Refract.} = \frac{a}{29{\cdot}6} \times \text{tan.} (z - 3.2r) \times 56''.9 \times \frac{500}{450} + h.$$

Where $a =$ the height of the barometer in inches; $h =$ the height in degrees of Fahrenheit's thermometer, and $r = 57''$ tan. z.

REFRACTION of *Altitude*, is an arc of a vertical circle, by which the altitude of a star is increased by the refraction.

REFRACTION of *Ascension* or *Descension*, is an arc of the equator, by which the ascension or descension, whether right or oblique, is increased or diminished by refraction.

REFRACTION of *Declination*, is the increase or decrease in the declination of a star by refraction.

REFRACTION, in *Latitude*, is the increase or decrease in the latitude of a star from refraction.

REFRACTION in *Longitude*, is the increase or decrease in the longitude of a star from refraction.

REFRANGIBILITY of *Light*, the property of the rays to be refracted, but more commonly employed with reference to the different degrees in which the different rays possess this property.

REGULAR *Figure*, in *Geometry*, is one that has all its sides and all its angles equal. If these are not both equal, the figure is irregular.

REGULAR *Bodies*, are those which have all their sides, angles, and faces, similar and equal.

Of these there are only 5, viz.

The *Tetraedon*, contained by 4 equilateral triangles;
The *Hexaedron* or *cube*, by 6 squares;
The *Octaedron*, by 8 triangles;
The *Dodecaedron*, by 12 pentagons; and
The *Icosaedron*, by 20 triangles.

RELATION, in *Mathematics*, is the same as RATIO.

REPULSION, in *Physics*, that property in bodies, whereby, if they are placed just beyond the spheres of each other's attraction of cohesion, they mutually recede and fly off. Thus, if any oily substance, lighter than water, be placed upon its surface, or if a piece of iron be laid upon mercury, the surface of the fluid will be depressed about the body which is laid on it; this depression is manifestly occasioned by a repelling power in the bodies, which prevents the approach of the fluid towards them. But it is possible, in some cases, to press or force the repelling bodies into the sphere of each other's attraction; and then they will mutually tend toward each other, as when we mix oil and water till they are incorporated. Dr. Knight defines repulsion to be that cause which makes bodies mutually endeavour to recede from each other, with different forces, at different times; and that such a cause exist in nature, he thinks evident for the following reasons. 1. Because all bodies are electrical, or capable of being made so; and it is well known that electrical bodies both attract and repel. 2. Both attraction and repulsion are very conspicuous in all magnetical bodies. 3. Sir Isaac Newton has shown from experience, that the surface of two convex glasses repel each other. 4. The same great philosopher has explained the elasticity of the air, by supposing its particles mutually to repel each other. 5. The particles of light are, in part at least, repelled from the surfaces of all bodies. 6, Lastly, it seems highly probable that the particles of light mutually repel each other, as well as the particles of air.

RESIDUAL *Analysis*, a branch of analysis invented by Landen, and applied by him to the solution of those problems which are more generally solved by the doctrine of fluxions. This method was called the residual analysis, because, in all cases where it is made use of, the conclusions are obtained by means of residual quantities. In this analysis a geometrical or physical problem is reduced to another purely algebraical; and the solution is then obtained without any supposition of motion, and without considering quantities as composed of infinitely small particles.

The residual analysis proceeds by taking the difference of the same function of a variable quantity in two different states of that quantity, and expressing the relation of this difference to the difference between the two states of the said variable quantity itself. This relation being first expressed generally, is then considered in the case when the difference of the two states of the variable quantity is $= 0$.

Residual *Quantity*, in *Algebra*, is a binomial connected by the sign $-$; thus $a-b$; $a-\sqrt{b}$, &c. are residual quantities.

RESISTANCE, in *Physics*, any power which acts in opposition to another, so as to destroy or diminish its effect.

Resistances are of various kinds, arising from the nature and properties of the resisting bodies, the circumstance in which they are placed, and the laws by which they are governed. These may be divided into the following cases: 1. The resistance between the surfaces of contiguous solid bodies, generally denominated friction. 2. The resistance between the contiguous particles of the same body, whether fluid or solid. 3. The resistance that solid bodies oppose to penetration. 4. The resistance of elastic and non-elastic fluids to the motion of bodies moving in them.

The resistance that a body experiences from the fluid medium through which it is impelled, depends on the velocity, form, and magnitude of the body, and on the inertia and tenacity of the fluid. For fluids resist the motion of bodies through them. 1. By the inertia of their particles. 2. By their tenacity, or the adhesion of their particles. 3. By the friction of the body against the particles of the fluid.

In perfect fluids the latter causes of resistance are very inconsiderable, and therefore are not commonly considered; but the first is always very considerable, and obtains equally in the most perfect, and in the most imperfect fluids. In what follows, and in all cases of a similar description, it will be necessary to distinguish between *resistance* and *retardation;* the former being the quantity of *motion*, and the latter the quantity of *velocity*, which is lost; therefore the retardations are as the resistances applied to the quantity of matter, and in the same body they have always the same constant ratio to each other. In fluids of uniform tenacity, the resistance from the cohesion of its particles is as the velocity with which the body moves. For, since the cohesion of the particles is constantly the same, in the same space, whatever may be the velocity, the resistance from this cohesion will be as the space described in a given time; that is, as the velocity. In a fluid whose parts yield easily without disturbing each other's motions, and which flows in behind as fast as a plane body moves forward, the resistance will be as the density of the fluid; for in this case the pressure on every part of the body is the same as if the body were at rest. And on the same hypothesis, the resistance from inertia will be as the square of the velocity. For the resistance must vary as the number of particles which strikes the plane in a given time, multiplied into the force of each against the plane, and both these quantities varying as the velocity, the resistance which is measured by this product must vary as the square of the velocity. We here suppose the plane of the body to be perpendicular to its direction; but if, instead of being so, it is inclined to it in any given angle, then the resistance of the plane in the direction of the motion will be diminished in the ratio of 1, to the sine cubed of the angle of inclination.

Hence, making the velocity $= v$, the area of the plane $= a$, the specific gravity of the fluid $= n$, the force of gravity $32\frac{1}{6}$ feet $= g$; then the altitude due to the velocity v being $\frac{v^2}{2g}$; the whole resistance or motive force m, will be expressed by the following formula:

$$m = a\,n \times \frac{v^2}{2g} = \frac{a\,n\,v^2}{2g}\,;\ \text{or} = \frac{a n\,v^2 s^3}{2g}$$

when the plane is not perpendicular to the line of its motion.

If w be made to represent the weight of the body, and f the retarding force; then on the same principles we derive

$$f = \frac{m}{w} = \frac{a\,n\,v^2 s^3}{2gw}$$

If the body be a cylinder moving in the direction of its axis, and the diameter of its base $= d$, or radius r, and $\pi = 3{\cdot}14159$, &c. then

$$f = \frac{m}{w} = \frac{\pi n d^2 v^2}{8gw} = \frac{\pi n r^2 v^2}{2gw}$$

But if the body be a cone, then the same notation remaining, only writing s for the sine of the angle of inclination of the side of the cone, then

$$f = \frac{m}{w} = \frac{\pi n d^2 v^2 s^2}{8gw} = \frac{\pi n r^2 v^2 s^2}{2gw}$$

For in this case the inclination has no effect in reducing the section opposed to the resistance of the fluid, this being the same as in the cylinder, and therefore will vary as s^2.

The same notation still remaining, it is found from a fluxional investigation that the resistance to the body, when terminated with a hemispherical surface, is,

$$f = \frac{m}{w} = \frac{\pi n d^2 v^2}{16gw} = \frac{\pi n r^2 v^2}{4gw}$$

that is, half what it is when the end is a plane surface.

Hence the resistance to a sphere impelled through any fluid is equal to half the direct resistance to a great circle of it, or to a cylinder of the same diameter.

Since $\frac{1}{6}\pi d^3$, is the magnitude of the globe; if N denotes its density or specific gravity, its weight $w = \frac{1}{6}\pi d^3 \mathrm{N}$, and therefore the retardive force

$$f = \frac{m}{w} = \frac{\pi n v^2 d^2}{16g} \times \frac{6}{\pi \mathrm{N} d^3} = \frac{3nv^2}{8g\mathrm{N}d} = \frac{v^2}{2gs}$$

where s is the space described; for $2fgs = v^2$, by the laws of accelerated or retarded motions.

Hence we have $s = \frac{\mathrm{N}}{n} \times \frac{4}{3}d$;

which is the space that would be described by the globe, while its whole motion is generated or destroyed by a constant force, which is equal to the force of resistance, if no other force acted on the globe to continue its motion. And if the density of the fluid were equal to that of the globe, the resisting force is such as, acting constantly on the globe without any other force, would generate or destroy its motion in describing the space $\frac{4}{3}d$, or $\frac{4}{3}$ of its diameter, by that accelerating or retarding force.

Hence the greatest velocity that a ball will acquire by descending in a fluid by means of its relative weight in that fluid, will be found by making the resisting force equal to that weight. For, after the velocity has arrived at such a degree, that the resisting force is equal to the weight that urges it, it will increase no longer, and the globe will then continue to descend with an uniform velocity.

Now N and n being the separate specific gravities of the globe and fluid, $N-n$ will be the relative gravity of the globe in the fluid, and therefore $w = \frac{1}{6}\pi d^3 (N-n)$ is the weight by which it is urged, $m = \frac{\pi n v^2 d^2}{16 g}$ is the resistance; consequently $\frac{\pi n v^2 d^2}{16 g} = \frac{1}{6}\pi d^3 (N-n)$ when the velocity becomes uniform; whence we obtain

$$v = \left(\surd\, 2g \times \frac{4}{3} d \times \frac{N-n}{n}\right)$$

for the uniform or greatest velocity of the globe.

Thus, for example, if a leaden ball 1 inch in diameter descend in water, and in air of the same density as at the earth's surface; the three specific gravities being, lead $= 11\frac{1}{3}$, water $= 1$, and air $= \frac{2}{2500}$; then

$$v = \surd\left(2 \times 32\tfrac{1}{6} \times \frac{4}{36} \times 10\tfrac{1}{3}\right) =$$

8·5944 feet per second, for the greatest velocity in water; and

$$v = \surd\left(4 + \frac{193}{12} \times \frac{4}{36} \times \frac{34}{3} \times \frac{2500}{3}\right)$$

$=$ 250 82 feet per second, for the greatest velocity in air.

But as this velocity, all other things being the same, varies as $\surd d$; it follows that a ball of $\frac{1}{100}$ of an inch diameter, would only acquire velocities $\frac{1}{10}$th of those given above.

It is obvious, however, that all the preceding results are deduced upon an hypothesis which cannot obtain in real practice; because it supposes first, that the medium in which the body moves, falls in behind the body in motion as fast as this moves forward, and that the body is therefore always in equilibrio with regard to the pressure of the medium, which it is evident cannot be the case except when the velocity is very small. It has moreover no reference to any other resistance than that which arises from the inertia of the particles of the fluid. And farther, these particles are supposed to be so constituted that after the body strikes them, the action of these particles entirely cease; whereas the particles after they are struck must necessarily diverge and act upon other particles behind them. On all of which accounts therefore there must necessarily be considerable difference between the theory and practice.

By means of these principles, Dr. Hutton investigates the height to which a body will ascend that is projected perpendicularly upwards with any given velocity, supposing the air to be uniformly of the same density as at the surface of the earth, from which he finds the greatest height is

$$h = \text{hyp. log.}\ \frac{v^2 - 150\,v + 21090\,d}{21090\,d}$$

$\times\ 330\,d$ where d is the diameter, and v the velocity of projection.

Hence, supposing the ball to be one belonging to the first table of resistances, its weight being 16 oz. 13 dr. or 1·05 lb. and its diameter 1·965 inches; and the velocity of the projection 2000 feet per second; the height to which it will rise is 2920 feet, whereas, without considering the resistance of the air, the projectile theory would give 12 miles for the height due to that velocity.

If the resistance, instead of depending upon the function of the velocity, be taken as the square of the velocity, the expression for the height becomes

$$h = \frac{w}{4ga} \times \text{hyp. log.}\ \frac{a v^2 + w}{w}$$

where w is the weight of the ball $a = \cdot 000025\frac{4}{9}$; g the force of gravity, and $v =$ the velocity of projection.

When the height is given and the time is required, this is found from the formula,

$$t = \frac{1}{29} \times \surd\,\frac{w}{a} \times \text{arc to tang.}$$

$\left(v \div \surd\,\frac{w}{a}\right)$ the letters indicating the same quantities as before.

Solid of Least Resistance.—This is one of the simples of the class of problems, commonly called *isoperimetrical.*

RESOLUTION of *Equations*, in *Algebra*, is the determination of the values of the unknown letters or quantities of which the equation is composed; in order to which, it is necessary, first to exterminate or eliminate all the unknown quantities but one out of the equation, and then the value of the remaining quantity is to be found by the proper rules for this purpose, *viz.* by the rules given for *Simple Quadratic*, *Cubic*, or *Biquadratic Equations*, according to which of these it may belong; or, by the general method of *Approximation.*—But all these cases having reference only to one unknown quantity, it will not be amiss in this place to explain some of those methods which are most commonly employed for reducing equations to this state.

First it may be observed, that in any determinate problem, there are always as many equations independent of each other as there are unknown quantities; if there are not so many, the question is *indeterminate;* and, if there be more, it is *impossible.*

We cannot, in a limited article like the present, give all the methods that may be employed for exterminating the unknown letters, which are extremely various, and depending much upon the practice and proficiency of the analyst himself, and the manner in which the most applicable and general methods of eliminating the unknown quantities are the three following:

1. Find the value of one and the same unknown quantity in each equation, and put all these values equal to each other, which will eliminate one of the quantities, and reduce the number of equations to one less. Then do the same in these new equations; and again in the last, and so on, till there be but one equation and one unknown quantity, the value of which must be found by the proper rules, as above referred to.

2. Find the value of one of the unknown quantities in one of the equations in terms of the other quantities; then substitute this value for that quantity in all the other equations. Again, find the value of one of the remaining quantities, and substitute its value as before, and so on, till there remain but one equation and one unknown quantity, whose value is to be found as before.

3. Multiply each of the equations by such a number as will render the co-efficients of one of the letters the same in all, then by adding or subtracting these equations, according as the equal co-efficients have unlike or like signs, the quantity whose co-efficients were equal will disappear; which being repeated again upon the remaining quantities, there will ultimately be found only one equation and one unknown quantity. If any of the unknown quantities have fractional co-efficients, the whole equations in which they are found should be multiplied by such a number as will convert these fractions into integers.

Thus, in the equations,

$$\begin{cases} \frac{3}{5}x + \frac{1}{15}y = 10 \\ \frac{1}{2}x + \ 3y = 95 \end{cases}$$

Multiply the first by 15 and the latter by 2, gives

$$\begin{cases} 9x + 2y = 150 \\ \ \ x + 6y = 190 \end{cases}$$

The solution of which, by each of the preceding rules, will be as follows.

Or putting letters instead of the above numerical co-efficients, in order to render the solutions more general, let there be given

$$\left.\begin{matrix} ax + by = c \\ dx + ey = f \end{matrix}\right\} \text{to find } x \text{ and } y.$$

1st *Method.*

$$\left.\begin{matrix} ax = c - by \\ dx = f - ey \end{matrix}\right\} \text{by transposition.}$$

$$x = \frac{c - by}{a} = \frac{f - ey}{d} \text{ by division.}$$

$$dc - dby = af - aey \text{ by multip.}$$

$$(ae - db)y = af - dc$$

$$y = \frac{af - dc}{ae - db}$$

And in the same manner we find

$$x = \frac{ec - bf}{ae - db}.$$

2d *Method.*

$$\left.\begin{matrix} ax + by = c \\ dx + ey = f \end{matrix}\right\} \text{to find } x \text{ and } y$$

$$x = \frac{c - by}{a} \text{ as above}$$

$$\frac{dc-dby}{a}+ey=f \text{ by substit.}$$

$$dc-dby+aey=af \text{ by mult.}$$

$$(ae-db)y=\ af-dc$$

$$\left.\begin{array}{l} y=\dfrac{af-dc}{ae-db} \\ x=\dfrac{ec-bf}{ae-db} \end{array}\right\} \text{as before.}$$

3d *Method.*

$$\left.\begin{array}{l} ax+by=c \\ dx+ey=f \end{array}\right\} \text{to find } x \text{ and } y$$

$$dax+dby=dc \text{ mult. by } d$$

$$adx+aey=af \text{ mult. by } a$$

$$(db-ae)y=dc-af$$

$$\left.\begin{array}{l} y=\dfrac{dc-af}{db-ae}=\dfrac{af-dc}{ae-db} \\ x=\dfrac{bf-ec}{db-ae}=\dfrac{ec-bf}{ae-db} \end{array}\right\} \text{as before.}$$

REST, in *Physics*, the continuance of a body in the same place, either absolutely or relatively.

RETARDATION, any force tending to diminish the velocity of moving bodies.

Retardation may arise either from the effect of resistance, or from the action of gravity. For that which arises from the former, and for the distinction between resistance and retardation, see **Resistance.**

Retardation, from *Gravity*, is peculiar to bodies projected upwards, which have their velocities diminished, by precisely the same laws as falling bodies have theirs accelerated.

Thus, if a body be projected perpendicularly upwards, with a velocity which would, independently of gravity, cause it to ascend a feet per second; it will, in consequence of the action of gravity, have its velocity so diminished, that, at the end of the first second, it will be only $(a-32)$ feet, at the end of the second it will be only $(a-64)$ feet, &c.

Hence to find the greatest height to which a body will ascend when projected perpendicularly upwards with any given velocity, the time of ascent, &c. it is only necessary to find the space through which a body must fall to generate that velocity, and the time it would be in descending through that space, which will be precisely the same as the height through which it will ascend, and the time of its ascent.

RETROGRADATION, or **Retrogression**, in *Astronomy*, is an apparent motion of the planets, by which they seem to move backward in the ecliptic, or in *antecedentia*, or contrary to the order of the signs. When a planet moves in consequentia, or according to the order of the signs Aries, Taurus, Gemini, &c. it is said to be *direct*. When it appears for a few successive days in the same place or point of the heavens, it is said to be *stationary*. And when it goes in antecedentia, or contrary to the order of the signs, it is said to be retrograde.

REVERSION of *Series*, in *Algebra*, is the method of finding the value of the root or unknown quantity, whose powers enter the terms of a finite or infinite series, by means of another series in which it does not enter.

Thus, if we have

$$x=ay+by^2+cy^3+dy^4+\&c.$$

and we can find

$$y=Ax+Bx^2+Cx^3+Dx^4+\&c.$$

the original series is said to be inverted.

The reversion of series was first proposed by Newton, in his "Analysis per Equationes Numero terminorum Infinitas," and has since engaged the attention of many of the most profound modern analysts; and accordingly different methods have been suggested for this purpose; but that of M. Arbogast, in his "Calcul. des Derivations," is the most complete, and we regret that the nature of this work will not admit of sufficient detail to enter into an explanation of his notation and operation; the reader, however, will find every necessary information on this head in Woodhouse's "Principles of Analytical Calculation," arts. 72 and 73, to which work he is therefore referred, and we shall merely give here the most simple but less general method of our own algebraists.

This consists in assuming a series of a proper form, for the required unknown quantity, and then substituting the powers of this series, instead of the powers of that quantity in the proposed series, and finally equating the co-efficients, whereby the values of the indeterminate or unknown co-efficient will be obtained; thus

Let $x = a\,y + b\,y^2 + c\,y^3 + \&c.$

Assume $y = \mathrm{A}\,x + \mathrm{B}\,x^2 + \mathrm{C}\,x^3 + \&c.$

then $a\,y = a\,\mathrm{A}\,x + a\,\mathrm{B}\,x^2 + a\,\mathrm{C}\,x^3 + \&c.$

$b\,y^2 = b\,\mathrm{A}^2\,x^2 + 2\,b\,\mathrm{A}\,\mathrm{B}\,x^3 + \&c.$

$c\,y^3 = c\,\mathrm{A}^3\,x^3 + \&c.$

&c. &c.

Now since $x = a\,y + b\,y^2 + c\,y^3 + \&c.$

And this latter series being also found in powers of x, it follows that the sum of all the co-efficients of x must be equal to 1, and the co-efficients of the higher powers equal to 0, which depend upon a known property, *viz.* that if $\alpha\,x + \beta\,x^2 + \gamma\,x^3 + \&c. = 0$, under every possible value of x, then $\alpha = 0$, $\beta = 0$, $\gamma = 0$, &c.

Hence we have
$$a\,\mathrm{A} = 1$$
$$a\,\mathrm{B} + b\,\mathrm{A}^2 = 0$$
$$a\,\mathrm{C} + 2\,b\,\mathrm{A}\,\mathrm{B} + c\,\mathrm{A}^3 = 0$$

Consequently $\mathrm{A} = \frac{1}{a}$, $\mathrm{B} = \frac{-b\,\mathrm{A}^2}{a} = \frac{-b}{a^3}$ $\mathrm{C} = \frac{-c\,\mathrm{A}^3}{a} - \frac{2\,b\,\mathrm{A}\,\mathrm{B}}{a} = \frac{2\,b^2 - a\,c}{a^5}$

Whence
$$y = \frac{1}{a}\,x - \frac{b}{a^3}\,x^2 + \frac{2\,b^2 - a\,c}{a^5}\,x^3, \ \&c.$$

Again, let $x = y - a\,y^3 + b\,y^5 - \&c.$ to find y in terms of x.

Assume $y = \mathrm{A}\,x + \mathrm{B}\,x^3 + \mathrm{C}\,x^5 + \&c.$

then $y = \mathrm{A}\,x + \mathrm{B}\,x^3 + \mathrm{C}\,x^5 + \&c.$

$-a\,y^3 = -a\,\mathrm{A}^3\,x^3 - 3\,a\,\mathrm{A}^2\,\mathrm{B}\,x^5 - \&c.$

$+b\,y^5 = +\,b\,\mathrm{A}^5\,x^5 + \&c.$

And hence $\mathrm{A} = 1$; $\mathrm{B} - a\,\mathrm{A}^3 = 0$; $\mathrm{C} - 3\,a\,\mathrm{A}^2\,\mathrm{B} + b\,\mathrm{A}^5 = 0$.

Consequently $\mathrm{A} = 1$; $\mathrm{B} = a$; $c = 3\,a^2 - b$, &c. therefore $y = x + a\,x^3 + (3\,a^2 - b)\,x^5 + \&c.$

REVERSION, in the doctrine of *Annuities*, is an annuity which is not to commence till after a certain number of years; and its present value is such a sum as, put out to interest, will provide for the several payments of the annuity as they become due; and in order to obtain this value, we must find the present value of each reversionary payment, and the sum of them will be the total present value of the reversion. Or, if we make

$p =$ the present value
$a =$ the annuity
$n = \left\{\text{the number of years the annuity is to continue}\right.$
$\nu = \left\{\text{the number of years before it becomes payable}\right.$
$r = \left\{\text{the } \textit{annual} \text{ rate of interest.}\right.$

Then $p = a \times$
$$\frac{(1+r)^{-\nu} - (1+r)^{-(\nu+n)}}{r}$$

$a = p \times$
$$\frac{r}{(1+r)^{-\nu} - (1+r)^{-(\nu+n)}}$$

$$n = \frac{\text{co. log.}\left\{1 - \frac{p}{a}\,.\ r.\,(1+r)^{\nu}\right\}}{\log.\,(1+r)}$$

$$\nu = \frac{\log.\left\{1 - (1+r)^{-n}\right\} - \log.\,\frac{p\,r}{a}}{\log\,(1+r)}$$

$$r = \frac{\left\{12\,\delta - (n^2 - 1)\,\beta\right\}\cdot\beta}{12\,\delta - 2\,(n^2 - 1)\,\beta}$$

In which last $\delta = 2\,\nu + 1$, and $\beta = \left(\frac{a\,n}{p}\right)^{\frac{2}{\delta}} - 1$

From which theorems any one of those quantities may be found when the other quantities are given. When the reversion is a perpetuity, then n becomes infinite, and the quantity $(1+r)^{-(\nu+n)}$ vanishes out of the general expressions, which then is reduced to

$p = a \times \frac{(1+r)^{-\nu}}{r} =$ present value

$a = p\,r\,(1+r)^{\nu} =$ annuity

$$\nu = \frac{\text{co. log.}\,\frac{p\,r}{a}}{\log.(1+r)} = \text{years deferred}$$

$$r = \frac{\left\{6 + (5\nu + 1)\,\beta\right\}\beta}{6 + 4\,(2\nu + 1)\,\beta} = \textit{ann.}\ \text{rate of interest.}$$

$$\text{whence } \beta = \left(\frac{p + a\,\nu}{p}\right)^{\frac{1}{\nu}} - 1.$$

It should be observed, that these theorems are formed on the supposition of yearly payments. If the payments are to be made quarterly, or half-yearly, then n and ν must represent the number of periods of payment, and r the rate of interest for such period of time.

For the investigation of these theorems, see Baily's "Doctrine of Interest and Annuities;" where this, and every case relating to these important subjects, are very fully and ably illustrated.

REVOLUTION, the motion of a body or line about a centre, which remains fixed.

Period of REVOLUTION, in *Astronomy*, is the time a planet, comet, &c. employs in passing from any point in its orbit to the same point again. This, with regard to the earth, is what determines the length of the year.

RHUMB *Line*, or *Loxodromia*, in *Navigation*, is a line prolonged from any point in a sea-chart, except in the direction of any of the four cardinal points, or it is the line described by a ship while her course is constantly directed towards one and the same point of the compass, except the four above mentioned, that is, while she crosses all the meridians at the same angle, providing this is not a right one, and this angle is called the angle of the rhumb; and that which it makes with the equator, or a parallel to the equator, is called the complement of the rhumb. If a vessel sail either north or south, it evidently describes a great circle of the sphere, or part of such a circle, and if her course is either due-east or west, she cuts all the meridians at right angles. But if her course is oblique to these principal points, then she no longer describes a circle, but a sort of spiral, the characteristic property of which is, that it cuts all the meridians at the same angle, and is thence denominated the loxodromia, or loxodromic curve, or rhumb line, which, though it continually approaches towards the pole, can never arrive at it, except after an infinite number of revolutions. A ship's way is, therefore, not to be computed, as if her course was made in a great circle of the sphere, but as made up of the successive arcs of this spiral.

RICOCHET *Firing*, in the practical part of gunnery, is a method of firing with small charges, and at small degrees of elevation, *viz.* from 3° to 6°, in consequence of which the ball is constantly bounding and rolling along, and thus destroying more men than that which has a charge sufficient for penetrating a column.

RING of *Saturn*, in *Astronomy*, is a broad opaque circular body encompassing the equitorial regions of that planet, at a considerable distance from him; which presents, under favourable circumstances, one of the finest telescopic objects in the heavens. An apparent irregularity was first observed in the form of Saturn by Galileo, but his telescope was not sufficiently powerful for him to discover the cause of it; this, however, was soon after effected by Huygens, who in consequence published his "New Theory of Saturn," in 1659.

This ring, which is very thin, not exceeding 4500 miles, is inclined to the plane of the ecliptic in an angle of 31° 19′ 12″; and revolves from west to east in $10^{h}\ 29^{m}\ 16''.8$, being nearly the time of the diurnal revolution of Saturn, and which is also found, from the laws of Kepler, to be the time in which a satellite would revolve about that planet at the mean distance of the ring; a very remarkable confirmation of the universality of the laws of the planetary motions. This rotation is performed about an axis perpendicular to the plane of the ring, and passing through the centre of the planet.

The ring being, as we observed above, very thin, it sometimes nearly disappears, that is, when its plane coincides with, or passes through the centre of the earth or sun, at which time it subtends an angle of not more than half a second, and can therefore only be discovered by the most powerful telescopes, through which it has then the appearance of a luminous line beyond the body of the planet. And as this plane is presented to the sun twice during each sidereal revolution of the planet, the disappearance of the ring will happen about every 15 years, and at nearly the same intervals it will appear to the greatest advantage.

When viewed in the most favourable position, with a magnifying power of 700, the ring is observed to be divided into two unequal portions, by a black cocentric line, which is now ascertained to be a real separation, and that what we call the ring of Saturn consists at least of two rings; and some astronomers have even supposed it to be still farther subdivided, and to consist of several circular parts, but this at present is little more than conjecture.

The dimensions of this double ring, as given by Dr. Herschel, are as follows:

	Eng. Miles
Diameter of the planet	76068
Inside diam. smaller ring	146345
Outside	184393
Inside diam. larger ring	190248
Outside	204883
Breadth of inner ring	20000
——— of outer ring	7200
Space between the rings	2839
——— between planet & ring	70277
Mean thickness of ring	4500

The intersection of the plane of the ring with the ecliptic is in 5^{s} 20°, and 11^{s} 20°, in which points, therefore, it disappears, and between these, viz. at 2^{s} 20°, and 8^{s} 20°, it appears most brilliant.

RISING, in *Astronomy*, the first appearance of the sun, moon, or other celestial body above the horizon.

RIVER, in *Geography*, a stream or current of fresh water flowing in a bed or channel from its source or spring into the sea.

The doctrine which relates to the flux, reflux, motion, and discharge of rivers, is a branch of hydraulics, and, as such, forms a part of the present work, though our limits will only admit of a slight sketch of the theory. Water running in open canals or rivers is accelerated in consequence of its depth, and of the declivity on which it runs, till the resistance increasing with the velocity, becomes equal to the acceleration, when the motion of the stream becomes uniform. But this resistance, it is obvious, can only be determined by experiment, and hence several philosophers have undertaken different courses of experiments for this purpose, amongst whom Buat seems to have met with the most complete success.

Let V represent the velocity of the stream per second in inches; R the quotient arising from the division of the section of the stream by its perimeter, *minus* the superficial breadth, all in inches; and S the cotangent of the inclination of the slope. Then the section and velocity being both supposed uniform,

$$V = \frac{307\sqrt{(R-\frac{1}{10})}}{S^{\frac{1}{2}} - \frac{1}{2}\,\text{hyp. log.}\,(S+\frac{16}{10})} - \frac{3}{10}\sqrt{\left(R-\frac{1}{10}\right)};\ \text{or}$$

$$V = \sqrt{\left(R-\frac{1}{10}\right)} \times \left\{\frac{307}{S^{\frac{1}{2}} - \frac{1}{2}\,h.\,l.\,(S+\frac{16}{10})} - \frac{3}{10}\right\}$$

which, when R is very great, and *s* small, may be reduced to

$$V = R^{\frac{1}{2}} \times \left\{\frac{307}{S^{\frac{1}{2}} - \frac{1}{2}\,h.\,l.\ S} - \frac{3}{10}\right\}$$

From which it appears that when the slope remains the same, the velocity varies as $\sqrt{(R-\frac{1}{10})}$, or as $\sqrt{R}$, when R is very great. Hence the velocity of two great rivers of the same declivity are as the square root of $\frac{bd}{b+2d}$, where *b* and *d* represent the breadth and depth of a transverse section in inches.

It follows also, from what is said

above, that if $R = \frac{1}{10}$, or is less than that, the velocity is zero, which agrees with the theory of capillary attraction; also the slope may be so small that the other factor may become zero, or

$$\frac{307}{S^{\frac{1}{2}} - \frac{1}{2}\,\mathrm{h.\,l.}\,(S + \frac{16}{10})} - \frac{3}{10} = 0$$

in which case likewise there will be no motion; this, however, can never happen, if the declivity be not less than $\frac{1}{10}$th of an inch in an English mile, as this will produce a sensible motion in the water.

In a river the greatest velocity is at the surface, and in the middle of the stream, from which it diminishes towards the bottom and sides, where it is least; and it has been found by experiment, that if $v =$ the velocity of the stream in the middle, in inches, then $v - 2\sqrt{v} + 1$, is the velocity at the bottom. Prin. d'Hydraul. par Du Buat, art. 67.

The mean velocity, or that with which (were the whole stream to move) the discharge would be the same as the real discharge, is equal to half the sum of the greatest and least velocities, as computed by the above formulæ. Hence the mean velocity is

$$v - \sqrt{v} + \tfrac{1}{2}.$$

Suppose that a river, having a rectangular bed, is increased by the junction of another river equal to itself, the declivity remaining the same; required the increase of depth and velocity.

Let the breadth of the river equal b, the depth before the junction d, and after it x; the velocity before v, and after it v'; then the quantity denoted by R, in the preceding formula, is $R = \frac{b\,d}{b + 2\,d}$ before the junction, and $R' = \frac{b\,x}{b + 2\,x}$ after the junction.

So $v = \frac{307\,R^{\frac{1}{2}}}{S^{\frac{1}{2}}}$, and $v' = \frac{307\,R'^{\frac{1}{2}}}{S^{\frac{1}{2}}}$, that is, by supposing the breadth of the river to be such, that we may reject the small quantity subtracted from R, R', in the above formulæ; whence substituting for R and R', we have

$$v = \frac{307}{S^{\frac{1}{2}}} \times \sqrt{\frac{b\,d}{b + 2\,d}}, \text{ and}$$

$$v' = \frac{307}{S^{\frac{1}{2}}} \times \sqrt{\frac{b\,x}{b + 2\,x}}.$$

Now multiplying these into the areas of the section $b\,d$, $b\,x$, we have the discharges; viz.

$$b\,d\,v = \frac{307}{S^{\frac{1}{2}}} \times b\,d\,\sqrt{\frac{b\,d}{b + 2d}}, \text{ and}$$

$$b\,x\,v' = \frac{307}{S^{\frac{1}{2}}} \times b\,x\,\sqrt{\frac{bx}{b + 2\,x}}.$$

Now the last of these is double the former; therefore

$$\frac{\overline{b\,x}|^{\frac{3}{2}}}{\sqrt{(b + 2\,x)}} = \frac{2\,\overline{b\,d}|^{\frac{3}{2}}}{\sqrt{(b + 2d)}}, \text{ or}$$

$$\frac{b^3\,x^3}{b + 2\,x} = \frac{2\,b^3\,d^3}{b + 2\,d}; \text{ whence}$$

$$x^3 - \frac{3\,d^3}{b + 2d}\,x = \frac{4\,b\,d^3}{b + 2\,d}$$

from which the equation x may be determined.

As an example, let $b = 10$ feet, $d = 1$, then

$$x^3 - \frac{2}{3}x = \frac{10}{3}, \text{ and } x = 1{\cdot}4882,$$

which is the depth of the increased river; hence we have $1{\cdot}488 \times v' = 2\,v$, and $1{\cdot}488 : 2 = v : v'$, or $v : v' = 37 : 50$ nearly.

When the water in the river receives a permanent increase, the depth and velocity, as in the example above, are the first quantities that are augmented, the increase in the velocity increases the action on the sides and bottom; in consequence of which the width is augmented, and sometimes, though rarely, the depth also. The velocity is thus diminished till the tenacity of the soil, or the hardness of the rock, affords a sufficient resistance to the force of the water. The bed of the river then changes only by insensible degrees, and is said to be permanent; though, in strictness, this is not applicable to the course of any river. When the sections of a river vary, the quantity of water remaining the same, the mean velocities are inversely as the areas of the sections; this being necessary to preserve a uniform discharge. Playfair's Outlines of Natural Philosophy.

ROD, or *Pole* a long measure of,

16½ linear feet, or a square measure of 272¼ square feet.

ROLLING, that motion of a body which is caused by its rectilinear motion being resisted, by the friction of some surface or otherwise; whereby its several parts come successively in contact with that plane or surface; such is the motion of a carriage-wheel upon the ground, &c.

ROOT, in *Arithmetic* and *Algebra*, denotes a quantity, which being multiplied a certain number of times into itself, produces another number, called a power, and of which power the original quantity is called the root. Roots are distinguished into square roots, cube roots, biquadratic roots, &c. or into 2d, 3d, 4th, 5th, &c. roots, which depends upon the number of multiplications necessary to generate the proposed power. If one multiplication only is necessary, or if two equal factors are multiplied together, it is called the square or second root; if three, the cube or third root; if four, the biquadratic or fourth root, &c.; thus

8 is the square root of 64
4 is the cube root of 64
2 is the sixth root of 64
&c. &c.

For the extraction of the roots of numbers. See *Extraction*.

ROOTS of an *Equation*, are those numbers or quantities which, substituted for the unknown quantity, render the whole equation equal to zero.

And of these there are always as many real or imaginary, as there are units in the highest power of the unknown quantity. So an equation of the 2d degree has two roots; one of the 3d degree, three; of the 4th degree, four, &c. See *Equations*.

The roots of an equation are either *positive*, *negative*, or *imaginary*.

A *Positive* ROOT, is an absolute number affected with the sign + *plus*.

A *Negative* ROOT, is also an absolute number, but is affected with the sign — *minus*.

An *Imaginary* ROOT, is one to which no absolute value can be attached, one part of it consisting always of the square root of a negative quantity; yet it is such, that when substituted for the unknown quantity, it is found to answer the conditions of the equation. See *Imaginary Roots* and *Quantities*.

ROTA *Aristotelica*, or *Aristotle's Wheel*, denotes a problem in mechanics proposed by Aristotle concerning the motion of a coach-wheel; viz. that the nave of a wheel describes by its motion, (supposing it to roll along a plane) a line of the same length as the circumference, by its motion on the ground; which was long considered paradoxical, nor was it clearly understood till M. Meyran, a Frenchman, sent a satisfactory solution of it to the Academy of Sciences, the principle of which is, that each point of the circumference of the nave, as it approaches the plane, is drawn forward over a space greater than itself, whereas every point and part of the circumference passes over a space exactly equal to itself.

ROTATION, the motion of the different parts of a solid body about an axis, called the *axis of rotation*, being thus distinguished from the progressive motion of a body about some distant point or centre; thus the diurnal motion of the earth is a motion of rotation, but its annual motion one of revolution. When a solid body turns round an axis, retaining its shape and dimensions unaltered, every particle is actually describing a circle round this axis, which axis passes through the centre of the circle, and is perpendicular to its plane. Moreover in any instant of the motion, the particle is moving at right angles with the radius vector, or line joining it with its centre of rotation; therefore, in order to ascertain the direction of any particle, we may draw a line from that particle perpendicular to the axis of rotation. This line will be in the plane of the circle of rotation of that particle, and will be its radius vector, and a line drawn from the particle perpendicular to its radius vector, will be a tangent to the circle of rotation, and will represent the direction of the motion of this particle.

The whole body being supposed to turn together, it is evident, that when it has made one complete rotation, each point has described the circumference of a circle, and the whole paths of the different particles will be in the ratio of these circumferences; and therefore of their radii, and this is also true of any portion of such circumferences; that is, the velocities of the different particles are proportional to their radii vectores, or to their distances from the axis of rotation. And all these motions are in parallel planes, to which the axis of rotation is perpendicular. Hence it follows, that when we compare the rotation of different bodies in respect of velocity, it is evident that it cannot be done by directly comparing the velocity of any particle in one of the bodies with that of any particle of the other; for as all the particles of each have different velocities, this comparison can establish no ratio. But we may familiarly compare such motions by the number of complete turns which they make in any equal portions of time. Therefore, as the length or number of feet described by a body in rectilinear motion is a proper measure of its progressive velocity, so the angle described by any particle of a whirling body, is a proper measure of its velocity of rotation; and in this manner may the rotation of two or more bodies be compared, and this velocity is, with propriety, called the angular velocity.

In what is stated above we have had principally in view, a fixed and permanent axis of rotation, the body not being supposed at liberty to revolve about any other; but it is obvious, that if any force is impressed upon a body, or system of bodies, in free space, (unless that force be exerted in a direction passing through the centre of gravity of the system) a rotatory motion will ensue about an axis passing through the centre of gravity of the system; and the centre, about which this motion is performed, is called the *centre* of *spontaneous rotation*. A body may begin to revolve on any line as an axis that passes through its centre of gravity, but it will not continue to revolve permanently about that axis, unless the opposite centrifugal forces exactly balance each other.

Thus a homogeneous sphere may revolve permanently on any diameter, because the opposite parts of the solid, being in every direction equal and similar, the opposite centrifugal forces must be equal; so that no force has a tendency to change the position of the axis. Hence also a homogeneous cylinder may revolve permanently about the line, which is its geometric axis; as it may also about any line that bisects that axis at right-angles, but it can revolve permanently about no other line, because then the centrifugal forces could not be equal, and the same is true with respect to any solid of rotation.

In every body, however irregular, there are three permanent axis of rotation, at right-angles to each other, on any one of which, when the body revolves, the opposite centrifugal force exactly balances, and therefore the rotation becomes permanent. These three axes have also this remarkable property, that the momentum of inertia, with respect to any of them, is either a maximum or a minimum; that is, is either greater or less than if the body revolved about any other axis.

At present we have considered those cases of rotation that are produced by a force impressed upon a body, either as supported on a fixed axis, about which, therefore, that system must necessarily revolve, or as in free space, in which case the system acquires a spontaneous centre of rotation, and finally, a permanent axis of rotation; but there are other circumstances which will produce a rotatory motion that are not included in either of the above, but which it will be proper to mention before we conclude this article; such are those which arise from a body descending down an inclined plane, having a ribbon or cord wound about it, one end of which is fixed at the upper part of the plane, which by preventing the body sliding freely causes a rotatory motion; the same effect also follows from the friction of the body against the plane; and the same may be imagined when there is no plane but the body left to fall freely, except so far as the cord wound about it shall produce a rotatory motion in its descent.

The force by which spheres, cylinders, &c. are caused to revolve, as they move down an inclined plane (instead of sliding), is the adhesion of their surfaces, occasioned by their pressure against the plane; this pressure is part of the weight of the body, for this weight being resolved into its component parts, one in the direction of the plane, the other perpendicular to it; the latter is the force of the pressure; and which, while the same body rolls down the plane, will be expressed by the cosine of the plane's elevation. Hence, since the cosine decreases, while the arc or angle increases; after the angle of elevation arrives at a certain magnitude, the adhesion may become less than what is necessary to make the circumference of the body revolve fast enough, and in this case it will proceed partly by sliding and partly by rolling; but the angle, at which this circumstance takes place, will evidently depend upon the degree of adhesion between the surfaces of the body and plane. This, however, will never happen, if the rotation is produced by the unwinding of a ribbon, and it is on this latter supposition that the following particular cases are deduced.

Let W be the weight of the body, s the space descended by a heavy body, falling freely, or sliding freely down a plane, then the spaces described by rotation in the same time, by the following bodies, will be in these proportions.

1. A hollow cylinder, or cylindrical surface $S = \frac{1}{2}s$, tension $= \frac{1}{2}W$.
2. A solid cylinder $S = \frac{2}{3}s$, tention $= \frac{1}{3}W$
3. A spheric surface $S = \frac{3}{5}s$, tension $= \frac{2}{5}W$.
4. A solid sphere $S = \frac{5}{7}s$, tension $= \frac{2}{7}W$.

RULE *of Three*, in *Arithmetic*, called by some authors the *Golden Rule*, is an application of the doctrine of proportion to arithmetical purposes, and is divided into two cases, *simple* and *compound*; now frequently termed *Simple*, and *Compound Proportion*.

Sliding RULE, a mathematical instrument serving to perform computations in gauging, measuring, &c. without the use of compasses, merely by the sliding of the parts of the instrument one by another, the lines and divisions of which give the answer or amount by inspection.

This instrument is variously contrived and applied by different authors, particularly Gunter, Partridge, Hunt, Everard, and Coggeshall, but the more usual and useful ones are those of the two latter.

Everard's Sliding RULE, is chiefly used in cask gauging. It is commonly made of box, 12 inches long, 1 inch broad, and $\frac{6}{10}$ of an inch thick. It consists of three parts, *viz.* the stock just mentioned, and two thin slips of the same length, sliding in small groves in two opposite sides of the stock; consequently, when both these pieces are drawn out to their full extent the instrument is 3 feet long.

On the first broad face of the instrument are four logarithmic lines in numbers; for the properties, &c. of which, see GUNTER'S *Line*. The first, marked A, consisting of two radii 1, 2, 3, 4, 5, 6, 7, 8, 9, 1; and then 2, 3, 4, 5, &c. to 10. On this line are four brass centre pins, two in each radius; one in each of them being marked M B, for malt bushel, is set at 215042, the number of cubic inches in a malt bushel; the other two are marked with A, for ale gallon, at 282, the number of cubic inches in an ale gallon. The 2d and 3d lines of numbers are on the sliding pieces, and are exactly the same with the first; but they are distinguished by the letter B. In the first radius is a dot marked S 1, at ·707, the side of a square inscribed in a circle whose diameter is 1. Another dot, marked *Se.* stands at ·886, the side of a square equal to the area of the same circle. A third dot, marked W, is at 231, the cubic inches in a wine gallon. And a fourth, marked C, at 314, the circumference of the circle, whose diameter is 1. The fourth line of numbers, marked M D, to signify malt depth, is a broken line of two radii, numbered 2, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 9, 8, 7, &c.; the number 1 being set directly against M B on the first radius.

On the second broad face, marked *c d*, are several lines: as, 1st, a line marked D, and numbered 1, 2, 3, &c. to 10. On this line are four centre pins; the first marked W G, for wine gauge, is at 17·15, the gauge point for wine gallons, being the diameter of a cylinder whose height is one inch, and content 231 cubic inches, or a wine gallon. The second centre pin, marked A G, for ale gauge, is at 18·95, the like diameter for an ale gallon. The third mark, M S, for malt square, is at 46·3, the square root of 2150·42, or the side of a square whose content is equal to the number of inches in a solid bushel. And the fourth, marked M R, for malt-round, is at 52·42, the diameter of a cylinder or bushel, the area of whose base is the same 2150·42, the inches in a bushel. 2dly, Two lines of numbers on the sliding piece, on the other side marked C. On these are two dots, the one marked *c*, at ·0795, the area of a circle whose circumference is 1; and the other marked *d*, at ·785, the area of the circle whose diameter is one. 3dly, Two lines of segments, each numbered 1, 2, 3, to 100, the first for finding the ullage of a cask, taken as the middle frustrum of a spheroid, lying with its axis parallel to the horizon; and the other for finding the ullage of a cask standing.

Again, on one of the narrow sides noted *c*, are 1st, a line of inches, numbered 1, 2, 3, &c. to 12, each subdivided into 10 equal parts. 2dly. A line by which, with that of inches, we find a mean diameter for a cask in the figure of the middle frustrum of a spheroid; it is marked spheroid, and numbered 1, 2, 3, &c. to 7. 3dly. A line for finding the mean diameter of a cask, in the form of the middle frustrum of a parabolic spindle, which gaugers call the second variety of casks; it is therefore marked second variety, and is numbered 1, 2, 3, &c.

4thly. A line by which is found the mean diameter of a cask of the third variety, consisting of the frustrums of two parabolic conoids, abutting on a common base, it is therefore marked third variety, and is numbered 1, 2, 3, &c.

On the other narrow face, marked *f*, are, 1st, a line divided into one hundred equal parts, marked F M. 2ndly. A line of inches, like that before mentioned, marked I M. 3dly. A line for finding the mean diameter of the fourth variety of casks, which is formed of the frustrums of two cones, abutting on a common base. It is numbered 1, 2, 3, &c. and marked F C, for frustrum of a cone.

On the back side of the two sliding pieces is a line of inches, from 12 to 36, for the whole extent of the 3 feet, when the pieces are put endways; and against that, the correspondent gallons, and 100th parts, that any small tub or the like open vessel will contain at 1 inch deep.

For the various uses of this instrument, see the authors mentioned above, and most writers on gauging.

Coggeshall's Sliding Rule, is chiefly used in measuring the superfices and solidity of timber, masonry, brick-work, &c.

This consists of two parts, each a foot long, which are united together in various ways. Sometimes they are made to slide by one another like glazier's rulers: sometimes a groove is made in the side of a common two-foot rule, and a thin sliding piece on one side, and Coggeshall's lines added on that side; thus forming the common or carpenter's rule: and sometimes one of the two rulers is made to slide in a groove made in the side of the other.

On the sliding side of the rule are four lines of numbers, three of which are double, that is, are lines of two radii, and the fourth is a single broken line of numbers. The first three, marked A, B, C, are figured, 1, 2, 3, &c. to 9; then 1, 2, 3, &c. to 10; the construction and use of them being the same as those on Everard's sliding rule.

The single line, called the girt line, and marked D, whose radius is equal to two radii of any of the other lines, is broken for the easier measuring of timber, and figured 4, 5, 6, 7, 8, 9, 10, 20, 30, &c. From 4 to 5 it is divided into 10 parts, and each 10th subdivided into 2, and so on from 5 to 10, &c.

On the back side of the rule are, 1st, a line of inch measure, from 1 to 12, each inch being divided and subdivided

2dly. A line of foot measure, consisting of one foot divided into 100 equal parts, and figured 10, 20, 30, &c.

The back side of the sliding piece is divided into inches, halves, &c. and figured from 12 to 24; so that when the slide is out there may be a measure of 2 feet.

In the carpenter's rule the inch measure is on one side continually all the way from 1 to 24, when the rule is unfolded and subdivided into 8th or half-quarters; on this side are also some diagonal scales of equal parts. And upon the edge, the whole length of two feet is divided into 200 equal parts, or 100ths of a foot.

S.

SAILING, in *Navigation*, denotes the act of conducting a vessel from one port to another, by means of the action of the wind upon her sails, being otherwise expressed by the more significant term navigating.

SAILING is distinguished into different cases, according to the principles upon which the computations are founded, as *Plane Sailing*, *Middle Latitude Sailing*, *Mercator Sailing*, *Globular Sailing*, &c.

Plane SAILING, is that which is performed on a supposition of the earth being an extended plane surface, and by means of plane charts, in which case the meridians are considered as parallel lines, the parallels of latitude at right angles to the meridians, and the lengths of the degrees on the meridians, equator, and parallels of latitude, as every where equal. Here the principal terms are the latitude distance, and departure; difference of latitude and rhumb, longitude having no place in plane sailing. It is obvious, however, that calculations conducted on these principles must be too erroneous to be depended upon in any case, and therefore it would be but wasting the reader's time to enter farther into an explanation of this case, which is now nearly if not wholly disused by navigators.

Traverse SAILING, may be defined compound plane sailing, being the method of working, or calculating traverse or compound courses, so as to reduce them into one. This is used when a ship having to sail from one port to another is, by reason of contrary winds, or other obstacles, obliged to tack and sail upon different courses, which are then to be brought into one; and hence the difference of latitude, departure, and other circumstances determined as in plane sailing.

Globular SAILING, is the method of estimating a ship's motion and run, upon principles drawn from the globular figure of the earth. In this, its most extended sense, globular sailing comprehends *Parallel Mercator*, *Middle Latitude*, and *Great Circle Sailing*; for a definition of each see the following articles.

Parallel SAILING, is the sailing on a parallel of latitude, or parallel to the equator, of which there are three cases.

1. Given the distance and difference of longitude; to find the latitude, which is performed by the following rule:

As the difference of long. = the distance = the radius : the cosine of the latitude.

2. Given the latitude and difference of longitude; to find the distance. Rule,

As radius : the cosine of the latitude = the difference of longitude : the distance.

3. The latitude and distance being given, to find the difference of longitude. Rule,

As cosine of latitude : radius = the distance : the difference of longitude.

Middle Latitude SAILING, is a method of resolving the cases of globular sailing, by means of the middle latitude between that departed from and that come to. This method is not accurate, being founded on the principles of plane and globular sailing conjointly; *viz.* on a supposition that the departure is reckoned as a meridional distance in that latitude, which is the middle parallel between the latitude sailing from, and the latitude come to; which would be correct if the cosine of a middle latitude was an arithmetical mean between the cosine of two extreme latitudes; and the departure between two places on an oblique rhumb, equal to the meridional distance in the middle latitude; but neither of these cases obtain. Yet when the parallels are near the equator, or near to each other, in any latitude, the error is not considerable.

This method seems to have been invented on account of the easy manner in which the several cases may be resolved by the traverse table, and when a table of meridional parts is not at hand, the computations may be made as follows; *viz.*

Take half the sum of the two given latitudes for the middle latitude, then say,

1. As cosine of mid. lat. : the radius = the departure : diff. of longitude.
2. As cosine of mid. lat.: tan. of course = diff. of lat. : diff. of longitude.

Mercator SAILING, or more properly *Wright's Sailing*, is the method of computing the cases of sailing on the principles of Mercator's chart, which principles were first laid down by Wright in the beginning of the 17th century. These consist in finding on a plane the motion of a ship upon any assigned course that shall be true, as well in longitude and latitude, as in distance, the meridians being all parallel, and the parallels of latitude straight lines.

Then, from the similarity of triangles, when three,

From the following analogies:

Rad. : sin. course = dis. : depart.

Rad. : cosin. course = dist. : dif. of lat.

Rad. : tan. course = merid. dif. lat. : dif. lon. any of the cases of Mercator sailing may be readily resolved.

Circular SAILING, or *Great Circle* SAILING, is the finding what places a ship must go through, and what courses to steer, that her path may be in an arc of a great circle on the globe, or nearly so, passing through the place sailed from, and that bound to.

As the solutions of the cases of Mercator sailing are performed by plane triangles, in this method they are resolved by means of spherical triangles, and present a great variety of cases; but the following is that which most commonly occurs, *viz.*

Given the latitude and longitude of two cases, to find their nearest distance on the surface, with the angles of position from either place to the other.

This problem involves six cases, *viz.* 1. when the places lie under the same meridian; 2. when they are both under the equator; 3. when only one of them is under the equator; 4. when they are both under the same parallel of latitude; 5. when the places are both on the same side of the equator; and 6. when they are on different sides; these cases, it is obvious, are all solved by means of the different cases of spherical trigonometry, and which it would therefore be useless to repeat in this place.

Spheroidical SAILING, is computing the cases of navigation, on the principles of the true spheroidical figure of the earth; on which subject see also Robertson's "Navigation," vol. ii. book 8. sect. 8.

Sailing is also used in a different sense, to denote the theory and practice of manœuvring of vessels, *viz.* as to the best position of the sails and rudder, their effect on the motion of the ship, &c. &c. on which subject we have several learned treatises by Bernouilli, Euler, Bouguer, Borelli, Juan, &c.

SATELLITE, in *Astronomy*, certain secondary planets moving round the primary planets, as the moon does about the earth; they are thus called because always found attending them, from rising to setting, and making the tour of the sun together with them.

The satellites, in their motion round their primaries, are governed by the same laws, as these are in their revolution round the sun; *viz.* they describe equal areas in equal times, and those belonging to the same planet have the squares of their periodic times, proportional to the cubes of their mean distances from the planet.

The number of satellites at present known in our system is eighteen, *viz.* the moon, the satellite of the earth; four belonging to Jupiter; seven to Saturn; and six to Uranus: but of these only the moon is visible to the naked eye, for the particulars of which see Moon.

Satellite of *Jupiter*. By the aid of a telescope we may discover four satellites revolving about this planet; the sidereal revolution of which, with their mean distances, &c. are given in the following table, as extracted from Laplace's "System du Monde," 3d edition.

Satellite.	Sidereal Revolution.		Mean Distance.	Mass.
I.	1^d 18^h $27'$ $33''$,5	1^d 769137788148	5·812964	·0000173281
II.	3 13 13 42 ,0	3 551181017849	9·248676	·0000232355
III.	7 3 42 33 ,4	7 154552783970	14·752401	·0000884972
IV.	16 16 31 49 ,7	16 688769707084	25·946860	·0000426591

First Satellite. The inclination of the orbit of this satellite does not differ much from the plane of Jupiter's orbit. Its eccentricity is insensible.

Second Satellite. The eccentricity of the orbit of this satellite is also insensible. The inclination of its orbit to that of its primary is variable, as well as the position of its nodes.

Third Satellite. This satellite has a little eccentricity, and the line of its apsides has a direct but variable motion; the eccentricity itself is also subject to very sensible variations. The inclination of its orbit to that of Jupiter, and the position of its nodes, are far from being uniform.

Fourth Satellite. The eccentricity of this satellite is greater than that of any of the other three, and the line of the apsides has an annual and direct motion of 42′ 58″ ,7. The incination of its orbit, with the plane of Jupiter's orbit, forms an angle of about 2° 25′ 48″; but this angle, although stationary about the middle of the last century, has lately begun to increase very sensibly. At the same time the motion of its nodes has begun to diminish.

The motions of the first three satellites are related to each other by a most singular analogy. For the mean sidereal or synodical motion of the first, added to twice that of the third, is constantly equal to three times the mean motion of the second. And the mean sidereal or synodical longitude of the first, minus three times that of the second, plus twice that of the third, is always equal to two right angles.

The satellites of Jupiter are liable to be eclipsed by passing through his shadow; and on the other hand, they are frequently seen to pass over his disc and eclipse a portion of his surface. This happens to the first and second satellite at every revolution; the third very rarely escapes in each revolution; but the fourth, (on account of its great distance and inclination) is seldom obscured.

These eclipses are of great utility in enabling us to determine the longitude of places by their observation; and they likewise exhibit some curious phenomena with respect to light.

From the singular analogy above alluded to, it follows that (for a great number of years at least) the first three satellites cannot be eclipsed at the same time; for in the simultaneous eclipses of the second and third, the first will always be in conjunction with Jupiter, and *vice versa*.

SATELLITES of *Saturn*. Seven satellites may be seen, by means of the telescope, to revolve about Saturn, the elements of which are but little known, on account of their great distance.

The orbits of the first six satellites appear to be in the plane of Saturn's ring; whilst the seventh varies from it very sensibly.

SATELLITES of *Uranus*. Six satellites revolve round Uranus; which, together with their primary, can be discovered only by the telescope.

All these satellites move in a plane which is nearly *perpendicular* to the plane of the planet's orbit, and contrary to the order of the signs.

SATURN, one of the primary planets, being the sixth in order of distance from the sun, of the old planets, but the tenth including the four new ones, being the outermost of all, except Uranus; and is marked with the character ♄, denoting an old man supporting himself with a staff, representing the ancient god Saturn.

Saturn shines with but a feeble light on account of his great distance, and his dull red colour, which latter is supposed to arise from a peculiarly dense atmosphere.

This planet is perhaps one of the most engaging objects that astronomy presents to our view, being surrounded by a double ring and seven satellites, all visible in a telescope of sufficient magnifying power, and which, in certain situations, present one of the most beautiful objects it is possible to conceive.

Saturn has also certain obscure zones, or belts, appearing at times across his disc, like those of Jupiter, which are changeable, and are probably obscurations in his atmosphere, which Dr. Herschel, Phil. Trans. 1790, has shown to be very dense. He revolves about an axis, which is perpendicular to the plane of the ring, and his figure is, like the other planets, an oblate spheroid, being considerably flattened at both poles.

Saturn performs his *sidereal revolution* in $10758^d\ 23^h\ 16'\ 34''$,2, or in 29·456 Julian years. But this period is subject to some inequalities. His mean synodical revolution is performed in about 378 days.

His *mean distance* from the sun is 9·539; that of the earth being considered as unity, which makes his mean distance above 890 millions of miles.

The *eccentricity* of his orbit is ·0562; half the major axis being taken as unity.

His *mean longitude* at the commencement of the present century, was in $4^s\ 15°\ 20'\ 31''$,5.

The longitude of his *perihelion* was at the same time in $2^s\ 29°\ 8'\ 57''$,9; but the line of the apsides has an apparent motion in longitude, according to the order of the signs of $1'\ 9''$,5 in a year, or $1°\ 55'\ 47''$,1 in a century.

His orbit is *inclined* to the plane of the ecliptic in an angle of $2°\ 20'\ 38''$,1; which is observed to decrease about $15''$,5 in a century.

His orbit, at the commencement of the present century, crossed the ecliptic in $3^s\ 21°\ 55'\ 40''$,5; but the place of the *nodes* has an apparent motion in longitude, according to the order of the signs, of $27''$,4 in a year, or $45'\ 43''$,5 in a century.

The *rotation* on his axis is performed in $10^h\ 16'\ 19''$,2: and the *axis* is inclined in an angle of $58°\ 41'$ to the plane of the ecliptic.

His mean diameter is 76068 miles; consequently he is nearly 1000 times as large as our earth. The axis of his poles is to his equatorial diameter as 11 to 12.

His *mass*, compared with that of the sun, considered as unity, is $\frac{1}{3334.02}$; but his *density* is ·45, that of water being one.

The proportion of *light* and *heat* received from the sun is ·011; that received by the earth being considered as unity.

As viewed from the earth, the motion of Saturn sometimes appears *retrograde*. The mean arc which he describes in this case is about 6° 18′; and its duration is nearly 139 days. This retrogradation commences, or finishes, when the planet is distant about 108° 54′ from the sun.

His mean *apparent diameter* is $17'',6$.

SCALE, in *Arithmetic*, is used to denote any particular division of numbers into periods; thus we have the binary, ternary, quaternary, &c. scale. See *Notation*.

SCALE, is also the name given to certain, mathematical instruments, consisting of several lines drawn on wood, brass, silver, &c. Such are the Diagonal Scale, Gunter Scale, Plain Scale, &c.

Diagonal **SCALE**, is projected thus: first draw eleven parallel lines at equal distances, the whole length of which being divided into a certain number of equal parts, according to the length of the scale, by lines parallel to its length, let the first division be again subdivided into ten equal parts, both above and below; then drawing the oblique lines from the first perpendicular below, to the first subdivision above, and from the first subdivision below to the second subdivision above, &c. the first space shall thereby be exactly divided into one hundred equal parts; for as each of these subdivisions is one tenth part of the whole first space or division, so each parallel above it is one-tenth of such subdivision, and, consequently, one hundredth part of the whole first space; and if there be ten of the larger divisions, one thousandth part of the whole scale. If, therefore, the larger divisions be accounted units, the first subdivisions will be tenth parts of an unit; and the second subdivisions, marked by the diagonals on the parallels, hundredth parts of a unit. Again, if the larger divisions be reckoned tens, the first subdivisions will be units, and the second subdivisions tenth parts; and if the larger divisions be accounted hundredths, the first subdivisions will be tens, and the second units, and so on.

The method of constructing the scales of chords, sines, tangents, and secants, usually engraven on instruments, for practice, is too obvious to require any particular explanation.

Gunter's **SCALE**, an instrument so called from Mr. Gunter, its inventor, is generally made of box: there are two sorts, the long Gunter, and the sliding Gunter, having both the same lines, but differently used, the former with the compasses, the latter by sliding. The lines now generally delineated on those instruments are the following; viz. a line of numbers, of sines, tangents, versed sines, sine of the rhumb, meridional parts, and equal parts, which are constructed after the following manner:—

The line of numbers is no other than the logarithmic scale of proportionals, wherein the distance between each division is equal to the number of mean proportionals contained between the two terms, in such parts as the distance between 1 and 10 is 1000, &c. equal the logarithm of that number. Hence it follows, that if the number of equal parts expressed by the logarithm of any number be taken from the same scale of equal parts, and set off from 1 on the line of numbers, the division will represent the number answering to that logarithm. Thus, if you take ·954, &c. (the logarithms of 9) of the same parts, and set it off from 1 towards 10, you will have the division standing against the number 9. In like manner, if you set off ·903, &c. ·845, &c. ·778, &c. the (logarithms of 8, 7, 6) of the same equal parts from 1 towards 10, you will have the divisions answering to the numbers 8, 7, 6.

After the same manner may the whole line be constructed.

The line of numbers being thus constructed, if the numbers answering to the natural sines and tangents of any arch, in such parts as the radius is 10,000, &c. be found upon the line of numbers, right against them will stand the respective divisions answering to the respective arches, or, which is the same thing, if the distance between the centre and that division of the line of numbers, which expresses the number answering to the natural sine or tangent of any arch, be set off on its respective line from its centre towards the left-hand, it will give the point answering to the sine or tangent of that arch; thus the natural sine of 30 degrees being 5000, &c. if the distance between the centre of the line of numbers (which in this case is equal to 10,000, &c. equal the radius) and the division, on the same line, representing 5000, &c. be set off from the centre, or 90 degrees, on the line of sines, towards the left-hand, it will give the point answering to the sine of 30 degrees. And after the same manner may the whole line of sines, tangents, and versed sines be divided.

The line of sines, tangents, and versed sines being thus constructed, the line sine of the rhumb, and tangent of the rhumb, are easily divided; for if the degrees and minutes answering to the angle which every rhumb makes with the meridian, be transferred from its respective line to that which is to be divided, we shall have the several points required; thus, if the distance between the radius or centre, and sine of 45 degrees, equals the fourth rhumb, be set off upon the line sine of the rhumb, we shall have the point answering to the sine of the fourth rhumb; and after the same manner may both these lines be constructed. The line of meridional parts is constructed from the table of meridional parts, in the same manner as the line of numbers is from the logarithms.

The lines being thus constructed, all problems relating to arithmetic, trigonometry, and their depending sciences, may be solved by the extent of the compasses only; and, as all questions are reducible to proportions, the general rule is, to extend the compasses from the first term to the second, and the same extent of the compasses will reach from the third to the fourth; which fourth term must be so continued as to be the thing required, which a little practice will render easy.

SCALENE, or **Scalenous**, is a term used to distinguish any figure or solid, when the line drawn from the vertex to the centre of the base is not perpendicular to the base.

SCHOLIUM, a note, annotation, or remark, occasionally made on some passage, proposition, or the like.

The term is much used in geometry, and other parts of mathematics; where, after demonstrating a proposition, it is common to point out how it might be done some other way; or to give some advice or precaution in order to prevent mistakes, or to add some particular use or application thereof.

SCIENCE, in *Philosophy*, denotes any doctrines deduced from self-evident principles. Sciences may be properly divided as follows. 1. The knowledge of things, their constitutions, properties, and operations: this, in a little more enlarged sense of the word, may be called natural philosophy, the end of which is speculative truth. 2. The skill of rightly applying these powers. The most considerable under this head is ethics, which is the seeking out those rules and measures of human actions that lead to happiness, and the means to practise them; and the next is mechanics, or the application of the powers of natural agents to the uses of life. 3. The doctrine of signs, the most usual of which being words, it is aptly enough termed logic. "This (says Mr. Locke) seems to be the most general as well as natural division of the objects of our understanding. For a man can employ his thoughts about nothing but either the contemplation of things themselves for the discovery of truth, or about the things in his own power, which are his actions, for the attainment of his own ends; or the signs the mind make use of both in the one and the other, and the right ordering of them for its clearer information. All which three, viz. things as they are in themselves knowable, actions as they depend on us in order to happiness, and the right use of signs in order to knowledge, being *toto calo* different they seem to be the three great provinces of the intellectual world, wholly separate and distinct one from another."

SCREW, one of the mechanical powers, or rather a combination of two of them, the inclined plane and the lever, principally used in pressing bodies together, or in lifting great weights, which may be conceived to be generated as follows:

Let a solid and a hollow cylinder of equal diameters be taken, and let there be a right-angled plane triangle, whose base is equal to the circumference of the solid cylinder, and applied to the latter in such a manner, that the base may coincide with the circumference of the base of the cylinder, and the hypotenuse will form a spiral thread on its surface. By applying to the cylinder triangles in succession similar and equal to this, in such a manner that their bases may be parallel to its base, the spiral thread may be continued; and supposing the threads to have thickness, or the cylinder to be protuberant where it falls, the external screw will be formed, in which the distances between two contiguous threads, measured in a direction parallel to the axis of the cylinder, is the perpendicular of the triangle.

Again, let the triangles be applied in the same manner, to the concave surface of the hollow cylinder, and where the thread falls let a groove be made, and the internal screw will be formed. The two screws being thus exactly adapted to each other, the solid or hollow cylinder, as the case requires, may be moved about the common axis by a lever, or in such other manner as the nature of the case may require. The external screw is sometimes called the *male* screw, and the internal the *female* screw.

When there is an equilibrium upon the screw, then the power is to the weight, as the distance between two contiguous threads, measured in a direction parallel to the axis, is to the circumference of the circle described by the power.

That is, if P represents the power, W the weight to be lifted, or the resistance to be overcome, also d the distance of two contiguous threads, and l the length of the lever; then $P : W = d : 6{\cdot}2832 \,.\, l$. calling 6·2832 the circumference of a circle to radius 1.

This results immediately, if we admit the equality in the momenta of the power and weight, for the velocity of the power is to the velocity of the weight, as the circumference of the circle described by the power is to the distance of the threads; but this principle has been justly objected to by some modern writers, and other demonstration adopted in its stead. The result, however, is the same in all cases; and so far as relates to the theory it is perfectly correct; but in practice, not only the weight, or resistance, but also the friction of the screw, is to be overcome, which in this machine is very great, in some cases equal to the weight itself, being frequently sufficient to sustain this after the power is removed.

Endless or *Perpetual* SCREW, is one which works in and turns a dented wheel, without the internal screw, and is thus called because it constantly preserves its motion, while that of the wheel is continued.

Archimides' SCREW, or the *Water-Snail,* is a machine for raising water, which consists either of a pipe wound spirally round a cylinder, or of one or more spiral excavations formed by means of spiral projections from an internal cylinder, covered by an external coating, so as to be water-tight. This screw is one of the most ancient, and, at the same time, ingenious machines we know. It operates thus:—

If we conceive that a flexible tube is rolled regularly about a cylinder from one end to the other, this tube or canal will be a screw or spiral, of which we suppose the intervals of the spires or threads to be equal. The cylinder being placed with its axis in a vertical position, if we put in at the upper end of the spiral tube a small ball of heavy matter, which may move freely, it is certain that it will follow all the turnings of the screw from the top to the bottom of the cylinder, descending always as it would have done had it fallen in a right line along the axis of the cylinder, only it would occupy more time in running through the spiral. If the cylinder were placed with its axis horizontally, and we again put the ball into one opening of the canal, it will descend, following the direction of the first demi-spire; but when it arrives at the lowest point of this portion of the tube it will stop. It must be remarked, that though its heaviness has no other tendency than to make it descend in the demi-spire, the oblique position of the tube with respect to the horizon, is the cause that the ball, by always descending, is always advancing from the extremity of the cylinder, whence it commenced its motion, to the other extremity. It is impossible that the ball can ever advance more towards the farther, or, as we shall call it, the second extremity of the cylinder, if the cylinder, placed horizontally, remains always immoveable; but if, when the ball is arrived at the bottom of the first demi-spire, we cause the cylinder to turn on its axis, without changing the position of that axis, and in such manner that the lowest point of the demi-spire, on which the ball presses, becomes elevated, then the ball falls necessarily from this point upon that which succeeds, and which becomes lowest; and since this second point is more advanced towards the second extremity of the cylinder than the former was, therefore, by this new descent, the ball will be advanced towards that extremity, and so on throughout, in such a manner that it will at length arrive at the second extremity, by always descending, the cylinder having its rotatory motion continued. Instead of the ball let us now consider water as entering by the lower extremity of the spiral canal, when immersed in a reservoir: this water descends at first in the canal solely by its gravity; but the cylinder being turned, the water moves on in the canal to occupy the lowest place, and thus, by the continual rotation, is made to advance farther and farther in the spiral, till at length it is raised to the upper extremity of the canal, where it is expelled. There is, however, an essential difference between the water and the ball; for the water, by reason of its fluidity, after having descended by its heaviness to the lowest point of the demi-spire, rises up on the contrary side to the original level; on which account more than half one of the spires may soon be filled with the fluid. This is an important particular, which, though it need not be regarded in a popular illustration, must be attended to in the more particular exhibition of the theory.

SEA, in a general sense, the great reservoir of water, into which the lakes and rivers empty themselves, and from which it is again drawn by evaporation, that moisture which, falling in showers of rain, fertilizes the earth, and supplies the waste of the springs and rivers.

The absolute quantity of sea-water cannot be ascertained, as its mean depth is unknown. Laplace has demonstrated that a depth of four leagues is necessary to reconcile the height to which the tides are known to rise in the main ocean by the Newtonian theory. If we suppose this to be the mean depth, the quantity of water in the ocean must be immense. Even on the supposition that its mean depth is not greater than the fourth part of a mile, its quantity, allowing its surface to be three-fourths of that of the superficies of the earth, would be 32 058,939¾ cubic miles.

According to the most accurate observations hitherto made, the surface of the sea is to the land as three to one; the ocean, therefore, extends over 128,235,759 square miles, supposing the superficies of the whole globe to be 170,981,012 square miles. To ascertain the depth of the sea is still more difficult than its superficies; both on account of the numerous experiments which it would be necessary to make, and the want of proper instruments for that purpose. Beyond a certain depth the sea has hitherto been found unfathomable; and though several very ingenious methods have been contrived to obviate this difficulty, none of them has completely answered the purpose.

We know, in general, that the depth of the sea increases gradually as we leave the shore; but if this continued beyond a certain distance, the depth in the middle of the ocean would be prodigious. Indeed, the numerous islands every where scattered in the sea demonstrate the contrary, by showing us that the bottom of the water is unequal, like the land; and that, so far from uniformly sinking, it sometimes rises into lofty mountains. If the depth of the sea is in proportion to the elevation of the land, as has generally been supposed, its greatest depth will not exceed four miles, for there is no mountain whose altitude exceeds this in perpendicular height. Along the coasts, where the depth of the sea is in general well known, it has always been found proportioned to the height of the shore; when the coast is high and mountainous, the sea that washes it is deep; when, on the contrary, the coast is low, the water is shallow. Whether this analogy holds at a distance from the shore, experiments alone can determine.

In order to compute the quantity constantly discharged into the sea, let us take a river whose velocity and quantity of water are known, the Po, for instance, discharges into the sea 200,000 cubic perches of water in an hour, or 4,800,000 in a day. A cubic mile contains 125,000,000 cubic perches; the Po, therefore, will take twenty-six days to discharge a cubic mile of water into the sea. Let us now suppose, what is, perhaps, not very far from the truth, that the quantity of water which the sea receives from the rivers in any country is proportioned to the extent of that country. The Po, from its origin to its mouth, traverses a country 380 miles long, and the rivers which fall into it on every side rise from sources about sixty miles distant from it. The Po, therefore, and the rivers which it receives, water a country of 45,600 square miles. Now, since the whole superficies of the dry land is about 42,745,253 square miles, it follows, from our supposition, that the quantity of water discharged by all the rivers in the world, in one day, is thirty-six cubic miles. If, therefore, the sea contains 32,058,939 cubic miles of water, it would take all the rivers in the world 2439 years to discharge an equal quantity. It may seem surprising that the sea, since it is continually receiving such an immense supply of water, does not visibly increase, and at last cover the whole earth. But our surprise will cease, if we consider that the rivers themselves are supplied from the sea, and that they do nothing more than carry back those waters which the ocean is continually lavishing upon the earth.

SEASONS, certain portions of the year, as Spring, Summer, Autumn, and Winter.

SECANT, in *Geometry*, a line which cuts another, whether right or curved.

SECANT, in *Trigonometry*, is a right line drawn from the centre of a circle to meet the upper or farther extremity of any tangent, to the same circle.

An arc and its supplement have their secants equal, only the latter one is accounted negative, being drawn the contrary way to the former. And thus the secants in the second and third quadrants are negative, while those in the first and fourth quadrants are positive.

The secant of an arc, which is the complement of the former arc, is called the cosecant, or the secant of its complement. The cosecants in the first and second quadrants are affirmative, but in the third and fourth negative.

The secant of an arc is reciprocally as the cosine, and the cosecant reciprocally as the sine, or the rectangle of the secant and cosine, and the rectangle of the cosecant and sine, are each equal to the square of the radius.

An arc a, to the radius r, being given, the secant s and cosecant σ, and their logarithms, or the logarithmic secant and cosecant, may be expressed in infinite series, as follows, *viz.*

$$s = r + \frac{a^2}{2r} + \frac{5a^4}{24r^3} + \frac{61a^6}{720r^5} + \frac{277\,a^8}{8064\,r^7} \text{ \&c.}$$

$$\sigma = \frac{r^2}{a} + \frac{a}{6} + \frac{7\,a^3}{300\,r^4} + \frac{31\,a^5}{15120\,r^4} + \frac{127\,a^7}{604800\,r^6} \text{ \&c.}$$

$$\text{log. } s = m \times \left(\frac{a^2}{2} + \frac{a^4}{12} + \frac{a^6}{45} + \frac{17a^8}{2520} \text{ \&c.}\right)$$

$$\text{log. } \sigma. = -\text{log. } a + m \times \left(\frac{a^2}{6} + \frac{a^4}{180} + \frac{a^6}{2835} + \frac{a^8}{37800} \text{ \&c.}\right)$$

where m is the modulus of the system of logarithms.

If a be any arc, then

$$\text{sec. } 2\,a = \frac{\text{sec. }^2 a}{2 - \text{sec. }^2 a}$$

$$\text{sec. } 3\,a = \frac{\text{sec. }^3 a}{4 - 3\text{ sec. }^2 a}$$

$$\text{sec. } 4\,a = \frac{\text{sec. }^4 a}{8 - 8\text{ sec. }^2 a + \text{sec. }^4 a}$$

$$\text{sec. } \tfrac{1}{2}\,a = \sqrt{\frac{2\text{ sec. } a}{1 + \text{sec. } a}}.$$

SECANTS, *Figure of.* See FIGURE.

SECOND, is the sixtieth part of a minute, both as it relates to the measure of angles or time.

SECTION, in *Geometry*, denotes a side or surface of one body or figure cut by another, or the place where lines, planes, &c. cut each other.

SECTIONS of a *Cone.* See CONIC *Sections.*

SECTOR, in *Geometry*, is a portion of a circle comprehended between any two radii and their intercepted arcs.

Similar SECTORS, are those whose radii include equal angles.

To find the Area of a Sector.—Say as 360° is to the degrees, &c. in the arc of the sector; so is the area of the whole circle to the area of the sector.

Or multiply the radius by the length of the arc, and half the product will be the area.

SECTOR also denotes a mathematical instrument of great use in finding the proportion between qualities of the same kind: as between lines and lines, surfaces and surfaces, &c. whence the French call it the compass of proportion.

The great advantage of the sector above the common scales, &c. is, that it is made so as to fit all radii, and all scales. By the lines of chords, sines, &c. on the sector, we have lines of chords, sines, &c. to any radius betwixt the length and breadth of the sector when open.

The sector is founded on the fourth proposition of the sixth book of Euclid; where it is demonstrated, that similar triangles have their homologous sides proportional.

SECULAR *Equations*, in *Astronomy*, are corrections required to compensate such inequalities in the motions of the heavenly bodies as are found to obtain in the course of a century.

SEGMENT of a *Circle*, is a part of a circle bounded by an arc and its chord, and is either greater or less than a semicircle. The following are their most remarkable properties: all angles in the same segment of a circle are equal to each other; if the segment is greater than a semicircle, the angle is less than a right-angle; if less than a semicircle, it is greater than a right-angle.

SEGMENT of a *Sphere*, is any part of a sphere cut off by a plane; the section of which, with the sphere, is always a circle.

To find the Superficies and Solidity of Spherical Segments.

Let d denote the diameter of the sphere, or the chord of half the circumference, and c the chord of half the arc of any segment, also a the altitude or versed sine of the same; then, $3\ 1416d^2$ is the surface of the whole sphere, and $3{\cdot}1416c^2$, or $3\,1416ad$, the surface of the segment.

To find the Solidity.—1. To three times the square of the radius of its base, add the square of its height; multiply the sum by the height, and the product by ·5236. Or, 2dly, From three times the diameter of the sphere, subtract twice the height of the frustrum; multiply the remainder by the square of the height, and the product by ·5236. That is, in symbols, the solid content is either $= {\cdot}5236\,a \times (3r^2 + a^2)$ or $= {\cdot}5236\,a^2 \times (3d - 2a)$; where a is the altitude of the segment, r the radius of its base, and d the diameter of the whole sphere.

SEMICIRCLE, is half a circle, or the area comprehended between a diameter and the semicircumference.

Semicircle, or *Graphometer*, is the name of an instrument much used in surveying; it is a sort of half theodolite, the only difference being, that the limb of the theodolite is an entire circle graduated from 1° to 360°, whereas in the semicircle the gradation is only carried to 180°. See Theodolite.

SEMIDIAMETER, the same as Radius.

SERIES, in *Algebra*, is a continued rank, or progression, of quantities connected together by the signs of + or —; usually proceeding according to a certain determinate law.

Such are the following:

$$1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9}\ \&c.$$

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \&c.$$

the former being the reciprocals of the odd numbers, and the latter a geometrical series, of which the ratio is $\frac{1}{2}$.

Series are of different forms, and arise in various ways, but more commonly from the expansion of some binomial expression; thus we find

$$\frac{1}{a+b} = (a+b)^{-1} = \frac{1}{a} - \frac{b}{a^2} + \frac{b^2}{a^3} - \frac{b^3}{a^4} + \&c.$$

$$\frac{1}{a-b} = (a-b)^{-1} = \frac{1}{a} + \frac{b}{a^2} + \frac{b^2}{a^3} + \frac{b^3}{a^4} + \&c.$$

which differ from each other only in the signs.

Series also receive several different denominations according to certain circumstances relating to their formation, the law which they observe, &c. as converging, diverging, &c. series.

Converging Series, is one in which the terms decrease, or become successively less and less, as

$$1 + \frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \frac{1}{5^4} + \&c.$$

Diverging Series, is one in which the terms continually increase, as

$$1 - 2 + 4 - 8 + 16 - 32 + \&c.$$

Neutral Series, is that in which all the terms are equal to each other; such is

$$1 - 1 + 1 - 1 + 1 - 1 + \&c.$$

which arises from the division of 1 by 1 + 1.

Indeterminate Series, is one whose terms proceed by the powers of an indeterminate quantity, as

$$x + \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 + \tfrac{1}{4}x^4 + \&c.$$

and this is either ascending or descending.

An *Ascending* Series, is that in which the powers of the indeterminate, or unknown quantity, continually increase, as

$$1 + ax + bx^2 + cx^3 + \&c.$$

A *Descending* Series, is that in which the powers of the indeterminate quantity continually decrease, as

$$1 + ax^{-1} + bx^{-2} + cx^{-3} + \&c. \text{ or}$$

$$1 + \frac{a}{x} + \frac{b}{x^2} + \frac{c}{x^3} + \&c.$$

Law of a Series, is used to denote that relation which subsists between the successive terms of a series, and by which their general term may be denoted; thus the series

$$1 + \frac{2}{3}x + \frac{8}{15}x^2 + \frac{16}{35}x^3 + \frac{128}{315}x^4 + \&c.$$

may be put under the form

$$1 + \frac{2}{3}x + \frac{2.4}{3.5}x^2 + \frac{2.4.6}{3.5.7}x^3 + \frac{2.4.6.8}{3.5.7.9}x^4, \&c.$$

where the law by which it may be indefinitely continued is manifest.

There are, beside these, various other denominations for particular series, or particular forms of series, as

Circular SERIES, which is that whose sum depends upon the quadrature of the circle.

Recurring SERIES, is one in which each term is some constant function of a certain number of the preceding terms. See RECURRING *Series.*

Summation of SERIES, is the method of finding the sum of a series, whether the number of its terms be finite or infinite.

We have already explained the method of finding the successive differences of the terms of a series, and therefore in this place we shall find the sum of a given series.

1. To find the nth term of any series a, b, c, d, e, &c. when the differences of any order become equal to each other.

Let d', d'', d''', d^{iv}, d^{v}, &c. be the first terms of the several order of differences, then will

$$a+\frac{n-1}{1}d'+\frac{(n-1)(n-2)}{1.2}d''+\frac{(n-1)(n-2)(n-3)}{1.2.3}d'''+\&c.$$

= the nth term required.

Thus, if it were required to find the 12th term of the series

2, 6, 12, 20, 30, &c.

Here we have

	2	6	12	20	30, &c.
1st diff.	4	6	8	10,	&c.
2d diff.	2	2	2,	&c.	
3d diff.	0	0,	&c.		

therefore, $a = 2$, $d' = 4$, $d'' = 2$, and $d''' = 0$; also, $n = 12$, whence

$$a+\frac{n-1}{1}d'+\frac{(n-1)(n-2)}{1.2}d''+\&c. = 2+11\,d'+55\,d''=156,$$

the 12th term.

Again, required the 20th term of the series of triangular numbers

1, 3, 6, 10, 15, &c.

	1	3	6	10	15 &c.
1st diff.	2	3	4	5,	&c.
2d diff.	1	1	1	&c.	
3d diff.	0	0,	&c.		

therefore $a = 1$, $d' = 2$, $d'' = 1$, and $n = 20$; whence

$$a+\frac{n-1}{1}d'+\frac{(n-1)(n-2)}{1.2}d''$$
$$= 1+19\,d'+171\,d''=210.$$

2. To find the sum of n terms of any series, of which any order of its differences are equal to each other.

Let d', d'', d''', d^{iv}, &c. be the first of the several order of differences, then will

$$a+\frac{n(n-1)}{1.2}d'+\frac{n(n-1)(n-2)}{1.2.3}d''+\frac{n(n-1)(n-2)(n-3)}{1.2.3.4}d'''+$$

&c. = the sum of n terms as required.

Required the sum of the series of squares,

1, 2^2, 3^2, 4^2, &c, to n^2.

	1	4	9	16
1st diff.	3	5	7	
2d diff.	2	2		
3d diff.	0			

therefore $a = 1$, $d' = 3$, $d'' = 2$; consequently

$$n+\frac{3\,n(n-1)}{1.2}+\frac{2.n(n-1)(n-2)}{1.2.3}$$
$$=\frac{(2n+1)(n+1)n}{6}=\text{sum of } n$$

terms.

Leibnitz gave several curious numerical series of a different kind from the former; among which may be reckoned the following:

$$\frac{1}{3}+\frac{1}{8}+\frac{1}{15}+\frac{1}{24}+\frac{1}{35}+\frac{1}{48}+\&c.$$

Or

$$\frac{1}{2^2-1}+\frac{1}{3^2-1}+\frac{1}{4^2-1}+\frac{1}{5^2-1}+\&c.$$

the sum of an infinite number of terms of which is equal to $\frac{3}{4}$. The sum of its odd terms being equal to $\frac{1}{2}$, and of the even terms to $\frac{1}{4}$, that is

$$\frac{1}{3}+\frac{1}{15}+\frac{1}{35}+\frac{1}{63}+\&c.=\frac{1}{2}, \text{ and}$$
$$\frac{1}{8}+\frac{1}{24}+\frac{1}{48}+\frac{1}{80}+\&c.=\frac{1}{4}$$

the sum of an infinite number of terms of the same series, omitting every three terms, from the 1st to the 5th, from the 5th to the 9th, &c. as

$$\frac{1}{3}+\frac{1}{35}+\frac{1}{99}+\frac{1}{195}+\frac{1}{323}+\&c.$$

is equal to the area of a circle, of which the inscribed square is $\frac{1}{2}$.

But if we begin at the second term, and thence omit every three as above, as

$$\frac{1}{8}+\frac{1}{48}+\frac{1}{120}+\frac{1}{224}+\frac{1}{360}+\&c.$$

its sum will be equal to the area or space included between the curve and asymptote of an equilateral hyperbola, or $\frac{1}{2}$ hyp. log. of 2.

SEXAGESIMALS, or **Sexagesimal** *Fractions*, are fractions whose denominators proceed by the powers of 60, instead of by the powers of 10, as in common decimal fractions. That is, in sexagesimals a prime or first minute, is $\frac{1}{60}$, a second, $\frac{1}{60^2}$, or $\frac{1}{3600}$; a third, $\frac{1}{60^3}$, or $\frac{1}{216000}$, and so on; but here, as in decimal fractions, the denominators being always the power of the same number, they are not put down in any operations performed on these numbers, their real value being indicated by the place they occupy in the result; thus 26°, 9′, 31″, 16‴, means $26° + \frac{9°}{60} + \frac{31°}{3600} + \frac{16°}{216000}$, and so on.

Anciently, there were no other than sexagesimals used in astronomical operations, for which reason they are sometimes called astronomical fractions; and they are still retained in many cases, as in the division of time and of the circle.

SEXTANS, a new constellation. See *Constellation*.

SEXTANT, is the sixth part of a circle, or an arc of 60°.

Sextant, is also the name of an instrument resembling a quadrant, differing from it only in its arc, consisting of 120° instead of 90°.

SHADOW, in *Optics*, a space deprived of light by the interposition of some opaque body before the luminous point whence the rays of light proceed.

SHILLING, an English silver coin. A shilling should weigh the 62d part of a pound Troy, or 3 dwts. 20 $\frac{28}{31}$ grs. at which rate the value of an ounce of silver is about 5*s.* 2*d.*

SHOT, in *Military Affairs*, includes all sorts of balls or bullets, great and small, whether they be iron or lead.

An iron shot of 4 inches diameter weighs 9lb. and

A lead shot of $4\frac{1}{4}$ inches diameter weighs 17lb.

Hence as the weight of similar bodies are to each other as the cubes of their diameters, putting *d* and *w* for the diameter and weight of an iron shot, and D and W for the same of a leaden shot, we have, as

$$\left.\begin{array}{l} 4^3 : 9 = d^3 : w \\ \sqrt[3]{9} : 4 = \sqrt[3]{w} : d \end{array}\right\} \text{iron shot}$$

$$\left.\begin{array}{l} \frac{17^3}{4^3} : 17 = D^3 : W \\ \sqrt[3]{17} : \frac{17}{4} = \sqrt[3]{W} : D \end{array}\right\} \text{lead shot.}$$

SIDE, in *Geometry*, is used for any line which forms one of the boundaries of a right-lined figure, as ths side, a triangle, square, &c. Similar figures are to each other as the squares of their like sides.

SIDEREAL, or *Siderial*, any thing relating to the stars.

SIDUS, *Georgium*. See **Uranus**.

SIGNS, in *Algebra*, are symbols employed to indicate certain operations, as addition, subtraction, multiplication, &c.

Signs, in *Astronomy*, are certain portions of the ecliptic, or zodiac, each being a twelfth part of the whole, and, therefore, containing 30°. The ancients first divided the zodiac into those twelve portions, commencing at the point where the equator and ecliptic intersected at that time, and proceeding forward from west to east, according to the course of the sun. These signs received the names of the several constellations which possessed those segments in the time of Hipparchus. But the constellations have since so changed their places, in consequence of the precession of the equinoxes, that Aries is now in the sign called Taurus, and Taurus in Gemini, &c.

Northern **Signs**, are those on the north side of the equator.

Southern **Signs**, are those on the south side.

These, with their farther distinguishing appellations, are as follow:

Northern.

Vernal or Spring signs.	Aries	♈
	Taurus	♉
	Gemini	♊
Estival, or Summer signs	Cancer	♋
	Leo	♌
	Virgo	♍

Southern.

Autumnal signs.	Libra	♎
	Scorpio	♏
	Sagittarius	♐
Brumal, or Winter signs.	Capricornus	♑
	Aquarius	♒
	Pisces	♓

The southern signs are sometimes called *ascending* signs, and the northern *descending* signs.

SIMILAR, is a word used to denote two things having the same disposition and conformation of parts, differing from each other only in quantity or magnitude.

Similar *Plane Figures*, in *Geometry*, are such as have the angles of the one respectively equal to the angles of the other, and the sides about those angles proportional; and such figures are to each other as the squares of their like sides.

All equiangular triangles are similar, as are also all regular plane figures of the same number of sides.

Similar *Sectors*, and *Segments* of *Circles*, are such as are contained under arcs, having the same measures, or being the same part or parts of their respective circles.

Similar *Solids*, are those which are contained under the same number of similar planes alike placed, or such as have their solids, angles, equal each to each; and all such bodies are to each other as the cubes of their like sides, or like linear dimensions.

Similar *Curves* are such, that any right-lined figure being inscribed within the one, a similar rectilinear figure may be inscribed within the other.

Similar *Ellipses, Parabolas,* and *Hyperbolas*, are such as have their axes and parameters proportional.

Similar *Bodies*, in *Physics*, are such as have the same component parts.

SIMILITUDE, the relation of things that are similar to each other.

Simple *Equation*, in *Algebra*, is that in which only the simple power of the unknown quantity enters, as $7ax=b$; $3x+ax=b$, &c.

Rule for the solution of Simple Equations. — Transpose all the terms containing the unknown quantity to the left-hand side of the equation, and all the known quantities to the other side. Then, if the unknown quantity be multiplied by any known quantity, the whole equation must be divided by it, and the quotient will be the value of the unknown quantity. If the unknown quantity be divided by any known quantity, the whole equation must be multiplied by that divisor. If the unknown quantity be involved in any radical, such a power of both sides of the equation must be taken, as will destroy the radicality; that is, if it be the square root, the known quantity must be squared; if it be the cube root, it must be cube, and so on. If the unknown quantity be involved in any power, such a root of both sides must be taken as will reduce the unknown quantity to a simple power. So that, in all cases, the unknown quantity is found by performing on the known side of the equation the reverse operation to that under which the unknown quantity enters; that is, if it be multiplied, you must divide; if it be divided, you must multiply; if it be any radical, you must involve; and if it be any power, you must extract the root of that power.

Note. In transposing any quantity from one side of the sign of equality to the other, the sign of it must be changed from + to —, or from — to +. Transposing a + quantity is the same as subtracting it from both sides of the equation, and transposing a — quantity the same as adding it to both.

Ex. Give $\frac{a}{1+x}+\frac{a}{1-x}=2b$, to find x.

Reduce the first two fractions to a common denominator, and we have $\frac{a-ax}{1-x^2}+\frac{a-ax}{1-x^2}2b$.

Multiply by $1-x^2$, then $2a=2b-2bx^2$.

By transposition, $2bx^2=2b-2a$.

Whence by division and extraction, $x=\sqrt{\left(\frac{2b-2a}{2b}\right)}=\sqrt{\left(\frac{b-a}{b}\right)}$

SIMPLE *Quantity*, in *Algebra*, consists but of one term.

SINE, or *Right Sine* of an arc, in trigonometry, is a right line drawn from either extremity of an arc, perpendicular to the radius joining the other extremity and the centre of the circle, or it is half the chord of the double arc.

Hence, the sine increases from 0 in the first quadrant to 90°, where it is the greatest, being then equal to radius, after which it decreases in the second quadrant to 180°; where it is again nothing; it then increases in the third quadrant, decreases in the fourth, and so on alternately; but, in the two latter, it is accounted negative.

The sine of 90°, or radius, is called the *whole sine*.

CO-SINE of an angle, is the sine of the complement of that angle, or of what it wants of 90°; the word *co*, being merely a contraction of complement.

The cosine, therefore, decreases in the first quadrant from radius to *zero*, then increases from zero in the second to radius; again, decrease in the third, increase in the fourth, and so on; being positive in the 1st and 4th quadrants, and negative in the 2d and 3d.

Versed SINE of an arc, is that part of the diameter intercepted between the sine and the extremity of the arc.

Arithmetic of SINES. The object of this branch of science is to exhibit the relation of the sines, cosines, tangents, &c. of arcs, multiple arcs, &c.

The notation used for this purpose is very simple, being as follows; viz. a being any angle, we denote

sine of a by sin. a

cosine of a by cos. a

sine of $2a$, $3a$, na, &c. by sin. $2a$, sin. $3a$, sin, n, a, &c.

cosine of $2a$, $3a$, n, a, &c. by cos. $2a$, cos. $3a$, cos. na.

And in a similar manner,

tan. of $a=$ tan. a, sec. of $a=$ sec. a.

tan. of $na=$ tan. na, sec. of na $=$ sec. na.

Again, the square, cube, &c. of the sine, cosine, tangent, &c. of an angle, are thus expressed:

square of the sine of $a=\text{sin.}^2\,a$

cube of the tangent $a=\text{tan.}^3\,a$

&c. &c.

This notation being understood, and the deduction drawn from one fundamental, geometrical proposition; then all the rest is deduced from pure analytical principles, and trigonometry is divested of all those formidable diagrams which are so extremely embarrassing in the ancient authors.

Making radius $=1$; cos. of one arch $=a$; cos. of a second arch $=b$; and cos. of their sum $=(a+b)$. Then we have, by similar triangles,

cos. $(a+b)=$ cos. a . cos. b — sin. a . sin. b. which is the fundamental theorem on which the whole of the arithmetic of sines, or analytical trigonometry depends, and from which all the rest may be deduced.

Squaring both sides of this equation, we have

$$\text{cos.}^2(a+b)=\text{cos.}^2 a.\ \text{cos.}^2 b+\text{sin.}^2 a\ .\ \text{sin.}^2 b$$
$$-2\,\text{cos.}\,a\ .\ \text{cos.}\,b.\ \text{sin.}\,a\ .\ \text{sin.}\,b$$

Now we know that $1=\text{sin.}^2 a+\text{cos.}^2 a$.

And subtracting this from the above, we have

Or

$$1-\text{cos.}^2(a+b)=\begin{cases}\text{sin.}^2 a\,(1-\text{sin.}^2 b)\\ +\,\text{cos.}^2 a\,(1-\text{cos.}^2 b)+2\ \text{cos.}\ a\ .\\ \text{cos.}\ b.\text{sin.}\ a.\ \text{sin.}\ b.\end{cases}$$

$$\text{sin.}^2(a+b)=\begin{cases}\text{sin.}^2 a\ .\ \text{cos.}^2 b+\text{cos.}\\ {}^2a\ \text{sin.}^2 b+2\,\text{sin.}\ a\ .\\ \text{cos.}\ b\ .\ \text{cos.}\ a\,\text{sin.}\ b=\\ (\text{sin.}\,a\,.\,\text{cos.}\,b+\text{cos.}\,a\ .\\ \text{sin.}\ b)^2.\end{cases}$$

Whence

sin. $(a+b)=$ sin. a . cos. b + cos. a. sin. b.

Or by making b in both of these negative, we have

$\cos. (a + b) = \cos. a \,.\, \cos. b - \sin. a \,.\, \sin. b$

$\cos. (a - b) = \cos. a \,.\, \cos. b + \sin. a \,.\, \sin. b$

$\sin. (a + b) = \sin. a \,.\, \cos. b + \cos. a \,.\, \sin. b$

$\sin. (a - b) = \sin. a \,.\, \cos. b - \cos. a \,.\, \sin. b$

And by means of the ambiguous sign ± these four fundamental theorems may be reduced to the two following; viz.

$\cos. (a \pm b) = \cos. a \,.\, \cos. b \mp \sin. a \,.\, \sin. b$

$\sin. (a \pm b) = \sin. a \,.\, \cos. b \mp \cos. a \,.\, \sin. b$

And from these we readily draw the following expression for the cosines and sines of multiple arcs, viz. by making $a = b$, we obtain the sin. and cos. of $2\,a$, and a similar substitution gives us the sines and cosines of other multiples, but as our limits will not admit of tracing these several transformations through all their steps, we shall merely state some of the most remarkable and useful results, but for their investigations we must refer the reader to the following works: viz. "Traité de Trigonometrie," by M. Cagnoli, translated from the Italian by M. Chompré; Bonnycastle's Treatise of Plane and Spherical Trigonometry, 2d edit.; Woodhouse's Trigonometry, and the article Arithmetic of Sines in the Brewster's Encyclopedia.

Some of the most useful formulæ relating to sines and cosines are as follow; similar expressions for tangents, secants, &c. will be found under the articles TANGENT and SECANT.

$\cos. (a - b) + \cos. (a + b) = 2 \cos. a \,.\, \cos. b$

$\cos. (a - b) - \cos. (a + b) = 2 \sin. a \,.\, \sin. b$

$\sin. (a + b) + \sin. (a - b) = 2 \sin. a \,.\, \cos. b$

$\sin. (a + b) - \sin. (a - b) = 2 \cos. a \,.\, \sin. b$

Or writing $a = \frac{1}{2}(p + q)$ and $b = \frac{1}{2}(p - q)$

$\cos. q + \cos. p = 2 \cos. \frac{1}{2}(p + q) \cos. \frac{1}{2}(p - q)$

$\cos. q - \cos. p = 2 \sin. \frac{1}{2}(p + q) \sin. \frac{1}{2}(p - q)$

$\sin. p + \sin. q) = 2 \sin. \frac{1}{2}(p + q) . \cos. \frac{1}{2}(p - q)$

$\sin. p - \sin. q = 2 \cos. \frac{1}{2}(p - q) . \sin. \frac{1}{2} p - q)$

Or writing $a = n\,p$, and $b = p$

$\cos. (n - 1)\, p + \cos. (n + 1)\, p = 2 \cos. p . \cos. n\,p$

$\cos. (n - 1)\, p - \cos. (n + 1)\, p = 2 \sin. p . \sin. n\,p$

$\sin. (n + 1)\, p + \sin. (n - 1)\, p = 2 \cos. p . \sin. n\,p$

$\sin. (n + 1)\, p - \sin. (n - 1)\, p = 2 \sin. p . \cos. n\,p$

From which are readily obtained the sine and cosine of any multiple arc, *viz.* by making $p = a$, sine $a = x$, cosine $a = y$, and $n = 1, 2, 3$, &c. we readily obtain the following formulæ:

$\cos. 2\,a = 2\,x \cos. 1\,a - \cos. 0\,a$
$\cos. 3\,a = 2\,x \cos. 2\,a - \cos. 1\,a$
$\cos. 4\,a = 2\,x \cos. 3\,a - \cos. 2\,a$
$\cos. 5\,a = 2\,x \cos. 4\,a - \cos. 3\,a$
&c. &c.

Where the law is obvious, and which may be brought under the following form:

$\cos. 1\,a = x$
$\cos. 2\,a = x^2 - y^2$
$\cos. 3\,a = x^3 - 3\,x\,y^2$
$\cos. 4\,a = x^4 - 6\,x^2 y^2 + y^4$
$\cos. 5\,a = x^5 - 10\,x^3 y^2 + 5\,x\,y^4$
$\cos. 6\,a = x^6 - 15\,x^4 y^2 + 15\,x^2 y^4 - y^6$
$\cos. n\,a = x^n - r\,x^{n-2} y^2 + s\,x^{n-4} y^4 + t\,x^{n-6} y^6$, &c.

Where r, s, t, &c. are the alternate co efficients of a binomial raised to the nth power.

And in a similar manner we have from formula 3,

$\text{Sin. } 2\,a = 2\,x \sin. 1\,a - \sin. 0\,a$
$\text{Sin. } 3\,a = 2\,x \sin. 2\,a - \sin. 1\,a$
$\text{Sin. } 4\,a = 2\,x \sin. 3\,a - \sin. 2\,a$
$\text{Sin. } 5\,a = 2\,x \sin. 4\,a - \sin. 3\,a$
&c. &c.

Or,

$\text{Sin. } 1\,a = y$
$\text{Sin. } 2\,a = 2\,x\,y$
$\text{Sin. } 3\,a = 3\,x^2 y - y^3$
$\text{Sin. } 4\,a = 4\,x^3 y - 4\,x\,y^3$
$\text{Sin. } 5\,a = 5\,x^4 y - 10\,x^2 y^3 + y^5$
$\text{Sin. } 6\,a = 6\,x^5 y - 20\,x^3 y^3 + 6\,y^6$
$\text{Sin. } n\,a = n\,x^{n-1} y - p\,x^{n-3} y^3 + q\,x^{n-5} -$ &c. where n, p, q are the alternate even terms of a binomial raised to the power n.

In the first series the upper sign is to be used when n is of the form $4\,m + 1$, and the lower when it is of the form $4\,m - 1$. And in the second the upper sign is to be used when n is of the form $4\,m$, and the lower when it is of the form $4\,m + 2$.

In all the preceding formulæ we have considered radius as unity; but in case it is necessary to introduce it, it is only necessary to make it enter into each term with such a power as will render all the terms homogeneous.

The following are some of the simplest cases in which the sines of certain angles are expressible in surd forms.

$\text{Sin. } 0 = 0$

$\text{Sin. } 9^\circ$

$= \frac{r}{4}\left\{\sqrt{(3+\sqrt{5})} - \sqrt{(5-\sqrt{5})}\right\}$

$\text{Sin. } 15^\circ = \frac{r}{4}(\sqrt{6} - \sqrt{2})$

$\text{Sin. } 18^\circ = \frac{r}{4}(1+\sqrt{5})$

$\text{Sin. } 27^\circ$

$= \frac{r}{4}\left\{\sqrt{(5+\sqrt{5})} - \sqrt{(3-\sqrt{5})}\right\}$

$\text{Sin. } 30^\circ = \frac{r}{2}$

$\text{Sin. } 36^\circ = \frac{r}{4}\sqrt{(10-2\sqrt{5})}$

$\text{Sin. } 45^\circ = \frac{r}{2}\sqrt{5}$

$\text{Sin. } 54^\circ = \frac{r}{4}(1+\sqrt{5})$

$\text{Sin. } 60^\circ = \frac{r}{2}\sqrt{3}$

$\text{Sin. } 63^\circ$

$= \frac{r}{4}\left\{\sqrt{(5+\sqrt{5})} + \sqrt{(3-\sqrt{5})}\right.$

$\text{Sin. } 72^\circ = \frac{r}{4}\sqrt{(10+2\sqrt{5})}$

$\text{Sin. } 75^\circ = \frac{r}{4}(\sqrt{6}+\sqrt{2})$

$\text{Sin. } 81^\circ$

$= \frac{r}{4}\left\{\sqrt{(3+\sqrt{5})} + \sqrt{(5-\sqrt{5})}\right.$

$\text{Sin. } 90^\circ = r.$

SINICAL *Quadrant*, is a quadrant made of wood or metal, with lines drawn from each side, intersecting each other, with an index divided by sines, also with 90 degrees on the limb, and sights at the edge.

SIPHON, or **Syphon**, is a bent tube used in drawing off wine, liquors, and other fluids; the principle of which may be illustrated as follows.

If one end of the siphon or bent tube be put into a vessel of water or fluid, and the other end without be lower than the surface of the water; then, if the air be drawn out, the water will ascend in the leg, in consequence of the pressure of the atmosphere on the external surface of the fluid, from which it will descend in the other leg by its own gravity, and be there discharged into any vessel placed to receive it. For the pressure of the air, to force the fluid back, is equal to the pressure of the air on the surface of that in the vessel, which force is exerted to drive the fluid in the contrary direction; at least, these forces are very nearly equal, the only difference being in their different altitudes, and consequently different densities of the air at those places, which must necessarily be extremeiy small. But the former pressure is opposed to the pressure or weight of a long column, and the latter by the pressure of a short one; the latter pressure of the air, therefore, being less opposed than the former, the fluid must move in the direction of the greater pressure.

When the siphon is very large, and several feet in height, as in drawing off water from a valley or pit, it will be best to proceed as follows. Stop the two orifices, or at least that at which the discharge is to take place, and then by means of an opening made at the top for the purpose, fill the siphon completely, and then stop this opening by means of a plug, or otherwise; after which, if the two ends be opened, the water will begin to flow, and so continue till either the water be exhausted, or till the surfaces of it at each end are in the same horizontal line. It must be observed, however, that this cannot be put in practice if the top of the siphon be more than 33 feet above the surface of the water to be drawn off.

SIRIUS, the *Dog Star*, a very bright star of the first magnitude, in the constellation Canis Major, or the Great Dog.

SITUS, in *Algebra* and *Geometry*, is used by Wolfius and Leibnitz to denote the situation of lines, surfaces, &c. It is, however, never used by modern authors.

SLIDING, in *Mechanics*, is the motion of a body along a plane, when the same face or surface of the moving body keeps in contact with the surface of the plane.

SLING, a string instrument, used in casting stones, &c. which it does with great violence. The motion of a stone discharged from a sling arises from its centrifugal force, when whirled round in a circle. The velocity with which it is discharged, is the same as that which it has in the circle, which much exceeds what could be communicated to it in any other way by the hand; and the direction in which it is discharged is that of the tangent of the circle at the point of discharge; whence its motion, flight, &c. may be computed the same as a common projectile.

SMOKE, a humid matter exhaled in the form of vapour by the action of heat, either externally or internally.

SNOW, a well-known meteor, formed by the freezing of the vapours of the atmosphere.

SOLID, in *Geometry*, is a body of three dimensions, having length, breadth, and thickness: being thus distinguished from a surface which has but two dimensions, and from a line which has but one.

Solid, in *Physics*, is that whose parts adhere to each other with a greater or less force; being thus distinguished from a fluid whose parts yield to the least external pressure.

Regular Solids, are those bounded by equal and regular plane figures.

Solid *Angle*, is that formed by three or more plane angles meeting in a point, like an angle of a die, or the point of a diamond, &c. Or, more generally, a solid angle is the angular space included between several plane surfaces.

Solid angles may be computed and compared with each other, as to quantity, by considering the angular point as the centre of a sphere, and the portion of its surface intercepted between the bounding planes as the measure of the angles. Assuming, then, any number as 2000 for expressing the whole spherical surface, or 1000 for half the same, being the limit of the greatest solid angle; the measure of the angles of any solid will be known by finding the measure of the spherical surface included between its bounding planes. This idea of comparing solid angles by means of spherical surfaces, seems to have been first given by Albert Girard, in his "Inventions Nouvelles en l'Algebra," and has been lately much extended and exemplified in vol. iii. of Dr. Hutton's "Course of Mathematics."

Solid *Numbers*, are those arising from the product of three prime factors.

Solid *Problem*, is that which cannot be constructed geometrically, *viz.* by means of right lines and circles, but requires the introduction of some curves of a higher order, as the ellipse, parabola, and hyperbola, which, being the sections of solids, gave rise to the term solid problem; it is not, however, often employed by modern writers.

SOLIDITY, in *Geometry*, denotes the quantity of space contained or occupied by a solid body, called also its solid content, being estimated by the number of solid or cubic inches, feet, yards, &c. which it contains. The rules for finding the content of all the most common solids are given under the respective articles in this work, and we shall, therefore, in this place confine ourselves to the investigation of the general methods of finding the solidity and surface of any solid of rotation, *viz.* any one whose generation may be conceived to arise from the revolution of any plane surface about a fixed line or axis; or, which amounts to nearly the same thing, by the parallel motion of a circle gradually expanding and contracting itself according to the nature of the generating plane.

To find the surfaces and solidities of solids of rotation.

Let $c = 3{\cdot}1416$, the circumference of a circle whose diameter is 1, and $a = {\cdot}7854$, the area of the same circle; put any variable absciss $= x$, the corresponding ordinate $= y$, and the arc $= z$. Also S the solidity, and s the surface required.

Then because $\dot{z} = \sqrt{(\dot{x} + \dot{y}^2)}$ we shall have

$\dot{s} = 2cy\dot{z} = 2cy\sqrt{(\dot{x} + \dot{y}^2)}$, also

$\dot{S} = 4ay^2\dot{x}$; therefore

$s =$ fluent of $2cy\sqrt{(\dot{x}^2 + \dot{y}^2)}$, and

$S =$ fluent of $4ay^2x$.

And hence is derived the following general rule, viz. from the given equation of the curve, find the value of x in terms of y, or y in terms of x, and substitute it for that quantity in the above fluxional values of $\dot{S}$ or $\dot{s}$, and the fluent of that expression will be the solidity or surface required.

Let the proposed solid be a sphere whose diameter is d, to find its surface and solidity.

1. *For its solidity.* By the equation to the circle we have $y^2 = dx - x^2$, and substituting this value of y^2 in the fluxional equation,

$\dot{S} = 4ay^2\dot{x}$, it becomes

$\dot{S} = 4adx\dot{x} - 4ax^2\dot{x}$

the fluent of which is,

$$S = 2adx^2 - \frac{4}{3}ax^3$$

a general expression for any segment of the sphere, which, when $x = d$, (as is the case in the complete sphere) becomes

$$2ad^3 - \frac{4}{3}ad^3 = \frac{2a}{3}d^3 = \cdot 5236\,d^3$$

the solidity required.

2. *For the surface.* Here again we have $y^2 = dx - x^2$; or,

$y = \sqrt{(dx - x^2)}$; the fluxion of which gives

$$\dot{y} = \frac{(d-2x)\dot{x}}{2\sqrt{(dx-x^2)}} = \frac{(d-2x)\dot{x}}{2y},$$

whence

$$\dot{y}^2 = \frac{d^2 - 4dx + 4x^2}{4y^2}\dot{x}^2 = \frac{d^2 - 4y^2}{4y^2}$$

$\dot{x}^2$ consequently

$$\dot{x}^2 + \dot{y}^2 = \frac{d^2\dot{x}^2}{4y^2} \text{ and } \dot{z} = \frac{d\dot{x}}{2y}$$

Substituting this value of $\dot{z}$, in the general fluxional equation $\dot{s} = 2cy\dot{z}$ gives $\dot{s} = cd\dot{x}$, and consequently $s = cdx$, and when $x = d$ this becomes cd^2, $= 3\cdot 1416\,d^2$ for the whole surface.

SOLSTICE, that time when the sun is in one of the solstitial points: that is, when he is at his greatest distance from the equator: thus called because he then appears to stand still, and not to change his distance from the equator for some time, an appearance arising from the obliquity of our sphere. The solstices are two in each year; viz. the estival, or summer solstice, and the hyemal, or winter solstice. The summer solstice is when the sun is in the tropic of Cancer, which is about the 21st of June, being then the longest day. The winter solstice is when he enters the first degree of Capricorn, about the 22d of December, being then the shortest day.

SOLSTITIAL *Points*, those two points in the ecliptic, which are farthest from the equator, and diametrically opposite to each other, being the first points of Cancer and Capricorn.

SOLUTION, in *Mathematics*, is the operation whereby the answer of a question or problem is determined.

SOLUTION, in *Physics*, is the reduction of any solid to a fluid state by means of some menstruum.

SOUND. To illustrate Sound, it is to be observed, 1st. That a motion is necessary in the sonorous body for the production of sound. 2dly. That this motion exists first in the small and insensible parts of the sonorous bodies, and is excited in them by their mutual collision against each other, which produces the tremulous motions so observable in bodies that have a clear sound, as bells, musical chords, &c. 3dly. That this motion is communicated to, or produces a like motion in the air, or such parts of it as are fit to receive and propagate it. Lastly. That this motion must be communicated to those parts that are the proper and immediate instruments of hearing. Now that motion of a sonorous body, which is the immediate cause of sound, may be owing to two different causes; either the percussion between it and other hard bodies, as in drums, bells, chords, &c. or the beating and dashing of the sonorous body and the air immediately against each other, as in flutes, trumpets, &c. But in both these cases, the motion which is the consequence of the mutual action, as well as the immediate cause of the sonorous motion which the air conveys to the ear is supposed to be an invisible, tremulous, or undulating motion, in the small and insensible parts of the body.

The sonorous body having made its impression on the contiguous air, that impression is propagated from one particle to another, according to the laws of pneumatics. A few particles, for instance, driven from the surface of the body, push or press their adjacent particles into a less space; and the medium, as it is thus rarefied in one place, becomes condensed in the other; but the air thus compressed in the second place is, by its elasticity, returned back again both to its former place and its former state; and the air immediately contiguous to it is compressed in the same manner; which, afterwards, by expanding produces another compression, and so on.

Therefore, from each agitation of the air, there arises a motion in it, analogous to the motion of a wave on the surface of the water, which is called a wave or undulation of air. In each wave the particles go and return back again through very short equal spaces, the motion of each particle being analogous to the motion of a vibrating pendulum while it performs two oscillations, and most of the laws of the pendulum, with very little alteration, being applicable to the former.

Sounds are as various as are the means that concur in producing them. The chief varieties result from the figure, constitution, quantity, &c. of the sonorous body; the manner of percussion, with the velocity, &c. of the consequent vibrations; the state and constitution of the medium; the disposition, distance, &c. of the organ; the obstacles between the organ and the sonorous object and the adjacent bodies. The more notable distinction of sounds arising from the various degrees and combinations of the conditions above mentioned, are into loud and low (or strong and weak), into grave and acute (or sharp and flat, or high and low), and into long and short. The management of which is the office ot music.

Euler is of opinion, that no sound making fewer vibrations than 30 in a second, or more than 7520, is distinguishable by the human ear. According to this doctrine the limit of our hearing, as to acute and grave, is an interval of eight octaves. The velocity of sound is the same with that of the aerial waves, and does not vary much, whether it go with the wind or against it. By the wind, indeed, a certain quantity of air is carried from one place to another, and the sound is accelerated while its waves move through that part of the air, it their direction be the same as that of the wind. But as sound moves much swifter than the wind, the acceleration it will hereby receive is but inconsiderable; and the chief effect we can perceive from the wind is that it increases and diminishes the space of the waves, so that, by help of it, the sound may be heard to a greater distance than it would otherwise be.

That the air is the usual medium of sound appears from various experiments in rarefied and condensed air. In an unexhausted receiver a small bell may be heard to some distance; but, when exhausted, it can scarcely be heard at the smallest distance. When the air is condensed, the sound is louder in proportion to the condensation, or quantity of air crowded in, of which there are many instances in Hauksbee's experiments, in Dr. Priestley's, and others. Besides, sounding bodies communicate tremors to distant bodies; for example, the vibrating motion of a musical string puts others in motion, whose tension and quantity of matter dispose their vibrations to keep time with the pulses of air, propagated from the string that was struck. Galileo explains this phenomenon by observing, that a heavy pendulum may be put in motion by the least breath of the mouth, provided the blasts be often repeated, and keep time exactly with the vibrations of the pendulum, and also by the like art in raising a large bell.

It is not air alone that is capable of the impressions of sound but water also, as is manifested by striking a bell under water; the sound may plainly enough be heard, only not so loud, and also a fourth deeper, according to good judges in musical notes. And Mersenne says, a sound made under water is of the same tone or note as if made in air, and heard under the water.

The velocity of sound, or the space through which it is propagated in a given time, has been very differently estimated by authors who have written upon this subject. Roberval states it at the rate of 560 feet in a second; Gassendus at 1473; Mersenne at 1474; Duhamel, in the History of the Academy of Sciences at Paris, at 1338; Newton at 968; Derham, in whose measure Flamstead and Halley acquiesce, at 1142. The medium velocity of sound, therefore, is nearly at the rate of a mile, or 3280 feet, in $4\frac{2}{3}$ seconds, or a league in 14 seconds, or 13 miles in a minute. But sea-miles are to land-miles nearly as 7 to 6; and, therefore, sound moves over a sea mile in seconds, or a sea league in 16 seconds. Farther, it is a common observation, that persons in health have about 75 pulsations, or beats of the artery at the wrist in a minute; consequently, in 75 pulsations, sound flies about 13 land-miles, or $11\frac{1}{7}$ sea-miles, which is about 1 land-mile in 6 pulses, or 1 sea-mile in 7 pulses, or a league in 20 pulses. And hence the distance of objects may be found by knowing the time employed by sound in moving from those objects to an observer. For example: on seeing the flash of a gun at sea, if 54 beats of the pulse, at the wrist, were counted before the report was heard, the distance of the gun will easily be found, by dividing 54 by 20, which gives 2·7 leagues, or about 8 miles.

SPACE, a simple idea, of which the modes are distance, capacity, extension, duration, &c. Space, considered with regard to length only, is the same idea as we have of distance. If it be considered in length, breadth, and thickness, it is the same as capacity. When considered between the extremities of matter, which fills the capacity of space with something solid, tangible, and moveable, it is called extension.

SPACE, in *Geometry*, is used to express the surface of any figure, or that which fills the interval or distance between the lines that terminate or bound it; thus we say hyperbolic space, parabolic space, &c.

SPECIES, in *Algebra*, are the letters, symbols, marks, or characters, which represent the quantities in any operation or equation.

SPECIFIC. See *Specific Gravity*.

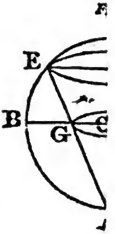
SPECTACLES, an optical instrument, consisting of two lenses set in a frame, and applied to assist in defects of the organs of sight. If the objects are seen through a perfectly flat glass, the rays of light pass through it from them to the eye, in a straight direction, and parallel to each other; and, consequently, the object appears very little either diminished or enlarged, or nearer or farther off, than to the naked eye; but if the glass they are seen through have any degree of convexity, the rays of light are directed from the circumference towards the centre, in an angle proportional to the convexity of the glass, and meet in a point, at a greater or lesser distance from the glass, as it is more or less convex. This point, where the rays meet, is called the focus, and this focus is nearer or farther off according to the convexity of the glass; for, as a little convexity throws it to a considerable distance, so when the convexity is much, the focus is very near. Its magnifying power is also in the same proportion to the convexity; for, as a flat glass scarcely magnifies at all, the less a glass departs from flatness, the less, of course, [illegible] Archimedes,

[illegible]

A Right or Direct Sphere, [illegible] the poles in the horizon, and the [illegible] the zenith and nadir.

In an Oblique Sphere, the equator [illegible] the horizon obliquely, and [illegible]

Parallel Sphere, the poles are in the zenith and nadir, and the equator coincides with the horizon.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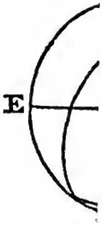
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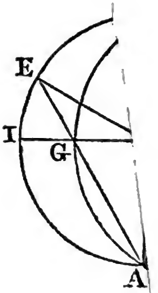
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it magnifies, and the more it approaches towards the globular figure, the nearer its focus is, and the more its magnifying power.

SPECTRUM, in *Optics*, denotes principally the image formed on any white surface, by the rays of the sun passing through a small hole in the window-shutter of a dark room when refracted by a glass prism.

SPECULUM, the same as *Mirror*.

SPHERE, or GLOBE, in *Geometry*, is a solid contained under one uniform surface, every point of which is equally distant from a point within, called the centre of the sphere, and may be conceived to be generated by the revolution of a semi-circle about its diameter, which remains fixed, and which is hence called the axis of the sphere.

The principal properties of the sphere are as follows:

1. A sphere is equal to two-thirds of its circumscribing cylinder; or it is equal to a pyramid or cone, whose base is equal to the whole surface of the sphere, and its altitude equal to half the diameter.

2. All spheres are similar figures, and are to each other as the cubes of their diameters or circumferences.

3. The surface of a sphere is equal to the area of four of its great circles, or to the curve surface of its circumscribing cylinder; and, therefore, the surfaee of different spheres are to each other as the squares of their diameters.

4. The surface of a sphere is equal to the area of a circle, whose radius is equal to the diameter of the sphere; and the curve surface of any segment of a sphere is equal to a circle, having for its radius the chord of half the arc of that segment.

5. The surface of any segment or zone of a sphere is equal to the curve surface of a corresponding portion of the circumscribing cylinder; that is, any two planes passing through the sphere and its circumscribing cylinder parallel to the base of the latter, the surface of the segment of the sphere and cylinder thus cut off will be equal to each other. Most of these properties we owe to Archimedes, being given by that celebrated geometrician in his treatise on the sphere and cylinder.

The several rules relating to the solidities and surfaces of spheres and their segments are contained in the following formulæ:

Let d represent the diameter, and c the circumference of a sphere; also s the surface, and S the solidity of the same, then

1 $s = c\,d = 3{\cdot}14159d^2 = {\cdot}3183c^2$

2. $S = \frac{1}{6}s\,d = {\cdot}5236\,d^3 = {\cdot}01688c^3$.

Spherical Segments, the same notation remaining, and r being put for the radius of the base, and h for the height of the segment

3. surface $= 3{\cdot}14159\,d\,h$

4. solidity $= {\cdot}5236\,h\,(3\,r^2 + h^2)$, or

5. solidity $= {\cdot}5236\,h\,(3\,d - 2\,h)$.

Spherical Zones. Put R and r for the radii of the ends, and h for the height, then

6. surface $= 3{\cdot}14159\,d\,h$

7. solidity $= 1{\cdot}5708\,(R^2 + r^2 + \frac{1}{3}\,h^2)$.

See Hutton's and Bonnycastle's "Treatises of Mensuration."

SPHERE of *Activity* of a *Body*, is used to denote the space through which the influence of any body, as a magnet, extends.

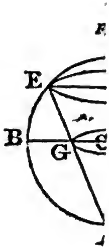
SPHERE, in *Astronomy*, that concave orb or expanse which appears to invest our globe, and in which all the heavenly bodies appear fixed.

Circles of the SPHERE, are certain circles supposed to be described about the sphere for the better determination of the several points, &c. about it, as the *equator*, *ecliptic*, &c. These circles of the heavens are transferred to the earth, and the disposition of these circles, with regard to each other, is what is called the *sphere* in *geography*, which varies in the different parts of the earth; and according to the position of these circles we have a right, oblique, and parallel sphere.

A Right or *Direct* SPHERE, is that which has the poles of the world in the horizon, and the equator in the zenith and nadir.

In an *Oblique* SPHERE, the equator cuts the horizon obliquely; and in a

Parallel SPHERE, the poles are in the zenith and nadir, and the equator coincides with the horizon.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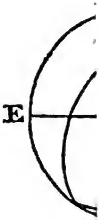
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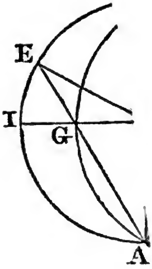
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SPHERICAL, something relating to a sphere, as spherical *angle*, *triangle*, *trigonometry*, for which see the articles **Triangle** and **Trigonometry**.

Spherical *Excess*, in *Trigonometry*, the excess of the sum of the three angles of any spherical triangle, above two right angles. Let A, B, C, be the angles of a spherical triangle, *a*, *b*, *c*, the sides opposite to the angles A, B, C, respectively, $\pi =$ two right angles, $r =$ the radius of a great circle of the sphere, and R'' the number of seconds comprised in radius; then the spherical excess may be ascertained by the following elegant theorem, first given by Simon Lhuillier of Geneva.

$$\text{Tan. } \tfrac{1}{2}(A + B + C - \pi) = \frac{1}{r}\sqrt{\left(\tan.\frac{a+b+c}{4}\tan.\frac{a+b-c}{4}\tan.\frac{a+c-b}{4}\tan.\frac{b+c-a}{4}\right)}.$$

M. Puissant gives the following theorems for ε the excess:

$$\varepsilon = \tfrac{1}{2}\left(\frac{R''}{2r^2}\right)a^2 \sin. 2C + \tfrac{1}{2}\left(\frac{R''}{2r^2}\right)c^2 \sin. 2A.$$

$\varepsilon = \tfrac{1}{2}\left(\frac{R''}{r^2}\right) bh$, where $b =$ the base, and $h =$ the height of the triangle. It will hence be easy to form a table, from which the spherical excess of any triangle measured may be learnt from its base and height.

Spherical *Polygon*, is a figure of more than three sides, formed on the surface of a sphere by the intersection of several of its great circles.

SPHERICITY, the quality of a sphere, or that by which a thing becomes spherical.

SPHERICS, the doctrine of the sphere, particularly of the several circles described on its surface, with the method of projecting the same on a plane.

SPHEROID, a solid body resembling a sphere, which is supposed to be generated by the revolution of any oval figure about an axis. This is the most general definition of a spheroid; but the term is generally meant only to denote a solid generated by the revolution of an ellipse about one of its axis, and, therefore, the latter is sometimes called an ellipsoid, to distinguish it from the more general figure spheroid.

When the ellipse revolve about its transverse axis, it is called an oblong or *prolate spheroid;* when about its conjugate axis, an *oblate spheroid;* and when about any other of its diameters, a *universal spheroid;* in which latter case its figure is somewhat resembling a heart.

The principal rules for finding the solidities, surfaces, &c. of spheroids generated from an ellipse and their segments, zones, &c. are contained in the following formulæ:

Let f be the fixed axis, and r the revolving axis, 3·14159, &c. $= p$, make $\frac{f^2 \backsim r^2}{f^2} = q$, then

1. solidity $= \frac{1}{6} f r^2 p$

2. surface $= f r p \left\{ 1 \mp \frac{1 . Aq}{2 . 3} - \frac{1 . 3 Bq}{4 . 5} \mp \frac{1 . 3 . 5 . Cq}{6 . 7} - \text{\&c.} \right\}$

The upper sign having place for the prolate, and the lower for the oblate sphere, also A, B, C, &c. the preceding terms.

"If also we make $\frac{r}{f} = z$, $\sqrt{(1 \backsim z^2)} = s$, $m =$ the degrees of the arc whose sine is s, likewise

$P = {\cdot}0\,1745329\, m$ in the prolate sphere.

$P = 2{\cdot}30285 \log. (s + z)$ in the oblate sphere.

3. surface $= \frac{Pf + rs}{2s} \times 3{\cdot}14159\, r.$

The area of a prolate spheriod, is less than four times the area of its generating ellipse, and the area of an oblate spheroid greater than four times the same, and the sphere, which is the intermediate limit of the two, is equal to four times its generating circle. For several other interesting properties of these solids, see Hutton's "Mensuration," p. 267, 2d edit.

Frustrums of Spheroids, or the parts cut off by two planes. Here $f, r, q, p,$ denoting the same as above, and *h* the height of the frustrum, also $\frac{4qh^2}{f^4} = s$; then the frustrum being cut off by two planes passing perpendicular to the fixed axis, one of them passing through the centre of the spheroid, we have

$$1.\ \text{surface} = prh \left\{ 1 \mp \frac{Az}{2.3} - \frac{1.3Bs}{4.5} \mp \frac{1.3.5Cz}{6.7} - \&c. \right\}$$

A, B, C, representing the same as above, and the signs having place as there stated.

For the solidity, make the diameter of the greater end $= D$, of the less d; then

$$1.\ \text{solidity} = \frac{1}{12}(2D^2 + d^2)h \times 3{\cdot}14159$$

Segments of Spheroids, or parts cut off by one plane. Here *f*, *r*, and *h* being still as above; then

1. When the base is parallel to the revolving axis.

$$\text{Solidity} = \frac{r^2}{f^2}(3f \backsim 2h)h^2 \times {\cdot}5236$$

2. When the base is perpendicular to the revolving axis.

$$\text{Solidity} = \frac{r^2}{f^2}(3r \backsim 2h)h^2 \times {\cdot}5236$$

See Hutton's "Mensuration."

SPHEROIDAL, or SPHEROIDICAL, having the form of a spheroid.

SPHERULE, a little globe or sphere.

SPINDLE, in *Geometry*, a solid generated by the revolution of a curve about its base or double ordinate, and is farther denominated elliptic, parabolic, hyperbolic, &c. according to figure from which it is generated. See ELLIPTIC, PARABOLIC, HYPERBOLIC *Spindle*.

SPIRAL, in *Geometry*, a curve line of the circular kind, which in its progress always recedes more and more from its centre. There are various kinds of spirals, according to the law by which the point recedes from the centre.

Archimedian SPIRAL. This is generated by supposing the arc of a circle to be divided into any number of equal parts, then radii being drawn from the centre to these points, divide one of them into the same number of equal parts, and make the several radii equal to 1, 2, 3, &c. of those parts, and the line drawn from the centre passing through these several points will be the spiral of Archimedes.

The same spiral may be otherwise generated by a continued motion, by making a point receded uniformly from the centre, while the radius is made to revolve uniformly about the same point.

Hyperbolic or *Reciprocal* SPIRAL. If from one of the extremities of the right line as a centre, an indefinite number of arcs be described so as to be all equal in length to each other, the curve drawn through their extremities is called the hyperbolic or reciprocal spiral, of which the right line is called the axis, and the point the centre.

SPRING, in *Natural History*, a fountain or source of water rising out of the earth. Various have been the opinions of philosophers concerning the origin of springs; as, 1. That the sea-water is conveyed through subterraneous ducts, or canals, to the places where the springs flow out of the earth. 2. The capillary hypothesis, or supposition of the water to rise from the depths of the sea through the porous parts of the earth, as it rises in capillary tubes, or through sand or ashes. 3. That the true sources of springs are melted snow, rain-water, dew, and vapours condensed.

Springs are either such as run continually, called perennial; or such as run only for a time, and at certain seasons of the year, and therefore called temporary springs. Others again are called intermitting springs, because they flow and then stop, and flow and stop again; and finally, reciprocating springs, whose waters rise and fall, or flow and ebb, by regular intervals.

SPRING, in *Mechanics*, is used to signify a body of any shape perfectly elastic.

Strength or *Force of a* SPRING, is used for the force or weight, which, when the spring is wholly compressed or closed, will just prevent it from unbending itself. Also the force of a spring partly

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bent or closed, is the force or weight which is just sufficient to keep the spring in that state, by preventing it from unbending itself any farther.

The theory of springs is founded on this principle: that the intensity is as the compressing force; or if a spring be any way forced or put out of its natural situation, its resistance is proportional to the space by which it is removed from that situation.

For elucidating this principle, on which the whole theory of springs depends, suppose a spring resting against any immoveable support, but otherwise lying in its natural situation, and at full liberty. Then if this spring be pressed inwards by any force p, through the space of one inch, and can be there detained by that force p, the resistance of the spring, and the force p, exactly counterbalancing each other; then will the double force $2p$ bend the spring through the space of two inches, and the triple force $3p$ through 3 inches, and the quadruple force $4p$ through 4 inches, and so on. The space through which the spring is bent, or by which its end is removed from its natural situation, being always proportional to the force which will bend it so far, and will just detain it when so bent. On the other hand, if the end be drawn outwards to any place, and be there detained from returning back by any force p, the space through which it is so drawn outwards, will be also proportional to the force p, which is just able to retain it in that situation.

The number of similar and equal springs a given body in motion can wholly close, is always proportional to the squares of the velocity of that body. And it is from this principle that the chief argument, to prove that the force of a body in motion is as the square of its velocity, is deduced.

Spring *Tides*. See Tides.

SQUARE, is a quadrilateral figure, having its four sides equal to each other, and its angles all right-angles; the area of which is found by multiplying its side by itself.

Square *Number*, is the product arising from a number being multiplied into itself, or a number formed of two equal factors, Square numbers have several curious properties, of which the following are the most remarkable.

1. Every square number is of one of the forms $4n$, or $4n+1$, that is, every square number when divided by 4 will leave either 0 or 1 for a remainder. From an examination of this there may be drawn several curious corollaries.

2. The sum of two odd squares cannot be a square.

3. An odd square taken from an even square cannot leave a square remainder.

4. If the sum of two squares is itself a square, one of the three squares is divisible by 5, and, therefore, by 25.

5. Square numbers must terminate with one of the digits 0, 1, 4, 5, 6, or 9.

6. The area of a rational right-angled triangle cannot be equal to a square.

7. The two following series are remarkable for being such as when reduced to improper fractions, the sum of the squares of the numerator and denominator of each are complete squares, or, which is the same, they are the sides of rational right-angled triangles. These series are as follow: *viz.*

$$1\tfrac{1}{3},\ 2\tfrac{2}{5},\ 3\tfrac{3}{7},\ 4\tfrac{4}{9},\ 5\tfrac{5}{11},\ \&c.$$
$$1\tfrac{7}{8},\ 2\tfrac{11}{12},\ 3\tfrac{15}{16},\ 4\tfrac{19}{20},\ 5\tfrac{23}{24},\ \&c.$$

8. The second difference of consecutive squares are all equal to each other; thus,

$$\begin{array}{lccccccc} \text{squares} & 1, & & 4, & & 9, & & 16,\ \&c. \\ \text{1st diff.} & & 3 & & 5 & & 7 & \&c. \\ \text{2d diff.} & & & 2 & = & 2 & = & 2,\ \&c. \end{array}$$

Square *Root*, in *Arithmetic*, is that number which, multiplied into itself, will produce the given number or square.

If an integral number has not an integral root neither has it a fractional one, and only an approximation towards the exact root can be obtained, which may be done by the following rule.

Rule. Divide the given number into periods of two figures each, beginning at the unit's place and proceeding to the left-hand in integers, and to the right in decimals. Find the greatest square in the first period on the left-hand, and set its root on the right-hand of the given number, after the manner of a quotient figure in division. Subtract the square thus found from the said period, and to the remainder annex the two figures of the next period for a dividend. Double the root above mentioned for a divisor; and find how often it is contained in the said dividend, exclusive of its right-hand figure: and set that quotient figure both in the quotient and divisor. Multiply the whole augmented divisor by this last quotient figure, and subtract the product from the said dividend; bringing down the next period of the given number, as before, for a new dividend. Repeat the same process for each period, and the number thus obtained will be the root required.

The root will necessarily consist of as many integers and decimals as there are periods in each respectively. And when the figures of the given number are all exhausted, the operation may be continued at pleasure by adding ciphers, two in each period.

STAFF, a name given to several different instruments, as the CROSS *Staff*, BACK *Staff*, FORE *Staff*, &c. for which see the several articles.

STAR, is a general name for all the heavenly bodies; but by English writers it is more commonly used to denote a fixed star, or one completely unconnected with the solar system, the others being distinguished by their particular names, as planets, comets, satellites, &c.

The stars are divided into different magnitudes, according to their apparent size, the largest being said to be of the first magnitude; the next largest of the second magnitude; and so on to the sixth magnitude, which class includes the least stars that are visible to the naked eye. All the stars beyond the sixth magnitude are called *telescopic* stars, and those which lie in spaces between (and therefore not included in) the constellations, are called *unformed* stars. The various catalogue founded on new observations, the time in which they were published, the number of stars which they contain, and the works in which the catalogues may be found, will be seen in the following enumeration.

Hipparchus	128	1022	In Ptolem. Almagest.
Tycho		777	Astron. Instaur. Progym.
Prince of Hesse	1593		Flamstead's Historia Celestis.
Riccioli	1665		Astron. Reform.
Bayer	1679	11,800	Uranometria
Hevelius	1690	950	Prodromus Astronomiæ
Flamstead	1712	2884	Historia Celestis.
La Caille	1757	397	Astron. Fundamenta.
La Caille	1750	1942	Southern, Cœlum Australe Stelliferum, p. 141.
La Caille	1765	515	Zodiacal.
Le Monnier	1751, &c.	400	
Tobias Mayer	1775	998	Mayer's Opera Inedita, and in Vince's Astron. vol. ii. p. 549.
Bradley	1773	389	Naut. Almanack, 1773.
Maskelyne	1770	36	
Zach	1800	381	Tabul. Mot. Solis, and in Vince's Astron. vol. iii. p. 523, 537.
Zach	1800	162	
Bode	1797 1801	17,000	Atlas Celeste. Berlin, 1797.
La Lande, sen. & jun.	1789 1801	50,000	Mem. Acad. Par. 1789, and 1790; and Hist. Celest. Franc. tom. I.

The relative positions of the stars in their respective constellations have been represented on the surface of globes, or on maps called planispheres, and atlases. The best planispheres are those of

Lenox; and the most correct celestial atlases are those of Flamstead, Fortin, and Bode, the last of which contains twenty sheets, each sheet being twenty-eight inches by twenty.

Distance and Parallax of the Fixed Stars.—When the fixed stars are examined through a good telescope, they do not seem to have any sensible disc; and their size rather diminishes by increasing the magnifying power of the instrument. This circumstance alone would have been a striking proof of the immeasurable distance of these celestial bodies, had we not been in possession of evidence still more convincing. If the earth's diameter had subtended any sensible angle at the nearest fixed star, astronomers would have been able to determine the distance by the observed change in its place, when viewed from the two extremities of the earth's diameter. This base, however, being found to be too small, they have attempted to discover a change in the position of the stars, when viewed from the earth in two opposite points of its orbit, or, in other words, to find the angle subtended by the diameter of the earth's orbit at the fixed stars, which is called their annual parallax. If a star is viewed, when the earth is in one extremity of its transverse axis, we should expect that it would appear in a different part of the heavens than when it is viewed from the other extremity. But notwithstanding all the attempts of astronomers to discover this change of position, or annual parallax, with the most accurate instruments, they have hitherto been unable to detect it. The observations which have been made for this purpose were so extremely exact, that the parallax of the stars, if it does exist, must be less than 1″, so that if we are unable to determine the distance of these celestial bodies, we can at least fix the limits beyond which they must lie. If we suppose, then, that the parallax of the stars is 1″, and that the mean distance of the earth from the sun, or the semidiameter of the earth's orbit, is 95,000,000 miles, we shall have a triangle whose base is 190,000,000, and the angle at its vertex 1″, to find its side, or the distance of the nearest star, which will be 20,159,665,000,000 miles, or 20 billions of miles, a distance so immensely great, that light cannot pass through it in less than three years. It is very probable, however, that the parallax of the stars is much less than 1″, and that the nearest of these bodies is placed at a much greater distance from us than 20 billions of miles. Some of them even are, perhaps, so remote, that since they were created, the first beam of light which they emitted has not yet reached the limits of our system; while others, which have disappeared, or have been destroyed for many ages, will continue to shine in the heavens till the last ray which they emitted has reached our earth.

By comparing the places of some of the fixed stars, as determined from ancient and modern observations, Dr. Halley discovered that they had a motion of their own, which could not arise from parallax, precession, or aberration.

The probability of a progressive motion of the sun was suggested upon theoretical principles by the late Dr. Wilson of Glasgow; and La Lande deduced a similar opinion from the rotatory motion of the sun, by supposing, that the same mechanical force which gave it a motion round its axis would also displace its centre, and give it a motion of translation in absolute space.

If the sun has a motion in absolute space, directed towards any quarter of the heavens, it is obvious that the stars in that quarter must appear to recede from each other, while those in the opposite region seem gradually approaching, in the same manner as when walking through a forest, the trees to which we advance are constantly widening, while the distance of those which we leave behind is gradually contracting. The proper motion of the stars, therefore, in those opposite regions, as ascertained by a comparison of ancient with modern observations, ought to correspond with this hypothesis; and Dr. Herschel found, that the greater part of them are nearly in the direction which would result from a motion of the sun towards the constellation Hercules, or rather to a part of the heavens whose right ascension is 250° 52′ 30″, and whose north polar distance is 40° 22′, Klugel found the right ascension of this point to be 260°, and Prevot makes it 230°, with 65° of north polar distance. Herschel supposes that the motion of the sun, and the system, is not slower than that of the earth in its orbit, and that it is performed round some distant centre.

While the stars exhibit these apparent variations they are affected with others of a different kind, which seem to arise from some great physical changes that are going on in these bodies. Several new stars have appeared for a time, and then vanished; some that are given in the ancient catalogues can no longer be found; while others are constantly and distinctly visible, which have not been described by the ancients, some stars like β in the Whale, have gradually increased in brilliancy; others like it, in the Great Bear, have been constantly diminishing in brightness, and a great number sustain a periodical variation in their brilliancy. In order to explain these singular changes, astronomers have supposed that the stars are suns, having parts of their surface occupied by large black spots, which, in the course of their rotation about an axis, present themselves to us, and thus diminish the brilliancy of the star.

Double STARS. These, when viewed by the naked eye, and some of them even by the help of a telescope of moderate power, have the appearance only of a single star; but, viewed through a good telescope, they are found to be double, and in some cases a very marked difference is perceptible, both as to their brilliancy and the colour of their light. These Dr. Herschel supposes to be so near each other as to obey reciprocally the power of each other's attraction, revolving about their common centre of gravity, in certain determined periods.

STATICS, is that branch of mechanics which treats of the equilibrium, weight, pressure, &c. of solid bodies when at rest; being thus distinguished from dynamics, which is the science of moving powers, or of the action of forces on solid bodies, when the result of that action is motion.

STATION, in practical *Geometry*, &c. is a place pitched upon to make an observation, or take an angle, or such like, as in surveying, measuring heights and distances, levelling, &c.

An accessible height is taken from one station; but an inaccessible height or distance is only to be taken by making two stations, from two places whose distance asunder is known. In making maps of counties, provinces, &c. stations are fixed upon certain eminences, &c. of the county, and angles taken from thence to the several towns, villages, &c.

STATISTICS, a modern term adopted to express a more comprehensive view of the various particulars constituting the natural and political strength and resources of a country than was usually embraced by writers on political arithmetic.

STEAM, is the name given in our language to the visible vapour which arises from all bodies which contain juices easily expelled from them by heats not sufficient for their combustion.

STEAM, and especially that which is raised from the volatilization of hot water, is employed as a first mover in that admirable contrivance the steam-engine. This steam, when raised with the ordinary heat of boiling water, is almost 3000 times rarer than water, or more than 3½ times rarer than air, and then has its elasticity equal to that of the common atmospheric air: by great heat, it has been found that the steam may be expanded into 14000 times the space of water, when it exerts a force of nearly five times the pressure of the atmosphere: and there is no reason to suppose this is the extreme limit; indeed, some accidents which have happened, prove clearly that the elastic force of steam may at least equal that of gunpowder.

The following table shows the force of this aqueous vapour, at several different degrees of temperature.

Temperature on Fahrenheit	Force of aqueous Vapour.	Temperature on Fahrenheit	Force of aqueous Vapour.
80	·0333	220	1·000
90	·0453	230	1·166
100	·0620	240	1·391
110	·0843	250	1·655
120	·1110	260	1·940
130	·1446	270	2·257
140	·1913	280	2·595
150	·2473	290	2·958
160	·3153	300	3·337
170	·4043	310	3·727
180	·5050	315	4·117
190	·6333	320	4·309
200	·7880	325	4·500
210	·9613	330	4·600

STEELYARD. The common steelyard consists of an iron beam in which is assumed a point at pleasure, on which is raised a perpendicular. On the shorter arm is hung a scale or bason to receive the bodies weighed: the moveable weight is shifted backward and forward on the beam, till it be a counterbalance to 1, 2, 3, 4, &c. pounds placed in the scale; and the points are noted where the constant weight balances these 1, 2, 3, 4, &c. pounds. From this construction of the steelyard, the manner of using it is evident.

Spring STEELYARD, is a kind of portable balance, serving to weigh any matter, from 1 to about 40 pounds.

STEREOGRAPHIC *Projection of the Sphere*, is that in which the eye is supposed to be placed in the surface of the sphere.

STEREOGRAPHY, the art of drawing the forms of solids upon a plane.

STRENGTH, in *Physiology*, the same with force.

On the Measure and Application of Animal Strength.—Men may apply their strength several ways in working a machine. A man of ordinary strength, turning a roller by the handle, can act for a whole day against a resistance equal to thirty pounds weight; and if he works ten hours a day, he will raise a weight of thirty pounds through three feet and a half in a second of time; or, if the weight be greater, he will raise it so much less in proportion. But a man may act, for a small time, against a resistance of fifty pounds or more. If two men work at a windlass, or roller, they can more easily draw up seventy pounds than one man can thirty pounds, provided the elbow of one of the handles be at right angles to that of the other. And with a fly, or heavy wheel, applied to it, a man may do one-third more work; and for a little while he can act with a force, or overcome a continual resistance, of eighty pounds; and work a whole day when the resistance is but forty pounds. Men used to bear loads, such as porters, will carry, some one hundred and fifty pounds, others two hundred or two hundred and fifty pounds, according to their strength. A man can draw but about seventy or eighty pounds horizontally; for he can but apply about half his weight. If the weight of a man be one hundred and forty pounds, he can act with no greater force in thrusting horizontally, at the height of his shoulders, than twenty-seven pounds. A horse is, generally speaking, as strong as five men. A horse will carry two hundred and forty or two hundred and seventy pounds. A horse draws to greatest advantage when the line of direction is a little elevated above the horizon, and the power acts against his breast: and he can draw two hundred pounds for eight hours a day, at two miles and a half an hour. If he draw two hundred and forty pounds, he can work but six hours, and not go quite so fast. And, in both cases, if he carries some weight, he will draw the better for it. And this is the weight a horse is supposed to be able to draw over a pulley out of a well. But in a cart, a horse may draw one thousand pounds, or even double that weight, or a ton weight.

Strength of Materials.—Emerson has particularly treated of the strength of bodies depending on their dimensions and weight. In the general scholium after his propositions on this subject, he adds: If a certain beam of timber be able to support a given weight; another beam, of the same timber, similar to the former, may be taken so great, as to be able but just to bear its own weight: while any larger beam cannot support itself, but must break by its own weight; but any less beam will bear something more. For, the strength being as the cube of the depth; and the stress being as the length and quantity of matter, is the fourth power of the depth; it is plain, therefore, that the stress increases in a greater ratio than the strength. Hence, there is a certain limit, in regard to magnitude, not only in all machines and artificial structures, but also in natural ones, which neither art nor nature can go beyond.

Hence, it is impossible that mechanic engines can be increased to any magnitude at pleasure. For, when they arrive at a particular size, their several parts will break and fall asunder by their weight. Neither can any buildings of vast magnitudes be made to stand, but must by their great weight go to ruin.

It is likewise impossible for nature to produce animals of any vast size at pleasure: except some sort of matter can be found, to make the bones of, which may be so much harder and stronger than any hitherto known: or else that the proportion of the parts be so much altered, and the bones and muscles made thicker in proportion; which will make the animal distorted, and of a monstrous figure, and not capable of performing any proper actions.

Fishes may indeed be produced to a larger size than land animals; because their weight is supported by the water. But yet even these cannot be increased to immensity, because the internal parts will press upon one another by their weight, and destroy their fabric.

On the contrary, when the size of animals is diminished, their strength is not diminished in the same proportion as the weight. For which reason, a small animal will carry far more than a weight equal to its own, whilst a great one cannot carry so much as its weight. And hence it is that small animals are more active, and run faster, jump farther, or perform any motion quicker, for their weight, than large animals: for the less the animal, the greater the proportion of the strength to the stress. And nature seems to know no bounds as to the smallness of animals, at least in regard to their weight.

It is likewise impossible, in the nature of things, that there can be any trees of immense size; if there were any such, their limbs, boughs, and branches, must break off and fall down by their own weight. Thus, it is impossible there can be an oak a quarter of a mile high; such a tree cannot grow or stand, but its limbs will drop off by their weight. And hence also smaller plants can better sustain themselves than large ones.

To form an idea of the strength, or resistance, or renitency, of the parts of bodies, suppose a cylindrical one suspended vertically by one end. Here all its parts, being heavy, tend downwards, and endeavour to separate the two contiguous planes or surfaces where the body is the weakest; but all the parts of them resist this separation by the force with which they cohere, or are bound together. Here then are two opposite powers; *viz.* the weight of the cylinder, which tends to break it; and the force of cohesion of the parts, which resists the fracture.

If now the base of the cylinder be increased, without increasing its length; it is evident that both the resistance and the weight will be increased in the same ratio as the base; and hence it appears that all cylinders, of the same matter and length, whatever their bases be, have an equal resistance, when vertically suspended.

But if the length of the cylinder be increased, without increasing its base, its weight is increased, while the resistance or strength continues unaltered; consequently the lengthening has the effect of weakening it, or increases its tendency to break.

Hence, to find the greatest length a cylinder of any matter may have, when it just breaks with the addition of another given weight, we need only take any cylinder of the same matter, and fasten to it the least weight that is just sufficient to break it; and then consider how much it must be lengthened, so that the weight of the part added, together with the given weight, may be just equal to that weight, and the thing is done. Thus, let l denote the first length of the cylinder, c its weight, g the given weight the lengthened cylinder is to bear, and w the least weight that breaks the cylinder l, also x the length sought;

then as $l : x :: c : \frac{cx}{l} =$ the weight of the longest cylinder sought; and this, together with the given weight g, must be equal to c, together with the weight w; hence then $\frac{cx}{l} + g = c + w$; therefore $x = \frac{c + w - g}{c} l =$ the whole length of the cylinder sought. If the cylinder must just break with its own weight, then is $g = 0$, and in that case $x = \frac{c + w}{c} l$ is the whole length that just breaks by its own weight. By this means, Galileo found that a copper wire, and of consequence any other cylinder of copper, might be extended to 4801 braccios or fathoms of six feet long.

If the cylinder be fixed by one end into a wall, with the axis horizontally; the force to break it, and its resistance to fracture, will here be both different; as both the weight to cause the fracture, and the resistance of the fibres to oppose it, are combined with the effects of the lever; for the weight to cause the fracture, whether of the weight of a beam alone, or combined with an additional weight hung to it, is to be supposed collected into the centre of gravity, where it is considered as acting by a lever equal to the distance of that centre beyond the face of the wall where the cylinder or other prism is fixed; and then the product of the said whole weight and distance will be the momentum or force to break the prism. Again, the resistance of the fibres may be supposed collected into the centre of the transverse section, and all acting there at the end of a lever equal to the vertical semidiameter of the section, the lowest point of that diameter being immoveable, and about which the whole diameter turns when the prism breaks; and hence the product of the adhesive force of the fibres multiplied by the said semidiameter will be the momentum of resistance, and must be equal to the former momentum when the prism just breaks.

Hence, to find the length a prism will bear, fixed horizontally, before it breaks, either by its own weight, or by the addition of any adventitious weight; take any length of such a prism, and load it with weights till it just breaks. Then, put

$l =$ the length of this prism,
$c =$ its weight,
$w =$ the weight that breaks it,
$a =$ distance of weight w,
$g =$ any given weight to be borne,
$d =$ its distance,
$x =$ the length required to break.

Then $l : x = c : \frac{cx}{l}$ the weight of the prism x, and $\frac{cx}{l} \times \frac{1}{2} x = \frac{cx^2}{2l} =$ its momentum; also $dg =$ the momentum of the weight g; therefore $\frac{cx^2}{2l} + dg$ is the momentum of the prism x and its added weight. In like manner $\frac{1}{2} cl + aw$ is that of the former or short prism, and the weight that brake it; consequently $\frac{cx^2}{2l} + dg = \frac{1}{2}cl + aw$, and $x = \sqrt{\frac{aw + \frac{1}{2}cl - dg}{c}} \times 2l$ is the length sought, that just breaks with the weight g at the distance d. If this weight g be nothing, then $x = \sqrt{\frac{aw + \frac{1}{2}cl}{c}} \times 2l$ is the length of the prism that just breaks with its own weight.

If two prisms of the same matter, having their bases and lengths in the same proportion, be suspended horizontally; it is evident that the greater has more weight than the lesser, both on account of its length, and of its base; but it has less resistance on account of its length, considered as a longer arm of a lever, and has only more resistance on account of its base; therefore, it exceeds the lesser in its momentum more than it does in its resistance, and consequently it must break more easily.

Hence appears the reason why, in making small machines and models, people are apt to be mistaken as to the resistance and strength of certain horizontal pieces, when they come to execute their designs in large, by observing the same proportions as in the small.

When the prism, fixed vertically, is just about to break, there is an equilibrium between its positive and relative weight; and consequently those two opposite powers are to each other reciprocally as the arms of the lever to which they are applied, that is, as half the diameter to half the axis of the prism. On the other hand, the resistance of a body is always equal to the greatest weight which it will just sustain in a vertical position, that is, to its absolute weight. Therefore, substituting the absolute weight of the resistance, it appears, that the absolute weight of a body, suspended horizontally, is to its relative weight, as the distance of its centre of gravity from the fixed point or axis of motion, is to the distance of the centre of gravity of its base from the same.

SUBCONTRARY *Position*, in *Geometry*, is when two equiangular triangles are so placed as to have one common angle at the vertex, and yet their bases not parallel.

SUBDUPLICATE *Ratio* of two quantities, is the ratio of their square roots, as $\sqrt{a} : \sqrt{b}$ is the subduplicate ratio of $a : b$.

SUBMULTIPLE. A submultiple number, or quantity, is that which is contained a certain number of times in another, and which, therefore, repeated a certain number of times, becomes exactly equal to it.

SUBMULTIPLE *Ratio*, is that between the quantity contained and the quantity containing.

SUBNORMAL, in *Geometry*, or subperpendicular, the distance upon the axis, between the foot of the ordinate and the perpendicular to the curve or its tangent. In all curves, the subnormal is a third proportional to the subtangent and the ordinate; and in the parabola it is a constant quantity, being equal to half the parameter of the axis.

SUBTANGENT of a *Curve*, the line that determines the intersection of a tangent with the axis; or that determines the point wherein the tangent cuts the axis prolonged.

The tangent, subtangent, and semiordinate, always make a right-angled triangle.

It is a rule in all equations, that if the value of the subtangent comes out positive, the point of intersection of the tangent and axis falls on that side of the ordinate, where the vertex of the curve lies, as in the parabola and paraboloids.

If it come out negative, the point of intersection will fall on the contrary side of the ordinate, in respect of the vertex or beginning of the absciss; as in the hyperbola and hyperboliform figures.

And universally, in all paraboliform and hyperboliform figures, the subtangent is equal to the exponent of the power of the ordinate, multiplied into the absciss; thus, in the common parabola, whose property is $px = y^2$; the subtangent is in length equal to x, the absciss multiplied by 2, the exponent of the power of y; that is, it is equal to twice the absciss;

and by the former rule for paraboliform figures, it must be taken above the ordinate in the axis produced.

Thus, also, in one of the cubical paraboloids, where $px^2 = y^3$; the length of the subtangent will be $\frac{3}{2}$ of the absciss: and in a parabola of any kind, the general equation being $a^m x^n = + y^{m+n}$, the subtangent is $= \frac{m+n}{n} \times x$, or its ratio to the absciss is constantly that of $m + n$ to n.

SUBTRACTION, in *Arithmetic*, is one of the first four fundamental rules of that science; and consists in finding a number equal to the difference of two other given numbers, which number is generally called the remainder.

Simple SUBTRACTION, is the method of ascertaining the difference between any two numbers having the same denomination.

Compound SUBTRACTION, is the method of finding the difference between two given quantities having different denominations.

SUBTRACTION of *Fractions*, is finding the difference between any two fractions of the same, or of different denominations.

SUBTRACTION of *Decimals*, is finding the difference between two quantities consisting partly of integers and partly of decimals, or of decimals only.

SUBTRACTION, in *Algebra*, is finding the difference between two algebraical quantities, and connecting those quantities together with their proper signs. And this is generally divided into the following cases:

1. When the indeterminate letters are the same and affected with the same sign.

Rule. Subtract the co-efficients of the several quantities from each other, and prefix before the remainder the proper sign, whether *plus* or *minus*. When the quantity to be subtracted is greater than the other, then the sign to be prefixed to the remainder must be changed from + to —; or from — to +.

2. When the quantities are the same, but affected with different signs.

Rule. Conceive all the signs of the lower line to be changed, and then add the quantities together, as in addition.

SUBTRACTION of *Algebraic Fractions*, is finding the difference of any two fractions expressed by indeterminate letters.

SUBTRACTION of *Surds*, is finding the difference between any two surd expressions.

Rule. Reduce the surds to their simplest forms; then, if the surd part is the same in both, subtract the co-efficients, and annex to the remainder the common surd. If they are not the same, the subtraction can only be effected by the sign *minus*.

SUBTRAHEND, that quantity which is to be subtracted, the other quantity from which it is to be taken being called the *Minuend*.

SUBTRIPLICATE *Ratio*, is the ratio of the cube root; thus $\sqrt[3]{a}$ to $\sqrt[3]{b}$ is the subtriplicate ratio of a to b.

SUCCESSION of *Signs*, in *Astronomy*, called also *consequentia*, is the order in which the sun enters them: as Aries, Taurus, Gemini, Cancer, &c.

SUCCULAR, in *Mechanics*, a bare axis or cylinder, with staves in it to move it round, but without any tympanum or peritrochium.

SUCKER, in *Mechanics*, a term vulgarly used for the piston of a sucking pump.

SUCKING *Pump*, the common pump working with two valves opening upwards.

SUM, the aggregate of several quantities, or that number which is found by adding several numbers or quantities together.

SUMMER, is one of the quarters of the year, beginning about the 21st of June, and ending on the 22d of September. Summer is also frequently used to denote one half of the year, *viz.* the time in which the sun passes through the northern signs; *viz.* from about the 21st of March, till about the 22d of September, exceeding the winter, or the time in which he passes through the southern signs, by about seven days. See EARTH and EQUINOX.

SUN, *Sol* ⊙, in *Astronomy*, the great source of light and heat to the solar system, about which, as a centre, the several planets perform their revolutions, and to the influence of which, combined with their sidereal and diurnal revolution, they owe the successive alternations of summer and winter, day and night.

The sun is the most considerable of all the heavenly bodies; its diameter is 111·454 times the mean diameter of the earth; whence its volume is 1384472 times greater than that of the earth, but its mass is only 337086 times greater, and its density is $\frac{1}{3\cdot9326}$, or about ¼ that of our globe.

It is surrounded by an atmosphere of great extent, and is frequently obscured by spots, some of which have been observed so large as to exceed the earth five or six times in diameter.

The observation of these spots shows that the sun moves on its axis, which is nearly perpendicular to the ecliptic, the duration of an entire sidereal rotation being about 25½ days, and consequently it is flattened at the poles, the same as all the other planetary bodies.

The solar equator is inclined 7° 30′ to the plane of the ecliptic.

A body which weighs one pound at the surface of the earth, would, if removed to the surface of the sun, weigh 27·933 pounds; and bodies would fall there with a velocity of 334·65 feet in the first second of time.

The sun, together with the planets, moves round the common centre of gravity of the system, which centre is nearly in the centre of the sun; and this motion changes the ecliptic orbits of the planets and comets into epicycloids.

The sun appears to have a particular motion, which carries our system towards the constellation of Hercules.

The apparent diameter of the sun, as seen from the earth, undergoes a periodical variation. It is greatest when the earth is in its perihelion; at which time it is 32′35″ ,6; and it is least when the earth is in its aphelion, when it is 31′ 31″ ,0. Its mean apparent diameter is therefore 32′ 3″ ,3.

His horizontal parallax, as determined by the transit of Venus, is 8¾″.

The greatest equation of his centre is 1° 55′ 27″ ,7, which diminishes at the rate of 16″ ,9 in a century.

These particulars relating to the sun are extracted from Laplace's "System du Monde," 3d edition, and may, therefore, be considered as the most accurate and authentic which have yet been published, having been computed by that eminent astronomer from the latest and most accurate observation.

When the sun is examined with a telescope of sufficient magnifying power, and the splendour of his light is intercepted by the interposition of a dark glass or otherwise, a number of black spots, as we have said above, of various forms and magnitudes, are frequently perceived upon his disc: these are sometimes so large that they have been seen by the naked eye. In the year 1779 Dr. Herschel perceived one which was about 50,000 miles in diameter, more than six times that of our earth, and consequently presenting a disc 36 times larger. The solar spots, when observed for two or three days in succession, seem to move across the body of the sun from east to west, and from this motion the duration of the time of rotation of this luminary has been determined as above stated.

Various hypotheses have been advanced as to the cause of these spots, and the nature of the luminary on which they appear. All these hypotheses are founded upon a supposition of the sun being in itself a hot and luminous body, which would be at best but gratuitous, were it not contradicted by many well-established facts in natural philosophy, which show that heat is produced by the sun's rays, only when they act on a calorific medium; they are only the cause of the production of heat, by uniting with the matter of fire, which is contained in the terrestrial atmosphere, or substances that are heated, as water poured upon lime is the cause of the heat which ensues; and one may easily imagine, that an animal living in such a medium, and under such circum stances, would, in the first instance, consider the water not merely as the cause of heat, but heat itself, as we do the solar rays. Numerous facts, however, may convince us of the contrary. On the tops of mountains of sufficient height, where clouds can seldom reach to shelter them from the direct rays of the sun, we find regions of perpetual snow. Now, if the solar rays themselves conveyed all the heat we find on our globe, it ought to be hottest where their course is the least interrupted; viz. on the tops of those mountains, which we know, from observation, to be in a constant state of congelation. The same has been observed by those who have ascended in balloons, *i. e.* the higher they ascend the greater degree of cold they experience; the sun itself appears diminished, both in splendour and magnitude; and the heavens, instead of the azure or blue, which we observe, approach more and more towards a total obscurity.

SUPERFICIES, or **Superfice**, in *Geometry*, the outside or exterior surface of any body. This is considered as of two dimensions, viz. length and breadth, but without thickness, and therefore forms no part of the substance, or solid content, or matter of the body.

The bounds or extremities of a superficies are lines, and it is said to be a *rectilinear* or *curvilinear* superficies, according as the bounding lines are right or curved.

Superficies are farther divided into *plane*, *concave*, and *convex*.

Plane Superficies, is that in which if a right-line touches it in two points, it touches it in every point; if not, it is curved, and is either concave or convex.

Concave Superficies, is one which sinks inwards, like the inside of a cup or hemisphere.

Convex Superficies, is that which rises outwards, as the outside of a sphere or hemisphere.

SUPPLEMENT of an *Arc*, in *Trigonometry*, is what it wants of 180°.

SURD, in *Arithmetic* and *Algebra*, denotes the root of any quantity, when that quantity is not a complete power of the dimension required; thus $\sqrt{2}$, $\sqrt[3]{4}$, $\sqrt[4]{7}$, &c: are surds or irrational quantities.

Surds are either simple or compound.

A *Simple* Surd, is that which consists of only one term, as $\sqrt{2}$, $\sqrt[3]{6}$, &c.

A *Compound* Surd, is that which consists of two or more terms or radicals, thus $\sqrt{3}+\sqrt{2}$; $\sqrt{3}-\sqrt{2}$, or $\sqrt{(3+\sqrt{3})}$, &c. are compound; and when there are only two terms they are also called *binomial* surds.

Surds, are again either irrational or imaginary; they are *irrational* when the quantity under the radical is a positive quantity, or if it be negative, and the radical be of odd dimensions; and they are *imaginary* when that quantity is negative, and the root to be extracted is of even dimensions.

Surds, are also farther distinguished into *finite* and *continued*; of the former kind are all those which we have given above, and the latter are those of the form $\sqrt{5+\sqrt{5+\sqrt{5+}}}$ &c.; and it is singular that the latter, though apparently so much more complex than the former, are frequently expressible in rational numbers.

To extract the square root of binomial surds.

Let it be proposed to find the square root of $a \pm \sqrt{b}$. Assume $\sqrt{(a \pm \sqrt{b})} = \sqrt{x} \pm \sqrt{y}$; then by squaring

$$a \pm \sqrt{b} = x + y \pm \sqrt{4xy}$$

this gives us $\left\{\begin{array}{l} x+y=a \\ 4xy=b \end{array}\right.$

$$\text{Whence we find } x = \frac{a+\sqrt{(a^2-b)}}{2}$$

$$\text{and } y = \frac{a-\sqrt{(a^2-b)}}{2}$$

which may therefore be considered as general formulæ for this purpose.

Suppose, for example, the $\sqrt{(12+2\sqrt{35})}$ was required, here $a=12$, and $b=140$, therefore,

$$x = \frac{12+\sqrt{(144-140)}}{2} = \frac{12+\sqrt{4}}{2} = 7$$

$$y = \frac{12-\sqrt{(144-140)}}{2} = \frac{12-\sqrt{4}}{2} = 5,$$

therefore the root is $\sqrt{7}+\sqrt{5}$.

To extract the cube root of a binomial surd.

Let $a \pm m\sqrt{b}$ be the proposed surd, $m\sqrt{b}$ being already in its simplest form.

Assume $\sqrt[3]{(a \pm m\sqrt{b})} = x \pm y\sqrt{b}$.

By cubing $a \pm m\sqrt{b} = x^3 \pm 3x^2y\sqrt{b} + 3y^2bx \pm y^3b\sqrt{b}$

Whence $\begin{cases} x^3 + 3y^2bx = a \\ 3x^2y + y^3b = m \end{cases}$ from which equations x and y may be found by exterminating one of those quantities; but, as they must necessarily be either integral or fractional when the extraction is possible, it will commonly be the readiest way to assume several values of y in the latter, and thence find the value, which, when substituted for x in the first equation, must give the true result, if y have been rightly assumed; if not, another value of y must be taken, till a proper value of x is obtained, and if this cannot be done the quantity proposed has no radical root.

Exam. Let it be proposed to find the cube root of $135 \pm 78\sqrt{3}$. Here $a = 135$, $b = 3$, and $m = 7$; and the equations are

$$x^3 + 9xy^2 = 135$$
$$3x^2y + 3y^3 = 78$$

Whence it obtain $y = 2$, and $x = 3$, therefore

$\sqrt[3]{(135 \pm 78\sqrt{3})} = 3 \pm 2\sqrt{3}$, as required.

SURFACE, in *Geometry*. See *Superficies*.

SURSOLID, a term given by the early algebraists to what we now more commonly call a fifth power.

Sursolid *Problem*, that whose solution depends upon the higher geometry, viz. on the conic sections and other curves; this term, however, is now seldom or never made use of.

SURVEYING, in a general sense, denotes the art of measuring the angular and linear distances of objects, whereby to delineate their several positions on paper, and to ascertain the superficial area or space between them. This is of two kinds, *Land surveying*, and *Marine surveying*; the former having generally in view the measure or content of certain tracts of land, and the latter the position of remarkable objects, as beacons, towers, shoals, coasts, &c. Those extensive operations which have in view the determination of the latitudes and longitudes of places, and the lengths of terrestrial arcs in different latitudes, are also placed under the general term surveying.

Of Land-surveying.—This consists of three distinct cases, viz. 1. The measuring of certain lines, angles, &c. 2. Protracting, or laying the same down on paper, so as to form a map of the estate or country. 3. The computation of the superficial content, as found by the preceding operations.

Various instruments are made use of for the purpose of taking the dimensions; the principal and most indispensable of which is the chain commonly called Gunter's chain, which is 22 yards in length, and divided into 100 links: 10 of these square chains, or 100000 square links is one acre, viz.

625 square links is 1 perch.
25000 square links, or 40 perches, 1 rood.
100000 square links, or 4 roods, 1 acre.

This is for taking the linear dimensions, beside which the surveyor must provide himself with 10 small arrows for marking the several lengths, and which are successively put down by the person leading the chain, and taken up by him who follows, and whose business it is to direct the survey. These with the chain are sufficient for surveying estates of considerable extent; but it will frequently save a great deal of labour to be furnished with proper instruments for measuring angles.

The following directions are useful in making a large survey:—

1. Before the survey is begun, walk over the estate two or three times, in order to get a perfect idea of it, till you can carry the form of it tolerably in your mind. And, to help your memory, draw an eye-draught of it on paper, or at least, of the principal parts of it, to guide you.

2. Choose two or more eminences in the estate for your stations, from whence you can see all the principal parts of it: and let these stations be as far distant from one

another as possible; as the fewer stations you have to command the whole, the more exact your work will be; and they will be fitter for your purpose, if these station-lines be in or near the boundaries of the ground, and especially if two or more lines proceed from one station.

3. Take angles between the stations, such as you think necessary, and measure the distances from station to station, always in a right-line; these things must be done, till you get as many angles and lines as are sufficient for determining all your points of station. And in measuring any of those station distances, mark accurately where these lines meet with any hedges, ditches, roads, lanes, paths, rivulets, &c. and where any remarkable object is placed, by measuring its distance from the station-line, and where a perpendicular from it cuts that line; and always mind, in any of these observations, that you be in a right line, which you will know by taking backsight and foresight, along your station-line. And thus as you go along any main station-line take offsets to the ends of all hedges, and to any pond, house, mill, bridge, &c. omitting nothing that is remarkable. And all these things must be noted down; for these are your data, by which the places of such objects are to be determined upon your plan. And be sure to set marks up at the intersections of all hedges with the station-line, that you may know where to measure from, when you come to survey these particular fields, which must immediately be done, as soon as you have measured that station-line, whilst they are fresh in memory. In this way all your station-lines are to be measured, and the situation of all places adjoining to them determined, which is the first grand point to be obtained.

4. As to the inner parts of the estate, they must be determined in like manner by new station-lines, for after the main stations are determined, and every thing adjoining to them, then the estate must be subdivided into two or three parts by new station-lines; taking inner stations at proper places where you can have the best view; measure these station-lines as you did the first, and all their intersections with hedges, and all offsets to such objects as appear. Then you may proceed to survey the adjoining fields by taking the angles that the sides make with the station-line at the intersections, and measuring the distances to each corner from the intersections. For every station-line will be a base to all the future operations, the situation of all parts being entirely dependant upon them; and, therefore, they should be taken of as great a length as possible; and it is best for them to run along some of the hedges or boundaries of one or more fields, or to pass through some of their angles. All things being determined for these stations, you must take more inner ones, and so continue to divide and subdivide, till at last you come to single fields, repeating the same work for the inner stations as for the outer ones, till all be done; and close the work as often as you can, and in as few lines as possible. And that you may choose stations the most conveniently, so as to cause the least labour, let the station-lines run as far as you can along some hedges, and through as many corners of the field and other remarkable points as you can; and take notice how one field lies by another, that you may not misplace them in the draught.

5. An estate may be so situated, that the whole cannot be surveyed together, because one part of the estate cannot be seen from another. In this case you may divide it into three or four parts, and survey the parts separately, as if they were lands belonging to different persons, and at last join them together.

6. As it is necessary to protract or lay down your work as you proceed in it, you must have a scale of a due length to do it by. To get such a scale you must measure the whole length of the estate in chains; then you must consider how many inches in length the

map is to be, and from these you will know how many chains you must have in an inch; then make your scale, or choose one already made accordingly.

7. The trees in every hedge-row must be placed in their proper situation, which is soon done by the plain table, but may be done by the eye without an instrument; and being thus taken by guess in a rough draught, they will be exact enough, being only to look at; except it be such as at any remarkable places, as at the ends of hedges, at stiles, gates, &c. and these must be measured. But all this need not be done till the draught is finished. And observe in all the hedges what side the ditch is on, and, consequently, to whom the fences belong.

8. When you have long stations you ought to have a good instrument to take angles with; and the plain table may very properly be made use of to take the several small internal parts, and such as cannot be taken from the main stations, as it is a very quick and ready instrument.

Trigonometrical SURVEYING. In the preceding part of this article our views have not extended beyond the surveying of fields, estates, &c. the purpose of which is principally to ascertain the exact quantity of land contained in the several fields, and their relative positions with regard to each other; and with regard to the latter the utmost accuracy is not required, and therefore no account is taken of the spherical figure of the earth, but the whole survey is considered as if carried on upon an extended plane surface. But, of late years, surveys of a much more extensive nature have been carrying on, under the patronage of most of the governments of Europe, particularly those of France and England. These have been undertaken principally for the accomplishment of one or other of these objects, viz. 1. For finding the difference of longitude between two noted meridians, as, for instance, between those passing through the observatories of Greenwich and Paris. 2. The accurate determination of the geographical position of the principal places of a country, with a view to give greater accuracy to maps and charts. 3. For the determination of the length of an arc of the meridian in different latitudes, from which to determine the true figure and magnitude of the earth.

When objects so important as these are to be attained, it is manifest, that, in order to insure the necessary degree of correctness in the results, the instruments employed, the operations performed, and the computations required, must each have the greatest possible degree of accuracy.

In the determination of distances of many miles, whether for the survey of a kingdom, or for the measurement of a degree of latitude or longitude, the whole distance between the two extreme points is not actually measured; for this, on account of the inequalities of the earth's surface, the interposition of mountains, rivers, &c. would be always very difficult, and frequently impossible. But a line a few miles in length is very carefully measured on some plain, heath, or marsh, which is so nearly level as to facilitate the measurement of an actual horizontal line, or rather an arc of a great circle of the terrestrial sphere; and this line being assumed as the base of the operations, certain hills, towers, &c. are selected; at which signals can be placed so as to be distinctly visible the one from the other; the lines joining those points constituting a double series of triangles, of which the assumed base forms the first side; the angles of these, that is, the angles made at each station or signal-staff by two other signal-staves, are very accurately measured by a theodolite, which is carried from one station to another; and thus the distance between the two extreme points may be found by the rules of trigonometry, in which, however, the spherical figure of the earth and other minute circumstances are to be introduced.

Thus far the principles of operation are extremely obvious, because we have supposed the whole series of triangles as formed upon an extended horizontal plane, but this is evidently not correct, as instead of such a plane, they form a part of a spherical surface, and therefore the computations must be carried on upon the principles of spherical trigonometry, or the base actual measured must be reduced to the chord of its arc, in which case plane trigonometry may be employed; on both of which methods we propose to offer a few observations.

Measurement of the base.—The first important operation to be attended to in these surveys is, the correct measurement of a base, because on this depend the whole accuracy of the results. The French philosophers used for this purpose, in their measurement for the determination of the length of the *metre*, rulers of platina, and of copper, forming metallic thermometers. The Swedish mathematicians Swanberg and Overbom, employed iron bars covered towards each extremity with plates of silver. The English, under the direction of General Roy, began their measurement with deal rods 20 feet long, but they were soon found not to be sufficiently accurate, and glass rods of the same length were employed, the quantity of whose expansions from any increase of temperature, was first found by experiment, and afterwards adjusted in the survey by means of thermometrical observations. The base thus measured was found to be 27404.08 feet, or about 5.19 miles. And several years afterwards the same base was remeasured by Colonel Mudge, with a steel chain 100 feet long, constructed by Ramsden, and jointed after the manner of the internal chain of a watch. This chain was always stretched to the same tension, supported on troughs laid horizontally, and allowances were made for changes in its length by reason of the variations in temperature, at the rate of ·0075 of an inch for each degree of heat from 62° of Fahrenheit's thermometer; the result of which measurement differed only 2¾ inches from General Roy's, which, considering the whole distance, 5·19 miles, was a remarkable confirmation of the accuracy of both measures.

After the important operation of the measurement of a base, which is to be the side of the first triangle, (which side is a spherical arc, if the measurement be conducted by spherical triangles; or a straight line, if by plane triangles of the chords of the spherical arcs), the angular distance between the objects that mark the several stations must be observed; but, as the objects are probably not situated in the same horizontal plane, nor on the same spherical surface, the angle observed is an oblique angle: but, as the lines, &c. to be computed, are supposed to be in the surface of the earth, horizontal angles are required.

Therefore, the oblique angle observed must be reduced to an horizontal angle.

If we could determine, independently of observation, the quantity by which the sum of the three angles ought to exceed 180°, we should then be able to judge of the accuracy of the observations, which it is desirable to do, since observations made on objects situated nearly in the horizon, are liable to some uncertainty. Now, the theorem concerning the area of a spherical triangle, given under that article, enables us to determine the quantity of the excess; for the area is equal to the difference of the sum of the three angles and 180°, and consequently, since the sides of the spherical triangle, described on the earth's surface, are nearly rectilinear, they may, with scarcely any error, be considered as rectilinear in the computation of the area.

This elegant application of the above rule of Albert Girard's was first made by General Roy, in the Phil. Trans. for 1790, where he gives the following rule, for computing what he calls the spherical excess: *viz.*

From the logarithm of the area of the triangle taken as a plane one in feet, subtract the constant

logarithm 0·3267737, *and the remainder is the logarithm of the excess above* 180° *in seconds nearly.*

This rule was communicated to General Roy by Mr. Dalby, now Professor of Mathematics at the Military College at High Wycomb, but at that time engaged with the General in conducting the survey. Delambre has also given a simple method of determining the spherical excess in Memoires of the Institute for 1806.

By this means the observer may examine the accuracy of his observations, and in some degree correct them when necessary; after which, he may proceed to calculate the sides of the triangles by the rules of spherical trigonometry; but these rules, although they must necessarily give the exact results, will be very deficient in point of expedition, which is an object of great importance where many hundred such operations are to be performed. Legendre has, therefore, furnished us with the following rule, which combines sufficient accuracy, with all the conciseness that can be expected in such cases: *viz.*

A spherical triangle being proposed, of which the sides are very small with regard to the radius of the sphere, if from each of its angles, one-third of the excess of the sum of its three angles above 180° *be subtracted, the angles so diminished may be taken for the angles of a rectilinear triangle, the sides of which are equal in length to those of the proposed rectilinear triangle.*

Triangles solved by either of these methods, *viz.* either by the direct process, or by Legendre's approximation, will be spherical triangles, their sides arcs of great circles, and the computed arc of the meridian also an arc of a great circle. But this is not absolutely necessary; Delambre in France, and Colonel Mudge in England, do not consider the line of the meridian as a curve, but as formed of the chords of curves; and they have accordingly resolved their triangles, not as spherical, but as rectilinear triangles, the sides of which are chords of arcs.

This method, therefore, requires theorems and formulæ different from those we have been explaining, though in some measure depending upon similar considerations. It requires, for instance, the oblique angles to be reduced, either instrumentally or by calculation, to horizontal angles; but these horizontal angles being, as above stated, spherical angles, must be reduced to the angles contained by the chords of the spherical sides of the triangle; that is, we must resolve the following problem.

Having given the angle contained by two spherical arcs of a spherical triangle, to find the angle contained by the chords of those arcs: which is resolved by the following formulæ:

Let the two given spherical arcs be denoted by a and b, the spherical angle by C, the required angle of the chords by θ; then

$$\cos.\theta = \cos.\frac{a}{2}\cdot\cos.\frac{b}{2}\cdot\cos.C + \sin.\frac{a}{2}\cdot\sin.\frac{b}{2}.$$

SURVIVORSHIP, the doctrine of reversionary payments depending upon certain contingencies, or contingent circumstances, as the extinction of some certain life or lives, or the survivorship of some particular life beyond or after other lives. This is one of the most important parts of the doctrine of life annuities, which our limits will not admit detailing; we can, therefore, only refer the reader to our article LIFE *Annuities*, and the works of the several authors there enumerated.

SUSPENSION, in *Mechanics*, the balancing or supporting any body from a point above it, which point is called the *point or centre of suspension.*

SUTTON's *Quadrant*, the name given to a quadrant invented by Mr. Sutton, but which is now seldom made use of, having been superseded by others of a more complete construction.

SYMETRICAL *Equations*, in *Algebra*, are those equations consisting of two or more unknown quantities, similarly involved, so that no difference will arise by writing these quantities reciprocally for each other.

SYNCHRONISM, the being or happening of several events *at* the same time, as *isochronism* denotes two or more things being performed *in* the same time.

SYNODIC, or SYNODICAL *Month*, is the period or interval of time in which the moon passes from one conjunction with the sun to another, and is, therefore, precisely one complete lunation, the mean duration of which is $29^d\,12^h\,44'\,2{\cdot}8''$.

SYNODIC *Revolution* of a *Planet*, is the time between two conjunctions, or two oppositions of the same planet and the sun.

SYNTHESIS, denotes a method of composition, as opposed to analysis, or resolution.

In the synthesis, or synthetic method, we pursue the truth by reasoning drawn from principles before established, or assumed, and propositions formerly demonstrated; thus proceeding by a regular process till we come to the conclusion; and is hence also called the direct method, and composition, in opposition to analysis or resolution.

Such is the method in Euclid's Elements, and most demonstrations of the ancient mathematicians, which proceed from definitions and axioms, to the demonstration of theorems, problems, &c. and from those theorems to the demonstration of others. See ANALYSIS.

SYNTHETICAL *Method*, the method by synthesis, or composition, or the direct method.

SYPHON. See SIPHON.

SYRINGE, in *Hydraulics*, a small simple machine, serving first to imbibe, or suck in a quantity of water, or other fluid, and then to expel the same with violence in a small jet.

The syringe is, in fact, a small single sucking-pump without a valve, the water ascending in it on the same principle as is explained under the article PUMP. It consists, like the pump, of a small cylinder, with an embolus or sucker, moving up and down in it by means of a handle, and fitting it very close within. At the lower end is either a small hole, or a smaller tube fixed to it than the body of the instrument through which the fluid or the water is drawn up, and expelled out again. Thus, the embolus being first pushed close down, introduce the lower end of the pipe into the fluid, then draw the sucker up, by the handle, and the fluid will immediately follow, so as to fill the whole tube of the syringe, and will remain there, even when the pipe is taken out of the fluid; but by thrusting forward the embolus, it will drive the water before it; and being partly impeded by the smallness of the hole, or pipe, it will hence be expelled in a smart jet, and to the greater distance, as the sucker is pushed down with the greater force, or the greater velocity.

SYSTEM, in a general sense, denotes an assemblage or concatenation of principles and conclusions; or the whole of any doctrine, the several parts of which are united together, and follow or depend on each other. As a system of astronomy, a system of the planets, a system of philosophy, a system of motion, &c.

SYSTEM, in *Astronomy*, denotes any hypothesis, or a supposition of a certain order and arrangement of several parts of the universe; by which astronomers explain all the phenomena or appearances of the heavenly bodies, their motions, changes, &c.

This is more peculiarly called the system of the world, and sometimes the solar system.

The most celebrated systems of the world are, the *Ptolomaic*, the *Copernican* or *Pythagorean*, and the *Tychonic;* the principles of each of which is as follows:

Ptolomaic SYSTEM, is so called from the celebrated astronomer Ptolomy. In this system the earth is supposed at rest in the centre of the universe, while the heavens are considered as revolving about it, from east to west, and carrying along with them all the heavenly bodies, the stars, and planets, in the space of 24 hours.

The principal assertors of this

system are, Aristotle, Hipparchus, Ptolomy, and many of the ancient philosophers, and by philosophers in general, for a great number of ages, and long adhered to in many universities and other places. But the modern improvements in philosophy and general science, have utterly exploded this erroneous system from the place it so long held in the minds of astronomers.

Copernican SYSTEM, is that system of the world which places the sun at rest in the centre of the world, and the earth and planets all revolving round him in their several orbits. See the article *Copernican System.*

Solar or *Planetary* SYSTEM is usually confined to narrower bounds; the stars, by their immense distance, and the little relation they seem to bear to us, being accounted no part of it. It is highly probable that each fixed star is itself a sun, and the centre of a particular system, surrounded with a certain number of planets, comets, &c. which in different periods, and at different distances, perform their courses round their respective suns.

The planetary system described under the article COPERNICAN, is the most ancient in the world. It was first introduced into Greece or Italy by Pythagoras; from whom it was called the *Pythagorean System.* It was followed by Plato, Archimides, &c. but it was lost under the reign of the Peripatetic philosophy; till happily retrieved about the year 1500 by Copernicus.

Tychonic SYSTEM supposes that the earth is fixed in the centre of the universe or firmament of stars, and planets revolve round the earth in 24 hours; but it differs from the *Ptolomaic System,* as it not only allows a menstrual motion to the moon round the earth, and that of the satellites about Jupiter and Saturn in their proper periods, but it makes the sun to be the centre of the orbits of the primary planets Mercury, Venus, Mars, Jupiter, &c. in which they are carried round the sun in their respective years, as the sun revolves round the earth in a solar year; and all these planets, together with the sun, are supposed to revolve round the earth in 24 hours. This hypothesis was so embarrassed and perplexed that very few persons embraced it; it was afterwards altered by Longomontanus and others, who allowed the diurnal motion of the earth on its own axis, but they denied its annual motion round the sun. This hypothesis, partly true and partly false, is called the *Semi-Tychonic System.*

SYZYGY, a term equally used for the conjunction and opposition of a planet with the sun. On the phenomena and circumstances of the syzygies, a great part of the lunar theory depends.

It is shown in physical astronomy that the force which diminishes the gravity of the moon in the syzygies, is double that which increases it in the quadratures; so that in the syzygies the gravity of the moon is diminished by a part which is to the whole gravity, as 1 to 89·36; for in the quadratures, the addition of gravity to the whole gravity, as 1 to 178·73.

In the syzygies, the disturbing force is directly as the distance of the moon from the earth, and inversely as the cube of the distance of the earth from the sun. And at the syzygies, the gravity of the moon towards the earth receding from its centre, is more diminished than according to the inverse ratio of the square of the distance from that centre. Hence in the moon's motion from the syzygies to the quadratures, the gravity of the moon towards the earth is continually increased, and the moon is continually retarded in her motion; but in the moon's motion from the quadratures to the syzygies, her gravity is continually diminished, and the motion in her orbit is accelerated.

Farther, in the syzygies, the moon's orbit about the earth is more convex than in the quadratures; for which reason she is less distant from the earth at the former than at the latter. Also, when the moon is in this position, her apses go backward, or retrogade, and her nodes move in antecedentia.

T.

TABLE, in *Mathematics*, is a series or system of numbers methodically arranged, for the more ready calculation of various problems in *Annuities*, *Astronomy*, *Trigonometry*, &c.

Astronomical Tables, are computations of the motions, positions, and other phenomena of the heavenly bodies The oldest of these kind of tables are those given by Ptolomy in his Almagest, but which are now of little or no use, as they no longer agree with the motions of the heavens. These tables were corrected and republished in 1252 by Alphonso, king of Castile, and were long held in high estimation.

Since the revival of the sciences in Europe, and particularly since the restoration of the true system of astronomy by Copernicus, the number of astronomical tables have been continually increasing, and repeated observations, and a very refined calculus, have now brought them to an astonishing degree of perfection. We cannot, within the limits of this article, enter into a minute detail of the several improvements that have been made in this department of astronomy, at the same time we shall endeavour to enumerate a few of the most important particulars, as also a catalogue of those tables, which, from their accuracy and arrangement, are esteemed the most valuable.

Copernicus, after thirty years of observation and calculation, published a new set of Astronomical Tables in 1543, in his celebrated work "De Revolutionibus orbium Celestium," which were republished in 1566, 1593, and 1617. These were again corrected and improved by the observations of other astronomers, and were certainly the most correct of any that appeared before the publication of the "Rudolphine Tables," the work of Tycho and Kepler; these were published at Lintz, in Upper Austria, in 1627, containing 115 pages of tables, and 125 of rules and precepts.

The same were afterwards, viz. in 1650, changed into another form by Maria Cunitia, whose astronomical tables, comprehending the effect of Kepler's physical hypothesis, are very easy, satisfying all the phenomena without the use of logarithms, and with little or no trouble of calculation; and Mercator made a like attempt in his Astronomical Institution, published in 1676.

Beside these we may enumerate the following, viz. Tabulæ motuum Celestium, &c.; Auct. Chr. Severino Longomontano, 1624, 1640, folio.

Christiani Reinharti, Tabulæ Astronomicæ, 1630, 4to.

Philippi Lansbergii Tabulæ motuum, &c. 1632, folio.

Ismaelis Bullialdi Tabulæ Philolaicæ, &c. 1645.

This great and important work contained the most perfect observations, methods, and tables, that had ever appeared prior to its publication.

I. B. Riccioli S. I. Tabulæ Novæ Astron. &c. 1665, folio.

There are here 102 tables, forming a complete collection of all those that astronomers have need of in their computations.

Tabulæ Lodoicaeæ de doctrina eclipsium, &c. 1656.

Astronomical Tables for the Meridian of London, by John Newton, 1657, 4to.

Tabularum Astronomicarum, &c. Ph. De La Hire, 1687, 4to. 1702, 1725, 1727, 1735.

These tables were long regarded as the best, being superior to all those which had preceded them.

Astronomical tables for computing the Places of the Sun, Moon, Planets, and Comets, by Edmund Halley; London, 1749, 4to.; 1752 8vo.; beside two editions in French. These tables were not excelled till the publication of Lalande's tables in 1771.

Besides the tables above enumerated, we might mention many others, as La Caille's Tables of the Sun; Mayer's Tables of the Moon, published by the Board of Longitude. Tables of the same luminary have also been computed by Charles Mason, which are very accurate, being used by the computors of the Nautical Almanack. Many other sets of astronomical tables have also been published by various astronomers and academies; and divers sets of them may be found in the several treatises of astronomy and navigation, of which, however, those of Lalande, given in his Astronomy, are esteemed the most perfect; the same, with some additions, are given by Professor Vince in his large work on the same subject. For a more particular detail of the several astronomical tables, see Montucla's Hist. des Math. vol. iv. p. 302, *et seq.*

TANGENCIES, *Problem* of. This general problem in geometry furnishes the subject of one of the twelve treatises described by Pappus, in the preface to the 7th book of his Mathematical Collections. In Dr. Halley's translation of Pappus the problem is thus enumerated: "E punctis rectis et circulis, quibuscunque tribus positione datis, circulum ducere per singula datæ puncta, qui, si fieri possit, contingat etium datas lineas." This is naturally subdivided into ten distinct propositions, which, if a point be represented by (.), a line by (1), and a circle by (0), may be stated very briefly according to the several data, in the following order: (..1), (.11), (110), (.10), (100), (..0), (.00), (000), (...), (111).

The treatise on tangencies was restored by Vieta, under the title of Apollonius Gallus, and many of his deficiencies were supplied by Marinus Ghetaldus. These have been translated, with the addition of a supplement, by Mr. Lawson, and a farther addition of Fermat's Treatise on Spherical Tangencies. And Leslie has given, in his Geometry, solutions to all except the 5th, 7th, and 8th of the preceding enumeration.

TANGENT, in *Geometry*, is a line that touches a circle or other curve without cutting it.

A tangent touches a curve only in a geometrical point, but at that point the direction of the curve is the same as that of the tangent.

In the circle, a tangent is perpendicular to the radius at the point of contact; also it is a mean proportional between the whole subtangent and the part of it without the circle.

All tangents drawn from the same point to touch the same circle are equal to each other. And, therefore, if a number of tangents be drawn to different points of the circle, and an equal length be set off upon each of them from the points of contact, the locus of all the points will be a circle having the same centre.

It is also a curious property of tangents, that if there be any three circles and tangents be drawn common to each two, the three points of intersection will be in one and the same straight line.

To draw a Tangent to a Circle through a given Point.

1. When the given point is in the circumference of the circle, join the point and the centre, and draw perpendicular, and it will be the tangent required. 2. When the point is not in the circumference, join the point and the centre, and describe a semicircle upon the line cutting the given circle, and through the point of section draw the tangent required.

TANGENT, in *Trigonometry*, is a right line touching one extremity of an arc, and limited between that point and its intersection with the secant passing through the other extremity.

Hence the tangents in the 1st and 3d quadrants are *positive*, in the 2d and 4th *negative*, that is, they are drawn in an opposite direction to the former. Also, the tangent of 0° is *zero* or *nothing;* but the tangent of 90° is infinite, so likewise the tangent of 180° is *zero*, and of 270° *infinite*, &c.

The tangent of an arc, and the tangent of its supplement, are equal, but affected with contrary signs; the one being positive, and the other negative.

*Co-*TANGENT of an angle, is the tangent of the complement of that angle; the letters *co* being only a contraction of the word complement.

Some of the principal trigonometrical formulæ relating to the tangents and co-tangents of angles are as below; *viz.*

$$\text{Tan. } a = \frac{1}{\text{cot. } a} = \frac{\text{sin. } a}{\sqrt{(1 - \text{sin.}^2 a)}} = \frac{\sqrt{(1 - \text{cos.}^2 a)}}{\text{cos. } a}$$

$$= \sqrt{(\text{sec.}^2 a - 1)} = \frac{\text{sec. } a}{\text{cosec. } a}$$

$$= \frac{\text{sin. } a \, \text{cos. } a}{\text{cot. } a}$$

$$= \frac{\text{sin. } a}{\text{cos. } a} = \frac{\text{cos. } a \, . \, \text{sec. } a}{\text{cot. } a}$$

$$\text{Cot. } a = \frac{1}{\text{tan. } a} = \frac{\sqrt{(1 - \text{sin.}^2 a)}}{\text{sin. } a}$$

$$= \frac{\text{cos. } a}{\sqrt{(1 - \text{cos.}^2 a)}}$$

$$= \frac{1}{\sqrt{(\text{sec.}^2 a - 1)}} \text{ \&c. \&c.}$$

being the reciprocals of the former.

All the above formulæ are immediately deducible from the preceding figure; the following relating to the tangent and co-tangents of the sums and difference of arcs, as also of the multiples and submultiples of arcs, we have extracted from Cagnoli's Treatise of Trigonometry; to which work we must refer the reader, who is desirous of following their investigations. See also Bonnycastle's Trigonometry.

1. $\text{Tan. } (a \pm b) = \dfrac{\text{tan. } a \pm \text{tan. } b}{1 \mp \text{tan. } a \, \text{tan. } b}$

2. $\text{Cot. } (a \pm b) = \dfrac{\text{cot. } a . \, \text{cot. } b \pm 1}{\text{cot. } . \, b \mp \text{cot. } a}$

3. $\text{Tan. } a \, . \, \text{tan. } b$
$= \dfrac{\text{cos. } (a \backsim b) - \text{cos. } (a + b)}{\text{cos. } (a + b) + \text{cos. } (a \backsim b)}$

4. $\text{Cot. } a \, . \, \text{cot } b$
$= \dfrac{\text{cos. } (a + b) + \text{cos. } (a \backsim b).}{\text{cos. } (a \backsim b) - \text{cos. } (a + b)}$

5. $\text{Tan. } \; a = \text{tan. } a$

6. $\text{Tan. } 2\,a = \dfrac{2 \text{ tan. } a}{1 - \text{tan.}^2 a}$

7. $\text{Tan. } 3\,a = \dfrac{3 \text{ tan. } a - \text{tan.}^3 a}{1 - 3 \text{ tan.}^2 a}$

8. $\text{Tan. } 4\,a$
$= \dfrac{4 \text{ tan. } a - 4 \text{ tan.}^3 a}{1 - 6 \text{ tan.}^2 a + \text{tan.}^4 a}$

9. $\text{Tan. } 5\,a$
$= \dfrac{5 \text{ tan. } a - 10 \text{ tan.}^3 a + \text{tan.}^5 a}{1 - 10 \text{ tan.}^2 a + 5 \text{ tan.}^4 a}$

&c. &c.

10. $\text{Cot. } \; a = \text{cot. } a$

11. $\text{Cot. } 2\,a = \dfrac{\text{cot.}^2 a - 1}{2 \text{ cot. } a}$

12. $\text{Cot. } 3\,a = \dfrac{\text{cot.}^3 a - 3 \text{ cot. } a}{3 \text{ cot.}^2 a - 1}$

13. $\text{Cot. } 4 \; = \dfrac{\text{cot.}^4 a - 6 \text{ cot.}^2 a + 1}{4 \text{ cot.}^3 a - 4 \text{ cot. } a}$

&c. &c.

14. $\text{Tan. } \tfrac{1}{2}\,a = \dfrac{\text{sin. } a}{1 + \text{cos. } a} = \dfrac{1 - \text{cos. } a}{\text{sin. } a} = \sqrt{\dfrac{1 - \text{cos. } a}{1 + \text{cos. } a}}$

15. $\text{Co-tan. } \tfrac{1}{2}\,a = \dfrac{\text{sin. } a}{1 - \text{cos. } a} = \dfrac{1 + \text{cos. } a}{\text{sin. } a} = \sqrt{\dfrac{1 + \text{cos. } a}{1 - \text{cos. } a}}$

16. $\text{Arc } a = \text{tan. } a - \tfrac{1}{3} \text{ tan.}^3 a + \tfrac{1}{5} \text{ tan.}^5 a - \tfrac{1}{7} \text{ tan.}^7 a$, &c.; which, when the arc is 45°, becomes
$\text{arc } 45° = 1 - \tfrac{1}{3} + \tfrac{1}{5} - \tfrac{1}{7} + \tfrac{1}{9} -$ &c.
radius being unity, to which we may add the following curious property of tangents, *viz.*

17. The sum of the tangents of any three arcs which are together equal to 360°, is equal to the product of the same, divided by radius squared.

We might have carried these formulæ to a much greater extent; but it is presumed that what are given include the most useful cases.

Artificial **TANGENTS**, a term used by some authors to denote the *logarithmic* tangents, in contradistinction to *natural* tangents, or the absolute tangents to any given radius.

Figure of the **TANGENTS**. See **FIGURE**.

Line of **TANGENTS**. See **GUNTER'S** *Scale*.

Sub-**TANGENT**, the line lying under the tangent; or that part of the axis included between the tangent and ordinate at the point of contact.

Method of **TANGENTS**, is a method of drawing tangents to any algebraical curve, or of determining the magnitude of the tangent and sub-tangent, the equation to the curve being given.

The method of tangents is nearly related to that of *maxima et minima*, and the same authors, who in the early state of algebra attempted one of those cases, never failed of touching also on the other.

Hence we have the methods of Des Cartes, Fermat, Roberval, Hudde, &c. We have already explained, under the article MAXIMA *et Minima*, the several methods of these authors relating to that subject, and as their method of tangents differ in no respect from this, we shall not repeat them in this place, but merely explain the principle which led to so intimate a connection between the two problems.

Des Cartes' Method of Tangents. —It has been shown under the article above referred to, that Des Cartes' method of maxima and minima depended upon his making two roots of his equation equal to each other, and the same principle led him also to the solution of his problem of tangents.

Let us conceive, for example, a curve described on an axis; and from any point in this axis as a centre, let there be described a circle, which shall cut the curve at least in two points, from which draw two ordinates, and which will therefore be necessarily common both to the circle and curve; let us now imagine the radius of this circle to decrease, while its centre remains fixed; and it is obvious, that thus the two points of intersection will approach each other, and finally coincide, in which case the circle will touch the curve in the point E, and the tangent at that point will be common to both, and perpendicular to the radius of the circle at that point. Thus the problem of determining the tangent to a curve, is reduced to finding the position of the perpendicular to the curve, drawn from any point in its axis.

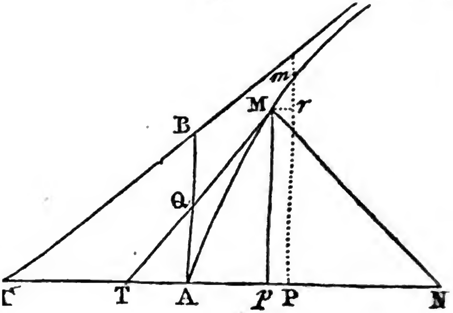
In order to which Des Cartes sought, in a general manner, the points of intersection in the curve, made by a circle described with a given radius from a given point in the axis. He thus arrived at an equation, which in the case of two intersections ought to contain two unequal roots, expressing the distance of the two ordinates from the vertex of the curve. But when the two points of intersection were united in one, as in the case of the circle touching the curve; then the two roots of the equation were necessarily equal to each other. His object, therefore, was in the equation first obtained, and of which the co-efficients were indeterminate, to give them such values that the two roots should be equal; for which purpose he compared the proposed equation with an equation of the same degree having two equal roots; and hence by equating the co-efficients, obtained the value of those in his first equation.

In order to illustrate this, let the curve be a parabola, and let a circle be drawn. Make one abscessa $= a$, another and less one $= x$, the radius $= r$, then the difference of the abscessa $= a - x$. Now since the ordinate belongs to the circle, we have $y^2 = r^2 -$ difference of abscessa $^2 - r^2 - (a - x)^2 = r^2 - a^2 + 2ax - x^2$; but the same ordinate belonging also to the parabola, we have from the known property of that curve $y^2 = px$, p being the parameter: therefore $r^2 - a^2 + 2ax - x^2 = px$, or $x^2 - (p - 2a)x + (a^2 - r^2) = 0$; which being an equation of the second degree, must necessarily have two roots or values of x, answering to the two abscisses, for we should arrive at the same conclusion if our equation had been deduced with reference to the point b; and it is obvious that these roots depend entirely upon the relation of the co-efficients $p - 2a$ and $a^2 - r^2$, or upon the ratio of the quantities a, p, and r, to each other; and consequently such values may be given to these quantities, that the two values of x may be equal.

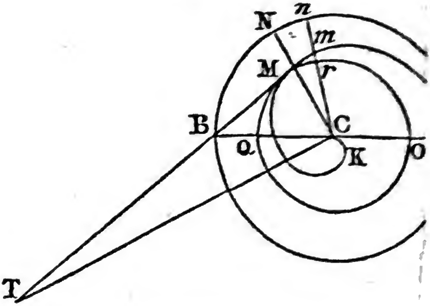
Now to find this ratio, Des Cartes formed an equation of the second degree having two equal roots, as $x^2 - 2ex + e^2 = 0$; *viz.* $(x - e)(x - e) = 0$, and comparing this with that found above, he obtained the equation $x - a = \text{CD}, = \frac{1}{2}p, = 0$; which shows that in the parabola, the subnormal is equal to half the parameter; whence it also

Tangents.

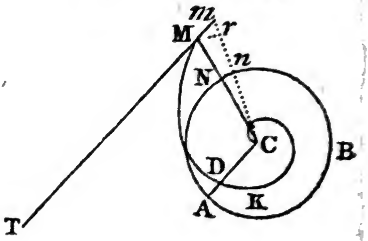
Fig. 1.



Spiral of Archimedes.



Logarithmic Spiral.



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ASTOR, LENOX AND
TILDEN FOUNDATIONS.

follows, that the sub-tangent is equal to double the absciss, which is the known property of the curve.

Besides this method of tangents, Des Cartes proposed another, a little different from it in practice, though the same in principle, which was as follows: he conceived a right line to revolve about a fixed point in the axis of the curve produced, which at first should cut the curve in a certain number of points; but by its revolution, these points of intersections approaching each other would finally coincide, and thus the revolving line become a tangent to the curve. For this purpose he also first obtained the general equation, which he equated with one having two equal roots, and thus determined the several relations of his indeterminate coefficients exactly as in the case above given.

Fermat's Method of Tangents.—It will be found by comparing the above method of tangents of Des Cartes', with that of the *maxima et minima*, that the two ultimately depend upon the same principle, *viz.* of making two roots of the equation equal to each other; and the coincidence of Fermat's methods for these two problems is still more obvious; in fact, he scarcely treats of them as distinct cases, but refers immediately for the solution of the case of tangents to that of his *maxima et minima.*

Barrow's Method of Tangents.—It is obvious from what is said above, and what has been stated under the article Maxima *et Minima*, that both the methods of tangents and of *maxima et minima*, as well by Des Cartes as by Fermat, and particularly those of the latter, were very nearly related to the present fluxional way of considering the same subjects; but with regard to tangents, a still nearer approach was made by Dr. Barrow. This accurate geometer considered the little triangle formed by the difference of the two ordinates, their distance, and the infinitely small part of the curve, as similar to that which is formed by the ordinate, the tangent, and sub-tangent. He then sought, by the equation of the curve, the ratio of the two sides of this triangle, when the difference of the ordinates is infinitely little; then said, as the one of their sides to the other, so is the ordinate to the sub-tangent required.

In the case of the parabola, for example, whose equation is $y^2=px$; supposing Pp the increase of the ordinate $=e$, and the corresponding increase ab of $y=a$; then the equation for the ordinate pb becomes

$$y^2+2ay+a^2=px+pe,$$

$$\left.\begin{array}{l}\text{subtracting}\\ \text{from both}\\ \text{sides}\end{array}\right\} y^2 \qquad\qquad =px$$

$$\text{remains} \qquad 2ay+a^2=pe$$

also, a being itself infinitely small, its square a^2 may be entirely neglected, and there results $2ay=pe$; therefore $a:e::p:2y$; but $a=ba$, and $e=\mathrm{B}a$, also $y=\sqrt{px}$; therefore, from the proposition stated above, *viz.*

$ab:a\mathrm{B}::$ ordin. : subtan. we have, as $p:2\sqrt{px}::\sqrt{px}:2x$, the *sub-tangent* required.

Inverse Method of Tangents This is the reverse of the foregoing, and consists in finding the nature of the curve that has a given sub-tangent.

The method of solution is to put the given sub-tangent equal to the general expression $\frac{y\dot{x}}{\dot{y}}$, which serves for every kind of curve; then the equation being reduced and the fluents taken, give the curve sought.

Thus, for example, to find the curve whose subtangent is $=\frac{2y^2}{a}$.

Here $\frac{2y^2}{a}=\frac{y\dot{x}}{\dot{y}}$; hence $2y\dot{y}=a\dot{x}$, and the fluent of this gives $y^2=ax$, the equation to the parabola; which is, therefore, the curve required.

To find the curve whose subtangent is $\frac{y^2}{2a-x}$.

Here making $\frac{y^2}{2a-x}=\frac{yx}{\dot{y}}$, we have $y\dot{y}=\dot{x}(2a-x)$; the fluent of which gives $y^2 = ax - x^2$, the equation to the circle.

By Fluxions.—Suppose the curve to be A M, (plate Tangents, fig. 1.) its axis, A P; its co-ordinates A P and P M; it is evident that to draw a tangent to the point M, we have only to determine the subtangent P T.

Let us imagine the arc M *m* to be infinitely small, draw the ordinate *m p*, infinitely near to A P, and suppose M *r* parallel to P *p*. Let, as usual, $\mathrm{AP} = x$, $\mathrm{PM} = y$, and we shall have P *p*, or $\mathrm{M}r = \dot{x}$, $mr = \dot{y}$; and by similar triangles M *r m*, T P M we have $mr : r\mathrm{M} = \mathrm{MP} : \mathrm{PT}$ or $\dot{y} : \dot{x} = y . \mathrm{PT} = \frac{y\dot{x}}{\dot{y}}$. Consequently we have only to throw the equation of the curve into fluxions, in order to obtain the value of $\frac{\dot{x}}{\dot{y}}$, and then to substitute this value in the formula for the subtangent just found, and P T will be determined.

The expression for the tangent M T is $\sqrt{(y^2 + \frac{y^2\dot{x}^2}{\dot{y}^2})} = \frac{y}{\dot{y}} \sqrt{(\dot{x}^2 + \dot{y}^2)}$; that of the subnormal P N is $\frac{y^2}{\mathrm{PT}}$, or $\frac{y\dot{y}}{\dot{x}}$; the normal M N $= \frac{y}{\dot{x}} \sqrt{(\dot{x}^2 + \dot{y}^2)}$; and if through the point A we draw the line A Q parallel to M P, we shall have PT : $\mathrm{AT} = \mathrm{MP} : \mathrm{AQ}$, that is $\frac{y\dot{x}}{\dot{y}} . \frac{y\dot{x}}{\dot{y}} - x = y : \mathrm{AQ} = y - \frac{x\dot{y}}{\dot{x}}$. These values of A Q and A T enable us to find the asymptotes of the curve A M if it have any; for if, after having substituted in these two values that of $\frac{\dot{y}}{\dot{x}}$ obtained from the equation of the curve, we suppose x infinite, there will be as many asymptotes as there are different values of the lines A Q and A T. The position of the asymptotes will always be determined by the points T and Q. We shall now apply these formula to a few examples.

The equation of the circle is $y^2 = a^2 - x^2$; therefore $y\dot{y} = - x\dot{x}$, and $\frac{y\dot{x}}{\dot{y}}$ or the subtangent $= \frac{-y^2}{x} = \frac{(a^2 - x^2)}{x}$. The sign $-$ indicates that the subtangent must be taken in the same direction as the abscissa, because in the construction of the formula it was taken in a contrary direction. If we had taken the vertex of the curve for the origin of the abscissæ, the equation would have been $y^2 = 2ax - xx$, and this would have given a positive result as in the formula.

The equation $y^2 = a^2 - x^2$ gives $\frac{y\dot{y}}{\dot{x}}$ or the subnormal $= - x$; and the normal $= \sqrt{(y^2 + \frac{y^2\dot{y}^2}{\dot{x}^2}} = \sqrt{(x^2 + y^2)} = a =$ the radius, as it evidently should be.

In the parabola $y^2 = px$; therefore $\frac{y\dot{y}}{\dot{x}} = \frac{1}{2}p$, the subnormal; and $\frac{y\dot{x}}{\dot{y}}$, or the subtangent $= \frac{2y^2}{p} = 2x$.

In the hyperbola $y^2 = \frac{b^2}{a^2} (2ax + xx)$; therefore $\frac{y\dot{y}}{\dot{x}} = \frac{b^2}{a^2} \times (a + x)$; and $\frac{y\dot{x}}{\dot{y}} = \frac{a^2\dot{y}^2}{b^2(a+x)} = \frac{2ax + xx}{a + x}$

We have also A T *(see preceding figure)* $= \frac{ax}{a + x}$ an expression which is reduced the quantity a, if we suppose x infinite. On the same supposition we find that AQ $= y - \frac{x\dot{y}}{\dot{x}} = y - \frac{b^2 x}{a^2 y} (a + x) = \frac{a^2y^2 - b^2 x(a + x)}{a^2 y} = \frac{b^2 x}{a y} = b \sqrt{\left(\frac{x}{2a + x}\right)}$, becomes simply $= b$.

These two values of AT and AQ give the position of the asymptotes the same as we found in article (49) conic sections.

In the logarithmic curve, we have $x = \mathrm{A} \log y$, and $\dot{x} = \frac{\mathrm{A}\dot{y}}{y}$. Therefore $\frac{y\dot{x}}{\dot{y}} = \mathrm{A}$. Consequently *its subtangent is always equal to the modulus.*

Let there be any curve BOC fig. 2, with another curve BMA, such, that if we prolong the ordinates OP of the first till they meet with the second curve, the line MO may be some function of arc BO; it is proposed to draw through the given point M the tangent MT.

Conceive the ordinate mp indefinitely near to MP, and Mr parallel to the tangent at the point O; if we make the arc $\mathrm{BO} = z$, $\mathrm{MO} = u$, we shall have $mr = \dot{u}$, $r\mathrm{M} = \mathrm{O}o = \dot{z}$, and $\dot{u} : \dot{z} = u : \mathrm{OT} = \frac{u\dot{z}}{\dot{u}}$. Now u being a function of z, we shall have $\frac{\dot{z}}{\dot{u}}$ by taking the fluxions of the equation which expresses the given relation; and consequently TO, or the point T will be determined, whence it will be easy to draw the tangent MT.

Suppose, for example, that $u = \frac{b}{a} z$, we shall have $\dot{u} = \frac{b\dot{z}}{a}$, and $\mathrm{OT} = z =$ arc BO. If BOC is a circular arc, then AMB, is a cycloid, and this construction is the same as we have already given.

In the quadratrix, if we compute the abscissæ from the centre, we have $y = \frac{x}{a} \cot \frac{cx}{a}$; therefore $\dot{y} = \frac{\dot{x}}{a} \cot \frac{cx}{a} - \frac{cx\dot{x}}{\sin^2 \frac{cx,}{a}}$ and $\frac{x\dot{y}}{\dot{x}} = \frac{x}{a} \cot \frac{cx}{a} - \frac{cxx}{\sin^2 \frac{cx}{a}}$. But $-\frac{x\dot{y}}{\dot{x}} = \mathrm{OT}$, as may be proved by the two similar triangles MOT, and the fluxional triangle formed by drawing an ordinate indefinitely near to MP. (We employ $-y$, because y diminishes, as x augments.)

Therefore $\mathrm{OT} = \frac{cxx}{\sin^2 \frac{cx}{a}} - \frac{x}{a} \cot \frac{cx}{a}$; and adding to each side $\mathrm{CO} = \mathrm{PM} = y = \frac{x}{a} \cot \frac{cx}{a}$, we shall have $\mathrm{CT} = \frac{cxx}{\sin^2 \frac{cx}{a}} = \frac{c}{aa} \mathrm{CM}^2$

When fig. 3. $\mathrm{CM} = \mathrm{CT}$, or at the point D, we have, as before explained, the base $\mathrm{CD} = \frac{aa}{c}$, and consequently $\mathrm{CT} = \frac{\mathrm{CM}^2}{\mathrm{CD}}$. We must therefore take CT a third proportional to the base CD and the radius CM; this will give the point T, through which and through the point M if we draw the line MT, it will be the tangent required.

To draw tangents to spirals, we must resolve the following problem. Let there be described a circle with any radius CA fig. 4, and let there be a curve CKM such, that drawing the radius CMN, the line CM may be any function of the arc ABN, it is required to draw through the given point M the tangent MT.

Conceive the two radii CMN, Cmn indefinitely near to each other, and the little arc Mr described from the centre C, and with the radius CM; and then draw CT perpendicular to CM.

This done, let $\mathrm{CM} = y$, $\mathrm{ABN} = x$, $\mathrm{CA} = a$; we shall then have

$$\mathrm{CA} : \mathrm{CM} = \mathrm{N}n : \mathrm{M}r$$

$$\text{or, } a : y = \dot{x} : \mathrm{M}r = \frac{y\dot{x}}{a},$$

and again,

$$rm : \mathrm{M}r = \mathrm{CM} : \mathrm{CT}$$

$$\text{or } \dot{y} : \frac{y\dot{x}}{a} = y : \mathrm{CT} = \frac{y^2\dot{x}}{a\dot{y}}.$$

For example, let $y = \frac{ax}{\pi}$, then CKM will be the spiral of Archimedes, and we have $\frac{\dot{x}}{\dot{y}} = \frac{\pi}{a}$, and

$$CT = \frac{y^2\pi}{aa} = \frac{axy}{aa} = \frac{xy}{a} = MQO'.$$

In the hyperbolic spiral whose equation is $xy = ab$, we shall have $x\dot{y} + y\dot{x} = o$, $y\dot{x}, = -x\dot{y}$, and CT $= -\frac{xy\dot{y}}{a\dot{y}} = -\frac{xy}{a} = -b$; as we before found it.

In the logarithmic spiral (fig. 5), in which the angle C M T is constant, conceive the radii C M, C m indefinitely near together, and from C as a centre and with any radius C N, describe a circle; make C M $= z$, C N $= a$; and marking on the circumference of the circle a fixed point A, suppose the abscissa A N $= x$, which will give this proportion, C N : N n = C M : M r

$$\text{or, } a : \dot{x} = z : Mr = \frac{z\dot{x}}{a}.$$

Let $t = \tan.\ Mmr$, we shall have $t = \frac{z\dot{x}}{a\dot{z}}$, or $\frac{\dot{x}}{at} = \frac{\dot{z}}{z} = \text{flux. } (\log. z)$; therefore $\log. z = \frac{x}{at}$, or $\frac{x}{at}$ + a constant quantity C; because the fluxion of the equation $\log. z = \frac{x}{at}$, is the same as that of $\log. z = \frac{x}{at} + C$.

Now the equation $\log. z = \frac{x}{at} + C$, shews 1°, that this spiral makes an infinite number of revolutions about its centre, as well in approaching towards it, as in receding from it; for in the place of x we may successively substitute $x + \pi$, $x + 2\pi$ $x + 3\pi$, &c. $-\pi + x$, $-2\pi + x$, &c. π being the circumference A N B.

2. That if we make $C = \log. C'$, we shall have $\log. \frac{z}{C'} = \frac{x}{at} = \frac{x}{at}$ $\log. e$, or $\frac{z}{C'} = e^{\frac{x}{at}}$, and $z\ C' e^{\frac{x}{at}}$; therefore at the point A where $x = o$, we have C D = C.

3. That if the abscissæ are taken in arithmetical progression as x, $2x$, $3x$, &c. the ordinates will form the geometrical progression $C'c^{\frac{x}{at}}$, $C'e^{\frac{2x}{at}}$, $C'e^{\frac{3x}{at}}$, &c.

4. That if $t = \infty$, we have $z = c'$, a property of the circle which, as we already know, cuts all its radii at right angles.

These examples will enable the student to draw tangents to all sorts of curves, whether geometrical or mechanical.

TANTALUS's *Cup*, in *Hydraulics*, a siphon so adapted to a cup, that the short leg being in the cup, the long leg may go down through the bottom of it.

The bended siphon is called Tantalus's cup, from the resemblance of the experiment made with an image, representing Tantalus in the fable, fixed up in the middle of the cup with a siphon concealed in his body, beginning in the bottom of his feet, and ascending to the upper part of his breast, where it makes a turn and descends through the other leg on which he stands, and thence down through the bottom of the cup, where it runs out, and causes the water to subside in the cup; as soon as it rises to the height of the siphon, or to the chin of the image, the water will begin to run through the siphon concealed in the figure, till the cup is emptied in the manner explained under the article Siphon.

TAURUS, the *Bull*, one of the twelve signs of the zodiac, denoted by the character ♉.

TAUTOCHRONE, a term sometimes applied to a cycloid with references to its theoretical property of isochronism.

TAYLOR's *Theorem*, in the higher mathematics, is a most elegant and fertile formula, given by Dr. Brook Taylor, in cor. 2, prop. 7, p. 23, of his Method of Increments. It is in purport as follows: If x and z be any two variable quantities, the relation of which is given, n while x by flowing uniformly ncreased by $\dot{x}$, z will be increas- by $\dot{z} + \frac{\ddot{z}}{1.2} + \frac{\dddot{z}}{1.2.3} +$ &c. in ich the value of $\dot{z}$, $\ddot{z}$, &c. are to determined from the given uation.

For the convenience of demonration the theorem may be thus pressed:

If z and x be cotemporaneous alues of two quantities any how lated, and $\underset{\cdot}{z}$ and $\dot{x}$ = fluxion of , cotemporaneous increments of hich x is uniformly generated; hen will

$$+ \underset{\cdot}{z} = z + \frac{\dot{z}}{1} + \frac{\ddot{z}}{1.2} + \frac{\dddot{z}}{1.2.3} + \ldots \frac{\overset{m.}{z}}{1.2\ldots m} + \text{\&c.}$$

when this series terminates or converges.

Demonstration.

$$\text{Let } \left.\begin{matrix} x, x+\underset{\cdot}{x}, x+2\underset{\cdot}{x} \ldots\ldots x+\dot{x} \\ z, z', z'', z''' \ldots\ldots z+\underset{\cdot}{z} \end{matrix}\right\} \text{be cotemporaneous values of } x \text{ and } z.$$

$$\text{Let also } \left.\begin{matrix} a, a', a'', a''', \text{\&c.} \\ b, b', b'', \text{\&c.} \\ c, c'', \text{\&c.} \\ d, \text{\&c.} \end{matrix}\right\} \text{be differences of the respective orders.}$$

Then, by the theorem for differences,

$$z + \underset{\cdot}{z} = z + n\,a + \frac{n.(n-1)}{1.2}\,b + \frac{n(n-1)(n-2)}{1.2.3}\,c + \text{\&c.}$$

where n is the number of successive values from x to $x + \dot{x}$, or from z to $z + \underset{\cdot}{z}$. Now, if n be increased *sine limite*, any assigned number of terms of this quantity approaches to the same number of terms in the series, $z + n\,a + \frac{n^2 b}{1.2} + \frac{n^3 c}{1.2.3} +$ &c. as its limit; or, because $n = \frac{x}{\underset{\cdot}{x}}$, to its equal $z + \frac{a}{\underset{\cdot}{x}}\dot{x} + \frac{b}{\underset{\cdot}{x}^2}, \frac{\dot{x}^2}{1.2} +$ &c.

But when n is so increased the limiting ratio of a to $\underset{\cdot}{x}$, or the limiting ratio of the increments of z and x is the ratio of the fluxions of z and x, and it follows therefore that when n is increased *sine limite*, the limiting value of $\frac{a}{\underset{\cdot}{x}} = \frac{\dot{z}}{\dot{x}}$.

Also, for the same reason $\frac{b}{\underset{\cdot}{x}} = \frac{\dot{a}}{\underset{\cdot}{x}}$, $\frac{c}{\dot{x}} = \frac{\dot{b}}{\underset{\cdot}{x}}$, &c. Whence the

$$\text{lim. val. of } \frac{b}{\underset{\cdot}{x}^2} = \frac{\dot{a}}{\dot{x}\underset{\cdot}{x}} = \frac{\ddot{z}}{\dot{x}^2},$$

$$\text{because } \frac{\dot{a}}{x} = \frac{\ddot{z}}{x}$$

$$\text{of } \frac{c}{\underset{\cdot}{x}^3} = \frac{b}{\dot{x}\underset{\cdot}{x}^2} = \frac{\dddot{z}}{\dot{x}^3},$$

$$\text{because } \frac{\dot{b}}{\underset{\cdot}{x}^2} = \frac{\dddot{z}}{\dot{x}^2} \text{ \&c. \&c.}$$

Whence the limiting value of $z + n\,a + \frac{n(n-1)}{1.2}\,b + \frac{n(n-1)(n-2)}{1.2.3}\,c +$ &c. when n is increased *sine limite* is $z + \frac{\dot{z}}{1} + \frac{\ddot{z}}{1.2} +$ &c.

And because when the former series terminates its value is $z + \underset{\cdot}{z}$; and when it converges its limit is also $z + \underset{\cdot}{z}$; therefore $z + \underset{\cdot}{z} = z + \frac{\dot{z}}{1} + \frac{\ddot{z}}{1.2} +$ &c. when the series terminates or converges. When it does not converge nothing can be asserted of it, because we cannot reason concerning a limit which does not exist.

The above demonstration is by Dr. Brinkley. Other demonstrations may be seen at p. 25, Calcul. Differential, &c. par Lacroix, and a synthetical one in the works of Frisi; but that given by L'Huilier in his "Principiorum Calculi," &c. is considered the most complete.

Taylor's theorem may be more generally expressed: for if z be a quantity composed of two or more independent quantities x, y, v, &c. then while x, y, v, &c. by flowing uniformly, become $x + \dot{x}$, $y + \dot{y}$, $v + v$, &c. z will become, $\dot{z} + \frac{\ddot{z}}{1.2} +$ &c.

See the same also in another form, under INCREMENTS, art. 4.

For the use of this theorem in finding fluxions *per saltum*, in approximating to the roots of equations, &c. see an ingenious paper by Dr. Brinkley in the seventh volume of the Transactions of the Irish Academy. And for an explanation of the cases in which the theorem is defective, and for the determination of its limits, see Francœur, Mathematiques Pures, tom. ii. p. 243, 254.

TEBET, or *Thevet*, the fourth month of the civil year of the Hebrews, answering to part of our months, December and January.

TEETH, in *Mechanics*, are certain projections on the extreme parts of wheels, and by means of which motion is communicated to the different parts of a machine.

Emerson, in his Mechanics, prop. 25, treats of the theory of teeth, *viz.* of their form, action, &c. maintaining that they ought to have the figure of epicycloids, for properly working one within the other, as was originally proposed by De la Hire, who affirmed that the pressure would be uniform if the teeth were formed into epicycloids; and Camus, in his Course of Mathematics, has pursued the same principle, and applied it to the various cases that are likely to arise in practice. The construction, however, is subject to a limitation; on which account a second method has been proposed, which secures the perfect uniformity of action without any such limitation. This method consists in making both teeth portions of circles.

TELEGRAPH, is a name very properly given to an instrument, by means of which information may be conveyed with the greatest expedition, from almost any distance.

It has been said that the ancients were in possession of certain methods of conveying intelligence to considerable distances, and some parts of a Greek play seems to countenance the supposition. Whatever perfection, however, the ancients might have attained in this respect, it was certainly lost to the moderns, and, therefore, the present telegraphs may be considered as totally a modern invention, of which the first hint is given by the Marquis of Worcester in his Century of Inventions, published 1663.

The next sketch we have on this subject was communicated to the Royal Society by Dr. Hooke, in 1684.

In this discourse he asserts the possibility of conveying intelligence from one place to another at the distance of 30, 40, 100, 120, &c. miles, "in as short a time almost as a man can write what he would have sent." He takes to his aid the then recent invention of the telescope, and explains the method by which characters exposed at one station may be rendered plain and distinguishable at the others. He directs, "first, for the stations; if they be far distant, it will be necessary that they should be high, and lie exposed to the sky, that there be no higher hill, or part of the earth beyond them, that may hinder the distinctness of the characters that are to appear dark, the sky beyond them appearing white: by which means also the thick and vaporous air near the ground will be passed over and avoided." "Next, the height of the stations is advantageous, upon the account of the refractions or inflections of the air." "Next, in choosing of these stations, care must be taken, as near as may be, that there be no hill that interposes between them, that is almost high enough to touch the visible ray; because in such cases, the refraction of the air of that hill will be very apt to disturb the clear appearance of the object." "The next thing to be considered is, what telescopes will be necessary for such stations." "One of these telescopes must be fixed at each extreme station, and two of them in each intermediate one; that a man for each glass, sitting and looking through them, may plainly discover what is done in the next adjoining station, and with his pen write down on paper the characters there exposed in their due order; there ought, therefore, to be two persons at each extreme station, and three at each intermediate one; so that, at the same time, intelligence may be conveyed forwards and backwards. Next, there must be certain times agreed on, when the correspondents are to expect; or else there must be set at the top of the pole, in the morning, the hour appointed by either of the correspondents, for acting that day; if the hour be appointed, pendulum clocks may adjust the moment of expectation and observing." "Next, there must be a convenient apparatus of characters, whereby to communicate any thing with great ease, distinctness, and secrecy. And there must be either day characters, or night characters." The day characters "may all be made of three slit deals:" the night characters "may be made with links, or other lights, disposed in a certain order." The doctor invented 24 simple characters, each constituted of right lines, for the letters of the alphabet; and several single characters, made up of semicircles, for whole sentences. He recommended that three very long masts or poles should be placed vertically, and joined at top by one strong horizontal beam; that a large screen should be placed at one of the upper corners of this frame, behind which all the deal board characters should hang, and by the help of proper cords should quickly be drawn forwards to be exposed, and then drawn back again behind the screen. By these means, says the doctor, "all things may be made so convenient that the same character may be seen at Paris, within a minute after it hath been exposed at London, and the like in proportion for greater distances; and that the characters may be exposed so quick after one another, that a composer shall not much exceed the exposer in swiftness." Among the uses of this contrivance, the inventor specifies these: "The first is for cities or towns besieged; and the second for ships upon the sea; in both which cases it may be practised with great certainty, security, and expedition." The whole of Dr. Hooke's paper was published in Derham's collection of his Experiments and Observations; from which it appears that he had brought the telegraph to a state of far greater maturity and perfection than M. Amonton's, who attempted the same thing about the year 1702; and, indeed, to a state but little inferior to several which have been proposed during the last twenty years.

It was not, however, till the French revolution that the telegraph was applied to useful purposes. Whether M. Chappe, who is said to have invented the telegraph first used by the French about the end of 1793, knew any thing of Hooke's or of Amonton's invention or not, it is impossible to say; but his telegraph was constructed on principles nearly similar. The manner of using this telegraph was as follows: At the first station, which was on the roof of the palace of the Louvre at Paris, M. Chappe, the inventor, received in writing from the Committe of Public Welfare, the words to be sent to Lisle, near which the French army at that time was. An upright post was erected on the Louvre, at the top of which were two transverse arms, moveable in all directions by a single piece of mechanism, and with inconceivable rapidity. He invented a number of positions for these arms, which stood as signs for the letters of the alphabet; and these, for the greater celerity and simplicity, he reduced in number as much as possible. The grammarian will easily conceive that sixteen signs may amply supply all the letters of the alphabet, since some letters may be omitted not only without detriment but with advantage. These signs, as they were arbitrary, could be changed every week; so that the sign of B for one day, might be the sign of M the next; and it was only necessary that the persons at the extremities should know the key.

The intermediate operators were only instructed generally in these sixteen signals; which were so distinct, so marked, so different the one from the other, that they were easily remembered.

The construction of the machine was such, that each signal was uniformly given in precisely the same manner at all times: it did not depend on the operator's manual skill; and the position of the arm could never, for any one signal, be a degree higher or a degree lower, its movement being regulated mechanically. M. Chappe having received at the Louvre the sentence to be conveyed, gave a known signal to the second station, which was at Mont Martre, to prepare. At each station there was a watch tower, where telescopes were fixed, and the person on watch gave the signal of preparation which he had received, and this communicated successively through all the line, which brought them all into a state of readiness. The person at Mont Martre then received, letter by letter, the sentence from the Louvre, which he repeated with his own machine; and this was again repeated from the next height, with inconceivable rapidity, to the final station at Lisle.

Various experiments were in consequence tried upon telegraphs in this country; and one was soon after set up by government, in a chain of stations from the Admiralty-office to the sea-coast. It consists of six octagon boards, each of which is poised upon an axis in a frame, in such a manner that it can be either placed vertically, so as to appear with its full size to the observer at the nearest station, or it becomes invisible to him by being placed horizontally, so that the narrow edge alone is exposed, which narrow edge is from a distance invisible. Six boards make thirty-six changes, by the most plain and simple mode of working; and they will make many more if more were necessary: but as the real superiority of the telegraph over all other modes of making signals consists in its making letters, we do not think that more changes than the letters of the alphabet, and the arithmetical figures, are necessary: but, on the contrary, that those who work the telegraphs should avoid communicating by words or signs agreed upon to express sentences; for that is the sure method never to become expert at sending unexpected intelligence accurately. It has been objected to it, that its form is too clumsy to admit of its being raised to any considerable height above the building on which it stands; and that it cannot be made to change its direction, and consequently cannot be seen but from one particular point; and in consequence several other telegraphs have been proposed to remedy these defects, and others to which the instrument is still liable. The dial-plate of a clock would make an excellent telegraph, as it might exhibit one hundred and forty-four signs, so as to be visible at a great distance. A telegraph on this principle, with only six divisions instead of twelve, would be simple and cheap, and might be raised twenty or thirty feet high above the building without any difficulty: it might be supported on one post, and therefore turn round; and the contrast of colours would always be the same.

TELESCOPE, an optical instrument employed in viewing distant objects.

The invention of the telescope is one of the most important acquisitions that the sciences, perhaps, ever acquired. At the same time that it unfolds to us the wonders of the heavens, it enables us to make such observations as furnish the most certain data for astronomical and nautical calculations, by which we are enabled to trace the laws and principles of the planetary motions, and to employ them with certainty in the most important sciences of navigation, geodesia, &c.

To whom we are indebted for the discovery of the powers of this noble instrnment, is not positively known. According to Wolfius, John Baptista Porta was the first who made a telescope, as he inferred from a passage in the

"Magia Naturalis" of that author, published in 1560; but this passage is too obscure to entitle him to the unqualified honour of this invention.

Thirty years afterwards, *viz.* in 1590, a telescope, 16 inches long, was made and presented to Prince Maurice of Nassau, by a spectacle-maker of Middleburg, but whose name is stated differently by different authors, some saying it was Lippersheim, and others Jansen, or, as Wolfius writes it, Hansen.

The telescopes, however, that were made before Galileo undertook their construction, were but toys to those which were formed by this celebrated philosopher, these being well calculated for astronomical observations. It is said that Galileo being at Venice, was told of a sort of optic-glass, made in Holland, which brought distant objects nearer; upon which, setting himself to think how it should be, he ground two pieces of glass into a form as well as he could, and fitted them to the two ends of an organ-pipe; and with these he showed at once all the wonders of the invention to the Venetians, on the top of the tower of St. Mark. The same author adds, that from this time Galileo devoted himself wholly to the improving and perfecting the telescope; and that he hence almost deserved all the honour usually done him of being reputed the inventor of the instrument, and of its being from him called *Galileo's tube*. Galileo himself, in his *Nuncius Sidereus*, published in 1610, acknowledges that he first heard of the instrument from a German; and that being merely informed of its effects, first by common report, and a few days after by a letter from a French gentleman, James Badovere, at Paris; he himself discovered the construction by considering the nature of refraction. He adds, in his "Saggiatore," that he was at Venice when he heard of the effects of Prince Maurice's instrument, but nothing of its construction; that the first night after his return to Padua he solved the problem, and made his instrument the next day, and soon after presented it to the Doge of Venice, who, in honour of his grand invention, gave him the ducal letters, which settled him for life in his lectureship at Padua, and doubled his salary, which then became treble of what any of his predecessors had enjoyed before And thus Galileo may be considered as the inventor of the telescope, though not the first inventor.

F. Mabillon, indeed, relates, in his Travels through Italy, that in a monastery of his own order, he saw a manuscript copy of the works of Commestor, written by one Conradus, who lived in the 13th century; in the third page of which was seen a portrait of Ptolomy, viewing the stars through a tube of four joints of draws; but he does not say that the tube had glasses in it. Indeed it is more probable, that such tubes were then used for no other purpose but to defend and direct the sight, or to render it more distinct, by singling out the particular object looked at, and shutting out all the foreign rays reflected from others whose proximity might have rendered the image less precise. And this conjecture is verified by experience; for we have often observed, that without a tube, by only looking through the hand, or even the fingers, or a pin-hole in a paper, the objects appear more clear and distinct than otherwise.

Be this as it may, it is certain that the optical principles, upon which telescopes are founded, are contained in Euclid, and were well known to the ancient geometricians; and it has been for want of attention to them, that the world was so long without that admirable invention; as doubtless there are many others lying hid in the same principles, only waiting for reflection, or accident, to bring them forth.

Telescopes are either refracting or reflecting; the former consist of different lenses, through which the objects are seen by rays refracted by them to the eye; and the latter, of specula, from which the rays are reflected and passed to the eye. The lens, or glass, turned to the object, is called the object-glass; and that next the eye, the eye-glass; and when the telescope consists of more than two lenses, all but that next the object are called eye-glasses.

The principal effects of telescopes depend upon this maxim, "that objects appear larger in proportion to the angles which they subtend at the eye; and the effect is the same, whether the pencils of rays, by which objects are visible to us, come directly from the objects themselves, or from any place nearer to the eye, where they may have been united, so as to form an image of the object, because they issue again from those points in certain directions, in the same manner as they did from the corresponding points in the object themselves. In fact, therefore, all that is effected by a telescope, is first to make such an image of a distant object, by means of a lens or mirror, and then to give the eye some assistance for viewing that image as near as possible; so that the angle, which it shall subtend at the eye, may be very large compared with the angle which the object itself would subtend in the same situation. This is done by means of an eye-glass, which so refracts the pencils of rays, as that they may afterwards be brought to their several foci by the natural humours of the eye. But if the eye had been so formed as to be able to see the image, with sufficient distinctness, at the same distance without an eye-glass, it would appear to him as much magnified as it does to another person who makes use of a glass for that purpose, though he would not, in all cases, have so large a field of view.

Although no image be actually formed by the foci of the pencil without the eye; yet if, by the help of an eye-glass, the pencils of rays shall enter the pupil, just as they would have done from any place without the eye, the visual angle will be the same as if an image had been actually formed in that place.

Telescopes are of several kinds, distinguished by the number and form of their lenses, or glasses, and denominated from their particular uses, &c. Such are the terrestrial, or land telescope; the celestial, or astronomical telescope; to which may be added, the Galilean, or Dutch telescope, the reflecting telescope, the achromatic telescope, &c.

We shall proceed to describe some of these in order to illustrate the leading principles.

1. *The refracting Telescope.*

This telescope differs from the microscope only in this, that the object is placed at so great a distance from it, that the rays of the same pencil flowing from thence, may be considered as falling parallel to one another upon the object-glass, and, therefore, the image made by that glass is looked upon as coincident with its focus or parallel rays.

There is a defect in all these kinds of telescopes, not to be remedied in a single lens by any means whatever, which was thought only to arise from hence, *viz.* that spherical glasses do not collect rays to one and the same point. But it was happily discovered by Sir Isaac Newton, that the imperfection of this sort of telescope, so far as it arises from the spherical form of the glasses, bears almost no proportion to that which is owing to the different refrangibility of light. This diversity in the refraction of rays is about a 28th part of the whole; so that the object-glass of a telescope cannot collect the rays which flow from any one point in the object into less than the circular space whose diameter is about the 56th part of the breadth of the glass.

Notwithstanding this imperfection, a dioptrical telescope may be made to magnify in any given degree, provided it be of sufficient length; for the greater the focal distance of the object-glass is, the less may be the proportion which the focal distance of the eye-glass may bear to that of the object-glass, without rendering the image obscure. Thus, an object-glass, whose focal distance is about four feet, will admit of an eye-glass whose focal distance shall be little more than an inch, and, consequently, will magnify almost 48 times; but an object-glass of 40 feet focus will admit of an eye-glass of only four inches focus, and will, therefore, magnify 120 times; and an object-glass of 100 feet focus will admit of an eye-glass of little more than six inches focus, and will, therefore, magnify almost 200 times.

The reason of this disproportion in their several degrees of magnifying is to be explained in the following manner. Since the diameter of the spaces, into which rays flowing from the several points of an object are collected, are as the breadth of the object-glass, it is evident that the degree of confusedness in the image is as the breadth of that glass; for the degree of confusedness will only be as the diameters or breadths of those spaces, and not as the spaces themselves. Now the focal length of the eye-glass, that is, its power of magnifying, must be as that degree, for if it exceeds it, it will render the confusedness sensible, and, therefore, it must be as the breadth or diameter of the object-glass. The diameter of the object-glass, which is as the square root of its aperture or magnitude, must be as the square root of the power of magnifying in the telescope, for unless the aperture itself be as the power of magnifying, the image will want light; the square root of the power of magnifying will be as the square root of the focal distance of the object-glass, and, therefore, the focal distance of the eye-glass must be only as the square root of that of the object-glass. So that in making use of an object-glass of a longer focus, suppose than one that is given, you are not obliged to apply an eye-glass of a proportionably longer focus than what would suit the given object-glass, but such a one only whose focal distance shall be to the focal distance of that which will suit the given object-glass, as the square root of the focal length of the object-glass you make use of, is to the square root of the focal length of the given one. And this is the reason that longer telescopes are capable of magnifying in a greater degree than shorter ones, without rendering the object confused or coloured.

3. But the inconveniency of very long telescopes is so great, that different attempts have been made to remove it. Of these the most successful have been by Dollond and Blair, and the general principles upon which these eminent opticians proceeded have been mentioned in our article Achromatic. It may be sufficient to observe, in addition to what has been already said, that the object-glasses of Blair's telescopes are composed of three distinct lenses, two convex and one concave, of which the concave one is placed in the middle. The two convex ones are made of London crown-glass, and the middle one of white flint-glass; and they are all ground to spheres of different radii, according to the refractive powers of the different kinds of glass and the intended focal distance of the object-glass of the telescope. According to Boscovich, the focal distance of the parallel rays for the concave lens is one-half, and for the convex-glass one third of the combined focus.

2. *The reflecting Telescope.*

The inconveniences arising from the great length of refracting telescopes, before Dollond's discovery, are sufficiently obvious; and these, together with the difficulties occasioned by the different refrangibility of light, induced Sir Isaac Newton to turn his attention to the subject of reflection, and endeavour to realize the ideas of himself and others concerning the possibility of constructing telescopes upon that principle.

In the best reflecting telescopes, the focus of the small mirror is never coincident with the focus of the great one, where the first image is formed, but a little beyond it (with respect to the eye); the consequence of which is, that the rays of the pencils will not be parallel after reflection from the small mirror, but converge so as to meet in points, where they would form a larger upright image than if the glass was not in their way; and this image might be viewed by means of a single eye-glass properly placed between the image and the eye; but then the field of view would be less, and, consequently, not so pleasant; for which reason the glass is still retained to enlarge the scope or area of the field.

To find the magnifying power of this telescope multiply the focal distance of the great mirror by the distance of the small mirror from the image next the eye, and multiply the focal distance of the small mirror by the focal distance of the eye-glass; then divide the product of the former multiplication by the product of the latter, and the quotient will express the magnifying power.

One great advantage of the reflecting telescope is, that it will admit of an eye-glass of a much shorter focal distance than a refracting telescope will, and, consequently, it will magnify so much the more; for the rays are not coloured by reflection from a concave mirror, if it be ground to a true figure, as they are by passing through a convex-glass, let it be ground ever so true.

The nearer an object is to the telescope, the more its pencils of rays will diverge before they fall upon the great mirror, and, therefore, they will be the longer in meeting after reflection; so that the first image will be formed at a greater distance from the large mirror, when the object is near the telescope, than when it is very remote. But as this image must be formed farther from the small mirror than its principal focus *n*, this mirror must be always set at a greater distance from the large one, in viewing near objects, than in viewing remote ones. And this is done by turning the screw on the outside of the tube until the small mirror be so adjusted that the object, or rather its image, appears perfect.

In looking through any telescope towards an object, we never see the object itself, but only that image of it which is formed next the eye in the telescope. For if a man holds his finger or a stick between his bare eye and an object, it will hide part, if not the whole, of the object from his view; but if he ties a stick across the mouth of a telescope before the object-glass, it will hide no part of the imaginary object he saw through the telescope before, unless it covers the whole mouth of the tube; for all the effect will be, to make the object appear dimmer, because it intercepts part of the rays. Whereas, if he puts only a piece of wire across the inside of the tube, between the eye-glass and his eye, it will hide part of the object which he thinks he sees, which proves that he sees not the real object, but its image. This is also confirmed by means of the small mirror E F, in the reflecting telescope, which is made of opaque metal, and stands directly between the eye and the object towards which the telescope is turned; and will hide the whole object from the eye at O, if the two glasses Z Z and S S are taken out of the tube.

Great improvements have been lately made in the construction of both reflecting and refracting telescopes, as well as in the method of applying those instruments to the purposes for which they are intended.

The chief consideration is to have a steady view of the distant object. This is unattainable, unless the axis of the instrument be kept constantly directed to the same point of it; for when the telescope is gently shifted from its position, the object seems to move in the same or in the opposite direction, according as the telescope inverts the object or shows it erect. This is owing to the magnifying power, because the apparent angular motion is greater than what we naturally connect with the motion of the telescope. This does not happen when we look through a tube without glasses.

All shaking of the instrument, therefore, makes the object dance before the eye; and this is disagreeable, and hinders us from seeing it distinctly. But a tremulous motion, however small, is infinitely more prejudicial to the performance of a telescope, by making the object quiver before us. A person walking in the room prevents us from seeing distinctly; nay, the very pulsation in the body of the observer agitates the floor enough to produce this effect, when the telescope has a great magnifying power; for the visible motion of the object is then an imperceptible tremor, like that of an harpsichord wire, which produces an effect precisely similar to optical indistinctness; and every point of the object is diffused over the whole space of the angular tremor, and appears co-existent in every part of this space, just as a harpsichord-wire does while it is sounding. The more rapid this motion is, the indistinctness is the more complete. Therefore the more firm and elastic, and well bound together, the frame-work and apertures of our telescope is, the more hurtful will this consequence be. A mounting of lead, were it practicable, would be preferable to wood, iron, or brass. This is one great cause of the indistinctness of the very finest reflecting telescopes of the usual constructions, and can never be totally removed. In the Gregorian form, it is hardly possible to prevent the elastic tremor of the small speculum, carried by an arm supported at one end only, even though the tube were motionless. But even the great mirror may vibrate enough to produce indistinctness. Refracting telescopes are free from this inconveniency, because a small angular motion of the object-glass round one of its own diameters, has no sensible effect on the image in its focus. They are affected only by an angular motion of the axis of the telescope or of the eye-glasses.

The high magnifying powers of Dr. Herschel's telescopes made all the usual apparatus for their support extremely imperfect. But his ingenuity soon avoided this imperfection. He has contrived for his reflecting telescopes stands, which have every property that can be desired. The tubes are all supported at the two ends. The motions, both vertical and horizontal, are contrived with the utmost simplicity and firmness. We cannot more properly conclude this article than with a description of his 40-feet telescope, the largest instrument, and possessing the highest magnifying power, of any that ever was constructed.

In Plate XIII. we have given a view of this noble instrument in a meridional situation, as it appears when seen from a convenient distance, by a person placed to the south-west of it. The foundation in the ground consists of two concentric circular brick walls, the outermost of which is 42 feet in diameter, and the inside one 21 feet. They are two feet six inches deep under ground, two feet three inches broad at the bottom, and one foot two inches at the top, and are capped with paving-stones about three inches thick, and twelve and three-quarters broad. The bottom frame to the whole apparatus rests upon these two walls, by twenty concentric rollers III, and is moveable upon a pivot, which gives a horizontal motion to the whole apparatus, as well as to the telescope.

The tube of the telescope A, though very simple in its form, which is cylindrical, was attended with great difficulties in the construction. This is not to be wondered at, when its size, and the materials of which it is made, are considered. Its length is 39 feet four inches; it measures four feet ten inches in diameter; and every part of it is of iron. Upon a moderate computation, the weight of a wooden tube must have exceeded an iron one at least 3000 pounds, and its durability would have been far inferior to that of iron. It is made of rolled or sheet iron, which has been joined together without rivets, by a kind of seaming well known to those who make iron funnels for stoves.

Very great mechanical skill is used in the contrivance of the apparatus by which the telescope is supported and directed. In order to command every altitude, the point of support is moveable, and its motion is effected by mechanism, so that the telescope may be moved from its most backward point of support to the most forward, and by means of the pulleys G G suspended from the great beam H, be set to any altitude up to the very zenith. The tube is also made to rest with the point of support in a pivot, which permits it to be turned sidewise.

The concave face of the great mirror is 48 inches in diameter, of polished surface. The thickness, which is equal in every part of it, remains now about three inches and a half; and its weight, when it came from the cast, was 2118 pounds, of which it must have lost a small quantity in polishing, and it magnifies with proper eye-glasses more than 6000 times. To put this speculum into the tube, it is suspended vertically by a crane in the laboratory, and placed on a small narrow carriage, which is drawn out, rolling upon planks, till it comes near the back of the tube; here it is again suspended and placed in the tube by a peculiar apparatus.

The method of observing by this telescope is what Dr. Herschel calls the front view; the observer being placed in a seat C, suspended at the end of it, with his back towards the object he views. There is no small speculum, but the magnifiers are applied immediately to the first focal image.

From the opening of the telescope, near the place of the eye-glass, a speaking-pipe runs down to the bottom of the tube, where it goes into a turning joint; and after several other inflections, it at length divides into two branches, one going into the observatory D, and the other into the work-room E. By means of the speaking-pipe the communications of the observer are conveyed to the assistant in the observatory, and the workman is directed to perform the required motions.

In the observatory is placed a valuable sidereal time-piece, made by Mr. Shelton. Close to it, and of the same height, is a polar distance piece, which has a dial-plate of the same dimensions with the time-piece; this piece may be made to show polar distance, zenith distance, declination, or altitude, by setting it differently. The time and polar distance-pieces are placed so that the assistant sits before them at a table, with the speaking-pipe rising between them; and in this manner observations may be written down very conveniently.

On the different Merits of Microscopes and Telescopes.

The advantages arising from the use of microscopes and telescopes depend, in the first place, upon this property of magnifying the minute parts of objects, so that they can by that means be more distinctly viewed by the eye; and secondly, upon their throwing more light into the pupil of the eye than what is done without them. The advantages arising from the magnifying power would be extremely limited, if they were not also accompanied by the latter; for if the same quantity of light is spread over a large portion of surface, it becomes proportionably diminished in force; and, therefore, the objects, though magnified, appear proportionably dim. Thus, though any magnifying-glass should enlarge the diameter of the object ten times, and consequently magnify the surface 100 times, yet if the focal distance of the glass was about eight inches (provided this was possible), and its diameter only about the size of the pupil of the eye, the object would appear 100 times more dim when we looked through the glass, than when we beheld it with our naked eyes; and this even on a supposition that the glass transmitted all the light which fell upon it, which no glass can do. But if the focal distance of the glass was only four inches, though its diameter remained as before, the inconvenience would be vastly diminished, because the glass could then be placed twice as near the object as before, and, consequently, would receive four times as many rays as in the former case, and, therefore, we should see it much brighter than before. Going on thus, still diminishing the focal distance of the glass, and keeping its diameter as large as possible, we shall perceive the object more and more magnified, and, at the same time, very distinct and bright. It is evident, however, that, with regard to optical instruments of the microscopic kind, we must sooner or later arrive at a limit which cannot be passed. This limit is formed by the following particulars. 1. The quantity of light lost in passing through the glass. 2. The diminution of the glass itself, by which it receives only a small quantity of rays. 3. The extreme shortness of the focal distance of great magnifiers, whereby the free access of the light to the object which we wish to view is impeded, and, consequently, the reflection of the light from it is weakened. 4. The aberrations of the rays, occasioned by their different refrangibility.

To understand this more fully, as well as to see how far these obstacles can be removed, let us suppose the lens made of such a dull kind of glass that it transmits only one half of the light which falls upon it. It is evident that such a glass of four inches focal distance, and which magnifies the diameter of an object twice, still supposing its own breadth equal to that of the pupil of the eye, will show it four times magnified in surface, but only half as bright as if it was seen by the naked eye at the usual distance; for the light which falls upon the eye from the object at eight inches distance, and likewise the surface of the object in its natural size, being both represented by 1, the surface of the magnified object will be 4, and the light which makes that magnified object visible only 2; because, though the glass receives four times as much light as the naked eye does at the usual distance of distinct vision, yet one half is lost in passing through the glass. The inconvenience in this respect can, therefore, be removed only as far as it is possible to increase the clearness of the glass, so that it shall transmit nearly all the rays which fall upon it; and how far this can be done hath not yet been ascertained.

The second obstacle to the perfection of microscopic glasses, is the small size of great magnifiers, by which, notwithstanding their near approach to the object, they receive a smaller quantity of rays than might be expected. Thus, suppose a glass of only $\frac{1}{16}$th of an inch focal distance; such a glass would increase the visible diameter 80 times, and the surface 6400 times. If the breadth of the glass could at the same time be preserved as great as that of the pupil of the eye, which we shall suppose $\frac{2}{10}$ths of an inch, the object would appear magnified 6400 times, at the same time that every part of it would be as bright as it appears to the naked eye. But if we suppose that this magnifying glass is only $\frac{1}{20}$th of an inch in diameter, it will then receive only $\frac{1}{4}$th of the light which otherwise would have fallen upon it; and therefore, instead of communicating to the magnified object a quantity of illumination equal to 6400, it would communicate only one equal to 1600, and the magnified object would appear four times as dim as it does to the naked eye. This inconvenience, however, is still capable of being removed, not indeed by increasing the diameter of the lens, because this must be in proportion to its focal distance, but by throwing a greater quantity of light on the object. Thus, in the above-mentioned example, if four times the quantity of light which naturally falls upon it could be thrown upon the object, it is plain that the reflection from it would be four times as great as in the natural way; and consequently the magnified image, at the same time that it was as many times magnified as before, would be as bright as when seen by the naked eye. In transparent objects this can be done very effectually by a concave speculum, as in the reflecting microscope; but in opaque objects the case is somewhat more doubtful; neither do the contrivances for viewing these objects seem entirely to make up for the deficiences of the light from the smallness of the lens and shortness of the focus. When a microscopic lens magnifies the diameter of an object 40 times, it hath then the utmost possible magnifying power, without diminishing the natural brightness of the object.

The third obstacle arises from the shortness of the focal distance in large magnifiers; but in transparent objects, where a sufficient quantity of light is thrown on the object from below, the inconvenience arises at last from straining the eye, which must be placed nearer the glass than it can well bear; and this entirely supersedes the use of magnifiers beyond a certain degree.

The fourth obstacle arises from the different refrangibility of the rays of light, and which frequently causes such a deviation from truth in the appearances of things, that many people have imagined themselves to have made surprising discoveries, and have even published them to the world; when, in fact, they have been only as many optical deceptions, owing to the unequal refractions of the rays. For this there seems to be no remedy, except the introduction of achromatic glasses into microscopes as well as telescopes. How far this is practicable, hath not yet been tried; but when these glasses shall be introduced (if such introduction is practicable), microscopes will then undoubtedly have received their ultimate degree of perfection.

With regard to telescopes, those of the refracting kind have evidently the advantage of all others, where the aperture is equal, and the aberrations of the rays are corrected according to Mr. Dollond's method; because the image is not only more perfect, but a much greater quantity of light is transmitted than what can be reflected from the best materials hitherto known. Unluckily, however, the imperfections of the glass set a limit to these telescopes, as hath already been observed, so that they cannot be made above three feet and an half long. On the whole, therefore, the reflecting telescopes are preferable in this respect, that they may be made of dimensions greatly superior; by which means they can both magnify to a greater degree, and at the same time throw much more light into the eye.

With regard to the powers of telescopes, however, they are all of them exceedingly less than what we should be apt to imagine from the number of times which they magnify the object. Thus, when we hear of a telescope which magnifies 200 times, we are apt to imagine, that, on looking at any distant object through it, we should perceive it as distinctly as we would with our naked eye at the 200th part of the distance. But this is by no means the case; neither is there any theory capable of directing us in this matter; we must therefore depend entirely on experience.

The best method of trying the goodness of any telescope is by observing how much farther off you are able to read with it than you can with the naked eye. But that all deception may be avoided, it is proper to choose something to be read where the imagination cannot give any assistance, such as a table of logarithms, or something which consists entirely of figures; and hence the truly useful power of the telescope is easily known. In this way Mr. Short's large telescope, which magnifies the diameter of objects 1200 times, is yet unable to afford sufficient light for reading at more than 200 times the distance at which we can read with our naked eye.

With regard to the form of reflecting telescopes, it is now pretty generally agreed, that when the Gregorian ones are well constructed, they have the advantage of those of the Newtonian form. One advantage evident at first sight is, that with the Gregorian telescope an object is perceived by looking directly through it, and consequently is found with much greater ease than in the Newtonian telescope, where we must look into the side. The unavoidable imperfection of the specula common to both also gives the Gregorian an advantage over the Newtonian form. Notwithstanding the utmost care and labour of the workmen, it is found impossible to give the metals either a perfectly spherical, or a perfectly parabolical form. Hence arises some indistinctness of the image formed by the great speculum, which is frequently corrected by the little one, provided they are properly matched. But if this is not done, the error will be made much worse; and hence many of the Gregorian telescopes are far inferior to the Newtonian ones; namely, when the specula have not been properly adapted to each other. There is no method by which the workman can know the specula which will fit one another without a trial; and therefore there is a necessity for having many specula ready made of each sort, that in fitting up a telescope those may be chosen which best suit each other.

The brightness of any object seen through a telescope, in comparison with its brightness when seen by the naked eye, may, in all cases, be easily found by the following formula. Let n represent the natural distance of a visible object, at which it can be distinctly seen; and let d represent its distance from the object-glass of the instrument. Let m be the magnifying power of the instrument; that is, let the visual angle subtended at the eye by the object when at the distance n, and viewed without the instrument, be to the visual angle produced by the instrument as 1 to m. Let a be the diameter of the object-glass, and p be that of the pupil. Let the instrument be so constructed, that no parts of the pencils are intercepted for want of sufficient apertures of the intermediate glasses. Lastly, let the light lost in reflection or refraction be neglected.

The brightness of vision through the instrument will be expressed by the fraction $\left(\frac{an}{mpd}\right)^2$, the brightness of natural vision being 1. But although this fraction may exceed unity, the vision through the instrument will not be brighter than natural vision. For when this is the case, the pupil does not receive all the light transmitted through the instrument.

In microscopes n is the nearest limits of distinct vision, nearly eight inches. But a difference in this circumstance, arising from a difference in the eye, makes no change in the formula, because m changes in the same proportion with n.

In telescopes, n and d may be accounted equal, and the formula becomes $\frac{a^2}{m^2p^2}$.

Achromatic TELESCOPE, a refracting telescope without colour. See ACHROMATIC.

Catadioptric TELESCOPE, the same as *Reflecting* TELESCOPE.

Dioptric TELESCOPE, the same as *Refracting* TELESCOPE.

TELESCOPICAL *Stars*, are such as are not visible to the naked eye, being only discernible by means of a telescope.

All stars less than those of the sixth magnitude are telescopic to an ordinary eye.

TENACITY, in *Natural Philosophy*, is that quality of bodies by which they sustain a considerable pressure or force without breaking, being the opposite quality to brittleness or fragility.

TENSION, that state which a chord, string, &c. is in when stretched beyond its natural length.

TERM, in *Geometry*, is the extreme of any magnitude, or that which bounds and limits its extent. So the terms of a line, are points; of a superficies, lines: of a solid, superficies.

TERMS, of an equation, or of any quantity, in *Algebra*, are the several members of which it is composed, separated from one another by the signs + or —. So, the quantity $ax + 2bc - 3ax^2$, consists of the three terms ax and $2bc$, and $3ax^2$.

In an equation, the terms are the parts which contain the several powers of the same unknown letter or quantity: for if the same unknown quantity be found in several members in the same degree or power, they form but for one term, which is called a compound one, in distinction from a simple or single term. Thus, in the equation $x^3 + (a - 3b)\,x^2 - acx = b^3$, the four terms are x^3, $(a - 3b)\,x^2$, acx and b^3; of which the second term $(a - 3b)\,x^2$ is compound, and the other three are simple.

TERMINAL *Velocity*, in the theory of projectiles, is the greatest velocity which a ball can acquire by descending vertically, in air, and with which, when attained, it would continue to descend uniformly if no solid obstacle destroyed the motion.

TERMINATOR, in *Astronomy*, a name sometimes given to the circle of illumination, from its properties of terminating the boundaries of light and darkness.

TERRA. See EARTH.

TERRA *Firma*, in *Geography*, is sometimes used for a continent in contradistinction to island.

TERRAQUEOUS, in *Geography*, an epithet applied to our earth when considered as consisting of land and water.

TERRESTRIAL, any thing relating to the earth, as terrestrial *globe*, *line*, &c.

TETRAEDRON, or *Tetrahedron*, in *Geometry*, one of the five regular or Platonic bodies or solids, comprehended under four equilateral and equal triangles. See BODY.

If a denote the linear edge or side of a tetraedron, b its whole superficies, c its solidity, r the radius of its inscribed sphere, and R the radius of its circumscribing sphere; then the general relation among all these is expressed by the following equations, *viz.*

$$a = 2r\sqrt{6} = \tfrac{2}{3}R\sqrt{\tfrac{1}{3}b\sqrt{3}} = \sqrt[3]{6c\sqrt{2}}.$$

$$b = 24r^2\sqrt{3} = \tfrac{8}{3}R^2\sqrt{3} = a^2\sqrt{3} = 6\sqrt[3]{c^2\sqrt{3}}.$$

$$c = 8r^3\sqrt{3} = \tfrac{8}{27}R^3\sqrt{3} = \tfrac{1}{12}a^3\sqrt{2} = \tfrac{1}{36}b\sqrt{2b\sqrt{3}}.$$

$$R = 3r = \tfrac{1}{4}a\sqrt{6} = \tfrac{1}{4}\sqrt{2b\sqrt{3}} = \tfrac{3}{2}\sqrt[3]{\tfrac{1}{2}c\sqrt{3}}.$$

$$r = \tfrac{1}{3}R = \tfrac{1}{12}a\sqrt{6} = \tfrac{1}{12}\sqrt{2b\sqrt{3}}\ \tfrac{1}{2}\sqrt[3]{\tfrac{1}{3}c\sqrt{3}}.$$

See Hutton's Mensuration, p. 248, &c. 2d edition.

TETRAGON, in *Geometry*, a quadrangle, or a figure having four angles. Such as a square, a parallelogram, a rhombus, and a trapezium.

A square is a *regular tetragon;* to which figure the term more commonly applies.

TETRAGONISM, is used by some old authors to denote the quadrature of the circle.

THEODOLITE, is an instrument employed by surveyors for measuring angles, whereby to compute the heights, distances, &c. of remote objects. There are various forms of this instrument arising out of the successive improvements of many eminent artists, but the principle of its operation is the same in all, and we shall, therefore, describe that of Mr. Sisson, which is one of the most usual construction.

In using this instrument, whether for horizontal or vertical angles, it must first be rendered perfectly horizontal by means of the adjusting screws and spiral level; then for horizontal angles observing one of the objects in the telescope, let it be brought to coincide very exactly with the centre of the cross wires of the object glass; and having then set the horizontal limb at 0, or 180, turn the telescope towards the other object, and get it exactly in the centre of the object glass as before; and the angle contained between the two objects will be shown on the limb of the instrument.

In measuring vertical angles, after having rendered the instrument horizontal as before directed, set the vertical limb at 0; then elevate the telescope till such time as the top of the object is seen in the centre of the cross wires; so shall the degrees, &c. cut on the line be the measure of the angle of elevation required.

THEOREM, in *Geometry*, is a proposition in which some truth is proposed to be demonstrated, being thus distinguished from a problem, in which something is proposed to be done. Proclus defines

A *Theorem*, something which is proposed to be demonstrated.

A *Problem*, something proposed to be done, and

A *Porism*, something proposed to be investigated.

Geometrical THEOREMS, are sometimes divided into different classes, as

Local THEOREM, is that which relates to a surface. As, that triangles of the same base and altitude are equal.

Plane THEOREM, is that which relates to a surface that is either rectilinear, or bounded by the circumference of a circle. As, that all angles in the same segment of a circle are equal.

Solid THEOREM, is that which considers a space terminated by a solid line, that is, by any of the three conic sections.

THEOREM, in *Algebra*, is used sometimes to denote a rule, particularly when that rule is expressed by algebraical symbols, or formulæ; such as those given under the articles *Annuities, Interest*, &c.

Algebraical THEOREMS, have also particular denominations, either from the subject to which they relate, or the names of the authors by whom they were invented.

THEORY, a doctrine which terminates in the sole speculation or consideration of its object, without any view to the practice or application of it.

To be learned in an art, &c. the theory is sufficient; to be a master of it, both the theory and practice are requisite.

Machines often promise very well in theory, but fail in the practice.

We say theory of the moon, theory of the rainbow, of the microscope, of the camera obscura, &c.

THEORY of *Numbers*, is a modern branch of analysis, which has for its object the investigation of certain properties of numbers, such as, that the sum of two odd squares cannot make a square number; that every number is composed of four or a less number of squares; that the sum of any number of the odd numbers 1, 3, 5, 7, &c. is equal to the square of the number of terms; that the sum of any number of the natural cubes 1, 8, 27, &c. is a square number, and a great variety of similar properties highly interesting to the speculative mathematician.

THEORY of the *Planets*, &c. is a system or hypothesis, according to which astronomers explain the reasons of the phenomena or appearances of them.

THERMOMETER, an instrument for measuring the degree of heat or cold in any body. The invention of the thermometer, like that of the telescope, and indeed most other useful instruments, has been claimed by different philosophers. Thus the invention is ascribed to Cornelius Drebbet of Alcmar, about the beginning of the 17th century, by his countryman Boerhaave (Chem. 1, p. 152, 156), and Musschenbroeck, (Introd. ad Phil. Nat. vol. ii. p. 625); Fulgenzio, in his Life of Father Paul, gives him the honour of the first discovery; Vincenzio Viviani (Vit. de l' Galil. p. 67, also Oper. di Galil. pref. p. 47) speaks of Galileo as the inventor of thermometers. But Sanctorino (Com. in Galen. Art. Med. p. 736, 842; Com. in Avicen. Can. Fen. 1. p. 22, 78, 219) expressly assumes to himself this invention: and Borelli (De Mot. Animal. 2 prop. 175) and Malpighi (Oper. Posth. p. 30) ascribe it to him without reserve. Upon which Dr. Martine remarks, that these Florentine academicians are not to be suspected of partiality in favour of one of the Patavinian school.

But whoever was the inventor, the first form of this instrument for measuring the degrees of heat and cold was the air-thermometer. It is a well known fact that air expands with heat so as to occupy more space than it does when cold, and that it is condensed by cold so as to occupy less space than when warmed, and that this expansion and condensation is greater or less according to the degree of heat or cold applied. The principle then on which the air-thermometer was constructed is very simple. The air was confined in a tube by means of some coloured liquor; the liquor rose or fell according as the air became expanded or condensed. What the first form of the tube was, cannot now, perhaps, be well known; but the following description of the air-thermometer will fully explain its nature. It consists of a glass tube, connected at one end with a large glass ball, and at the other end immersed in an open vessel, or terminating in a ball, with a narrow orifice, which vessel or ball contains any coloured liquor that will not easily freeze. Aqua-fortis, tinged of a fine blue colour, with a solution of vitriol or copper, or spirit of wine tinged with cochineal, will answer this purpose. But the ball must be first moderately warmed, so that a part of the air contained in it may be expelled through the orifice; and then the liquor pressed by the weight of the atmosphere will enter the ball, and rise, for example, to the middle of the tube, at a mean temperature of the weather; and in this state the liquor, by its weight, and the air included in the ball, by its elasticity, will counterbalance the weight of the atmosphere. As the surrounding air becomes warmer, the air in the ball and upper part of the tube, expanding by heat, will drive the liquor into the lower ball, and, consequently, its surface will descend; on the contrary, as the ambient air becomes colder, that in the ball is condensed, and the liquor pressed by the weight of the atmosphere will ascend; so that the liquor in the tube will ascend or descend more or less according to the state of the air contiguous to the instrument. To the tube is affixed a scale of the same length, divided upwards and downwards from the middle, into 100 equal parts, by means of which the ascent and descent of the liquor in the tube, and consequently the variations in the cold or heat of the atmosphere, may be observed.

The air being found improper for measuring with accuracy the variations of heat and cold according to the form of the thermometer which was first adopted, another fluid was proposed about the middle of the seventeenth century, by the Florentine Academy. This fluid was spirit of wine, or alcohol, as it is now generally named. The alcohol being coloured, was inclosed in a very fine cylindrical glass tube, previously exhausted of its air, having a hollow ball at one end, and hermetically sealed at the other end. The ball and tube are filled with rectified spirit of wine to a convenient height, when the weather is of a mean temperature, which may be done by inverting the tube into a vessel of stagnant coloured spirit, under a receiver of the air-pump, or in any other way. When the thermometer is properly filled, the end is heated red-hot by a lamp, and then hermetically sealed, leaving the included air of about one-third of its natural density, to prevent the air which is in the spirit from dividing it in its expansion. To the tube is applied a scale, divided from the middle into 100 equal parts, upwards and downwards. As spirit of wine is capable of a very considerable degree of rarefaction and condensation by heat and cold, when the heat of the atmosphere increases the spirit dilates, and consequently rises in the tube; and when the heat decreases, the spirit descends, and the degree or quantity of the motion is shown by a scale.

This was evidently an improvement on the air-thermometer, but was itself not free from objections. The liquor could not easily be obtained of the same strength, and hence different tubes filled with it, when exposed to the same degree of heat, would not correspond. Another defect was the want of some fixed guide as a standard to commence the graduation. Philosophers soon saw that some fixed and unalterable point must be found, by which all thermometers might be accurately adjusted. Dr. Halley proposed that thermometers should be graduated in a deep pit, where the temperature in all seasons was nearly the same. This, however, could not generally be practised. He thought of the boiling point of water, of mercury, and of spirit of wine, but preferred the latter. At length Sir Isaac Newton determined this important point, on which the accuracy and value of the thermometer depends. He chose, as fixed, those points at which water freezes and boils; the very points which the experiments of succeeding philosophers have determined to be the most fixed and convenient. Sensible of the disadvantages of spirit of wine, he tried another liquor, which was homogeneous enough, and capable of a considerable rarefaction, several times greater than spirit of wine. This was linseed-oil. It has not been observed to freeze even in very great colds, and it bears a heat very much greater than water before it boils. With these advantages it was made use of by Sir Isaac Newton, who discovered by it the comparative degree of heat for boiling water, melting wax, boiling spirit of wine, and melting tin; beyond which it does not appear that this thermometer was applied. The method he used for adjusting the scale of this oil-thermometer was as follows: supposing the bulb, when immerged in thawing snow, to contain 10,000 parts, he found the oil expand by the heat of the human body so as to take up one thirty-ninth more space, or 10,256 such parts; and by the heat of water boiling strongly 10,725; and by the heat of melting tin 11,516. So that reckoning the freezing point as a common limit between heat and cold, he began his scale there, marking it 0, and the heat of the human body he made 12°; and, consequently, the degrees of heat being proportional to the degrees of rarefaction, or $256 : 725 = 12 : 34$, this number 34 will express the heat of boiling water; and, by the same rule, 72 that of melting tin. This thermometer was constructed in 1701. To the application of oil as a measure of heat and cold there are insuperable objections. It is so viscid, that it adheres too strongly to the sides of the tube. On this account it ascends and descends too slowly in case of a sudden heat or cold. In a sudden cold, so great a portion remains adhering to the sides of the tube after the rest has subsided, that the surface appears lower than the corresponding temperature of the air requires. An oil-thermometer is therefore not a proper measure of heat and cold. All the thermometers hitherto proposed were liable to many inconveniencies, and could not be considered as exact standards for pointing out the various degrees of temperature. This led Reaumur to attempt a new one, an account of which was published in the year 1730, in the Memoirs of the Academy of Sciences. This thermometer was made with spirit of wine. He took a large ball and tube, the dimensions and capacities of which were known; he then graduated the tube, so that the space from one division to another might contain 1,000th part of the liquor; the liquor containing 1,000 parts when it stood at the freezing point. He adjusted the thermometer to the freezing point by an artificial congelation of water; then putting the ball of his thermometer and part of the tube into boiling water, he observed whether it rose 80 divisions; if it exceeded these, he changed his liquor, and by adding water lowered it, till upon trial it should just rise 80 divisions; or if the liquor, being too low, fell short of 80 divisions, he raised it by adding rectified spirit to it. The liquor thus prepared suited his purpose, and served for making a thermometer of any size, whose scale would agree with his standard. At length a different fluid was proposed, by which thermometers could be made free from most of the defects hitherto mentioned. This fluid was mercury, and seems first to have occurred to Dr. Halley, but was not adopted by him on account of its having a smaller degree of expansibility than the other fluids used at that time.

The honour of this invention is generally given to Fahrenheit, of Amsterdam, who presented an account of it to the Royal Society of London in 1724. That we may judge the more accurately of the propriety of employing mercury, we will compare its qualities with those of the fluids already mentioned, air, alcohol, and oil. Air is the most expansible fluid, but it does not receive nor part with its heat so quickly as mercury. Alcohol does not expand much by heat. In its ordinary state it does not bear much greater heat than 175° of Fahrenheit; but when highly rectified it can bear a greater degree of cold than any other liquor hitherto employed as a measure of temperature. At Hudson's Bay, Mr. Macnab, by a mixture of vitriolic acid and snow, made it to descend to 69° below nothing of Fahrenheit. There is an inconvenience, however, attending the use of this liquor; it is not possible to get it always of the same degree of strength. As to oil, its expansion is about 15 times greater than that of alcohol; it sustains a heat of 600°, and its freezing point is so low that it has not been determined; but its viscosity renders it useless.

Mercury is far superior to alcohol and oil, and is much more manageable than air. 1. As far as the experiments already made can determine, it is of all the fluids hitherto employed in the construction of thermometers, that which measures most exactly equal differences of heat by equal differences of its bulk: its dilatations are, in fact, very nearly proportional to the augmentations of heat applied to it. 2. Of all liquids it is the most easily freed from air. 3. It is fitted to measure high degrees of heat and cold. It sustains a heat of 600° of Fahrenheit's scale, and does not congeal till it falls 39 or 40° below 0. 4. It is the most sensible of any fluid to heat and cold, even air not excepted. Count Rumford found, that mercury was heated from the freezing to the boiling point in $58'$, while water took $2^{m}\,13'$, and common air 10^{m} and $17'$. 5. Mercury is a homogeneous fluid, and every portion of it is equally dilated or contracted by equal variations of heat. Any one thermometer, made of pure mercury, is *cæteris paribus*, possessed of the same properties with every other thermometer made of pure mercury. Its power of expansion is, indeed, about six times less than that of spirit of wine, but it is great enough to answer most of the purposes for which a thermometer is wanted. The fixed points, which are now universally chosen for adjusting thermometers to a scale, and to one another, are the boiling and freezing water points. The boiling water point, it is well known, is not an invariable point, but varies some degrees, according to the weight and temperature of the atmosphere. In an exhausted receiver, water will boil with a heat of 98° or 100°; whereas, in Papin's digester, it will acquire a heat of 412°. Hence it appears, that water will boil at a lower point, according to its height in the atmosphere, or to the weight of the column of air which presses upon it. In order to ensure uniformity, therefore, in the construction of thermometers, it is now agreed, that the bulb of the tube be plunged in the water when it boils violently, the barometer standing at 30 English inches, and the temperature of the atmosphere 55°.

As artists may be often obliged to adjust thermometers under very different pressures of the atmosphere, philosophers have been at pains to discover a general rule which might be applied on all occasions. M. de Luc, from a series of experiments, has given an equation for the allowance on account of this difference, in Paris measure, which has been verified by Sir George Schuckburg; also Dr. Horsley, Dr. Maskelyne, and Sir George Schuckburg, have adapted the equation and rules to English measures, and have reduced the allowances into tables, for the use of the artists. Dr. Horsley's rule, deduced from De Luc's, is this:

$$\frac{99}{8990000}\log. z - 92.804 = h.$$

Where h denotes the height of a thermometer plunged in boiling water above the point of melting ice, in degrees of Bird's Fahrenheit, and z the height of the barometer in 10ths of an inch. From this rule he has computed the following table, for finding the heights to which a good Bird's Fahrenheit will rise, when plunged in boiling water, in all states of the barometer, from 27 to 31 English inches; which will serve, among other uses, to direct instrument makers in making a true allowance for the effect of the variation of the barometer, if they should be obliged to finish a thermometer at a time when the barometer is above or below 30 inches; though it is best to fix the boiling point when the barometer is at that height.

Equation of the Boiling Point.

Barometer.	Equation.	Difference.
31.0	+1.57	0.78
30.5	+0.79	0·79
30.0	0.00	0.80
29.5	—0.80	0.82
29.0	—1.62	0.83
28.5	—2.45	0.85
28.0	—3.31	0.86
27.5	—4.16	0.88
27.0	—5.04	

The numbers in the first column of this table express heights of the quicksilver in the barometer, in English inches and decimal parts: the second column shows the equation to be applied, according to the sign prefixed to 212° of Bird's Fahrenheit, to find the true boiling point for every such state of the barometer. The boiling point, for all intermediate states of the barometer, may be had, with sufficient accuracy, by taking proportional parts, by means of the third column of differences of the equations.

The method of constructing Fahrenheit's thermometer, which is now in general use in this country, is the following: a small ball is blown on the end of a glass tube, of an uniform width throughout. The ball and part of the tube are then to be filled with quicksilver, which has been previously boiled to expel the air. The open end of the tube is then to be hermetically sealed. The next object is to construct the scale. It is found, by experiment, that melting snow, or freezing water, is always at the same temperature. If, therefore, a thermometer be immersed in the one or the other, the quicksilver will always stand at the same point. It has been observed, too, that water boils under the same pressure of the atmosphere at the same temperature. A thermometer, therefore, immersed in boiling water, will uniformly stand at the same point. Here, then, are two fixed points, from which a scale may be constructed, by dividing the intermediate space into equal parts, and carrying the same divisions as far above and below the two fixed points as may be wanted. Thus, thermometers constructed in this way may be compared together; for if they are accurately made, and placed in the same temperature, they will always point to the same degree on the scale. The fluid, as we have seen employed, is quicksilver, and it is found to answer best, because its expansions are most equable. The freezing point of Fahrenheit's thermometer is marked 32°, and the reason of this is said to have been, that this artist thought that he had produced the greatest degree of cold, by a mixture of snow and salt; and the point at which the thermometer then stood, in this temperature, was marked zero. The boiling point in this thermometer, is 212°, and the intermediate space between the boiling and freezing points, is therefore divided into 180°. This is the thermometer that is commonly used in Britain.

In determining the choice of tubes, it is best to have them exactly cylindric through their whole length. Capillary tubes are preferable to others, because they require less bulbs, and they are also less brittle, and more sensible. Those of the most convenient size for common experiments are such as have their internal diameter about the fourth of a line; and those made of thin glass are better than others, as the rise and fall of the mercury may be more distinctly perceived. The length of nine inches will serve for all common occasions; but for particular purposes the length both of the tubes and of the divisions should be adapted to the uses for which they are designed.

In determining the best size of the balls or bulbs, it has been usual to compare new tubes with such thermometers as are well proportioned. But M. Durand has proposed a formula for finding the proportion which the balls ought to bear to their respective tubes. With this view he expresses the length of the tube, measured in diameters of itself, by a; the whole capacity of the ball and tube by c; the capacity of the fundamental interval, expressed in the same parts with the whole capacity, by d; the number of degrees of the fundamental interval by m; the number of other degrees which the scale is to contain, besides those of the fundamental interval both above and below it, by n; and the diameter of the ball measured in diameters of the tube by b: and $b = \sqrt[3]{\left(\frac{2}{3} a \times \frac{cm}{d \times (m \times n)} - 1.\right)}$ For two cylinders having equal bases being as their heights, $m : n = d : \frac{dn}{m}$, which is the capacity of that part of the tube which exceeds the fundamental interval, to which adding d, that interval, we have the total capacity of the tube $= \frac{dn}{m} + d$, or $\frac{dn + dm}{m}$. Subtracting this from c, we shall have the capacity of the ball $= c - \frac{dn + dm}{m} = \frac{cm - dm - dn}{m}$. If this quantity be divided by the capacity of the tube, the quotient will show how often the capacity of the ball contains that of the tube; and this quotient is $= \frac{cm - dm - dn}{dm + dn}$. Consequently the ball is equal to as many cylinders, having a diameters of the tube, for their respective height, and 1 diameter for the base, as are contained in this last quotient; and, therefore, its cylindric solidity expressed in the cylindric solidities of the tube will be $= a \times \frac{cm - dm - dn}{dm + dn}$. But the diameter of this ball is equal to the base of the cylinder in which it may be inscribed, and the solidity of this cylinder is equal to $\frac{2}{3}$ the solidity of the circumscribing sphere. Consequently the solidity of this cylinder will be $= \frac{2}{3} a \times \frac{cm - dm - dn}{dm + dn}$; and the diameter of its base, equal to the diameter of the ball, will be $= \sqrt[3]{\left(\frac{2}{3} a \frac{cm - dm - dn}{dm + dn}\right)} = \sqrt[3]{\left(\frac{2}{3} a \frac{cm}{d \times (m + n)} 1\right)}$

Two unalterable points of temperature, *viz.* the first where ice becomes water, and the second where water becomes vapour, have been universally adopted by the various constructors of thermometers for the graduation of those instruments; but the space between them has been divided differently by different persons, and this difference gives the different names of thermometers, or rather of their graduations; such as Reaumur's thermometer, Fahrenheit's thermometer, &c. Reaumur divides the space between the above-mentioned two points into 80 equal parts or degrees; placing the 0 at freezing, and the 80th degree at the boiling point. Fahrenheit, as above remarked, divides it into 180 degrees or equal parts, but he places the 0 thirty-two degrees below the freezing point; so that the freezing point is at 32, and the boiling point is at 212 degrees.

Other persons have adopted other divisions, which have been suggested by supposed advantages or fanciful ideas.

Most of those graduations are at present out of use, but they are to be met with in various, not very recent, publications; we have therefore thought it necessary to set them down in the following table, which contains: 1st. The name of the person or society that has used each particular division: 2dly. The degree which has been placed, by each of them, against the freezing point; 3dly. The degree which has been placed against the boiling-point; and, 4thly. The number of degrees lying between those two points.

Tabular Comparison of some of the more remarkable Thermometers.

	Freezing point.	Boiling point.	Degrees between the preceding two points.
Fahrenheit's, which is generally used in Great ritain	32	212	180
Reaumur's, which is generally used in France id other parts of the continent	0	80	80
Centigrade or *Celsius's,* which has been used niefly in Sweden, hence it is also called the wedish thermometer. It has been lately dopted by the French chemists, under the ame of centigrade thermometer	0	100	100
The *Florentine* thermometers, which were nade and used by the members of the famous cademy *del' Cimento,* being some of the first nstruments of the sort, were vaguely graduated, some having a great many more degrees than thers. But two of their most common graduations seem to be	20 $13\frac{1}{2}$	174 $81\frac{5}{8}$	154 $68\frac{1}{8}$
The *Parisian* thermometer, *viz.* the *ancienne thermometre* of the Academy of Sciences, seems to have been graduated nearly thus	25	239	214
De la Hire's thermometer, which stood in the observatory at Paris above sixty years, was graduated thus	28	$199\frac{1}{3}$	$171\frac{1}{3}$
Amonton's	$51\frac{1}{2}$	73	$21\frac{1}{2}$
Poleni's	$47\frac{3}{10}$	$62\frac{9}{10}$	$15\frac{6}{10}$
De L' Isle's thermometer is graduated in an inverted order	150	0	150
Sir Isaac Newton's	0	34	34
Hale's	0	163	163
The *Edinburgh* thermometer, formerly used, seems to have been graduated thus	$8\frac{1}{5}$	47	$31\frac{4}{5}$

These are the chief thermometers that have been used in Europe; and the temperatures indicated by the principal of them may be reduced into the corresponding degrees on any of the others, by means of the following simple theorems; in which R signifies the degrees on the scale of Reaumur, F those of Fahrenheit, and S those of the Swedish thermometer.

1. To convert the degrees of Reaumur into those of Fahrenheit; $\frac{R \times 9}{4} + 32 = F.$

2. To convert the degrees of Fahrenheit into those of Reaumur; $\frac{(F - 32) \times 4}{9} = R.$

3. To convert the Swedish degrees into those of Fahrenheit; $\frac{S \times 9}{5} + 32 = F.$

4. To convert Fahrenheit's into Swedish; $\frac{(F - 32) \times 5}{9} = S.$

5. To convert Swedish degrees into those of Reaumur; $\frac{S \times 4}{5} = R.$

6. To convert Reaumur's degrees into the centigrade; $\frac{R \times 5}{4} = S$

The common contrivance for a self-registering thermometer, consists simply of two thermometers, one mercurial, and the other of alcohol, having their stems horizontal; the former has for its index a small bit of magnetical steel wire, and the latter a minute thread of glass, having its two ends formed into small knobs, by fusion in the flame of a candle.

The magnetical bit of wire lies in the vacant space of the mercurial thermometer, and is pushed forward by the mercury whenever the temperature rises, and pushes that fluid against it; but when the temperature falls, and the fluid retires, this index is left behind, and, consequently, shows the maximum. The other index, or bit of glass, lies in the tube of the spirit thermometer immersed in the alcohol; and when the spirit retires, by depression of temperature, the index is carried along with it, in apparent contact with its interior surface; but, on increase of temperature, the spirit goes forward and leaves the index, which therefore shows the minimum of temperature since it was set. As these indexes merely lie in the tubes, their resistance to motion is altogether inconsiderable. The steel index is brought to the mercury by applying a magnet on the outside of the tube, and the other is duly placed at the end of the column of alcohol, by inclining the whole instrument.

Differential THERMOMETER. Professor Leslie, the well-known author of a curious and interesting treatise on heat, invented a thermometer for indicating and measuring very minute differences of temperature, which he calls the differential thermometer. It consists of two tubes, each terminating in a small bulb of the same dimensions, joined by the blow pipe, and bent in the form of a U, a small portion of dark-coloured liquor having previously been introduced into one of the balls, as, for example, a solution of carmine in concentrated sulphuric acid, which is generally preferred. By managing the included air with the heat of the hand, this red liquor is made to stand at the required point of the opposite tube. This is the zero of a scale fastened to that tube, and divided into equal parts above and below that point. The instrument is then fixed on a stand. It is manifest that when the liquor is at rest, or points at zero, the column is pressed in opposite directions by two portions of air equal in elasticity, and containing equal portions of caloric. Whatever heat, therefore, may be applied to the whole instrument, provided both bulbs receive it in the same degree, the liquor must remain at rest. But if the one ball receives the slightest excess of temperature, the air which it contains will be proportionally expanded, and will push the liquid against the air in the other bulb, with a force varying as the difference between the temperatures of those two portions of air: thus the equilibrium will be destroyed, and the fluid will rise in the opposite tube. The degrees of the scale through which it passes will mark the successive augmentations in the temperature of the ball which is exposed to the greatest heat; so that this instrument is a balance of extreme delicacy for comparing the temperatures of its two scales.

When thermometers are contrived to measure very great degrees of heat, by the expansions they produce in substances, or on the contrary, the expansions corresponding to different temperatures, they are characterised by the name of pyrometers.

THERMOSCOPE, an instrument showing the changes happening in the air with respect to heat and cold. The word thermoscope is generally used indifferently with that of thermometer, though there is some difference in the literal import of the two; the first signifying an instrument that shows, or exhibits, the changes of heat, &c. to the eye: and th an
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THUNDER, the noise occasioned by the explosion of a flash of lightning passing through the air; or it is that noise which is excited by a sudden explosion of electrical clouds, which are therefore called thunder-clouds.

The rattling in the noise of thunder, which makes it seem as if it passed through arches, is, probably, owing to the sound being excited among clouds, hanging over one another, and the agitated air passing irregularly between them.

The explosion, if high in the air, and remote from us, will do no mischief; but when near, it may destroy trees, animals, &c. This proximity, or small distance, may be estimated nearly by the interval of time between seeing the flash of lightning, and hearing the report of the thunder, estimating the distance after the rate of 1142 feet per second of time, or $3\frac{2}{3}$ seconds to the mile. Dr. Wallis observes, that commonly the difference between the two is about seven seconds, which, at the rate above mentioned, gives the distance almost two miles. But sometimes it comes in a second or two, which argues the explosion very near us, and even among us. And in such cases the doctor assures us, he has sometimes foretold the mischiefs that happened.

Although in this country thunder may happen at any time of the year; yet the months of July and August are those in which it may almost certainly be expected; but its time of duration is very uncertain; sometimes only a few peals will be heard at any particular place during the whole season; at other times the storm will return at the interval of three or four days, for a month, six weeks, or even longer; not that we have violent thunder in this country, directly vertical in any one place, so frequently in any year, but in many seasons it will be perceptible that thunder-clouds are formed in the neighbourhood even at these short intervals. Hence it appears, that during this particular period there [illegible] be some natural cause oper[illegible] the production of this [illegible]hich does not take place at other times. This cannot be the mere heat of the weather, for we have often a long tract of hot weather without any thunder; and besides, though not common, thunder is sometimes heard in the winter also. As, therefore, the heat of the weather is common to the whole summer, whether there be thunder or not, we must look for the causes of it in those phenomena, whatever they are, which are peculiar to the months of July, August, and the beginning of September. Now it is generally observed, that from the month of April, an east or south-east wind generally takes place, and continues with little interruption till towards the end of June. At that time, sometimes sooner and sometimes later, a westerly wind takes place; but as the causes producing the east wind are not removed, the latter opposes the westerly wind with its whole force. At the place of meeting, there is naturally a most vehement pressure of the atmosphere, and friction of its parts against one another; a calm ensues, and the vapours brought by both winds begin to collect and form dark clouds, which can have little motion either way, because they are pressed almost on all sides. For the most part, however, the west wind prevails, and what little motion the clouds have is towards the east; whence the common notion in this country that thunder-clouds move against the wind. But this is by no means universally true; for if the west wind happens to be excited by any temporary cause before its natural period when it should take place, the east wind will very frequently get the better of it; and the clouds, even although thunder is produced, will move westward. Yet in either case the motion is so slow, that the most superficial observers cannot help taking notice of a considerable resistance in the atmosphere. When lightning acts with extraordinary violence, and breaks or shatters any thing, it is called a thunderbolt, which the vulgar, to fit it for such effects, suppose to be a hard body, and even a stone. But that we need not have recourse to a hard solid body to account for the effects commonly attributed to the thunderbolt, will be evident to any one who considers those of gunpowder, and the several chemical fulminating powders; but more especially the astonishing powers of electricity, when only collected and employed by human art, and much more when directed and exercised in the course of nature.

TIDES, two periodical motions of the waters of the sea every 24 hours; called also the *flux* and *reflux*, or the *ebb* and *flow*.

1. The waters of the sea alternately approach to, and recede from, our shores. This is produced by an elevation and subsequent depression of the surface of the ocean between determinate limits, which are those of high-water and low-water. The interval between two high-waters is about 12^h 25′, the half of the moon's daily circuit round the earth; so that we have two tides of flood and two of ebb in 24^h 50′, a period which may be called a lunar day. The gradual subsidence of the water is such, that the square of the heights are nearly as the squares of the times from high-water. The same may be said of the subsequent rise of the water in the next flood. The time of low-water is nearly half way between the two hours of high-water; not indeed exactly, for it is generally observed that the flood-tide commonly takes 10 minutes less than the ebb-tide.

2. The tides have a particular reference to the position and phases of the moon; for it is always high-water when the moon is on a determinate point of the compass (S. W. nearly) and the highest tides happen about the time of the full or change. At Brest, where an accurate register of the phenomena of the tides was made about the beginning of the last century, it was observed that the highest tide happens about a day and a half after full or change. If the time of high-water happen at the very time of the new or full moon, the third high-water after that is the highest of all: this is called the *Spring-tide*. From this period the tides gradually decrease, until the third high-water after the moon's quadrature, which is the lowest of all, and is called the *Neap-tide*. After this the tides increase, until the next spring-tide, and so on continually. The higher the tide of flood rises, the lower the ebb-tide generally sinks on that day. The total magnitude of the tide is estimated by the difference of high-water and low-water.

3. The tides have also a reference to the distance of the moon from the earth, being so much the greater as the moon is nearer. The highest spring-tides happen when the moon is in *perigee;* and the next spring-tide is the smallest, because then the moon is nearly in *apogee*. This makes a difference of 2¾ feet from the medium height of the spring-tide at Brest, and, therefore, a difference of 5½ feet between the greatest and least.

4. The tides also depend on the sun's distance, but not so much as on that of the moon. In our winter the spring-tides are greater than in summer, and the neap-tides smaller.

5. The tides depend very much upon the position of the sun and moon in respect of the equator. The phenomena, however, are more intricate than those we have already described.

6. All the phenomena are modified by the latitude of the place of observation; and some happen in high latitudes, which are not at all seen near the equator. In particular, when the observer and moon are both on the same side of the equator, the tide which happens when the moon is above the horizon, is greater than that which happens on the same day when she is below it; and the contrary happens when the observer and the moon are on opposite sides of the equator. If the observer's distance from the pole be equal to the moon's declination, he will see but one tide in the day, containing 12 hours flood, and 12 hours ebb.

7. All the phenomena we have described are greatly modified by local circumstances; such as the position of the shores, the width of the channels through which the

waters pass, the extent of the seas, winds, &c.

If the parts of the earth gravitate towards its centre; then as this gravitation far exceeds the action of the moon, and much more exceeds the difference of her actions on different parts of the earth, the effect that results from the inequalities of these actions of the moon, will be only a small diminution of the gravity of those parts of the earth which it endeavoured in the former supposition to separate from its centre; that is, those parts of the earth which are nearest to the moon, and those that are farthest from her, will have their gravity toward the earth somewhat abated; to say nothing of the lateral parts. So that supposing the earth fluid, the columns from the centre to the nearest and to the farthest parts must rise, till by their greater height they be able to balance the other columns, whose gravity is less altered by the inequalities of the moon's action. And thus the figure of the earth must still be an oblong spheriod.

Let us next consider the earth, instead of falling toward the moon by its gravity, as projected in any direction, so as to move round the centre of gravity of the earth and moon: it is evident that in this case, the several parts of the fluid earth will still preserve their relative positions; and the figure of the earth will remain tne same as if it fell freely toward the moon; that is, the earth will still assume a spheroidal form, having its longest diameter directed toward the moon.

From the above reasoning it appears, that the parts of the earth directly under the moon, and also the opposite parts, will have the flood or high-water at the same time; while the parts, at 90° distance, or where the moon appears in the horizon, will have the ebbs or lowest waters at that time.

Hence, as the earth turns round its axis from the moon to the moon again in 24 hours 48 minutes, this oval of water must shift with it; and thus there will be two tides of flood and two of ebb in that time.

But it is farther evident, that by the motion of the earth on her axis, the most elevated part of the water is carried beyond the moon in the direction of the rotation. So that the water continues to rise after it has passed directly under the moon, though the immediate action of the mooon there begins to decrease, and comes not to its greatest elevation till it has got about half a quadrant farther. It continues also to descend after it has passed at 90° distance from the point below the moon, to a like distance of about half a quadrant. The greatest elevation therefore is not in the line drawn through the centres of the earth and moon, nor the lowest points where the moon appears in the horizon, but all these about half a quadrant removed eastward from these points, in the direction of the motion of rotation. Thus in open seas, where the water flows freely, the moon M is generally past the north and south meridian, as at *p*, when the high-water is at Z and at *n*: the reason of which is plain, because the moon acts with the same force after she has passed the meridian, and thus adds to the libratory or waving motion, which the water acquired when she was in the meridian; and therefore the time of high-water is not precisely at the time of her coming to the meridian, but some time after, &c.

Besides, the tides answer not always to the same distance of the moon, from the meridian, at the same places; but are variously affected by the action of the sun, which brings them on sooner when the moon is in her first and third quarters, and keeps them back later when she is in her second and fourth; because, in the former case, the tide raised by the sun alone would be earlier than the tide raised by the moon, and in the latter case later.

2. We have hitherto adverted only to the action of the moon in producing tides; but it is manifest that, for the same reasons, the inequality of the sun's action on dif

ferent parts of the earth would produce a like effect, and a like variation from the exact spherical figure of a fluid earth. So that in reality, there are two tides every natural day, from the action of the sun, as there are in the lunar day from that of the moon, subject to the same laws; and the lunar tide, as we have observed, is somewhat changed by the action of the sun, and the change varies every day, on account of the inequality between the natural and the lunar day. Indeed, the effect of the sun in producing tides, because of his immense distance, must be considerably less than that of the moon, though the gravity toward the sun be much greater: for it is not the action of the sun or moon itself, but the inequalities in that action, that have any effect; the sun's distance is so great, that the diameter of the earth is but as a point in comparison with it, and therefore the difference between the sun's actions on the nearest and farthest parts, becomes vastly less than it would be if the sun were as near as the moon. However, the immense bulk of the sun makes the effect still sensible, even at so great a distance; and, therefore, though the action of the moon has the greatest share in producing the tides, the action of the sun adds sensibly to it when they conspire together, as in the full aud change of the moon, when they are nearly in the same line with the centre of the earth, and therefore unite their forces; consequently, in the syzygies, or at new and full moon, the tides are the greatest, being what are called the spring-tides. But the action of the sun diminishes the effect of the moon's action in the quarters, because the one raises the water in that case where the moon depresses it; therefore the tides are the least in the quadratures, and are called neap-tides.

Newton has calculated the effects of the sun and moon respectively upon the tides, from their attractive powers. The former he finds to be to the force of gravity, as 1 to 12868200, and to the centrifugal force at the equator as 1 to 44527. The elevation of the waters by this force is considered by Newton as an effect similar to the elevation of the equatorial parts above the polar parts of the earth, arising from the centrifugal force at the equator; and as it is 44527 times less, he finds it to be 24½ inches, or 2 feet and ½ an inch.

To find the force of the moon upon the water, Newton compares the spring-tides at the mouth of the river Avon below Bristol, with the neap-tides there, and finds the proportion as 9 to 5; whence, after several necessary corrections, he concludes that the force of the moon to that of the sun, in raising the waters of the ocean, is as 4.4815 to 1; so that the force of the moon is able of itself to produce an elevation of 9 feet 1⅞ inch, and the sun and moon together may produce an elevation of about 11 feet 2 inches, when at their mean distances from the earth, or an elevation of about 12¾ feet, when the moon is nearest the earth. The height to which the water is found to rise upon coasts of the open and deep ocean, is agreeable enough to this computation.

3. It must be observed, that the spring-tides do not happen precisely at new and full moon, nor the neap-tides at the quarters, but a day or two after, because, as in other cases, so in this, the effect is not greatest or least when the immediate influence of the cause is greatest or least. As *e. g.* the greatest heat is not on the solstitial day, when the immediate action of the sun is greatest, but some time after.

That this may be more clearly understood, let it be considered, that though the actions of the sun and moon were to cease this moment, yet the tides would continue to have their course for some time; for the water, where it is now highest, would subside, and flow down on the parts that are lower, till, by the motion of descent, being there accumulated to too great a height, it would necessarily return again to its first place, though in a less measure, being retarded by the resistance arising from the attraction of its parts.

hus it would for some time continue in an agitation like to that in which it is at present. The waves of the sea, that continue after a storm ceases, and every motion almost of a fluid, may illustrate this.

4. The different distances of the moon from the earth produce a sensible variation in the tides. When the moon approaches the earth, her action on every part increases, and the differences of that action, on which the tides depend, increase. For her action increases as the squares of the distances decrease; and though the differences of the distances themselves be equal, yet there is a greater disproportion betwixt the squares of less, than the squares of greater quantities; *e. g.* 3 exceeds 2 as much as 2 exceeds 1, but the square of 2 is quadruple of the square of 1, whilst the square of 3 (*viz.* 9) is little more than double the square of 2 (*viz.* 4).

Thus it appears, that by the moon's approach, her action on the nearest parts increases more quickly than her action on the remote parts; and the tides, therefore, increase in a higher proportion as the distances of the moon decrease. Sir Isaac Newton shows, that the tides increase in proportion as the cubes of the distances decrease, so that the moon, at half her present distance, would produce a tide eight times greater.

The moon describes an ellipse about the earth, and in her nearest distance produces a tide sensibly greater than at her greatest distance from the earth: and hence it is, that two great spring-tides never succeed each other immediately; for if the moon be at her nearest distance from the earth at the change, she must be at her greatest distance at the full, having in the intervening time finished half a revolution; and, therefore, the spring-tide then will be much less than the tide at the change was: and for the same reason, if a great spring-tide happens at the time of full moon, the tide at the ensuing change will be less.

5. The spring-tides are greatest about the time of the equinoxes, *i. e.* about the latter end of March and September, and least about the time of the solstices, *i. e.* toward the end of June and December; and the neap-tides are least at the equinoxes, and greatest at the solstices; so that the difference betwixt the spring and the neap-tides is much less considerable at the solstitial than at the equinoctial seasons. In order to illustrate and evince the truth of this observation, it is manifest, that if either the sun or moon was in the pole, they could have no effect on the tides, for their action would raise all the water at the equator to the same height; and any place of the earth, in describing its parallel to the equator, would not meet, in its course, with any part of the water more elevated than another, so that there could be no tide in any place.

6. The effect of the sun or moon is greatest when in the equinoctial; for then the axis of the spheroidal figure, arising from their action, moves in the greatest circle, and the water is put into the greatest agitation; and hence it is that the spring-tides produced, when the sun and moon are both in the equinoctial, are the greatest of any, and the neap-tides are the least of any about that time.

But the tides produced when the sun is in either of the tropics, and the moon in either of her quarters, are greater than those produced when the sun is in the equinoctial, and the moon in her quarters, because, in the first case, the moon is in the equinoctial; and, in the latter case, the moon is in one of the tropics; and the tide depends more on the action of the moon than on that of the sun, and is, therefore, greatest when the moon's action is greatest.

However, it is necessary to observe, that because the sun is nearer the earth in winter than in summer, the greatest spring-tides are after the autumnal, and before the vernal equinox.

7. Since the greatest of the two tides happening in every diurnal revolution of the moon is that wherein the moon is nearest the zenith, or nadir; for this reason,

while the sun is in the northern signs, the greater of the two diurnal tides in our climates is that arising from the moon above the horizon; when the sun is in the southern signs, the greatest is that arising from the moon below the horizon.

In proof of this observation, let it be considered, that when the moon declines from the equator toward either pole, one of the greatest elevations of the water follows the moon, and describes nearly the parallel on the earth's surface which is under that which the moon, on account of the diurnal motion, seems to describe; and the opposite greatest elevation, being antipodal to that, must describe a parallel as far on the other side of the equator; so that while the one moves on the north side of the equator, the other moves on the south side of it, at the same distance.

Such are the general principles on which these interesting phenomena depend. But it may be expedient to touch upon a few of the more particular appearances, which at first sight appear to contradict some parts of the preceding theory. Now, it is evident, that to allow the tides their full motion, the ocean in which they are produced ought to be extended from east to west 90 degrees at least; because that is the distance between the places where the water is most raised and depressed by the moon. Hence it appears, that it is only in the great oceans that such tides can be produced, and why in the larger Pacific ocean they exceed those in the Atlantic ocean. Hence also it is obvious, why the tides are not so great in the torrid zone, between Africa and America, where the ocean is narrower, as in the temperate zones on either side; and hence we may also understand why the tides are so small in islands that are very far distant from the shores. It is farther manifest, that in the Atlantic ocean, the water cannot rise on one shore but by descending on the other; so that at the intermediate islands it must continue at a mean height between its elevations on those two shores. But when tides pass over shoals and through straits into bays of the sea, their motion becomes more various, and their height depends on many circumstances.

To be more particular. The tide that is produced on the western coasts of Europe, in the Atlantic, corresponds to the situation of the moon already described. Thus it is high-water on the western coasts of Ireland, Portugal, and Spain, about the third hour after the moon has passed the meridian: from thence it flows into the adjacent channels, as it finds the easiest passage. One current from it, for instance, runs up by the south of England, and another comes in by the north of Scotland; they take a considerable time to move all this way, making always high water sooner in the places to which they first come; and it begins to fall at these places while the currents are still going on to others that are farther distant in their course. As they return, they are not able to raise the tide, because the water runs faster off than it returns, till, by a new tide propagated from the open ocean, the return of the current is stopped, and the water begins to rise again. The tide propagated by the moon in the German ocean, when she is three hours past the meridian, takes twelve hours to come from thence to London bridge; so that when it is high-water there, a new tide is already come to its height in the ocean; and in some intermediate place it must be low-water at the same time. Consequently when the moon has north declination, and we should expect the tide at London to be the greatest when the moon is above the horizon, we find it is least; and the contrary when she has south declination.

At several places it is high-water three hours before the moon comes to the meridian; but that tide, which the moon pushes as it were before her, is only the tide opposite to that which was raised by her when she was nine hours past the opposite meridian.

It would be endless to recount

all the particular solutions, which are easy consequences from this doctrine; as, why the lakes and seas, such as the Caspian sea and the Mediterranean sea, the Black sea and the Baltic, have little or no sensible tides: for lakes are usually so small, that when the moon is vertical she attracts every part of them alike, so that no part of the water can be raised higher than another; and having no communication with the ocean, it can neither increase nor diminish their water, to make it rise and fall; and seas that communicate by such narrow inlets, and are of so immense an extent, cannot speedily receive and empty water enough to raise or sink their surface.

Sir R. Phillips ascribes the cause of this very remarkable phenomenon to the re-action of the earth and moon, denying altogether the principle of attraction. "The tides (says he,) are simple and palpable phenomena of motion, and these motions of the waters are a necessary consequence of other motions. To prove that the motions of the waters, and their regular variations, are caused by the motions of the earth, and by known and regular changes in those motions, it is necessary to consider the motions of the earth and the variations which take place in its position, relatively to the sun and moon; and to shew that the effect and the sufficient causes are always coincident.

1. The earth and moon move round a centre of their momenta, or their mechanical fulcrum.

2. The fulcrum alone describes the uniform or true orbit round the sun, and in it lies the centre of the solar forces, on the system of the two bodies of the earth and moon.

3. When the moon is in its quarters, or half illumined, then the earth, the moon, and the fulcrum are all in the line of the orbit, and then only.

4. When the moon is performing the superiour part of its orbit, then the earth describes an inferiour orbit, nearer the sun than the fulcrum. And *vice versâ*.

5. The fulcrum is constantly in the right line, which joins the centres of the earth and moon.

6. The earth also revolves simultaneously on its axis.

7. The lever which communicates the motions of the earth and moon to each other, is *the gas*, which fills space.

8. But as gas radiates motion, the arms of this gaseous lever are of such length, that their squares multiplied by the masses are equal, *i.e.* the fulcrum is about 26,000 miles from the centre of the earth; and, to this distance on each side of the true orbit, the earth is impelled by the re-action of the moon, at the conjunction and opposition.

9. The earth too is not a homogeneous sphere of waters, but its great oceans are separated by the two continents, stretching from north to south.

10. The maximum of the solar forces, which carries round the earth and moon, is concentrated and adjusted in the fulcrum; and this is an essential characteristic of this fulcrum as a fulcrum.

These facts being understood (and they are recognized by every system of astronomy), it will not be difficult to comprehend the necessary causes of the phenomena of the tides, without having recourse to any fanciful and arbitrary power like that of attraction, or universal gravitation.

If we supposed the moon to be in its quarters, and that it would for ever go before or follow the earth, without mutual action and re-action, then there would never be any tides.

But the earth and moon are in relative action and re-action, and drive or impel each other, in orbits round a centre or fulcrum.

If then the moon, impelled by the earth, ascend in its orbit from the sun 800 miles, and the earth be forced by the re-action of the moon to descend towards the sun 100 miles; the earth's centre is thus so far removed from the line of the orbit, wherein lies the maximum or centre of the solar forces, *i.e.* in the fulcrum of the gaseous lever, which joins the earth and moon.

But though the solid and connected parts of the earth obey this action of the moon, and do so descend from the centre of the solar forces; yet the moveable waters accommodate themselves to the two forces, *i. e.* the ordinary mundane force, which always retains them on the earth; and the force with which the whole earth would return to its orbit, but for the action of the moon. The waters therefore, so to speak, seek the focus of the solar forces on the two bodies, or rise towards the fulcrum. While the earth revolving at the same time, they rise on every shore which they approach, or, to an inhabitant of the earth, appear themselves to move.

When the moon advances another 800 miles, the earth advances on the opposite side of their fulcrum another 100 miles; and the re-acting force being still greater, the waters rise still higher—and so on, till the centre of the earth is carried 26,000 miles from the line of its true orbit, where the centre of the solar forces acts—and now the re action being the greatest, the waters rise to their maximum, and form spring-tides.

The return to the quarters presents a decreasing series of phenomena, till the forces again coincide in the orbit, and then there is no tide except the continuance of the former oscillations or neap-tide.

The like phenomena, from like causes, attend the earth's progress through its superiour orbit, while the moon is performing its inferiour orbit.

Such are the circumstances of the earth and moon. They move, in the medium of space, round a common centre, or fulcrum, the arms or distances of the gaseous lever being in the inverse duplicate ratio of their distances and of their quantities of matter; and the mundane fluids, as fluids, in respecting the centre of motion, rise towards the centre of motion, or towards the fulcrum of the mutual revolution of the two bodies, which is necessarily in the right line joining the centres of the earth and moon.

But, while this joint revolution of the earth and moon is performing round the fulcrum of the gaseous lever of space, the earth is turning on its axis by a separate motion; and, as all parts of its surface are successively presented to the moon, or to the common fulcrum, so the successive portions of water are elevated towards the moon or fulcrum; and hence the phenomena arise which we call tides, governed necessarily in successive rotations by the times which the moon, or the common fulcrum, passes the meridian of any place.

The fulcrum of the earth and moon is the point about which both revolve, and is the centre of their reciprocal momenta. It necessarily lies in the line which joins the centres of the earth and moon; and, being the centre of their joint momenta, is the point acted upon by the sun's impulses on the medium of space, in producing the orbicular motion; and is, therefore, constantly in the earth's orbit; while the centres of the earth and moon constantly revolve around it, by their mutual action and re-action on each other through the medium of space.

Suppose the earth to be turning on its axis, with the fulcrum vertical over the meridian of central Africa, where, as there is no sea, there will be no apparent tide—though the seas of the Antarctic Ocean would be slightly affected. In an hour, the rotation carries the shores of the Atlantic under the moon or fulcrum, and the waters being capable of rushing to restore the equilibrium, in consequence of the disturbance already explained, they rise towards the fulcrum or centre of lunar and mundane gyration, and the elevation continues as long as the Atlantic is passing under the moon, or fulcrum.

But, when the Atlantic shores of America arrive opposite the moon or fulcrum, the waters rise along the coast, and fill the entrances of the rivers; but when the continent is presented to the moon, or fulcrum, then no tide is raised.—Nevertheless, the waters were brought to the shores and left there—what then becomes of them? The cause which produced their elevation has departed over another meridian. Is it not astonishing, says Sir R. Phillips, that no one ever asked this question?—Must they not swing back like the oscillation of a pendulum, and is not this the true cause of the second tide? Is not this a necessary cause of the second tide on the African and European shores; and is not such second tide a necessary consequence of the waters having been so accumulated on the shores of America? Would not the constant succession of this sufficient cause produce the same equable effect in the secondary as in the primary tide? Would not such constant succession produce a species of oscillation of the waters in the beds of the ocean, just such as exists in the tides?

We need not accompany the phenomena to the shores of the Pacific, where the oscillations are less, because the liquid pendulum is larger and heavier—but where exactly the same causes produce a primary and secondary tide. That is, as soon as the moon, or fulcrum, arrives over the western coasts of America, the waters rise and follow the moon, or fulcrum, till it reaches the shores of Asia; and then, being left there, they oscillate back, producing a second tide across that ocean.

In a word, the second tide arises from the re-actions of the first tide against the visible continents and invisible rocks which bound and fill the ocean; and which re-actions concur with the departure of the force over-land to other meridians, so that a returning tide is the returning stroke of a primary one. And, in confirmation of this theory of the secondary tide, is it not notorious, that in certain parts of the South Pacific, where few or no disturbances are created by re-actions or reflections of land, there is in many places but one tide in twenty-four hours?

The other peculiarity of the tides—their neap and spring, depending on the relative positions of the sun, moon, and earth, are susceptible of explanations equally clear and simple. In the quarters, the line joining the centres of the earth and moon coincides with the line of the earth's orbit, or with the direction of the earth's orbicular force; and the tide is then produced solely by the revolution of the earth round the fulcrum of the earth and moon. But, as soon as the moon departs in its orbit, towards the solar conjunction or opposition, from the line of the earth's orbit, the centre of the earth is carried simultaneously on the opposite side of the line of its orbit, and the lines of the rotatory and orbicular forces no longer coincide. This disturbance the waters are able to restore; and herein is a new cause of tide, which at the oppositions and conjunctions, not only coincides with the direction of the lunar fulcrum, but becomes itself a maximum, because the earth's centre is then removed the farthest from the line of the orbicular force. Hence the spring tides at the new and full moon; and hence all the degrees of tide, as the centres of the earth and moon approach the direction of the orbit. The fulcrum is of course also in the orbit, or the point which describes the true orbit.

In fine, he ascribes *the tides primarily* to the revolution of the earth round the fulcrum, or centre of the momentum, of the earth and moon, which fulcrum is always in the line which joins the centres of the earth and moon; and, as the moveable waters accumulate, or are accumulated, opposite that fulcrum, so they have the appearance of being attracted, as it is called, by the moon.

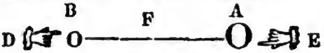
He ascribes *the double tide in every twenty-four hours* to the departure of the force from the sea over the next land, owing to the intervention of the great continents which separate the two great oceans from each other.

And he ascribes *the variable heights of the tides*, as apparently connected with the age of the moon, to the variable distance of the body of the earth from the line of the orbicular force, during its revolution round the lunar and mundane centre of motion.

It is, at the same time, most evident, that no attraction of the moon is either required or concerned; that there is no mutual attraction of the earth and moon, and no innate gravitation of the waters towards the moon. But it is true, that the tides are dependant on, and connected with, the position of the moon, the waters under it being constantly raised towards it; but this is not a consequence of any attraction, but a consequence of the two bodies being, so to speak, at the two ends of a gaseous lever, around whose fulcrum they turn, and towards which fulcrum they rise by their mobility, with an effort to maintain the equilibrium, presenting an *appearance* of being attracted. The earth and moon are in fact a system of two bodies; the fulcrum is equivalent to the centre of percussion, and the waters on the earth being mobile, they rise towards that centre of percussion, or in this case towards the mechanical fulcrum, which power of adjustment in them cause the phenomena of the tides.

It follows, as corollaries, that, as the action of the earth on the moon is 64 times that of the moon on the earth, or the orbit of the moon round the fulcrum is 64 times greater than that of the earth; if there were waters on the moon, they would be raised 64 times higher than the tides on the earth; consequently, it is not, on that and other accounts, to be believed that any water exists on the moon.

And further, that, as the centres of the several forces nearly coincide at the quarters when the earth and moon move in a common orbit, so the cause of the tides there ceases; and hence their diminution at those periods, when the slight tides are mere oscillations, continued from the great oscillations produced at the new and full moon. As the preceding theory of the tides excludes any principle of attraction, and that principle is universally admitted and taught, and is an essential ingredient of the Newtonian Physics, Sir R. Phillips has published the following demonstration of what he calls "the superstitious absurdity of the very principle of any attractive, or other force, being exerted between inert distant masses."



Suppose A, B, to be balls of lead, wood, or cork, or A is the earth, and B a heavy body above it, or A the sun and B the earth; or A the earth, and B the moon.

By hypothesis A is supposed to be much larger than B.

By experiment they are found to move towards each other, and to meet at F; FB being to FA reciprocally as A is to B; so that $B \times BF = A \times AF$.

But if A move towards F, the direction of the impelling force on A must be from the parts E — and in like manner the impelling force which moves B to F, must be from the parts D.

But A is said, by the principle of attraction, to be the cause of B moving towards F, and B is the cause of A moving to F, that is, A must act at D upon B, where it is not, and B must act on A at E where it is not—which is absurd.

The motion of A is therefore not created by B, nor the motion of B by A, or in other words, B does not attract A, nor the earth any body near it, nor the sun the earth, nor the earth the moon; but the force which moves B from the parts D towards F, and A from E, must be sought in some other operative cause, and not in their own mutual affections or pretended attractions.

And by further consequence, as the sun does not attract the earth, and the earth does not attract the moon, the principle of universal gravitation, and of lunar attraction on the earth's waters, are erroneous, and the phenomena must be otherwise explained.

TIMBER *Measure*, is the method employed by artificers in measuring trees, joists, beams, &c.; and as these always fall under one or other of the regular solids which have been already treated of under the several articles, it would seem unnecessary to repeat here any rules for the measuration of timber; but the fact is, that an erroneous rule has been adopted by persons concerned in this line of business, which common practice has so established, that it is rather to be wished than expected, it should be replaced by some other, either perfectly true, or approaching towards the truth; for according to the present rule, a tree frequently contains one-fourth more timber than it is estimated at, which, at the modern price of that article, is a matter of some importance, and merits the attention of the timber grower, as well as the merchants.

The rule, as it is at present employed, by all persons concerned in the buying or selling of timber, is as follows:

Multiply the square of the mean quarter girt, or $\frac{1}{4}$ circumference, by the length of the tree, for the content; which, when the dimensions are taken in feet, will be also feet; and this divided by 50, the number of feet in a load, will give the number of loads.

Such is the rule commonly used by persons concerned in buying and selling of timber, on which we intend to make a few remarks, in order to point out its inaccuracy, which is not so generally known as it ought to be. Suppose, for instance, we take a balk 24 feet long, and a foot square throughout, and consequently its solidity 24 feet. Now, if this piece of timber be slit exactly in two, from end to end, making each piece 6 inches, or $\frac{1}{2}$ a foot broad, and 12 inches, or a foot thick; it is evident that the true solidity of each piece will be 12 feet. But, by the quarter girt method, they would amount to much more; for, the false quarter girt being equal to half the sum of the breadth and thickness, in this case will be 9 inches, or $\frac{3}{4}$ of a foot; the square of which is $\frac{9}{16}$, and therefore $\frac{9}{16} \times 24 = 13\frac{1}{2}$ feet for the solidity of each part, making the two pieces together 27 feet, instead of 24 feet, which is the true content.

Again, suppose this balk to be so cut, that the breadth of the one piece may be only 4 inches, or $\frac{1}{3}$ of a foot; and that of the other 8 inches or $\frac{2}{3}$ of a foot. Here the true content of the less piece will be 8 feet, and that of the greater 16 feet. But, proceeding by the other method, we have the quarter girt of the less piece $\frac{2}{3}$ of a foot, and of the other piece $\frac{5}{6}$ of a foot. Whence the content of the less piece will be found $= \frac{4}{9} \times 24 = 10\frac{2}{3}$ feet, instead of 8 feet; and the content of the greater piece will be $16\frac{2}{3}$ feet instead of 16, making the sum of the two $27\frac{2}{3}$ feet, instead of 24 feet. Farther, if the less piece be cut only 2 inches broad, and consequently the greater 10 inches, the true content of the less piece would be 4 feet, and that of the greater 20 feet. Whereas, by the other method, the quarter girt of the less piece would be 7 inches, or $\frac{7}{12}$ of a foot; and $\frac{49}{144} \times 24 = 8\frac{1}{2}$ feet, instead of 4 feet, for the content; and, by the same method, the content of the greater piece would be $20\frac{1}{6}$ feet, instead of 20, and their sum $28\frac{1}{6}$ feet, instead of 24.

Hence it is obvious, that the greater the proportion is between the breadth and the depth, the greater will be the error, by using the false method; and the sum of the two parts, by the same method, is greater, as the difference of the same two parts is greater; and, consequently, the sum is least when the two parts are equal to each other; or, when the balk is cut equally in two; and, finally, when the sides of a piece of timber differ not above an inch or two from each other, the quarter girt may be used without any very sensible error. To avoid, therefore, this inconsistency in the result, the following method should be employed, *viz. Multiply the length, breadth, and depth continually together, and the product will be the true content in all cases of this kind.*

With regard to round timber the error is of a different kind. The area of a circle is found by squaring the circumference, and multiplying that square by ·07958, and therefore, if a quarter of the circumference is used, we must multiply its square by $\cdot07958 \times 16 = 1\cdot27328$. Hence, to find the true content of a piece of cylindrical timber, we ought to multiply the square of the quarter girt by the constant number 1·27328, and that product by the length; instead of which the constant multiplier is omitted, and consequently the solidity is returned about $\frac{27}{100}$ parts less than it is. But, as the utmost accuracy is not necessary in those cases, the following rule might be used, which is as simple as can be desired, *viz. Multiply the square of* ¼ *of the mean girt by double the length for the content, which is not far from the truth.*

Another error to which timber measure is always subject, is the way in which the mean girt is assumed in tapering trees, which, as we have before stated, is done, either by taking the girt in the middle, or half the sum of the extreme girts, both of which are equally false, so obviously so, that a tree of certain dimensions will measure more after a part of it has been cut off than it did before. This being the case, it will not be amiss to show the extreme inaccuracy of the method, and the folly in persisting in it by the solution of the following problems, which have been taken from Dr. Hutton's Mensuration.

Prop. 1. To find where a tapering timber must be cut, so that the two parts measured separately shall measure the most possible, or be greater than if it were cut in any other two parts, and greater than the whole.

Put $G =$ the greatest girt, $g =$ the least girt, $x =$ the girt at the section, $z =$ the length of the part to be cut off, and L the whole length of the timber. Then by similar figures $L : z = G - g : x - g$; hence $x = \frac{Gz - gz}{L} + g$. But $(g + x)^2 . z + (G + x)^2 . (L - z)$, is to be a *maximum;* which being put into fluxions and reduced gives $z = \frac{1}{2} L$.

Therefore a tree being cut exactly in the middle into parts, will measure more than if it were cut in any other two parts, and more than the whole tree. If a tree, of which the greater girt is 14 feet, and less girt 2 feet, and length 32 feet, be thus cut in two parts; the measure of the two parts will exceed the measure of the whole tree by 18 feet.

Prop. 2. To find where a tree must be cut, so that the part next the greater end may measure the greatest possible.

Here, by using the same notation as in the last problem, we have also, $x = \frac{Gz - gz}{L} + g$; and $(G + x)^2 . (L - x)$ a *maximum*, which put into fluxions as before, gives $z = \frac{G - 3g}{G - g} \times \frac{1}{3} L$.

Therefore, from the greater girt subtract three times the less girt, and that difference divided by the difference of the girts, and multiplied by ⅓ of the whole length, will be the length to be cut off.

Prop. 3. To find where a tree must be cut, so that the part next the greater end may measure the same as the whole tree before it was cut.

Using still the same notation, and writing, besides s for the sum of the two girts, and d for their difference; we shall have $s^2 L = (L - z)(G + x)^2$, or substituting instead of x, its value $\frac{Gz - gz}{L} + g$, or $\frac{dz}{L} + g$, we obtain $z = \frac{L}{2d} \times \left\{\sqrt{(4s^2 + d)} - 2s + d\right\}$ which length being cut off, the remaining part will measure the same as the whole tree.

These results, which are the necessary consequence of the preceding rules, are so obviously incongruous and inconsistent, that they speak for themselves, and therefore require no farther comment.

TIME, a succession of phenomena in the universe; or a mode of duration, marked by certain periods or measures, chiefly by the motion and revolution of the sun.

Time is distinguished into *absolute* and *relative*.

Absolute TIME, is time considered in itself, and without any relation to bodies, or their motions. This flows equally, *i.e.* never proceeds faster or slower, but glides on in a constant equable tenor.

Relative TIME, is the sensible measure of any duration, by means of motion. For, since that equable flux of time does not effect our senses, nor is any way immediately cognizable thereby, there is a necessity for calling in the help of some nearly equable motion to a sensible measure, whereby we may determine its quantity by the correspondency of the parts of this with the former.

Astronomical TIME, is that measured by the motion of the heavenly bodies only.

Civil TIME, is the former time accommodated to civil uses, and formed and distinguished into years, months, days, &c.

Astronomical TIME, is, again, either *apparent solar*, or true; *mean*, or *sidereal.*

Apparent Solar TIME, otherwise, though improperly, called *True* TIME, is that which is estimated by the sun's passage over the meridian of any place, and which is sometimes as much as 16 minutes sooner or later than that shown by a good clock. See EQUATION of *Time.*

Mean TIME, is the apparent solar time corrected according to the method described under EQUATION of *Time.* This mean time is still not exactly the same as sidereal time, the part added or subtracted still partaking of the inequalities it is meant to correct; it approaches, however, so near the truth, that no error is to be feared, except in cases of the most extreme delicacy.

Sidereal TIME, is that which is estimated by the passage of the same star over the meridian of any place, and which (except for some trifling inequalities in the motion of the stars, which astronomers are able to correct when necessary) is the most exact measure of time we are acquainted with, the most accurate observations having never detected in it any inequality, though it is suspected it may be subject to some, which must, however, if there be any, be extremely minute.

TIME *Keepers,* for measuring time, being otherwise called CHRONOMETERS.

TOPOGRAPHY, is a description of some particular place or small tract of land, differing from geography as a part does from a whole.

TORNADO, a sudden and violent gust of wind arising suddenly from the shore, and afterwards veering round all the points of the compass like a hurricane; these are very frequent on the coast of Guinea.

TORPEDO, the name of a fish, possessing an electric property.

TORRICELLIAN *Experiment,* an experiment made by Torricelli, by which he demonstrated the pressure of the atmosphere, in opposition to the doctrines of suction, &c. which pressure he shew, was able to support only a certain length of mercury, or any other fluid, in an inverted glass tube.

TORRID *Zone,* is the middle of the five zones into which the earth is supposed to be divided, extending to 23½ degrees on each side of the equator.

TORSION, *Force* of, a term applied by Coulomb in some of his experiments, to denote the effort made by a thread which has been twisted to untwist itself.

TRACTION, in *Mechanics,* is the drawing of one body towards another.

TRAJECTORY, a term generally used for the path of any body moving either in a void, or in a medium which resists its motion; or even for any curve passing through a given number of points. Thus Newton, Princip. lib. i, prob. 22, purposes to describe a trajectory that shall pass through five given points.

TRAJECTORY of a *Comet,* is its path or orbit, or the line it describes in its motion. This path was thought by Hevelius, in his Cometographia, to be very nearly a right line; but Dr. Halley considers it to be, as it really is, a very eccentric ellipsis; though its place may often be well computed on the supposition of its being a parabola. Newton, in prop. 41 of his 3d book, shows how to determine the trajectory of a comet from three observations; and in his last prop. how to correct a trajectory graphically described.

TRAMMELS, in *Mechanics*, an instrument used by artificers to describe ellipses. See *Elliptic* COMPASSES. The engines for turning ovals are constructed on the same principles with the trammels; the only difference being, that in the latter the board is at rest and the pencil moves, whereas in the former the tool which supplies the place of the pencil is at rest, and the board moves against it.

TRANSCENDENTAL, is a term applied to any equation, curve, or quantity, which cannot be represented or defined by an algebraical equation of a finite number of terms, with numeral and determinate indices.

TRANSCENDENTAL *Quantities*, therefore, include all exponential, logarithmic, and trigonometrical lines, because there is no finite algebraical formulæ by which these quantities can be expressed.

Hence a^x, x^x, log. x, sin. x, cos. x, tan. x, &c. are *transcendental quantities;* and any equation into which such quantities enter, are called *transcendental equations;* and every curve defined by such an equation is, for the same reason, termed a *transcendental curve.*

TRANSCENDENTAL *Equations*, are, however, sometimes defined to be such fluxional equations as do not admit of finite algebraical fluents, being only expressible by means of some curve, logarithm, or infinite series; as is the case in all those relating to the trigonometrical lines as above stated; thus

$$y = \text{fluent of } \frac{\dot{x}}{\sqrt{(a^2-x^2)}} \text{ and } y = \text{fluent } \frac{\dot{x}}{\sqrt{(ax-x)^2}},$$

&c. are, according to this definition, transcendental equations.

And for the same reason any curve, of which the equation is expressed by means of such fluents, is a transcendental curve; thus the cycloid whose equation is $y = \sqrt{(2ax-x^2)} + \text{fluent of } \frac{\dot{x}}{\sqrt{(2ax-x^2)}}$ is a transcendental curve; such also is the CATENARY, ELASTIC *Curve*, and several others.

TRANSFORMATION, in *Geometry*, is nearly the same as reduction, being used to denote the changing of any proposed figure to a different one of equal surface or solidity.

TRANSFORMATION of *Equations*, in *Algebra*, is a method of changing an equation to another form, but of equal value.

This is of various kinds, as 1. To transform an equation to another whose roots are greater or less than the roots of the proposed equation. 2. To transform an equation to another whose roots shall be some multiple or quotient of the proposed equation. 3. To transform an equation to another that shall want the second or any other term. 4. To transform or change the signs of the roots of an equation, from + to −, or from − to +, &c.

1. To transform an equation to another, whose roots shall be greater or less than the roots of the proposed equation.

Let $x^n + ax^{n-1} + bx^{n-2} + \&c. = 0$ be any equation; and let it be proposed to transform it to another whose roots shall be less than those of the above by a given quantity d.

Assume y to represent the unknown quantity of the new equation, and as this is to be less than x by the quantity d, we have $y = x - d$, or $x = y + d$; now substituting this value for x, we obtain,

$$\begin{aligned} x^n &= y^n + ndy^{n-1} + \frac{n.(n-1)}{1.2} d^2 y^{n-2} + \&c. \\ ax^{n-1} &= \qquad ay^{n-1} + a(n-1) d y^{n-2} + \&c. \\ bx^{n-2} &= \qquad\qquad by^{n-2} + \&c. \end{aligned}$$

Whence, by adding together the co-efficients of the like powers of y, we have

$$y^n + \left.\begin{matrix} nd \\ a \end{matrix}\right\} y^{n-1} + \left.\begin{matrix} \frac{n.(n-1)}{1.2} \\ a(n-1)d \\ b \end{matrix}\right\} \begin{matrix} y^{n-2} + \\ \&c. \end{matrix}$$

for the equation required.

If the roots of the equation are to be increased, then we must assume $y = x + d$.

As an example, let it be proposed

he roots of the equa-
$x^2 + 11x - 6 = 0$, by
Assume $y = x + 1$, or
then

$$\begin{array}{rrrr} y^3 - 3y^2 + & 3y - & 1 \\ -6y^2 + & 12y - & 6 \\ & 11y - & 11 \\ & - & 6 \\ \hline y^3 - 9y^2 + & 26y - & 24 = 0 \end{array}$$

ation sought.
ultiply or divide the roots
ation by any given quan-

$ax^{n-1} + bx^{n-2} + \&c. = 0$
roposed equation; which
transformed to another,
oots shall be some multiple
above.
ne $y = mx$, m being the
d multiple; then $x = \frac{y}{m}$,
bstituting this value for x in
oposed equation, we have

$$a\frac{y^{n-1}}{m^{n-1}} + b\frac{y^{n-2}}{m^{n-2}} + \&c. = 0$$

multiplying by m^n

$$m\,ay^{n-1} + m^2\,by^{n-2} + \&c. = 0;$$

h is an equation, whose roots
qual to the given multiple m
e roots of the proposed equa-

Hence we see, that to multi-
the roots of an equation by
quantity m, it is only necessary
multiply the several co-effi-
nts; beginning at the first by
terms of the geometrical pro-
ssion 1, m, m^2, m^3, m^4, &c. ob-
rving only that if any term in
e original equation be wanting,
must be introduced in its proper
lace, having zero for its co-effi-
ient; thus

Let it be proposed to transform
he equation

$$x^4 + 5x^3 + 3x + 7 = 0$$

to another equation, whose roots are the doubles of these roots.

Here, supplying the third term, which is wanting,

$$\begin{array}{llllll} \text{proposed equat.} & & & & & \\ x^4 + & 5x^3 + & 0x^2 + & 3x + & 7 = 0 \\ \text{geom. series} & & & & & \\ 1 & 2 & 4 & 8 & 16 \\ \hline y^4 + & 10y^3 & & + 24y + & 112 = 0 \end{array}$$

which is the equation required, the unknown quantity being changed from x to y merely for the sake of distinction.

When the roots of the equation are to be divided by any quantity m, we must substitute $y = \frac{x}{m}$, or $x = my$, and substituting this value of x we have

$$m^n y^n + am^{n-1} y^{n-1} + bm^{n-2} y^{n-2} + \&c. = 0;$$

or, dividing by m^n,

$$y^n + \frac{a}{m} y^{n-1} + \frac{b}{m^2} y^{n-2} + \&c. = 0;$$

that is, we must divide the co-efficients by the same series 1, m, m^2, m^3, m^4, &c.

3. In a similar way an equation may be transformed to another, whose roots are the reciprocals of the roots of the first.

Let $x^n + ax^{n-1} + bx^{n-2} + \&c. = 0$ be the proposed equation; and let it be required to transform this to an equation, whose roots shall be the reciprocals of these roots.

Assume $y = \frac{1}{x}$, or $x = \frac{1}{y}$, and substitute this value of x in the proposed equation; and we have

$$\frac{1}{y^n} + a\frac{1}{y^{n-1}} + b\frac{1}{y^{n-2}} \ldots w, \&c. = 0;$$

or $\quad 1 + ay + by^2 \ldots wy^n, \&c. = 0;$

or, dividing by w, and inverting the order of the equation,

$$y^n \ldots \frac{b}{w} y^2 + \frac{a}{w} y + \frac{1}{w} = 0,$$

the equation required.

That is, we must invert the order of the co-efficients, and divide by the absolute quantity of the proposed equation; observing here, as in the preceding case, to supply any terms of the equation that may be wanting, by prefixing a cipher to them for a co-efficient.

Transform the equation

$$x^4 + 3x^3 + 2x + 4 = 0$$

to another whose roots shall be the reciprocal of these.

Here, supplying the deficient term, and inverting the order of the co-efficients, we have

$$4 + 2x + 0x^2 + 3x^3 + x^4;$$

then dividing by 4, and introducing y, we have

$$y^4 + \frac{2}{4} y^3 + \frac{0}{4} y^2 + \frac{3}{4} y + \frac{1}{4} = 0$$

or $y^4 + \frac{1}{2} y^3 + \frac{3}{4} y + \frac{1}{4} = 0$, as required.

4. To change the signs of the roots of an equation.

Let $x^n + ax^{n-1} + bx^{n-2} + cx^{n-3} + \&c. = 0$ be an equation, the signs of whose roots are to be changed.

Assume $y = -x$, or $x = -y$, and substitute this for x in the proposed equation; so shall the roots of the proposed equation be changed as required.

Here it is obvious, that only the signs of the odd powers of x will be changed, because the even powers of a negative quantity have the same signs as a positive one; and since, also, no change takes place in the value of an equation, when all the signs of it are changed; we may reduce what has been said above to the following general rule. To change the signs of the roots of an equation, supply such terms as are wanting, and then change the signs of all the even terms, *viz.* the 2d, 4th, 6th, &c. from the left hand; and the equation will be transformed as required.

Let it be required to change the signs of the roots of the equation.

$$x^5 - 3x^4 + 7x^2 + 6x + 4 = 0$$

First, supplying the third term, which is wanting, the equation becomes

$$x^5 - 3x^4 + 0x^3 + 7x^2 + 6x + 4 = 0$$

Changing the alternate terms, and writing y for x, we have

$$y^5 + 3y^4 + 0y^3 - 7y^2 + 6y - 4 = 0,$$

or $y^5 + 3y^4 - 7y^2 + 6y - 4 = 0$ the equation sought.

5. To transform an equation to another that shall want the second, or any other term of the equation. Let $x^n + ax^{n-1} + bx^{n-2} + \&c. = 0$, be any equation which is to be transformed into another that shall want any required term.

Assume $x = y + d$, and substitute this value for x, and we obtain

$$\begin{aligned} x^n &= y^n + ndy^{n-1} + \frac{n(n-1)}{2}d^2 y^{n-2} + \&c. \\ ax^{n-1} &= ay^{n-1} + a(n-1)d\, y^{n-2} + \&c. \\ bx^{n-2} &= by^{n-2} + \&c. \end{aligned}$$

Now equate the co-efficient of that power of y which is to be exterminate to zero, and hence find the numerical value of d, which substituted for d will give the equation required.

As from the nature of the binomial theorem, the second term involves the quantity d in only the *first* degree, the third term in the *second* degree, the fourth term in the *third* degree, &c. it follows, that to exterminate the second term, the equation will be a *simple* one, and may therefore always be effected in rational numbers; but to exterminate the third term, we must solve a *quadratic;* and to exterminate the fourth, a *cubic,* and so on.

Thus in the above equation, to exterminate the second term, we have

$$nd + a = 0, \text{ or } d = \frac{-a}{n}.$$

To exterminate the third term,

$$\frac{n(n-1)}{2}d^2 + a(n-1)d + b = 0$$

or

$$d^2 + \frac{2a}{n}d = \frac{-2b}{n^2 - n}.$$

Whence

$$d = -\frac{ad}{n} \pm \sqrt{\left(\frac{a^2 d^2}{n^2} - \frac{2b}{n^2 - n}\right)}$$

and so on for any other term.

TRANSIT, in *Astronomy*, signifies the passage of any planet over a fixed star, or the sun; and of the moon in particular, covering or moving over any planet.

The transits of Mercury and Venus over the sun's disc are very interesting phenomena, not merely by reason of their rare and singular appearance, but because of their use in determining the parallax of the sun, and thence the real dimensions of the earth's orbit.

The transits of Mercury always occur either in May or in November; but they happen much the most frequently in the latter month. The difference depends upon the position of the elliptic projection of Mercury's orbit upon the plane of the ecliptic. This ellipse is now so placed that it presents to us its perihelion during the winter, and its aphelion during the summer; and, as it is very eccentric, Mercury is much nearer the sun in the month of November than in the month of May. Now, if we consider the luminous cone formed by the visual rays drawn from the earth to the sun, this cone is contracted in the vicinity of the earth while it is enlarged near the sun, the disc of which serves for its base: Mercury ought therefore to cut it more readily when it is near the sun than when it is remote from it; and, consequently, the transits of Mercury ought to occur most frequently in the winter part of the year.

Transits of Venus over the sun's disc happen much less frequently than those of Mercury, because Venus is more distant from the sun. The following are all that have occurred or will occur between 1631 and 2110.

1631 ... Dec. 6.
1639 ... Dec. 4.
1761 ... June 5.
1769 ... June 3.
1874 ... Dec. 8.
1882 ... Dec. 6.
2004 ... June 7.
2109 ... Dec. 10.

Now the chief use of these conjunctions is accurately to determine the sun's distance from the earth, or his parallax, which astronomers have in vain attempted to find by various other methods; for the minuteness of the angles required easily eludes the nicest instruments. But in observing the ingress of Venus into the sun, and her egress from the same, the space of time between the moments of the internal contacts, observed to a second of time, that is, to $\frac{1}{15}$ of a second, or $4'''$ of an arch, may be obtained by the assistance of a moderate telescope and a pendulum clock, that is consistent with itself exactly for 6 or 8 hours. Now, from two such observations rightly made in proper places, the distance of the sun within a 500th part may be certainly concluded, &c.

Transit *Instrument*, in *Astronomy*, is a telescope fixed at right angles to a horizontal axis; this axis being so supported that the line of collimation may move in the plane of the meridian.

The axis, to the middle of which the telescope is fixed, should gradually taper toward its ends, and terminate in cylinders well turned and smoothed; and a proper weight or balance is put on the tube, so that it may stand at any elevation when the axis rests on the supporters. Two upright posts of wood or stone, firmly fixed at a proper distance, are to sustain the supporters to this instrument; these supporters are two thick brass plates, having well smoothed angular notches in their upper ends, to receive the cylindrical arms of the axis; each of the notched plates is contrived to be moveable by a screw, which slides them upon the surfaces of two other plates immoveably fixed to the two upright posts: one plate moving in a vertical direction, and the other horizontally, they adjust the telescope to the planes of the horizon and meridian; to the plane of the horizon, by a spirit level hung in a position parallel to the axis, and to the plane of the meridian, in the following manner. Observe the times, by the clock, when a circumpolar star, seen through this instrument, transits both above and below the pole; then, if the times of describing the eastern and western parts of its circuit be equal, the telescope is then in the plane of the meridian; otherwise, the notched plates must be gently moved till the time of the star's revolution is bisected by both the upper and lower transits, taking care at the same time that the axis keeps its horizontal position.

When the telescope is thus adjusted, a mark must be set up, or made, at a considerable distance (the greater the better) in the horizontal direction of the intersection of the cross wires, and in a place where it can be illuminated in the night-time by a lantern near it, which mark, being on a fixed object, will serve at all times afterwards to examine the position of the telescope, by first adjusting the transverse axis by the level.

To adjust a clock by the sun's transit over the meridian, note the times by the clock, when the preceding and following edges of the sun's limb touch the cross wires: the difference between the middle time and 12 hours, shows how much the mean, or clock time, is faster or slower than the apparent or solar time, for that day; to which the equation of time being applied, it will show the time of mean noon for that day, by which the clock may be adjusted.

TRANSMISSION, in *Optics*, is used to denote the passage of the rays of light through transparent bodies.

TRANSPARENCY, or **TRANSLUCENCY**, in *Physics*, a quality in certain bodies, by which they give passage to the rays of light.

TRANSPOSITION, in *Algebra*, is the bringing any term of an equation over to the other side of it, in which case the sign of the quantity thus transposed must be changed, *viz.* from *plus* to *minus*, or from *minus* to *plus*.

For example, if $a + x = b$, then transposing a and changing its sign, we have $x = b - a$; and if, on the other hand, we had $x - a = b$, then by transposition we have $x = a + b$.

This operation is performed in order to bring all the known terms to one side of the equation, and all those that are unknown on the other side, whereby the value of these last became determined.

TRANSVERSE *Axis*, or *Diameter*, in the *Conic Sections*, is the first or principal diameter passing through both the foci of the curve.

In the ellipse the transverse is the longest of all the diameters, in the hyperbola it is the shortest, and in the parabola it is like all the other diameters, infinite in length.

TRAPEZIUM, in *Geometry*, a plane figure contained under four right lines, of which neither of the opposite sides are parallel. If a quadrilateral have both its pairs of opposite sides parallel, it is called a *parallelogram;* if only one of its pairs of opposite sides are parallel, it is called a *trapezoid;* and when neither pair are parallel, it is called a *trapezium.*

The trapezium has several remarkable properties, of which the following are the most important:

1. Any three sides of a trapezium are greater than the fourth side.

2. The sum of the four inward angles of a trapezium is equal to four right angles.

3. The diagonals of a trapezium divide it into four proportional triangles.

4. If the four sides of a trapezium be bisected, and the adjacent points of bisection be joined, it will form a parallelogram, the sides of which are parallel to the corresponding diagonals of the trapezium; and the area of the former is equal to half the area of the latter.

5. The sum of the four lines formed by the intersection of the two diagonals of a trapezium, is less than the sum of any other four lines drawn from a point within the trapezium to its four angles.

6. The sum of the opposite angles of a trapezium, which can be inscribed in a circle, is equal to two right angles, and if one side of such a figure be produced, the external angle is equal to the internal and opposite angle.

7. Also, in this case, the rectangle of its two diagonals is equal to the sum of the rectangles of its opposite sides.

TRAPEZOID, a quadrilateral figure, having two of its opposite sides parallel; the area of which is equal to half a parallelogram, whose base is equal to the sum of the two parallel sides, and its altitude equal to the perpendicular distance between them.

TRAVERSE, in *Navigation*, is the variation or alteration of a ship's course, occasioned by the shifting of the winds, currents, &c. or a traverse may be defined to be a compound course, consisting of several courses and distances.

TRIANGLE, in *Geometry*, a figure bounded by three sides, and consequently containing three angles, whence it derives its name.

Triangles are of different kinds, as *plane* or *rectilinear*, *spherical*, and *curvilinear*.

A *Plane* or *Rectilinear* TRIANGLE, is that which is bounded by right lines.

A *Spherical* TRIANGLE, is that which is bounded by three arcs of great circles of the sphere.

A *Curvilinear* TRIANGLE, is that which is bounded by any three curve lines.

And each of these three distinct classes of triangles receive other distinguishing denominations, according to the relation of their sides and angles.

Of *Plane* TRIANGLES. These are distinguished as follows:

1. An *Equilateral* TRIANGLE, is that which has its three sides equal, and consequently its angles, whence it is sometimes also called an *equiangular triangle*.

2. An *Isosceles* TRIANGLE, is that which has only two of its sides equal. The angles at the base of an isosceles triangle are equal to each other.

3. A *Scalene* or *Oblique* TRIANGLE, is that which has no two of its sides equal. This is termed a *scalene triangle* with reference to its sides; and oblique with regard to its angles.

Triangles also receive other denominations with reference to their angles.

4. A *Right-angled* TRIANGLE, is that which has a right angle. Here the side opposite the right angle is called the *hypothenuse*, and the other two sides the *base* and *perpendicular*, or sometimes the *legs*.

5. An *Oblique-angled* TRIANGLE, is that which has not a right angle; and is either *acute* or *obtuse*.

6. An *Acute angled* TRIANGLE, is that which has *three* acute angles.

7. An *Obtuse-angled* TRIANGLE, is that which has one obtuse angle.

8. *Similar* TRIANGLES, are such as have the angles of the one equal to the angles of the other, each to each.

Some of the principal properties of plane triangles are as follow:

1. The greater side of a triangle is opposite the greater angle, and the less side opposite the less angle.

2. Any side of a triangle is less than the sum, but greater than the difference of the other two sides.

3. The sum of the three angles of a triangle is equal to two right angles; and the external angle formed by producing one of its sides is equal to the sum of the two internal and opposite angles.

4. In a right-angled triangle the square of the hypothenuse is equal to the sum of the squares of the other two sides.

5. Or more generally, if a, b, and c, represent the three sides of a triangle, and C the angle contained by a and b; then if

$$\begin{cases} a^2+b^2 = c^2, \text{ the } \angle C = 90^\circ \\ a^2+ab+b^2 = c^2, \text{ the } \angle C = 120^\circ \\ a^2-ab+b^2 = c^2, \text{ the } \angle C = 60^\circ. \end{cases}$$

6. In any triangle the rectangle of any two sides is equal to the rectangle of the perpendicular on the third side, and the diameter of the circumscribing circle.

7. The square of a line bisecting any angle of a triangle, together with the rectangle of the segments of the opposite sides, is equal to the rectangle of the two sides, including the bisected angle.

8. Triangles, when their bases are equal, are to each other as their altitudes; and when their altitudes are equal they are to each other as their bases; and when neither are equal they are to each other in the compound ratio of their bases and altitudes.

9. Similar triangles have their like sides proportional, and their areas are to each other as the squares of the like sides.

10. The line which bisects any angle of a triangle divides the opposite side into two segments, which are in proportion to the adjacent sides.

11. The line which is drawn parallel to one side of a triangle divides the other two sides proportionally.

12. The three lines which bisect the three angles of a triangle intersect in one common point, as also do the three lines which bisect the three sides perpendicularly; the former point being the centre of the inscribed circle, and the latter the centre of the circumscribing one.

13. If perpendiculars be let fall from the three angles of a triangle to the opposite sides, they will intersect in one common point; as will also the three lines drawn from the three angles to the middle of the opposite side, which latter point is the centre of gravity of the triangle.

14. In a right-angled triangle, the perpendicular let fall from the right angle to the opposite side or base, is a mean proportional be-

tween the segments of the base, and each of the sides about the right angle is a mean proportional between the base and the adjacent segment.

There are also several curious problems in the Diophantine Analysis relating to right-angled triangles; viz.

1. The number which expresses the area of a rational right-angled triangle cannot be equal to a square number.

2. Let b represent the base of a right-angled triangle, p the perpendicular, and h the hypothenuse; then if we assume

$$b = r^2 - s^2, \text{ and } p = 2rs,$$

we shall have

$$p = r^2 + s^2;$$

that is, the triangle will be a rational one, r and s being assumed at pleasure.

3. Or if in the fraction $\frac{2rs}{r^2 - s^2}$, or $\frac{r^2 - s^2}{2rs}$, the numerator and denominator be taken for the sides of a right-angled triangle, it will be a rational one. And as we may here give any values to r and s at pleasure, we have, in the first instance, by assuming $r = s + 1$, and taking successively

$$s = 1, 2, 3, 4, 5, \&c.$$

the following remarkable series; viz.

$$\frac{2rs}{r^2 - s^2} = s + \frac{s}{2s + 1} =$$

$$1\tfrac{1}{3},\ 2\tfrac{2}{5},\ 3\tfrac{3}{7},\ 4\tfrac{4}{9},\ 5\tfrac{5}{11},\ \&c.$$

each of which terms, reduced to an improper fraction, will give the sides of a rational right-angled triangle.

4. And in the second form, by making $s = 1$, and $r = n + 2$, it becomes

$$\frac{r^2 - s^2}{2rs} = n + \frac{4n + 3}{4n + 4}, \text{ and assuming, here}$$

$$n = 1, 2, 3, 4, \&c.$$

we have this other series,

$$1\tfrac{7}{8},\ 2\tfrac{11}{12},\ 3\tfrac{15}{16},\ 4\tfrac{19}{20},\ 5\tfrac{23}{24},\ \&c.$$

which has the same property as the former.

To find the Area of a Triangle.—Let a, b, c, represent the three sides of the triangle; A, B, C, the angles opposite those sides respectivly; p the perpendicular falling upon the base b; then

1. $\text{Area} = \tfrac{1}{2}\, p\, b.$

2. $\text{Area} = \tfrac{1}{2}\, a b\, .\, \text{sin. C} = \tfrac{1}{2}\, a c\, .\, \text{sin. B} = \tfrac{1}{2}\, b c\, .\, \text{sin. A}.$

Make $a + b + c = s$; then

3. $\text{Area} = \sqrt{\left\{\tfrac{1}{2}s\, (\tfrac{1}{2}s - a)\, (\tfrac{1}{2}s - b)\, (\tfrac{1}{2}s - c)\right\}}$

4. $\text{Log. of area} = \tfrac{1}{2}\left\{\text{log. } \tfrac{1}{2}s + \text{log. } (\tfrac{1}{2}s - a) + \text{log. } (\tfrac{1}{2}s - b) + \text{log. } (\tfrac{1}{2}s - c)\right\}$

Spherical TRIANGLES. A spherical triangle is that which is formed by the intersecting arcs of three great circles of a sphere. Spherical triangles are divided into *right-angled, oblique, equilateral, isosceles, scalene,* &c. the same as plane triangles, and in the same cases, and therefore require no separate definition. They are also said to be *quadrantal,* when they have one side a quadrant.

Two sides, or two angles, are said to be of the *same affection,* when they are at the same time either both greater or both less than a quadrant, or 90°; and of *different* affections, when one is greater than a quadrant, and one less.

Spherical TRIANGLES have several useful properties, of which the following are the most important.

1. In spherical triangles, as well as in plane triangles, equal angles are subtended by equal sides; the greater angle is subtended by the greater side; and the less angle by the less side.

2. Any side or angle of a spherical triangle is less than a semicircle, or 180°.

3. The sum of any two sides of a spherical triangle is greater than the third side; and their difference is less than the third side.

4. The difference of any two sides of a spherical triangle is less than a semicircle, or 180°; and the sum of the three sides is less than 360°.

5. The snm of the three angles of any spherical triangle, is greater than two right angles, or 180°; and less than six right angles, or 540°.

6. The sum of any two angles of

a spherical triangle is greater than the supplement of the third angle.

7. If the three sides of a spherical triangle be equal to each other, the three angles will also be equal; and vice versâ.

8. If any two sides of a spherical triangle be equal to each other, their opposite angles will also be equal; and vice versâ.

9. If the sum of any two sides of a spherical triangle be equal to 180°; the sum of their opposite angles will also be equal to 180°; and vice versâ.

10. If the three angles of a spherical triangle be all acute, or all right, or all obtuse, the three sides will be accordingly all less than 90°, or all equal to 90°, or all greater than 90°; and vice versâ.

11. Half the sum of any two sides of a spherical triangle, is of the same kind as half the sum of their opposite angles; or the sum of any two sides is of the same kind, with respect to 180°, as the sum of their opposite angles.

To these may also be added the following properties of the polar triangle, by which the data in any case may be changed from sides to angles, and from angles to sides.

If the three arcs of great circles be described from the angular points of any spherical triangle as poles, or at 90° distance from them, the sides and angles of the new triangle so formed, will be the supplements of the opposite angles and sides of the other; and vice versâ.

To find the Area of a Spherical Triangle.—The area, or surface, of a spherical triangle, is equal to the difference between the sum of its three angles, and two right angles, multiplied by the radius of the sphere.

The area of any spherical triangle, which is small with regard to the whole sphere, (such as those used in geodetical operations) is *very nearly equal* to the area of a rectilinear triangle, of which the sides are equal in length to the former, and whose angles are each less than the corresponding angles of the former, by one-third of the spherical excess.

Arithmetical TRIANGLE, is a kind of table of numbers, arranged in the form of a triangle, formerly employed in arithmetical calculations; but it is now rendered useless, by the numerous improvements of modern times, and therefore any minute description of it seems useless.

TRIANGULAR *Compasses*, are those which have three legs, by means of which a triangle may be taken off at once, which renders them very useful in the construction of maps, charts, &c.

TRIBOMETER, the name given by Musschenbroek to an instrument invented by him for measuring the friction of metals.

TRIDENT, a term used by Des Cartes for a particular kind of parabola employed by him for the construction of equations of the sixth degree.

TRIGONOMETRY, the art of measuring the sides and angles of triangles, either plane or spherical, or rather of determining certain parts which are unknown from others that are given.

This is a science of the greatest importance in almost every branch of mathematics, and particularly in astronomy, navigation, surveying, dialling, &c. and has been accordingly practised by mathematicians of all ages.

Trigonometry is called *plane* and *spherical*, according to the species of triangles to which it is applied.

In plane trigonometry every circle is supposed to be divided into 360 equal parts, or degrees, every degree into 60 minutes, and each minute into 60 seconds, and so on; and the measure or quantity of an angle is estimated by the number of degrees, minutes, &c. contained in the arc by which it is bounded; the degrees being denoted by a small °, the minutes by a dash, the seconds by two dashes, and so on; thus 70 degrees, 16 minutes, 34 seconds, are denoted by 70° 16′ 34″.

The *complement* of an arc or angle is what it wants of 90°, or of a quadrant. And the *supplement* of an angle is what it wants of a semicircle or of 180°. Thus, if an angle measures 50°, its complement is 40°, and its supplement 130°.

Solution of the Cases of Plane Triangles.

Any right-lined triangle may be inscribed in a circle; and this being done, each side of the triangle will be the chord of an arc double of that which measures the opposite angle; that is, double the sine of that angle measured in the circle; therefore the sides of the triangle are to each other as the sines of the opposite angles measured in the same circle, and consequently as the sines of the same angles measured in the circle whose radius is that of the tables. Hence the following proposition of such frequent use in the practice of trigonometry.

In any triangle the sines of the angles are proportional to the sides opposite those same angles.

Therefore, 1st, in any right-angled triangle BAC (Plate Trigonometry, fig. 1,) the sine of the right-angle A, or radius, is to the hypothenuse B C = sin. C . A B = sin. B : A C.

Second: Since in all right-angled triangles the sine of one acute angle C, is the cosine of the other acute angle B, we have sin. C = cos. B and reciprocally.

Therefore, instead of the proportion sin. B : sin C = A C : A B, we may always substitute sin. B : cos. B = A C : A B.

But since sin. B cos B = tang. B : R, we have A C : A B tang. B : R = cot. C : R

These principles suffice to solve all the cases of the right-angled triangle A B C, when besides the right-angle A, we know any two of the five parts B, C, A B, A C, B C; provided they are not the two angles. In this latter case we can only determine the ratios of the three sides.

The Solution of right-angled Triangles.

Given.	Required.	Formulas.
A B, A C	B C B C	$BC = \sqrt{AB^2 + AC^2}$ A B : B C = R : tang. B A C : A B = R : tang. C
A B, B C	A C B C	$AC = \sqrt{BC^2 - AB^2}$ B C : A B = R : cos. B B C : A B = C : sin. C
A C, B C	A B B C	$AB = \sqrt{BC^2 - AC^2}$ B C : A C = R : sin B B C : A C = R : cos. C
AB, B	A C B C	R : tang. B = A B : A C cos. B : R = A B : B C
A C, B	A B B C	R : cot. B = A C : A B sin. B : R = A C : B C
A B, C	A C B C	R : cot C = A B : A C sin. C : R = A B : B C
A C, C	A B B C	R : tang. C = A C : A B. cos. C : R = A C : B C
B C, B	A B A C	R : cos. B = B C : A B R : sin. B = B C : A C
B C, C	A B A C	R sin. C = B C : A B R : cos. C = B C : A C

Trigonometry.

Fig. 1.

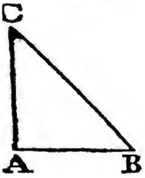


Fig. 2.

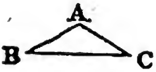


Fig. 3.

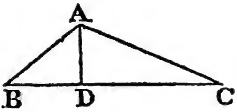


Fig. 4.

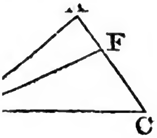


Fig. 5.

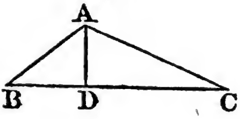


Fig. 6.

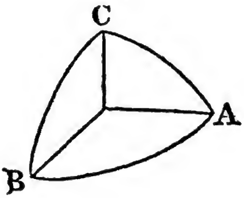


Fig. 7.

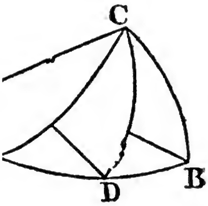


Fig. 8.

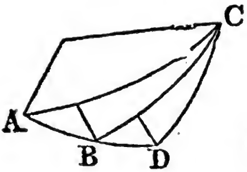


Fig. 9.

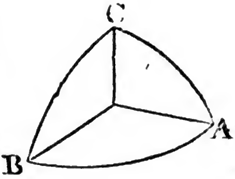


Fig. 10.

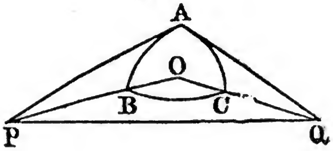
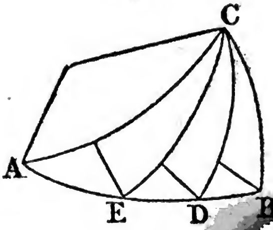


Fig. 11.



With regard to *oblique* angled triangles, or those which have no right-angle, the solution of them may be reduced to the four following.

CASE 1. Given any two angles B, A, (fig. 2.) and a side B C, to find the other sides B A, A C.

Solution. Construct the proportion sin A : B C = sin B : A C = $\frac{\text{B C sin B}}{\text{sin. A}}$ = sin C : A B, or = sin (A + B) : A B, (because sin C = sin (A + B).)

If B A had been the side given, we should have had sin (A + B) : B A = sin B : A C = sin A : B C.

CASE 2. Given two sides, and an angle opposite to one of the given sides, to find the third side and the other two angles, supposing that we know of what species they are.

Solution. Suppose that the sides A B, A C (fig. 2.) are given, and the angle B; we shall first obtain the angle C by this proportion A C : sin B = A B : sin C, which will give the third angle. And we may then obtain the side B C by the proportion sin B : A C = sin A : B C.

But to find the side B C at once, draw the perpendicular AD (fig. 3.) on the side B C; call A B, a; A C, b; sin B, s; cos B, c; and we shall have R : A B (a) = sin B (s) : A D $= \frac{as}{\text{R}}$ = cos B (c) : B D $= \frac{ac}{\text{R}}$

Therefore D C $= \sqrt{\text{A C}^2 - \text{A D}^2} = \sqrt{bb - \frac{a^2 s^2}{\text{R}^2}}$, and D C + B D = B C $= \frac{ac}{\text{R}} + \sqrt{bb - \frac{a^2 s^2}{\text{R}^2}}$

CASE 3. Given two sides and the included angle, to find the other two angles and the remaining side.

Solution. Let the angle A (fig. 2.) and the two sides A B, A C be given, and the angles B and C, and the side B C be required.

First we have,

A C : A B = sin B : sin C, or by composition A C + A B : A C − A B = sin B + sin C : sin B − sin C = $\frac{\text{sin B + sin C}}{\text{sin B − sin C}}$: 1

But (19)

$$\frac{\text{sin B + sin C}}{\text{sin B − sin C}} = \frac{\text{tang } \frac{1}{2}\text{ (B + C)}}{\text{tang } \frac{1}{2}\text{ (B − C)}}$$

Therefore,

A C + A B : A C − A B = tang $\frac{1}{2}$ (B + C) tang $\frac{1}{2}$ (B − C)

And since B + C = 180° − A, we have tang $\frac{1}{2}$ (B + C) = tang (90° − $\frac{1}{2}$ A) = cot $\frac{1}{2}$ A. And therefore,

A C + A B : A C − A B = cot $\frac{1}{2}$ A : tang $\frac{1}{2}$ (B − C)

And consequently since we know B − C and B + C, it is easy to obtain the angles B and C; and we shall then have sin B : A C = sin A : B C, which will give the third side B C.

This third case may also be solved in the following manner:

Draw the perpendicular B F (fig. 4.) upon the side A C, and let A B $= a$, A C $= b$, sin A $= s$, cos A $= c$, we shall have R : $a = s$: B F $= \frac{as}{\text{R}} = c$: A F $= \frac{ac}{\text{R}}$.

Therefore F C $= b - \frac{ac}{\text{R}}$. But in the right-angled triangle B F C, we have $\left(\text{F C} \left(b - \frac{ac}{\text{R}}\right) : \text{B F} \left(\frac{as}{\text{R}}\right)\right)$ = R : tang C $= \frac{a\text{R}s}{b\text{R} - ac}$

Similarly tang B $= \frac{b\text{R}s}{a\text{R} - bc}$

If the angle A is obtuse, C becomes negative, and we have tangent C $= \frac{a\text{R}s}{b\text{R} + ac}$, and tang B $= \frac{b\text{R}s}{a\text{R} + bc}$

The right-angled triangle B F C gives B C $= \sqrt{\left\{\frac{a^2 s^2}{\text{R}^2} + \left(b - \frac{ac}{\text{R}}\right)^2\right\}} = \sqrt{\left(a^2 + b^2 - \frac{2abc}{\text{R}}\right)}$; or $\sqrt{\left(a^2 + b^2 + \frac{2abc}{\text{R}}\right)}$, if the angle A is obtuse.

From the proportion A C + C B : A C − C B = tang $\frac{1}{2}$ (B + C) : tang

$\frac{1}{2}(B - C)$ it follows that in any triangle the sum of any two sides is to their difference, as the tangent of half the sum of the two angles opposite these sides, is to the tangent of half the difference of these same angles.

Case. 4. Given the three sides of a triangle, required the three angles.

Let B C (fig. 5.) $= a, \ldots$ A B $= b \ldots$ A C $= d \ldots$ R $= 1 \ldots$ and draw A D perpendicular on B C.

By prop. 47, Book 1, Euclid's Elements $AB^2 - BD^2 = AC^2 - CD^2 = AC^2 - (BC - BD)^2$

Whence

$$BD = \frac{AB^2 + BC^2 - AC^2}{2\,BC}, \text{ or } BD = \frac{a^2 + b^2 - d^2}{2a}$$

Again in the right-angled triangle B D A, we have

$$AB\,(b) : BD\left(\frac{a^2 + b^2 - d^2}{2a}\right) = \text{rad}\ (1) : \cos B = \frac{a^2 + b^2 - d^2}{2ab}$$

Therefore

$$1 - \cos.\ B = \frac{d^2 - a^2 + 2ab - b^2}{2ab} = \frac{d^2 - (a - b)^2}{2ab} = 2 \sin^2 \tfrac{1}{2} B\ (18)$$

From this expression we may deduce the following proportion:

$$ab : \frac{d - a + b}{2} \times \frac{d + a - b}{2} = R^2 : \sin^2 \tfrac{1}{2} B;$$

which, calling q the semiperimeter becomes

$ab : (q - a)(q - b) = R^2 : \sin^2 \frac{1}{2} B$; and by this expression we usually calculate the value of an angle from the three sides.

We may also calculate it by means of the formula $\cos B = \frac{R}{2ab}(a^2 + b^2 - d^2)$, which gives

$$2ab : a^2 + b^2 - d^2 = R : \cos B.$$

b B

That is, to find an angle B in a triangle, of which we know the three sides, we must first find the sum of the squares of the two sides containing the angle required, and subtract from this sum the square of the third side; this will leave a remainder. We must then construct this proportion. The double of the rectangle of the side containing the angle required is to this remainder, as radius to the cosine of the angle required.

If the angle B is obtuse, we shall have

$$\cos B = \frac{R}{2ab}(d^2 - a^2 - b^2)$$

A formula equally convenient for logarithmic calculation, may be obtained as follows:

$$1 - \cos B = 2 \sin^2 \frac{B}{2} = \frac{2.\,(q - a)(q - b)}{ab}$$

$$\text{And } 1 + \cos B = 1 + \frac{a^2 + b^2 - d^2}{2ab} = \frac{(a+b)^2 - d^2}{2ab} = \frac{(a+b+d)(a+b-d)}{2ab} = \frac{2.\,q\,(q - d}{ab} = 2 \cos^2 \frac{B}{2} \text{ by (18)}$$

: consequently we have

$$\frac{\sin^2 \frac{B}{2}}{\cos^2 \frac{B}{2}} = \text{tang}^2 \frac{B}{2} = \frac{\frac{(q - a)(q - b)}{ab}}{\frac{q(q - d)}{ab}} = \frac{(q - a)(q - b)}{(q)(q - d)}$$

if radius $= 1$; or $\text{tang}^2 \frac{B}{2} = \frac{R^2 (q - a)(q - b)}{q\,(q - d)}$, if radius $=$ R.

N. B. This formula must not be used, when the angle sought for is nearly 180°.

Such are the principles of plane trigonometry. Other problems concerning triangles might be proposed; but not to multiply examples, we shall add only the following one:

Given one of the acute angles of a right-angled plane triangle, together with its surface; required the three sides.

Let A be the given angle, x the opposite side, y the other side, and s the surface of the triangle. By a well known theorem $xy = 2s$; and again, $\sin A : \cos A = x : y$.

$$\text{Therefore } xy = \frac{y^2 \sin A}{\cos A} = 2s.$$

Hence

$$y = \sqrt{\frac{2\,s\,\cos A}{\sin A}} = \sqrt{\frac{2\,s\,\cot A}{R}}$$

$$x = \sqrt{\frac{2\,s\,\sin A}{\cos A}} = \sqrt{\frac{2\,s\,\mathrm{tang}\,A}{R}},$$

and the hypothenuse $= \sqrt{x^2 + y^2} =$

$$\sqrt{\frac{2\,s\,R^2}{\cos A \sin A}} = 2\sqrt{\frac{R\,s}{\sin 2\,A}}$$

Properties of Spherical Triangles.

1. The arc contained between the pole of a circle and any point of its circumference, is always 90°.

2. The measure of any spherical angle is the arc of a great circle comprised between its sides at 90° distance from the summit of that angle.

3. Every spherical angle is less than 180°.

4. Any side of a spherical triangle is less than 180°.

5. The sum of the sides of any spherical triangle is always less than 360°.

6. If from the three angles of any spherical triangle, taken as poles, we describe three arcs, whose meeting forms a new spherical triangle, the angles and sides of this second triangle will be reciprocally the supplements of the sides and angles opposite to them in the first triangle.

7. The sum of the angles of a spherical triangle may vary from 180° to 540° exclusively.

8. Two spherical triangles are equal, when their angles are respectively equal.

9. In every spherical triangle, equal sides are opposite to equal angles.

10. In any right-angled spherical triangle, each of the oblique angles is of the same species as the side opposite to it.

11. In a spherical triangle, if the two sides of the right-angle are of the same species, the hypothenuse will be less than 90°; and if they are of different species, the hypothenuse will be greater than 90°.

12. In every right-angled spherical triangle, radius is to the sine of the hypothenuse, as the sine of one of the oblique angles is to the sine of the side opposite to it.

13. In any spherical triangle, the sines of the angles are proportional to the sines of the sides.

14. In any right-angled spherical triangle, radius is to the cosine of one of the sides of the right-angle, as the cosine of the other side is to the cosine of the hypothenuse.

15. If an oblique-angled spherical triangle be divided into two right-angled triangles by an arc perpendicular to its base, we shall always have the cosines of the segments of the base proportional to the cosines of the two adjacent sides.

16. In any oblique-angled spherical triangle, on the base of which we have let fall a perpendicular arc, and which lies within the triangle, the tangent of the half base is to the tangent of half the sum of the other two sides, as the tangent of the half difference of these sides is to the tangent of half the difference of the segments of the base.

17. In any right-angled spherical triangle, radius is to the cosine of one of the sides of the right-angle, as the sine of the oblique angle opposite to the other side is to the cosine of the other oblique angle.

18. If we let fall an arc perpendicularly on the base of an oblique-angled triangle, the sines of the angles at the summit will be proportional to the cosines of the angles of the base.

19. In any right-angled spherical triangle, radius is to the cosine of one of the oblique angles, as the tangent of the hypothenuse is to the tangent of the side opposite to the other angle.

20. If we let fall an arc perpendicularly on the base of an oblique-angled spherical triangle, the cosines of the angles at the summit will always be reciprocally proportional to the tangents of the adjacent sides.

21. In any right-angled triangle, radius is to the sine of one of the sides of the right-angle, as the tangent of the oblique angle opposite to the other side is to the tangent of this last side.

22. In every right-angled spherical triangle, radius is to the cosine of the hypothenuse, as the tangent of one of the oblique angles is to the cotangent of the other angle.

Solution of all the possible cases of a Spherical Triangle A B C, right-angled at A.

Given.	*Required.*	*Proportions.*	*The part required is less than 90°.*
B C B	A C	R : sin B C = sin B : sin A C	If B ∠ 90°
	A B	R : cos B = tang B C : tang A B	If B C and B are of the same species
	C	R : cos B C = tang B : cot C	If B C and B are of the same species
B C C	A B	R : sin B C = sin C : sin A B	If C ∠ 90°
	A C	R : cos C = tang B C : tang A C	If B C and C are of the same species
	B	R : cos B C = tang C : cot B	If B C and C are of the same species
A C C	A B	R : sin A C = tang C : tang A B	If C ∠ 90°
	B C	R : cos C = cot A C : cot B C	If A C and C are of the same species
	B	R : cos A C = sin C : cos B	If A C ∠ 90°
A C B	A B	tang B : tang A C = R : sin A B	Ambiguous
	B C	sin B : sin A C = R : sin B C	Ambiguous
	C	cos A C : cos B = R : sin C	Ambiguous
A C B C	A B	cos A C : cos B C = R : cos A B	If B C and A C are of the same species
	B	sin B C : sin A C = R : sin B	If A C ∠ 90°
	C	tang B C : tang A C = R : cos C	If B C and A C are of the same species
A B C	A C	tang C : tang A B = R : sin A C	Ambiguous
	B C	sin C : sin A B = R : sin B C	Ambiguous
	B	cos A B : cos C = R : sin B	Ambiguous
A B B	A C	R : sin A B = tang B : tang A C	If B ∠ 90°
	B C	R : cos B = cot A B : cot B C	If A B and B are of the same species
	C	R : sin B = cos A B : cos C	If A B ∠ 90°
A B A C	B C	R : cos A B = cos A C : cos B C	If A C and C are of the same species
	B	R : sin A B = cot A C : cot B	If A C ∠ 90°
	C	R : sin A C = cot A B : cot C	If A B ∠ 90°
A B B C	A C	cos A B : cos B C = R : cos A C	If B C and A B are of the same species
	B	R : tang A B = cot B C : cos B	If B C and A B are of the same species
	C	sin B C : sin A B = R : sin C	If A B ∠ 90°
B C	A B	sin B : cos C = R : cos B	If C ∠ 90°
	A C	sin C : cos B = R : cos A C	If B ∠ 90°
	B C	R : cot B = cot C : cos B C	If B and C are of the same species

Solution of the different Cases of Oblique-angled Spherical Triangles.

Here there are as many varieties as there are different combinations among the six parts of a triangle, taken four by four. But there are fifteen of these combinations, and each admits of three different cases. Consequently, there are 45 problems to solve relating to oblique-angled triangles. But, as the resolution of one frequently includes that of several others, they may be reduced to the twelve following cases:

Case 1. In an oblique-angled spherical triangle A B C, (fig. 6.) given the angles B and A, and the side opposite to the angle A, to find the side A C opposite to the other angle.

Case 2. Given the two angles A, (fig. 7, 8.) and B, with the side B C, opposite to the angle A, to find the third angle.

Let fall the perpendicular arc C D, and in the triangle B C D we shall have

R : cos B C = tang B : cot B C D

and then

cos B : cos A = sin B C D : sin A C D.

Adding these two angles, or subtracting one from the other, according as the given angles A and B are of the same or of different species, we shall have the angle C required.

Case 3. Given two angles A and B, fig. 7 or 8, with the opposite side B C, as in the preceding case, to find the side contained between these two angles.

The perpendicular arc C D forms the two right-angled triangles A C D and D C B, the last of which gives

R : cos B = tang B C : tang B D

Besides we have

tang A : tang B = sin B D : sin A D.

Consequently, we shall have A B = A D + D B, according as the angles given are of the same or of different species.

Case 4. Given the two angles A and C, fig. 7 or 8, with the side on which these angles are formed, to find the third angle B

First,

R : cos A C = tang A : cot A C D

And then,

sin A C D : sin B C D = cos A : cos B.

The angle B C D is found by subtracting A C D from A C B, when the arc falls perpendicularly within the triangle; but when this arc falls without, we must subtract the angle A C B from the angle A C D, in order to find B C D.

Case 5. Given as before, the two angles A and C, with the contained side A C, to find the two other sides, B C for example.

First,

R : cos A C = tang A : cot A C D

And then,

cos B C D : cos A C D = tang A C : tang B C.

Case 6. Suppose any two sides given, A C and B C fig. 9, with one of the angles opposite to those sides, such as the angle A; to find the angle B, opposite to the other side.

This problem may be resolved by the well known proportion

sin B C : sin A C = sin A : sin B

This problem falls within the first case; they differ only by an inversion of the terms in the proportion which resolves both.

Case 7. The same things being given, required the third side A B.

R : cos A = tang A C : tang A D

cos A C : cos B C = cos A D : cos B D.

Case 8. On the same suppositions, to find the angle C, contained between the given sides A C and B C.

R : cos A C = tang A : cot A C D.

and,

tang B C : tang A C = cos A C D : B C D.

Whence we deduce A C B = A C D ± B C D, according as the sides A C and B C are of the same or of different species.

The supplemental triangle is equally proper to resolve this problem by reducing it to the third case.

Case 9. Given the two sides A C and A B, and the angle A, contained between them, to find the other side B C.

R : cos A = tang A C : tang A D

cos A D : cos B D = cos A C : cos B C.

Case 10. The same things being given, to find one of the other two angles, the angle B, for example.

R : cos A = tang A C : tang A D

sin B D : sin A D = tang A : tang B.

Case 11. If the three sides were given, how should we find one of the three angles, A for example?

We know that

$$\text{tang}\,\frac{A+B}{2} : \text{tang}\,\frac{AC+BC}{2} = \text{tang}\,\frac{AC-BC}{2} : \text{tang}\,\frac{AD-DB}{2}$$

This first proportion will give us the segment A D, and the following one will give the angle A.

tang A C : tang A D = R : cos A.

This last case may be resolved by one proportion, which may be investigated in the following manner, and which will serve to exemplify the use of the trigonometrical formulas.

In the oblique-angled spherical triangle A B C fig. 10, call the side B C, *a*; A C, *b*; and the side A B, *c*.

Imagine O to be the centre of the sphere, and from the point A, draw A P, A Q, tangents to the arcs A B, A C. Then by Euclid, def. 6, Book XI, the angle Q A P is equal to the spherical angle A, and the angle at O is measured by the arc B C. Draw O Q and O P, then will O A be the secant of the arc A C, and O P that of the arc A B.

We have in general

$$\cos A = \frac{AQ^2 + AP^2 - PQ^2}{2\,AP \times AQ} \text{ or,}$$

$$\cos A = \frac{\text{tang}^2 b + \text{tang}^2 c - PQ^2}{2\,\text{tang}\,b\,\text{tang}\,c}$$

whence

$PQ^2 = \text{tang}^2 b + \text{tang}\,c - 2\,\text{tang}\,b\,\text{tang}\,c\cos A$.

For similar reasons,

$PQ^2 = \sec^2 b + \sec^2 c = 2\sec b\sec c\cos a$, and subtracting the preceding equation from this latter one, we have $0 = 1 + 1 + 2\,\text{tang}\,b\,\text{tang}\,c\cos A - 2\sec b\sec c\cos a$, because $\sec^2 - \text{tang}^2 = R^2 = 1$. Whence by transposition,

$$\cos A = \frac{\cos a\sec b\sec c - 1}{\text{tang}\,b\,\text{tang}\,c} = \frac{\dfrac{\cos a}{\cos b\cos c} - 1}{\text{tang}\,b\,\text{tang}\,c}$$

$$= \frac{\cos a - \cos b\cos c}{\cos b\cos c\,\text{tang}\,b\,\text{tang}\,c}$$

(because $\sec b = \frac{1}{\cos b}$).

Again, since $\cos b\,\text{tang}\,b = \sin b$, the expression becomes

$$\cos A = \frac{\cos a - \cos b\cos c}{\sin b\sin c}$$

Consequently $1 + \cos A =$

$$\frac{\cos a - (\cos b\cos c - \sin b\sin c)}{\sin b\sin c}$$

$$= \frac{\cos a - \cos(b+c)}{\sin b\cos c} \text{ by (11)}$$

But in general (17) we have $\cos Q - \cos P = 2\sin\frac{P+Q}{2} \times \sin\frac{P-Q}{2}$ and therefore $1 + \cos A =$

$$\frac{2\sin\left(\frac{a+b+c}{2}\right) \times \sin\left(\frac{b+c-a}{2}\right)}{\sin b\sin c}$$

$$= \frac{2\sin S.\sin(S-a)}{\sin b\sin c}$$

by taking $S = \frac{a+b+c}{2}$.

But we have shown that $1 + \cos A = 2\cos\frac{{}^2A}{2}$. Therefore

$\cos\frac{{}^2A}{2} = \frac{\sin S.\sin(S-a)}{\sin b\sin c}$ and

$\cos\frac{A}{2} = \sqrt{\frac{\sin S.\sin(S-a)}{\sin b\sin c}}$ if radius $= 1$

or, $\cos\frac{A}{2} = \sqrt{\frac{R^2.\sin S.\sin(S-a)}{\sin b\sin c}}$

if radius $=$ R

Case 12. Given the three angles, to find a side, A C fig. 11, for example. This case resolved by the supplemental triangle would not differ from the preceding problem. But it may also be resolved by the two following proportions, if we suppose that the arc C E bisects the angle A C B. For we have first

$\cot\frac{A+B}{2} : \text{tang}\,\frac{B-A}{2} = \text{tang}\,\frac{1}{2}$ C : tang D C B

and then,

tang A : cot A C D = R : cos A C.

The first proportion gives the value of the angle D C E, which being added to the angle A C E, will shew the angle A C D. And this once known, we directly obtain the required side A C, by the second proportion.

TRILATERAL, *three-sided*, a term applied to any figure of three sides.

TRILLION, in *Arithmetic*, according to the English notation, is one million billions; but according to the French it is a thousand billions, their billion itself being a thousand millions; so that with English arithmeticians a trillion occupies the 19th place of figures, but with the French only the 13th. See BILLION.

TRINOMIAL, in *Algebra*, any quantity of three dimensions, as $x^2 + xy + az$; or $a + b - c$, &c.

TRIPLICATE *Ratio*, the ratio of the cubes of two quantities; thus the ratio of a^3 to b^3 is triplicate of the ratio of a to b.

TRISECTION, as the name imports, is the division of any thing into three equal parts, as a line, angle, &c.

TRISECTION of *Angle*, is a problem which has engaged the attention of mathematicians from the earliest period of authentic history; that is, the trisection of an angle geometrically, or with the ruler and compasses only, being in point of difficulty, or rather, perhaps, of impossibility, on a footing with the other two celebrated ancient problems, *viz.* the duplication of the cube, and the quadrature of the circle. Analytically the problem involves no difficulty, for since the

$$\sin. 3a = 3 \sin. a - 4 \sin.^3 a,$$

we have

$$\sin.^3 a - \tfrac{3}{4} \sin. a = -\tfrac{1}{4} \sin. 3a$$

whence by Cardan's formula

$$\sin. a = \tfrac{1}{2} \sqrt[3]{\left\{-\sin. 3a + \sqrt{(\sin.^2 3a - 1)}\right\}}$$
$$\tfrac{1}{2} \sqrt[3]{\left\{-\sin. 3a - \sqrt{(\sin.^2 3a - 1)}\right\}}$$

TROCHOID, the same as cycloid.

TROPICS, in *Astronomy* and *Geography*, are two circles supposed to be drawn on each side of the equinoctial and parallel thereto. That on the north side of the line is called the tropic of Cancer, and the southern tropic has the name of Capricorn, as passing through the beginning of those signs. These are distant from the equinoctial 23° 27′ 46″, the measure of the obliquity of the ecliptic.

TROY *Weight*, an English weight, used in buying and selling gold, silver, and other articles of jewellery.

TRUMPET, the name of an instrument used either for the purpose of being heard at a great distance, or to apply to the ear, for increasing sound, and rendering it distinct, the former being called a *speaking* and the latter a *hearing* trumpet.

Speaking **TRUMPET**, is a tube of considerable length, *viz.* from six feet to 12, and even more, used for speaking with, to make the voice heard to a greater distance. In a trumpet of this kind the sound in one direction is supposed to be increased, not so much by its being prevented from spreading all round, as by the reflection from the sides of the trumpet. The figure best suited for a speaking trumpet is that which is generated by the rotation of a parabola, about a line parallel to the axis.

Hearing **TRUMPET**, is an instrument to assist the hearing of persons who are deaf. Instruments of this kind are formed of tubes, with a wide mouth, and terminating in a small canal, which is applied to the ear.

TRUNCATED *Pyramid* or *Cone*, is used to denote the bottom part of either of those solids when cut by a plane passing parallel to its base.

TUBE, a hollow cylinder.

TWILIGHT, in *Astronomy*, is that faint light which is perceived before the sun-rising, and after sun-setting. The twilight is occasioned by the earth's atmosphere refracting the rays of the sun, and reflecting them among its particles. On a medium, about 18° of the sun's depression will serve tolerably well for our latitude, for the beginning and end of twilight, and according to which it is easy to compute the duration of twilight for any latitude and declination.

To find the time of shortest twilight at any given place, say as radius to the sine of the latitude, so is the tangent of 9° to the sine of the sun's declination at the time required. The declination of the sun and the latitude of the place must be of contrary kinds.

TWINKLING *of the Stars*, denotes that tremulous motion which is observed in the light proceeding from the fixed stars.

V

VACUUM, in *Physics*, a space empty or devoid of all matter.

Boileanum VACUUM, is used to express that approach towards a real vacuum, which we arrive at by means of the air-pump. Hence, any thing put in a receiver so exhausted, is said to be *in vacuo;* and the experiments thus carried on are said to be performed *in vacuo*, or in *vacuo Boileano.*

Torricellian VACUUM, is that made in the barometer-tube, between the upper end and the top of the mercury. This is, perhaps, never a perfect and entire vacuum; as all fluids are found to yield or to rise in elastic vapours, on the removal of the pressure of the atmosphere.

VALVE, in *Hydraulics*, *Pneumatics*, &c. is a kind of lid or cover to a tube, vessel, or orifice, contrived to open one way; but which, the more forcibly it is pressed the other way, the closer it shuts the aperture, like the clapper of a pair of bellows: so that it either admits the entrance of a fluid into the tube or vessel, and prevents its return, or permits it to escape, and prevents its re-entrance.

VANES, in *Mathematical* and *Philosophical Instruments*, are sights made to slide and move upon cross staves, fore staves, quadrants, &c.

VANISHING *Fractions*, are fractions in which, by giving a certain value to the variable quantity or quantities which enter into them, both numerator and denominator become zero, and the fraction itself $\frac{0}{0}$.

We have the following rule for finding the value of vanishing fractions.

1. If both the terms of the given fraction be rational, divide each of them by their greatest common measure; then, if the hypothesis which is found to reduce the original expression to the form $\frac{0}{0}$ be applied to the result, it will give the true value of the fraction in the state under consideration.

2. When any part of the fraction is irrational, observe what the unknown quantity is equal to, when the numerator and denominator both vanish, and put it equal to that quantity $+$ and $-i$; then if this be substituted for the unknown quantity, and the roots of the surds be extracted to a sufficient number of places, the result, when i is put equal to 0, will give the true value of the fraction.

From which rule the author obtains the following results:

1. $\frac{x^2-a^2}{x-a}=2a$, when $x=a$
2. $\frac{x-x^5}{1-x}=4$, when $x=1$
3. $\frac{b(x-\sqrt{ax})}{x-a}=\frac{1}{2}b$, when $x=a$
4. $\frac{x^m-a^m}{x-a}\ ma^{m-1}$, when $x=a$

&c. &c.

VAPOUR, in *Meteorology*, a thin humid matter, which being rarified to a certain degree by the action of heat, ascends to a height in the atmosphere, where it is suspended until it returns in the form of dew, rain, snow, &c.

VARIABLE *Quantities*, denote such as are either continually increasing or diminishing; in opposition to those which are constant, remaining always the same.

Ratio of VARIABLE *Quantities.*—In the investigation of the relation which varying and dependent quantities bear to each other, the conclusions are more readily obtained by expressing only two terms in each proportion than by retaining the four.

1. One quantity is said to *vary directly* as another, when their magnitudes depend wholly upon each other, and in such a manner, that if one be changed, the other is changed in the same proportion. Let A and B be mutually dependent upon each other in such a way, that if A be changed to any other value a, B must be changed to another value b, such that $A : a = B : b$, then A is said to vary directly as B, which is denoted by the symbol $\propto$ placed between the

two quantities. Thus, for example, while the altitude of a triangle remains constant, the area *varies directly* as the base or the area $\propto$ base. For if the base be increased or diminished, the area is increased or diminished in the same proportion.

2. One quantity is said to *vary inversely*, as another, when one cannot be changed in any manner, but the reciprocal of the other is changed in the same proportion. A varies inversely as B, or $A \propto \frac{1}{B}$ if when A is changed to a, B be changed to b, in such a manner that $A : a = \frac{1}{B} : \frac{1}{b}$ or $A : a = b : B$.

For example; if the area of a triangle be given, the base varies inversely as the perpendicular altitude.

Let A and a represent the altitudes, B and b, the bases of two triangles equal in area; then $\frac{A \times B}{2} = \frac{a \times b}{2}$ or $A \times B = a \times b$; therefore, $A : a = b : B$ or $A : a = \frac{1}{B} : \frac{1}{b}$.

3. One quantity is said to *vary* as *two others jointly*, if when the former is changed in any manner, the product of the other two are changed in the same proportion. That is, A varies as B and C jointly, or $A \propto BC$, when A cannot be changed to a, but the product BC must be changed to bc, such that $A : a = BC : bc$. The area of a triangle, for example, varies as its base and perpendicular altitude jointly. For let A, B, P represent the area, base, and perpendicular of one triangle; a, b, p the same in another, then $BP = 2A$ in the first, and $bp = 2a$ in the second, therefore, $\frac{A}{a} = \frac{BP}{bp}$, or

$$A : a = BP : bp.$$

4. One quantity is said to vary *directly* as a second, and *inversely* as a third, when the first cannot be changed in any manner, but the second multiplied by the reciprocal of the third, is changed in the same proportion. That is, $A \propto \frac{B}{C}$ when $A : a = \frac{B}{C} : \frac{b}{c}$, A, B, C; a, b, c, being corresponding values of the three quantities.

For example, the base of a triangle varies as the area directly, and the perpendicular altitude inversely. For, as in the preceding example, $\frac{BP}{bp} = \frac{A}{a}$, or multiplying both sides by $\frac{p}{P}$ we have $\frac{B}{b} = \frac{pA}{Pa}$, whence

$$B : b = \frac{A}{P} : \frac{a}{p}.$$

The following are some of the principal propositions relating to the ratio of variable quantities.

If $A \propto B$, and $B \propto C$, then $A \propto C$

If $A \propto B$, and $B \propto \frac{1}{C}$, then $A \propto \frac{1}{C}$

If $A \propto B$, and $B \propto C$, then $A \pm B \propto \sqrt{AB} \propto C$

If $A \propto B$, and m is any given number, then $A \propto mB$

If $A \propto B$; then $A^n \propto B^n$, or $A^{\frac{1}{n}} \propto B^{\frac{1}{n}}$

If $A \propto B$, and $M \propto m$; then $AM \propto am$

If $A \propto BC$; then $B \propto \frac{A}{C}$ and $C \propto \frac{A}{B}$

If AB be constant; then $A \propto \frac{1}{B}$ and $B \propto \frac{1}{A}$

If $A \propto B$ and B and $C \propto D$; then $AC \propto BD$. Wood's Algebra.

VARIATION of *Curvature*, is used for that inequality or change which takes place in the curvature of all curves except the circle, by which their curvature is more or less in different parts of them. And this variation constitutes the quality of the curvature of any line.

From the definition may be derived practical rules for the variation of curvature, as follows:

1. Find the radius of curvature, or rather its fluxion; then divide

this fluxion by the fluxion of the curve, and the quotient will give the variation of curvature; exterminating the fluxions when necessary, by the equation of the curve, or by expressing their ratio by help of the tangent, or ordinate, or subnormal, &c.

2. Since $\frac{\dot z^3}{-\dot x \ddot y}$, or $\frac{\dot z^3}{-\ddot y}$ (putting $\dot x = 1$) denotes the radius of curvature of any curve z, whose absciss is x, and ordinate y; if the fluxion of this is divided by $\dot z$, and $\dot z$ and $\ddot z$ are exterminated, the general value of the variation will come out $\frac{-3 \dot v \ddot y^2 + \dddot y (1 + \dot y^2):}{\ddot y^2}$ then, substituting the values of $\dot y$, $\ddot y$, $\dddot y$, (found from the equation of the curve) into this quantity, it will give the variation sought.

Variation, or *Declination* of the *Magnetic Needle*, is the distance of the magnetic from the true meridian in degrees measured upon the horizon. Inclination is the angle which the dipping-needle makes with the horizon, as measured on a vertical or azimuth circle. The changes in the declination and inclination may be represented by the words *variation* and *alteration* respectively.

After the discovery of that most useful property of the magnet, or loadstone; namely, the giving hardened iron and steel a polarity, the compass was for many years used without knowing that its direction in any wise deviated from the poles of the world; and about the middle of the sixteenth century, so certain were some of its inflexibly pointing to the north, that they treated with contempt the notion of the variation, which about that time began to be suspected. However, careful observations soon discovered, that in England and its neighbourhood, the needle pointed to the eastward of the true north; but the quantity of this deviation being known, mariners became as well satisfied as if the compass had none, because they imagined that the true course could be obtained by making allowance for the true declination.

From successive observations made afterwards, it was found that the deviation of the needle from the north was not a constant quantity, but that it gradually diminished; and, at last, about the year 1658 or 1660, it was found at London that the needle pointed due-north, and has ever since been getting to the westward; and now the declination is more than 24° to the westward of the north; so that in any one place it may be suspected the declination has a kind of libratory motion, traversing through the north to unknown limits eastward and westward. But the settling of this point must be left to time.

During the time of the said observations, it was also discovered that the declination of the needle was different in different parts of the world, it being west in some places when it was east in others; and in places where the declination was of the same name, yet the quantity of it greatly differed. It was therefore found necessary, that mariners should every day, or as often as they had opportunity, make, during their voyage, proper observations for an amplitude or azimuth; whereby they might be enabled to find the declination of the compass in their present place, and thence correct their courses.

M. Euler first examines the case wherein the two magnetic poles are diametrically opposite; second, he places them in the two opposite meridians, but at unequal distances from the poles of the world; third, he places them in the same meridians. Finally, he considers them situated in two different meridians. These four cases may become equally important; because, if it is determined that there are only two magnetic poles, and that these poles change their situations, it may some time hereafter be discovered that they pass through all the different positions.

Since the needle of the compass ought always to be in the plane which passes through the place of observation and two magnetic poles, the problem is reduced to the discovery of the angle contained between this plane and the plane of the meridian. M. Euler, after having examined the different cases, finds that they also express the earth's magnetism, represented in the chart published by Messrs. Mountain and Dodson in 1774, particularly throughout Europe and North America, if the following principles are established.

Between the arctic pole and the magnetic pole 14° 53′.

Between the antarctic pole and the other magnetic pole 29° 23′.

53° 18′ the angle at the north-pole, formed by the meridian's passing through the two magnetic poles.

250° the longitude of the meridian, which passes over the northern magnetic pole.

As the observations which have been collected with regard to the variation are for the most part loose and inaccurate, it is impossible to represent them all with precision; and the great variations observed in the Indian ocean seem to require, says M. Euler, that the three first quantities should be 14, 35, and 65 degrees.

Calculus of VARIATIONS, is a particular calculus, by which having given an expression, or function, of two or more quantities of which the ratio is expressed by a determinate law, we find what this function must become, when we suppose this law itself to experience an infinitely small variation, occasioned by the variation of one or more of the quantities by which it is expressed.

This calculus offers the only general and frequently the only possible means of solving those problems generally termed isoperimetrical.

Here every problem requires us ultimately to find the maximum or minimum of a formula, such as $\int Z\,dx$, where Z is a function of x with constant quantities; of x and y, or of x, y, and z, or of a greater number of variables; or Z may even contain certain integrals, as $\int V$, or integrals of integrals, as $\int V \int v$, &c. the general solution of which, as we observed above, we owe exclusively to Lagrange, although certain cases had been previously considered, and some formulæ of solution already established by Euler, in his celebrated work, "Methodus inveniendi curvas, &c." For a more circumstantial account of the successive improvements of this calculus, as well as for an illustration of its principles, and application to various problems, we refer the reader to Woodhouse's "Treatise on Isoperimetrical Problems, and the Calculus of Variations," 8vo. 1810. See also Lagrange's "Théorié des Fonctions Analytiques," and the "Leçons sur la Calcul des Fonctions," of the same author.

VAULT, in *Architecture*, an arched roof, so constructed that the stones which form it sustain each other.

VELOCITY, or SWIFTNESS, or CELERITY, in *Mechanics*, is that affection of motion, by which a moving body passes a certain space in a certain time. It is always proportional to the space passed over in a given time when the velocity is uniform, or constant during that time.

VELOCITY, is either uniform or variable. Uniform, or equal velocity, is that with which a body passes always over equal spaces in equal times. And it is variable, or unequal, when the spaces passed over in equal times are unequal; in which case it is either accelerated or retarded velocity; and this acceleration, or retardation, may also be equal or unequal, *i. e.* uniform or variable, &c. See ACCELERATION and MOTION.

VELOCITY is also either absolute or relative. Absolute velocity is that we have hitherto been considering, in which the velocity of a body is considered simply in itself, or as passing over a certain space in a certain time. But relative or respective velocity is that with which bodies approach to, or recede from one another, whether they both move, or one of them be at rest. Thus, if one body move with the absolute velocity of two feet per second, and another with that of six feet per second; then if they move directly towards each other, the relative velocity with which they approach is that of eight feet per second; but if they move both the same way, so that the latter overtake the former, then the relative velocity with which that overtakes it, is only that of four feet per second, or only half of the former; and consequently it will take double the time of the former before they come in contact together.

Initial VELOCITY, the velocity with which a body begins to move.

Vertual VELOCITY of a point solicited by any force, is the element of the space which it would describe in the direction of the power, when the system is supposed to have undergone an indefinitely small derangement.

Principle of Vertual VELOCITIES, in *Mechanics*, is much employed by the foreign mathematicians, and is thus enunciated: if any system whatever is solicited by powers in equilibrio, and there be given to this system any small motion, in virtue of which every point describes an indefinitely small space, the sum of the products of each power, multiplied by the space that the point where it is applied would describe, according to the direction of the same power, will be always equal to zero; regarding as positive the small spaces described in the sense of the powers; and as negative, those described in the opposite sense.

This principle is due to Galileo. For its history and demonstration, see Mechanique de La Grange, p. 8.

VENTILATOR, a machine by which the noxious air of any close place may be discharged and changed for fresh air.

VENUS, the second planet of our system, in order from the sun, revolving between the orbits of Mercury and the earth, her mean distance being ·723, that of the earth being considered as unity; whence her distance in English miles is about 68 millions. She performs her sidereal revolution in 224 days, 16 hours, 49′ 11″ ·2, and her mean synodical revolution in about 584 days.

The eccentricity of her orbit is ·0069; half the axis major being considered as unity, which is the least eccentricity of all the planets.

Her mean longitude, at the commencement of the present century, was in 0^s 10° 44′ 35″, and the longitude of her *perihelion* was, at the same time, in 4^s 8° 37′ 0″,9. The line of her apsides has a sidereal motion, contrary to the order of the signs, of 4′ 27″,8 in a century. But, if referred to the ecliptic, this motion will appear (on account of the precession of the equinoxes) to proceed according to the order of the signs at the rate of 47″, 4 in a year, or 1° 19′ 2″,2 in a century. Her orbit is *inclined* to the plane of the ecliptic in an angle of 3° 23′ 32″,7; (which angle decreases about 4″,6 in a century;) and at the commencement of the present century it crossed the ecliptic in 2^s 14° 52′ 33″,9. But the *nodes* have an apparent motion in longitude of 31″,4 in a year, or 52′ 20″,2 in a century.

The *rotation* on her axis is performed in 23^h 21′ 7″,2; but the inclination of her *axis* is not known.

The *diameter* of Venus is 7702 miles: consequently, she is nearly as large as the earth; and her *mass*, compared with that of the sun considered as unity, is $\frac{1}{356632}$, her density being $5\frac{11}{13}$, that of water being 1.

Venus is surrounded by an *atmosphere*, the refractive powers of which differ very little from those of our atmosphere.

As viewed from the earth, this is the most brilliant of all the planets; and may sometimes be seen with the naked eye at noon day. She is known as the morning and evening star; and never recedes far from the sun. Her *elongation*, or angular distance, varies from 45° to 48°.

Her motions sometimes appears *retrograde*. The mean arc, which she describes in such case, is about 16° 12′; and her mean duration is about 42 days. This retrogradation commences, or finishes, when she is about 28° 48′ distant from the sun.

Venus changes her *phases*, like the moon, according to her position with respect to the sun and the earth; which causes a very considerable difference in her brilliancy.

Her mean *apparent diameter* is $27'',0$; her greatest apparent diameter is about $57'',3$.

VERNIER, is a scale, or a division, well adapted for the graduation of mathematical instruments, so called from its inventor Peter Vernier, who communicated the discovery to the world.

Vernier's method is derived from the following principle. If two equal right lines, or circular arcs, A, B, are so divided, that the number of equal divisions in B is one less than the number of equal divisions of A, then will the excess of one division of B above one division of A be compounded of the ratios of one of A to A, and of one of B to B.

For let A contain 11 parts, then one of A to A is as 1 to 11, or $\frac{1}{11}$. Let B contain 10 parts, then one of B to B is as 1 to 10, or $\frac{1}{10}$. Now

$$\frac{1}{10} - \frac{1}{11} = \frac{11-10}{10 \times 11} = \frac{1}{10 \times 11} = \frac{1}{10} \times \frac{1}{11}.$$

Or if B contains n parts, and A contains $n+1$ parts; then $\frac{1}{n}$ is one part of B, and $\frac{1}{n+1}$ is one part of A. And

$$\frac{1}{n} - \frac{1}{n+1} = \frac{(n+1)-n}{n \times \overline{n+1}} = \frac{1}{n} \times \frac{1}{n+1}.$$

The most commodious divisions, and their aliquot parts, into which the degrees on the circular limb of an instrument may be supposed to be divided, depend on the radius of that instrument.

Let R be the radius of a circle in inches, and a degree to be divided into n parts, each being $\frac{1}{p}$th part of an inch.

Now the circumference of a circle, in parts of its diameter 2 R inches, is $3,1415926 \times 2\,R$ inches.

Then $360° : 3,1415926 \times 2\,R : 1° : \frac{3,1415926}{360} \times 2\,R$ inches.

Or, $0,01745329 \times R$ is the length of one degree in inches.

Or, $0,01745329 \times R \times p$ is the length of 1°, in pth parts of an inch.

But as every degree contains n times such parts, therefore $n = 0,01745329 \times R \times p$.

The most commodious perceptible division is $\frac{1}{8}$ or $\frac{1}{10}$ of an inch.

VERSED *Sine* of an *Arc*, in *Trigonometry*, is the part of the diameter intercepted between the sine and the commencement of the arc.

VERTEX of an *Angle*, in *Geometry*, is the angular point.

Vertex of a *Figure*, is the uppermost point.

Vertex of a *Curve*, is the extremity of the axis or diameter.

Vertex of a *Glass*, in *Optics*, the same as its pole.

Vertex is also used in astronomy for the point of the heavens vertically or perpendicularly over our heads, also called the zenith.

VERTICAL, something relating to the vertex or highest point. As,

Vertical *Angles*, in *Geometry*, are such as have their legs or sides continuations of each other.

Vertical *Point*, in *Astronomy*, is the same with vertex, or zenith. —Hence a star is said to be vertical, when it happens to be in that point which is perpendicularly over any place.

Vertical *Circle*, is a great circle of the sphere, passing through the zenith and nadir of a place. The vertical circles are also called aziumphs. The meridian of any place is a vertical circle, *viz.* that particular one which passes through the north or south point of the horizon. All the vertical circles intersect one another in the zenith and nadir.

The use of the vertical circles is to estimate or measure the height of the stars, &c. and their distances from the zenith, which is reckoned on these circles; and to find their eastern and western amplitude, by observing how many degrees the vertical, in which the star rises or sets, is distant from the meridian.

Prime **Vertical**, is that vertical circle, or azimuth, which passes through the poles of the meridian; or which is perpendicular to the meridian, and passes through the equinoctial points.

Vertical of the *Sun*, is the vertical which passes through the centre of the sun at any moment of time. Its use is, in dialling, to find the declination of the plane on which the dial is to be drawn, which is done by observing how many degrees that vertical is distant from the meridian, after marking the point or line of the shadow upon the plane at any times.

Vertical *Line*, in *Dialling*, is a line in any plane perpendicular to the horizon. This is best found and drawn on an erect and reclining plane, by steadily holding up a string and plummet, and then marking two points of the shadow of the thread on the plane, a good distance from one another, and drawing a line through these marks.

Vertical *Line*, in *Conics*, is a line drawn on the vertical plane, and through the vertex of the cone.

Vertical *Plane*, in *Conics*, is a plane passing through the vertex of a cone, and parallel to any conic section.

VERTICITY, that property of the magnet or of a needle touched with it, by which it directs itself to some particular point as its pole.

VESTA, one of the four new planets discovered since the commencement of the present century. From their regularity observed in the distances of the old planets from the sun, some astronomers supposed that a planet existed between the orbits of Jupiter and Mars. The discovery of Ceres confirmed this happy conjecture; but the opinion which it seemed to establish respecting the harmony of the solar system, appeared to be completely overturned by the discovery of Pallas and Juno. Dr. Olbers, however, imagined that these small celestial bodies were merely the fragments of a large planet, which had been burst asunder by some internal convulsion, and that several more might yet be discovered between the orbits of Mars and Jupiter. He therefore concluded, that though the orbits of all these fragments might be differently inclined to the ecliptic, yet, as they must have all diverged from the same point, they ought to have two common points of union, or two nodes in opposite regions of the heavens through which all the planetary fragments must sooner or later pass. One of these nodes Dr. Olbers found to be in Virgo, and the other in the Whale, and it was actually in the latter of these regions that Mr. Harding discovered the planet Juno. With the intention, therefore, of detecting other fragments of the supposed planet, Dr. Olbers examined thrice every year, all the little stars in the opposite constellations of the Virgin and the Whale, till his labours were crowned with success on the 29th March, 1807, by the discovery of a new planet in the constellation Virgo, to which he gâve the name of Vesta.

As soon as this discovery was made known in England, the planet was observed at Blackheath, on the 26th April, 1807, by S. Groombridge, Esq. an ingenious and active astronomer, who has successfully devoted his leisure and his fortune to the advancement of astronomy. He continued to observe it with his excellent astronomical circle, till the 20th May, when from its having ceased to become visible on the meridian, he had recourse to equatorial instruments. On the 11th August, Mr. Groombridge resumed his meridional observations, from which he has computed part of the elements of its orbit; and he had the good fortune to observe the ecliptic opposition of the planet, on the 8th September, 1808, at $7^h\ 30'$, in longitude $11^s\ 15°\ 51'\ 26''$. His observations were continued till the beginning of November, 1808, and he expected to have found the planet again at its opposition in February, 1810;

but from a continuance of cloudy weather, and probably from errors in the elements, he did not succeed.

The planet Vesta is of the fifth or sixth magnitude, and may be seen in a clear evening by the naked eye. Its light is more intense, pure, and white, than any of the other three; and it is very similar in its appearance to the Georgium Sidus. It is not surrounded with any nebulosity; and even with a power of 636, Dr. Herschel could not perceive its real disc. The orbit of Vesta cuts the orbit of Pallas, but not in the same place where it is cut by that of Ceres.

According to the observations of Schroeter, the apparent diameter of Vesta is only 0.488 of a second, one half of what he found to be the apparent diameter of the fourth satellite of Saturn; and yet it is very remarkable that its light is so intense, Schroeter saw it several times with his naked eye.

M. Burckardt is of opinion, that Le Monnier had observed this planet as a fixed star, since a small star situated in the same place, and noticed by that astronomer, has since disappeared.

The following are the elements of the orbit of Vesta, computed by Mr. Groombridge, from his own observations.

Revolution • • • •	$3^y\ 66^d\ 4^h$
Place of perihelion	$0^s\ 3^\circ\ 0'\ 0''$
Place of ascending node • • •	$3^s\ 14^\circ\ 38'\ 0''$
Inclination of orbit	$7^\circ\ 8'\ 20''$
Mean distance • •	$2'\ 163$
Eccentricity in parts of the earth's radius	0.0953

The following elements are given by Burckardt in the Connoisance de Temps for 1809, from the most recent observations on the continent.

Place of ascending node • • • • • •	$3^s\ 13^\circ\ 1'\ 0''$
Place of perihelion	$8^s\ 0^\circ\ 42'\ 53''$
Inclination of orbit	$7^\circ\ 8'\ 46''$
Mean distance • • •	2.373000
Eccentricity • • • •	0.093221

VIBRATION, in *Mechanics*, the regular reciprocating motion of a body, as a pendulum, musical chord, &c.

Vibration is also a term used by some authors to explain certain natural phenomena. Sensation, for instance, is supposed to be performed by means of the vibratory motions of the contents of the nerves, begun by external objects, and thence propagated to the brain.

This doctrine has been particularly illustrated by Dr. Hartly, who has extended it farther than any other writer, in establishing a theory of our mental operations.

The same ingenious author also applies the doctrine of vibrations to the explanation of muscular motion, which he thinks is performed in the same general manner as sensation, and the perception of ideas.

The several sorts and rays of light, Newton conceives to make vibrations of divers magnitudes, which, according to those magnitudes, excite sensations of several colours; much after the same manner of air, according to their several magnitude, excite sensations of several sounds.

Heat, according to the same author, is only an accident of light occasioned by the rays putting a fine, subtile, ethereal medium, which pervades all bodies, into a vibrative motion, which gives us that sensation.

From the vibrations or pulses of the same medium, he accounts for the alternate fits of easy reflexion, and easy transmission of the rays.

VINCULUM, in *Algebra*, a mark or character either drawn over, or including, or some other way accompanying a factor, divisor, dividend, &c. when it is compounded of several letters, quantities, or terms, to connect them together as one quantity, and show that they are to be multiplied, or divided, &c. together.

Vieta first used the bar or line over the quantities, for a vinculum, thus $\overline{a+b}$; and Albert Girard the parenthesis thus $\overline{(a+b)}$. Thus $\overline{a+b} \times c$, or $(a+b) \times c$, denotes the product of c, and the sum $a+b$ considered as one quantity. Also $\sqrt{a+b}$, or $\surd(a+b)$, denotes the square root of the sum $a+b$. Sometimes the mark : is set before a compound factor, as a vinculum,

especially when it is very long, or an infinite series; thus $3a \times 1 - 2x + 3x^2 - 4x^3 + 5x^5$, &c. though the more usual method adopted by modern algebraists is to include the expression, when long, in a parenthesis; thus
$3a\,(1 - 2x + 3x^2 - 4x^3 + 5x^5$, &c.$)$
or if it should happen, as it frequently may, that the quantity within the parenthesis consist itself of other parenthesis, then the outward one is made larger, or thus, $\Big\{ \qquad\qquad \Big\}$ in order to distinguish the whole factor from any of its parts; thus

$$3a\left\{1 + 3x\,(a+b) - 4x^2(a+b)^2 + \text{\&c.}\right\}$$

VIS, a Latin word, signifying force or power; adopted by writers on physics, to express divers kinds of natural powers or faculties.

The term vis is either active or passive: the *vis activa* is the power of producing motion; the *vis passiva* is that of receiving or losing it. The *vis activa* is again subdivided into *vis viva* and *vis mortua*.

Vis *Absoluta*, or *Absolute Force*, is that kind of centripetal force which is measured by the motion that would be generated by it in a given body, at a given distance, and depends on the efficacy of the cause producing it.

Vis *Acceleratrix*, or *Accelerating Force*, is that centripetal force which produces an accelerated motion, and is proportional to the velocity which it generates in a given time; or it is as the motive or absolute force directly, and as the quantity of matter moved inversely.

Vis *Impressa*, is defined by Newton to be the action exercised on any body to change its state, either of rest, or moving uniformly in a right line.

This force consists altogether in the action; and has no place in the body after the action is ceased: for the body perseveres in every new state by the vis inertiæ alone.

This vis impressa may arise from various causes; as from percussion, pression, and centripetal force.

Vis *Inertiæ*, or *Power of Inactivity*, is defined by Newton to be a power implanted in all matter, by which it resists any change endeavoured to be made in its state, that is, by which it becomes difficult to alter its state, either of rest or motion.

Vis *Motrix*, or *Moving Force* of a centripetal body, is the tendency of the whole body towards the centre, resulting from the tendency of all the parts, and is proportional to the motion which it generates in a given time; so that the vis motrix is to the vis acceleratrix as the motion is to the celerity; and as the quantity of motion in a body is estimated by the product of the velocity into the quantity of matter, so the vis motrix, from the vis acceleratrix, multiplied into the quantity of matter.

The followers of Leibnitz use the term vis motrix for the force of a body in motion in the same sense as the Newtonians use vis inertiæ; this latter they allow to be inherent in a body at rest, but the former of vis motrix a force inherent in the same body only while in motion, which actually carries it from place to place, by acting upon it always with the same intensity in every physical part of the line which it describes.

Vis *Motua*, or *Dead Force*, a term used by Leibnitz to denote the power of pressure in a body at rest; whereas

Vis Viva, or *Living Force*, is used by the same author to denote the force or power of a body in motion.

VISIBLE, any thing that is an object of sight or vision, or any thing whereby the eye is affected, so as to produce the sensation of sight.

VISION, the perception of external objects as conveyed to the brain by means of the organs of sight.

Take an ox's eye while it is fresh, and having cut off the three coats from the back part, quite to the vitreous humour, put a piece of white paper over that part and hold the eye towards any bright object, and you will see an inverted image of the object upon the paper.

By what change in the conformation of the eye, we are enabled to see objects distinctly at different distances, is not fully ascertained; but the effect itself is sufficiently manifest, although authors differ as to the manner in which it is produced. It is supposed by some, that the general figure of the eye is altered; that when the object to be viewed is near, the length of the eye measured along the axis, is increased by the lateral pressure of external muscles; and on the contrary, when the object is remote, that the length of the eye is diminished, by the relaxation of that pressure. Others suppose the effect to be produced by a change in the place, or figure of the crystalline humour; some by an alteration in the diameter of the pupil; while others ascribe the effect to a change in the curvature of the cornea.

When the distance of an object is diminished, supposing no alteration to take place in the eye, the divergency of the extreme rays of the pencil incident upon the pupil, is increased; and therefore, if the image of the object in the first situation be formed upon the retina, in the latter it will be formed behind it; but an increase in the curvature of the cornea will increase the convergency of the refracted rays, or bring them sooner to a focus; and thus, by a proper change in this coat of the eye, the rays will again be brought to a focus upon the retina, and the object be still seen distinctly. The least distance at which objects can be seen distinctly by common eyes, is about seven or eight inches. The greater distance cannot be so easily or accurately ascertained. It seems that the generality of eyes are capable of collecting parallel rays upon the retina, or so near to it, as to produce distinct vision; and thus, the greatest distance at which objects can be distinctly viewed, is unlimited. For this reason, in adopting optical instruments to common eyes, and calculating their powers, we suppose the parts to be so arranged, that the rays in each pencil may, when they fall upon the cornea, be parallel.

If the humours of the eye be too convex, parallel rays, and such pencils as diverge from points at any considerable distance, are collected before they reach the retina; and objects to be seen distinctly, must be brought nearer to the eye. This inconvenience may be remedied by a concave glass, whose focal length is so adjusted as to give the rays, proceeding from a distant object, such a degree of divergency as the eye requires.

Direct or *Simple* VISION, is that which is performed by direct rays or by rays passing directly from the object to the eye.

Reflected VISION, is that which is performed by reflected rays, that is, by means of rays reflected from mirrors or specula. See MIRROR and REFLECTION.

Refracted VISION, is that which is performed by means of refracted rays; or rays turned out of their direct course, by passing through glass, or some other medium.

VISUAL, relating to sight or seeing, as visual angle, visual rays, &c.

VISUAL *Angle* is the angle under which an object is seen, or which it subtends at the eye.

VISUAL *Rays*, are rays of light conceived to come from an object to the eye, and by which it becomes visible.

ULTIMATE *Ratios*. Let there be two quantities, one fixed and the other varying, so related to each other, that first the varying quantity continually approaches to the fixed quantity. Secondly, that the varying quantity does never reach or pass beyond that which is fixed. Thirdly, that the varying quantity approaches nearer to the fixed quantity than by any assigned difference, then is such a fixed quantity called the limit of the varying quantity; or in a looser way of speaking, the varying quantity may be said to be ultimately equal to the fixed quantity. These three properties may be otherwise expressed more distinctly, thus: First, the difference between the varying quantity, and the fixed quantity, must continually decrease. Secondly, this difference must never become either nothing or negative. Thirdly, this difference must become less in respect to the fixed quantity than by any assigned ratio; or the difference between the two quantities must become a less part of the fixed quantity than any fractional part that is assigned, how small soever the fraction expressing such part may be. Wherever these properties are found, the fixed quantity is called the limit of the varying quantity, or the varying quantity is said to be ultimately equal to the fixed quantity. This last phrase must not be taken in an absolute literal sense, there being no ultimate state, no particular magnitude, that is the ultimate magnitude of such a varying quantity. Under the word quantity in this definition must be included not only numbers, lines, &c. but more especially ratios considered as a peculiar species of quantity.

UNDECAGON, a polygon of eleven sides.

UNDULATORY *Motion*, is applied to denote a motion in any fluid, by which its parts are agitated like the waves of the sea, and is particularly applied to the motion of the air in the propagation of sound.

VOLCANO, a burning mountain, hollow below, and communicating, perhaps, with cavities still deeper than its own, from which it is supplied with fire, and ignited materials, which it usually throws up, after uncertain intervals, through one or more external apertures or spiracles.

Volcanoes constitute, without doubt, the most striking and formidable geognostic phænomenon which nature has presented to our view. They are not, indeed, so destructive to the lives of the human race as earthquakes; but they offer to the eye something much more terrific. Their number is very considerable, nearly two hundred having been reckoned by different writers. There is an immense range of them running from north to south on the continent of America, and occupying the summits of many of the Andes, as well as of the Mexican and Californian ridges. There is also a considerable number which spread along the eastern coast of Asia, and sprinkle the Indian islands. Iceland alone contains eight volcanoes.

Lunar VOLCANOES. As the moon has on its surface mountains and valleys in common with the earth, some modern astronomers have discovered a still greater similarity, *viz.* that some of these are really volcanoes, emitting fire as those on earth do. An appearance of this kind was discovered some years ago by Don Ulloa, in an eclipse of the sun.

VORTEX, or *Whirlwind*, in *Meteorology*, a sudden, rapid, violent motion of the air, in circular whirling directions.

VORTEX, is also used for an eddy or whirlpool, or a body of water, in certain seas and rivers, which runs rapidly round, forming a sort of cavity in the middle.

VOUSSOIRS, *Vault Stones*, are the stones which form immediately the arch of a bridge, vault, &c. being cut somewhat in the form of a truncated pyramid, their under side forming the intrados, to which their joints or ends should be every where in a perpendicular direction.

The length of the middle voussoir, or key-stone, and which is the least of all, should be about $\frac{1}{15}$ or $\frac{1}{16}$ of the span of the arch, whence these stones should be made to increase in size all the way down to the impost; in order that they may the better sustain the great weight which rests upon them, without being crushed or broken, and that they may also bind the firmer together.

URANUS, *Herschel*, or *Georgium Sidus*, the name of the planet discovered by Dr. Herschel, the 13th of March, 1781.

From certain inequalities in the motion of Jupiter and Saturn, which could not be accounted for from the mutual action of these planets, it was inferred by some astronomers that there existed beyond the orbit of Saturn another planet, by whose action these irregularities were produced, which happy conjecture was confirmed by Dr. Herschel on the day above stated, and which, in compliment to his late majesty George III. he called the *Georgium Sidus*, though on the continent it is more commonly, for the sake of analogy, called *Uranus*, and frequently the *Herschel*. This new planet, which is supposed to have been formerly observed as a small star by Flamstead, Mayer, and Le Monnier, and introduced into their catalogues of the fixed stars, is situated beyond the orbit of Saturn, at the distance of 1800 million miles from the centre of the solar system, and performs its sidereal revolution about the sun in 83 years, 150 days, and 18 hours. Its diameter is about 4½ times larger than that of the earth, being nearly 35112 English miles. When seen from the earth its apparent diameter, or the angle it subtends at the eye, is $3''\,32'''$, and its mean diameter, as seen from the sun, is $4''$. As the distance of this planet from the sun is double that of Saturn, it can scarcely be perceived with the naked eye; when, however, the sky is serene, it appears like a fixed star of the sixth magnitude with a bluish white light, and a brilliancy between that of Venus and the moon, but with a power of 200 or 300 its disc is visible and well defined.

Uranus is attended by six satellites, all of which were discovered by Dr. Herschel. The first is $25''\cdot5$ distant from its primary, and revolves round it in $5^d\,21^h\,25'$. The second satellite is $33''\cdot9$ distant from the planet, and performs its revolution in $8^d\,17^h\,1'\,19''$. The distance of the third satellite is $38''\,57$, and its periodic time $10^d\,23^h\,4'$. The distance of the fourth satellite is $44''\,22$, and the time of its periodic revolution $13^d\,11^h\,5'\,1''\,5$. The distance of the fifth satellite is $1'\,28''\cdot44$, and its revolution is completed in $38^d\,1^h\,49'$. The sixth satellite is placed at the distance of $2'\,56''\cdot38$, from the primary, and will, therefore, require $107^d\,16^h\,40'$ to complete one revolution.

The second and fourth of these satellites were discovered by Dr. Herschel, on the 11th of January, 1787; and the other four in the years 1790 and 1794; but their distances and periodic times have not been so accurately ascertained as the other two. It is a remarkable circumstance with regard to these satellites, that they all move in a retrograde order, and in orbits all lying in the same plane, and almost perpendicular to the ecliptic.

W.

WAGGONER, a popular name given to the Great Bear.

WATER, a transparent fluid, without colour, smell, or taste, in a very slight degree compressible, when pure, not liable to spontaneous change; liquid in the common temperature of our atmosphere, assuming a solid form at 32° Fahrenheit, and a gaseous at 212°, but returning unaltered to its liquid state on resuming any degree of heat between these points; it is capable of dissolving a greater number of natural bodies than any other fluid whatever, and especially those known by the name of the saline; performing the most important functions in the vegetable and animal kingdoms, and entering largely into their composition as a constituent part.

Water thus exists in three different states, in the solid state, or state of ice, in the liquid, and in the state of vapour or steam. It assumes the solid form, as was said, when cooled down to the temperature of 32°, in which state it increases in bulk, and exerts a prodigious expansive force, owing to the new arrangement of its particles, which assume a crystalline form, the crystals crossing each other at an angle of 60° or 120. The specific gravity of ice is thus less than that of water. When ice is exposed to a temperature above 32°, it is converted into a liquid state, or that of water. At the temperature of 42°·5 water is at its maximum of density; and according to some accurate experiments upon water in this state, a French cubic foot of it weighs 70 pounds 223 grains French, which is equal to 529452·9402 troy grains. An English cubic foot, at the same temperature, weighs 437102·4946 grains troy. By professor Robinson's experiments it is ascertained that a cubic foot of water, at the temperature of 55°, weighs 998·74 avoirdupois ounces, of 437·5 grains troy each, or about $1\frac{1}{4}$ ounce less than 1000 ounces avoirdupois, which latter, however, is the usual estimate.

When water is exposed to the temperature of 212°, it boils; and if this temperature be continued, the whole is converted into elastic vapour or steam. In this state it expands to about 1800 times its bulk, when in the state of water, which shows what an astonishing expansive force it must exert when it is confined; and hence its application to the steam engine, of which it is the moving power.

WATER-*Mill*, an engine which is put in motion by the action of water, of which there are four kinds: overshot-mills, breast-mills, undershot-mills, and mills with horizontal wheels; of which, however, the latter kind is by far the least common, being very disadvantageous and deficient in point of utility. In a breast-mill, the water falls down upon the wheel at right-angles to the float-boards or buckets placed all round the wheel to receive it: if float-boards are used, the water acts only by its impulse; but if buckets, it acts also by the weight of water in the buckets in the under quadrant of the wheel, and this is considerable. In the undershot wheel float-boards only are used, and the wheel is turned merely by the force of the current running under it, and striking upon the boards. In the overshot wheel the water is poured over the top, and thus acts principally, though not altogether, by its weight, for the fall upon the upper part cannot be very considerable, lest it should dash the water out of the buckets. Hence it is evident that an undershot wheel must require a much larger supply of water than the other; the breast-mill requires the next, unless the fall is very great; and an overshot-mill requires the least.

It was long believed that the float-boards of an overshot wheel ought to be so proportioned that when one of them was in a vertical position, or at the middle of its immersion, the next board should be just entering the water; but it is now well known that the more float-boards such a wheel has, the greater and more uniform will be its effects. According to the experiments made by M. Bossut on this subject, a wheel furnished with 48 float-boards produced a much greater effect than one furnished with 24; and the latter a greater effect than one with 12, their immersion in the water being equal.

If a stream of water impinge on the float-boards of an undershot wheel, and escape from it the very instant after it has made its impact, the quantity of water which actually impinges against the wheel, will be to the whole quantity which passes by it in a given time, as the difference between the velocities of the water and of the wheel to the absolute velocity of the water.

The force of the impinging water is as the square of the difference between the velocities of the wheel and the water. For the force is as the relative velocity into the quantity of impinging matter, and the latter is manifestly as the relative velocity; therefore the force will be as the square of the relative velocity.

If W be a weight fastened to a line which is wound round the horizontal axis of an undershot water-wheel. A the altitude of a column of water, equal to the force of the impact of the water on the wheel, when the wheel is quiescent; V the velocity with which the water impinges on the float-boards, v the velocity of the circumference of the wheel, R the radius of the wheel, and r the radius of the axle: then will the velocity of the wheel be $v = V - V \sqrt{\frac{wr}{AR}}$.

For the relative velocity with which the water strikes the wheel is $V - v$; whence, because the force of the stroke is as the square of the relative velocity, we have

$V^2 : (V - v)^2 = A : F$, the force of the water to turn the wheel when the velocity is v; and

$F^2 = A\left(\frac{V - v}{V}\right)^2$, or $v = V - V \sqrt{\frac{F}{A}}$. But the acceleration of the wheel will cease when the force of the water to turn the wheel is equal to the force of the weight which opposes it; that is, $FR = wr$, or $F = \frac{wr}{R}$. Hence, substituting this value of F for it in the preceding value of v, there results $v = V - V \sqrt{\frac{wr}{AR}}$, for the velocity of the wheel when its acceleration ceases.

This conclusion, it should be observed, is true only on the hypothesis, that the water escapes from the wheel as fast as it impinges.

If the weight W vary, its momentum will be the greatest possible, when the wheel has acquired its uniform velocity, if $W = \frac{4AR}{9r}$.

The greatest momentum generated in the ascending weight will be $= \frac{4}{27} AV$.

When the momentum of the ascending weight W is a maximum, that weight will be $\frac{4}{9}$ of the weight, which would, if suspended from the axle, balance the force of the stream.

When the momentum of the ascending weight is a maximum, the velocity of the wheel will be $\frac{1}{3}$ of the absolute velocity of the stream.

When the uniform velocity of the ascending weight W is a maximum, the radius of the wheel will be $= \frac{9wr}{4A}$. For the radius of the wheel being R, and that of the axle r, the uniform velocity of the ascending weight will be found by multiplying $V - V \sqrt{\frac{wr}{AR}}$ into the fraction $\frac{r}{R}$; that is, the velocity of w will be $= \frac{r}{R} \quad V - V \sqrt{\frac{wr^3}{AR^3}}$ which, supposing R variable, and making the fluxion $= 0$, gives $R = \frac{9wr}{4A}$.

If the velocity of the stream be given, the greatest effect will be as the quantity of water expended. For by cor. 2, last prop. the greatest effect is as $\frac{4}{27} AV$; that is, since $\frac{4}{27}$ is constant, the effect is in a ratio compounded of the force of impact, and the velocity of the stream; but the force of the impact is as the quantity of water expended and velocity conjointly; consequently the effect is as the quantity of water expended, and the square of the velocity; or, if the velocity be given, as the quantity of water expended.

When the expense of water is the same, the greatest effect will be as the square of the velocity.

The expense of water being the same, the effect will be as the height of the head of water. For $V = \sqrt{2gh}$, whence $h \propto V^2$.

The aperture being the same, the effect will be as the cube of the velocity. For the effect is as Q (the quantity of water) into V^2; and when the aperture is given $Q \propto V$, whence, effect $\propto V^3$.

If all the water which passes by an undershot wheel be supposed to impinge against it, the force of the stream will be simply in the direct ratio of the relative velocity. Because the number of particles which strike the wheel in a given time is given, whatever be the velocity of the wheel.

According to this hypothesis, v the velocity of the wheel will be equal to $V - - V \quad \frac{wr}{AR}$. For in this case, we have $V : V - = A : F$, whence $v = V \quad \frac{VF}{A}$; which, by substituting for F its value $\frac{wr}{R}$, becomes $v = V - \frac{Vwr}{AR}$.

Retaining the same hypothesis,

if W vary its momentum will be $= \frac{A\,R}{2\,R}$, when it is a maximum. For since the uniform velocity of the circumference of the wheel is $= V - \frac{V\,w\,r}{A\,R}$, which, multiplied into $\frac{r}{R}$, gives $\frac{V\,r}{R} - \frac{V\,r^2 w}{A\,R^2}$ for the uniform velocity of the ascending weight *w*; hence its momentum is $\frac{V\,r\,w}{R} - \frac{V\,r^2\,w^2}{A\,R^2}$, which being a maximum, we make its fluxion = 0, considering *w* as variable, whence results $w = \frac{A\,R}{2\,r}$.

On the same supposition, the velocity of the wheel will be half the velocity of the stream, when the effect is a maximum. For, by cor 1. $v = V - \frac{V\,w\,r}{A\,R}$; and in the case of the last cor. $w = \frac{A\,R}{2\,r}$. Putting this for *v* in the former equation, it is transformed to $v = V - \frac{V\,A\,R\,r}{2\,A\,R\,r} = \frac{1}{2}\,V$.

Still retaining the same hypothesis the greatest momentum generated in the ascending weight, will be $= \frac{1}{4}\,A\,V$. For $\frac{1}{2}\,V \times \frac{r}{R} = \frac{V\,r}{2\,R}$ is the uniform velocity of the ascending weight; and the weight moved, *w* is by cor. $2 = \frac{A\,R}{2\,r}$

Consequently $\frac{V\,r}{2\,R} \times \frac{A\,R}{2\,r} = \frac{1}{4}\,A\,V$, the momentum.

In practice, the velocity of the wheel when the machine is in its greatest perfection, will be between one-third and one-half of the velocity of the stream.

WATER-*Spout*, a remarkable kind of meteor, consisting of a moving column or pillar of water. Its first appearance is in the form of a deep cloud, the upper part of which is white, and the lower black. From the lower part of this cloud there hangs, or rather falls down, what is properly called the spout, resembling a conical tube, largest at the top. Under this tube is always observed a great agitation of the water, flying up in the manner of a *jet d'eau*. For some yards above the surface the water stands like a pillar or column, from the extremities of which it spreads, and goes off in a kind of mist or smoke. This cone frequently descends as low as the middle of the column, and continues for sometime contiguous to it, though frequently it only points to it at some distance, either in a perpendicular or in an oblique direction. Frequently it can be scarcely distinguished whether the cone or the column appears the first, both rising suddenly against each other. At other times the water boils up from the sea to a great height, without any appearance of a spout pointing to it in any direction; indeed, generally speaking, the boiling or flying up of the water has the priority, this always preceding its being formed into a column. For the most part the cone does not appear hollow till towards the end, when the sea-water is violently thrown up along its middle, as smoke up a chimney; soon after this, the spout or canal breaks and disappears; the boiling up of the water, and even the pillar, continuing to the last, and for some time afterwards; sometimes till the spout form itself again, and appear anew, which it will do several times in a quarter of an hour.

WATER-*Wheel*, the wheel of a water-mill, on which the water acts as a first mover.

WAVE, in *Physics*, a volume of water elevated by the action of the wind, &c. upon its surface, into a state of fluctuation, and accompanied by a cavity. The extent from the bottom, or lowest point of one cavity, and across the elevation, to the bottom of the next cavity, is the breadth of the wave. Waves are considered as of two kinds, which may be distinguished from one another by the names of natural and accidental waves. The natural waves are those which are regularly proportioned in size to the strength of the wind which produces them.

The accidental waves are those occasioned by the winds re-acting by repercussion from hills or high shores, and by the dashing of the waves themselves, otherwise of the natural kind, against rocks and shoals; by which means those waves acquire an elevation much above what they can have in their natural state.

Waves, whose breadth is equal to $39\frac{1}{8}$ inches or $\frac{325}{96}$ feet, will advance through a space equal to their breadth in one second of time; and therefore in one minute they will go over a space of $195\frac{5}{8}$ feet; and in an hour a space of 11737 feet nearly, or two miles and almost a quarter. And the velocity of greater or less waves will be augmented or diminished in the subduplicate ratio of their breadth.

WEATHER, the state of the atmosphere, with regard to heat and cold, dryness and moisture, wind, rain, hail, &c.

Mr. Kirwan, in vol. v. of the Transactions of the Irish Academy, has laid down the following rules, as the result of a careful examination of observations which had been made in England, during a period of 112 years.

1. When no storm has either preceded or followed the vernal equinox, the succeeding summer is generally dry, or at least so, five times out of six.

2. If a storm happen from an easterly point, on the 19th, 20th, or 21st day of May, the ensuing summer will, four times in five, be also dry. The same event generally takes place, if a storm arise on the 25th, 26th, or 27th days of March, in any point of the compass.

3. Should there be a storm, either at south-west, or at west-south-west, on the 10th, 20th, 21st, or 22d of March, the following summer is wet, five times out of six.

In England, if the winters and springs be dry, they are mostly cold; but, if moist, they are generally warm: on the contrary, dry summers and autumns are usually hot; as moist summers are cold. Thus, if the humidity or dryness of a particular season be determined, a tolerably correct idea may be formed respecting its temperature. To these indications may be added the following maxims; which, being the result of observations made by accurate inquirers, may so far be depended upon, as they will afford a criterion of the mildness or severity, and of the dryness or moisture, of future seasons.

1. A moist autumn, succeeded by a mild winter, is generally followed by a dry and cold spring; in consequence of which, vegetation is greatly retarded.

2. Should the summer be uncommonly wet, the succeeding winter will be severe; because the heat or warmth of the earth will be carried off by such unusual evaporation. Farther, wet summers are mostly attended with an increased quantity of fruit on the white-thorn and dog-rose; nay, the uncommon fruitfulness of these shrubs is considered as the presage of an intensely cold winter.

3. A severe winter is always indicated by the appearance of cranes and other birds of passage at an early period in autumn; because they never migrate southwards, till the cold season has commenced in the northern regions.

4. If frequent showers fall in the month of September, it seldom rains in May; and the reverse.

5. On the other hand, when the wind often blows from the south-west, during either summer or autumn; when the air is unusually cold for those seasons, both to our sensations, and by the thermometer; at the same time, the mercury being low in the barometer;—under these conditions, a profuse fall of rain may be expected.

6. Great storms, rains, or other violent commotions of the clouds, produce a kind of crisis in the atmosphere, so that they are attended with a regular succession, either of fine or of bad weather, for some months.

Lastly, an unproductive year mostly succeeds a rainy winter; as a rough and cold autumn prognosticates a severe winter.

WEDGE, one of the five mechanical powers, or simple engines, being a geometrical wedge, or very acute triangular prism, applied to the splitting of wood, rocks, or raising great weights.

The wedge is the most powerful of all the simple machines, having an almost unlimited advantage over all the other simple mechanical powers; both as it may be made vastly thin, in proportion to its height, in which consists its own natural power; and as it is urged by the force of percussion, or smart blows, which is a force incomparably greater in a given time than any mere dead weight or pressure, such as is employed upon other machines. And accordingly we find it produces effects vastly superior to those of any other power whatever; such as the splitting and raising the largest and hardest rocks; or even the raising and lifting the largest ship, by driving a wedge below it; which a man can do by the blow of a mallet.

To the wedge may be referred all edge-tools, and tools that have a sharp point, in order to cut, cleave, slit, split, chop, pierce, bore, or the like; as knives, hatchets, swords, bodkins, &c.

In the wedge, the friction against the sides is very great, at least equal to the force to be overcome; because the wedge retains any position to which it is driven; and therefore the resistance is at least doubled by the friction.

Authors have been of various opinions concerning the principle from whence the wedge derives its power. Aristotle considers it as two levers of the first kind, inclined towards each other, and acting opposite ways. Guido Ubaldi, Mersenne, &c. will have them to be levers of the second kind. But De Lanis shows, that the wedge cannot be reduced to any lever at all. Others refer the wedge to the inclined plane. And others again, with De Stair, will hardly allow the wedge to have any force at all in itself; ascribing much the greatest part to the mallet which drives it.

The doctrine of the force of the wedge, according to some writers, is contained in this proposition: "If a power directly applied to the head of a wedge, be to the resistance to be overcome, as the breadth of the back is to the height; then the power will be equal to the resistance; and if increased, it will overcome it."

But Desaguliers has proved that, when the resistance acts perpendicularly against the sides of the wedge, the power is to the whole resistance, as the thickness of the back is to the length of both the sides taken together.

WEIGHT, in *Physics*, a consequence of gravity, being that tendency in natural bodies downwards or towards the earth's centre.

WEIGHT, in *Mechanics*, denotes any thing to be raised, sustained, or moved by a machine, as distinguished from the power, or that by which the machine is put in motion.

WEIGHT, in *Commerce*, denotes a body of a known weight, appointed by law to be the standard of comparison between different quantities of merchandise of certain descriptions; the weight itself being usually of lead, iron, brass, or other metal.

The great diversity of weights and measures, in all nations, for different kinds of commodities, has always been a just subject of regret and complaint; being the cause of various disputes and deceptions, which it is almost impossible to avoid under present circumstances. And it is therefore much to be wished, though, perhaps, little to be expected, that one uniform system of weights should be adopted as applicable to all kinds of substances; an attempt at which was made in France during the revolution, but which it seems has been lately laid aside by an imperial decree, in consequence of the repeated representations of people in trade; so difficult is it to overcome prejudices and customs long established, however advantageous the change may be, when properly understood.

By the twenty-seventh chapter of Magna Charta, the weights all over England are to be the same; but for different commodities, there are two different sorts, *viz.* troy weight and avoirdupois weight. The origin from which they are both raised, is a grain of wheat gathered in the middle of the ear.

In troy-weight, twenty-four of these grains make one pennyweight sterling; twenty pennyweights make one ounce; and twelve ounces one pound.

By this weight we weigh gold, silver, jewels, and liquors. The apothecaries also use the troy pound, ounce, and grain; but they differ from the rest in the intermediate divisions. They divide the ounce into eight drachms; the drachm into three scruples, and the scruple into twenty grains.

In avoirdupois weight the pound contains sixteen ounces, but the ounce is less by near one-twelfth than the troy ounce; this latter containing 490 grains, and the former only 448. The ounce contains 16 drachms; 80 ounces avoirdupois are only equal to 73 ounces troy; and 17 pounds troy equal to 14 pounds avoirdupois.

By avoirdupois weight are weighed meat, grocery wares, base metals, wool, tallow, hemp, drugs, bread, &c.

Table of Troy Weight, as used by the Goldsmiths.

Grains.			
24	Pennyweight.		
480	20	Ounce.	
5760	240	12	Pound.

Apothecaries.

Grains.				
20	Scruple.			
60	3	Drachm.		
480	24	8	Ounce.	
5760	288	96	12	Pound.

Table of Avoirdupois Weight.

Dram.					
16	Ounce.				
256	16	Pound.			
7168	448	28	Quarter.		
28672	1792	112	4	Cwt.	
573440	35840	2240	80	20	Ton.

Comparison between Troy and Avoirdupois Weight.

175 troy pounds are equal to 144 avoirdupois pounds.

175 troy ounces are equal to 192 avoirdupois ounces.

1 troy pound contains 5760 grains.

1 avoirdupois pound contains 7000 grains.

1 avoirdupois ounce contains 437½ grains.

1 avoirdupois dram contains 27.34375 grains.

1 troy pound contains 13 oz. 2.651428576 drs. avoirdupois.

1 avoirdupois lb. contains 1 lb. 2 oz. 11 dwts. 16 gr. troy.

Therefore the avoirdupois lb. is to the lb. troy as 175 to 144, and the avoirdupois oz. is to the troy oz. as 437½ is to 480.

Goldsmiths and jewellers, &c. have a particular class of weights for gold and precious stones, *viz.* carat and grain; and for silver, the pennyweight and grain. In the mint they have also a peculiar subdivision of the troy grain: thus, dividing

the grain into 20 mites.
the mite into 24 droits,
the droit into 20 periots,
the periot into 24 blanks.

The dealers in wool have likewise a particular set of weights, *viz.* the sack, weigh, tod, stone, and clove; the proportions of which are as below; *viz.*

the sack containing 2 weighs,
the weigh .... 6½ tods,
the tod .... 2 stones,
the stone .... 2 cloves,
the clove .... 7 pounds.
Also 12 sacks make a last, or 4368 pounds.

Farther,
56lb. of old hay, or 60 lb. new hay, make a truss.
40 lb. of straw make a truss.
36 trusses make a load of hay or straw.
14 lb. make a stone.
5 lb. of glass a stone.

Other nations have also certain weights peculiar to themselves: thus, Spain has its arrobas, containing 25 Spanish pounds, or one-fourth of the common quintal: its quintal macho, containing 150 pounds, or one-half of the common quintal, or 6 arrobas; its adarme, containing one-sixteenth of its ounce. And for gold, it has its castillan, or one-hundredth of a pound; and its tomin, containing 12 grains, or one-eighth of a castillan. The same are in use in the Spanish West Indies.

Portugal has its arroba, containing 32 Lisbon arratals, or pounds. Savary also mentions its faratelle, containing 2 Lisbon pounds; and its rottolis, containing about 12 pounds. And for gold, its chego, containing four carats. The same are used in the Portuguese East Indies.

Italy, and particularly Venice, have their migliaro, containing four mirres; the mirre containing 30 Venice pounds; the saggio, containing a sixth part of an ounce. Genoa has five kinds of weights, *viz.* large weights, whereby all merchandises are weighed at the custom-house; cash weights for piastres, and other specie; the cantara, or quintal, for the coarsest commodities; the large balance for raw silks, and the small balance for the fine commodities. Sicily has its rottolo, 32 and a half pounds of Messina.

Germany, Flanders, Holland, the Hanse-towns, Sweden, Denmark, Poland, &c. have their schippondt, which, at Antwerp and Hamburgh, is 300 pounds; at Lubeck, 320; and at Koningsberg, 400 pounds. In Sweden the shippondt for copper is 320 pounds: and the shippondt for provisions 400 pounds. At Riga and Revel, the schippondt is 400 pounds; at Dantzic, 340 pounds; in Norway, 300 pounds; at Amsterdam, 300; containing 20 lyspondts, each weighing 15 pounds

In Moscow, they weigh their large commodities by the bercheroct, or berkewits, containing 400 of their pounds. They have also the poet or poede, containing 40 pounds, or one-tenth of the bercheroct.

In order to show the proportion of the several weights used throughout Europe, we shall add a reduction of them to one standard, *viz.* the London pound.

1. Proportion of the weights of the principal places of Europe.

The 100 lb. of England, Scotland, and Ireland, are equal to

lb.	oz.	
91	8	of Amsterdam, Paris, &c.
96	8	of Antwerp or Brabant.
88	0	of Rouen, the viscounty weight.
106	0	of Lyons, the city weight.
90	9	of Rochelle.
107	11	of Toulouse and Upper Languedoc.
113	0	of Marseilles or Provence.
81	7	of Geneva.
93	5	of Hamburgh.
89	7	of Frankfort, &c
96	1	of Leipsic, &c.
137	4	of Genoa.
132	11	of Leghorn.
153	11	of Milan.
152	0	of Venice.
154	10	of Naples.
97	0	of Seville, Cadiz, &c.
104	13	of Portugal.
96	5	of Liege.
112	$\frac{2}{3}$	of Russia.
107	$\frac{3}{24}$	of Sweden.
89	$\frac{1}{2}$	of Denmark.

Proportion of the Weights of several Places in Europe compared with those of Amsterdam.

The 100 lb of Amsterdam, are equal to

lb.	
105	of Antwerp.
100	of Bayonne and Paris.
95½	of Bergen in Norway.
151	of Bologna.
103	of Bremen.
125	of Breslaw.
105	of Cadiz.
107	of Copenhagen.
97	of Dublin and Edinburgh.
109	of London.
114	of Madrid.
169	of Naples.
109	of Riga.
146	of Rome.
110	of Stettin.
182	of Venice.

Universal Standard of Weight and Measure.—It has been shown in the preceding article that the French mathematicians have employed for their standard of measure, the ten millioneth part of a quadrant of the meridian, contained between the equator and north pole; and thence have deduced their unit or standard of weight. This, however, is only one of many different standards that have been at different times proposed for this purpose; nor is it, we conceive, the best that might have been adopted, being subject to this important objection; that it depends upon an accurate measure of a quarter of a meridian of the earth, which is obviously a very difficult, not to say an impossible, task to perform, particularly as there is every reason to suppose that meridians differ essentially amongst themselves. What therefore has been said in approbation of this system in the preceding article, must be understood as only applying to the division according to the decimal scale, and not to the standard itself.

Amongst the various projects for a universal measure (on which that of weight obviously depends), there are indeed but few that deserve any particular notice, except those that are founded on experiments with pendulous bodies, in some assumed position on the surface of the earth, or on the mensuration of some of the lines or circles measured on its surface after the manner adopted by the French, and which has been already explained. We have therefore, in this place, only to attend to those depending on the former principle, and of these we shall only mention one or two of the most plausible.

Huygens, in his "Horologium Oscillatorium," was, as far as we can ascertain, the first who proposed the distance from the point of suspension to the centre of oscillation, of a pendulum vibrating seconds, as the length of a universal standard yard, the third part of which was to be denominated *a horary foot*, to which all measures were to be referred. This standard is liable to two objections; for, 1st, it is not invariable, the length of a second's pendulum, like the space descended in the first second from quiescence, being different at different distances from the equator. And 2dly, the difficulty of exactly measuring the distance from the point of suspension to the centre of oscillation is such, that it is probable no two measures would come to the same result.

Several expedients were proposed after this of M. Huygens, but none that deserved any attention, until the year 1779, when a Mr. Hatton, in consequence of a premium offered by a society of Arts and Manufactures, "for obtaining invariable standards for weights and measures communicable at all times and to all nations," proposed a plan which consisted in the application of a moveable point of suspension to one and the same pendulum, in order to produce the full and absolute effect of two pendulums, the difference of whose lengths was the intended measure.

Here also the ratio of their lengths was easily determined, from observing the number of vibrations performed in a given time at each point of suspension. Whence there being two equations and two unknown quantities, the actual length of the pendulums themselves, might be easily deduced by simple algebraic rules. The late ingenious Mr. Whitehurst much improved upon Mr. Hatton's original notion, in his essay published in 1787, under the title of "An Attempt towards obtaining invariable Measures of Length, Capacity, and Weight, from the Mensuration of Time," &c.

Mr. Whitehurst's proposal is to obtain a measure of the greatest length that convenience will permit, from two pendulums whose vibrations are in the ratio of 2 to 1, and whose lengths coincide with the English standard in whole numbers. His numbers were chosen with considerable judgment and skill. On a supposition that the length of the second's pendulum in the latitude of London is 39·2 inches, the length of one vibrating 42 times in a minute must be 80 inches; and that of another vibrating 84 times in a minute must be 20 inches; their difference 60 inches, or 5 feet, in his standard measure. The difference in the lengths of the two pendulums, however, resulting from his experiments, was 59·892 inches, instead of 60, the discrepancy being occasioned by an error in the original assumption of 39·2 inches instead of 39·128, or 39⅛ inches, as it is very nearly. Still Mr. Whitehurst has accomplished a principal part of his grand design, by showing how an invariable standard may always be found for the same latitude. But this is by no means all that is wanted.

Perhaps the least objectionable way would be to take for the length of the metre the length of a simple pendulum vibrating seconds at the equator, at a certain height above the surface of the sea, when the thermometer is at a fixed medium temperature; the length of the metre would then be about 39·027 English inches, instead of 39·37023, the metre of the new French system. The magnitude of the arc, the stere, the gramme, &c. (or any other terms we thought proper to introduce for similar purposes), might have the same relations to the metre as in the French system. Thus should we possess a standard taken from the gravitating force of the globe we inhabit, and which might be safely considered as invariable, so long as the constitution of the earth and its time of rotation remains the same.

The material standard itself might be chosen of some shape that should possess the double advantage of being little affected by changes of temperature, and being a pendulum whose distance between the point of suspension and centre of oscillation, should be exactly equal to a fixed dimension of the pendulum that might readily be measured with exactness. Such a body we have in a right angled cone, or one, the diameter of whose base is equal to its altitude; for when this cone is suspended by its vertex as a centre of motion, the centre of oscillation is in the centre of its base; and when it is suspended by its base, the centre of oscillation coincides with the vertex of the solid: the length of the isochronous simple pendulum being in both cases equal to the altitude of the cone, or to the diameter of its base.

WHEEL, in *Mechanics*, a simple machine, consisting of a circular piece of wood, metal, or other matter, which revolves on an axis. This is otherwise called wheel and axle, or axis in peritrochio, as a mechanical power, being one of the most frequent and useful of any. In this capacity of it, the wheel is a kind of perpetual lever, and the axle another lesser one; or the radius of the wheel and that of its axle may be considered as the longer and shorter arms of a lever, the centre of the wheel being the fulcrum or point of suspension. Whence it is, that the power of this machine is estimated by this rule, as the radius of the axle is to the radius of the wheel, or as the circumference to the circumference, so is any given power to the weight it will sustain.

Wheels, as well as their axles, are frequently dented, or cut into teeth, and are then of use upon innumerable occasions; as in jacks, clocks, mill-work, &c.; by which means they are capable of moving and acting on one another, and of being combined together to any extent; the teeth either of the axle or circumference working in those of other wheels or axles; and thus, by multiplying the power to any extent, an amazing effect is produced.

To compute the power of a combination of wheels; the teeth of the axle of every wheel acting on those in the circumference of the next following. Multiply continually together the radii of all the axles, as also the radii of all the wheels; then it will be, as the former product is to the latter product, so is a given power applied to the circumference, to the weight it can sustain. Thus, for example, in a combination of five wheels and axles, to find the weight a man can sustain, or raise, whose force is equal to 150 pounds, the radii of the wheels being 30 inches, and those of the axles 3 inches.
Here $3\times 3\times 3\times 3\times 3=243$,
and $30\times30\times30\times30\times30=24300000$,
therefore as $243 : 24300000 = 150 : 15000000$lb. the weight he can sustain, which is more than 6696 tons weight. So prodigious is the increase of power in a combination of wheels!

But it is to be observed, that in this, as well as every other mechanical engine, whatever is gained in power, is lost in time; that is, the weight will move as much slower than the power, as the force is increased or multiplied, which, in the example above, is 100000 times slower.

Hence, having given any power, and the weight to be raised, with the proportion between the wheels and axles necessary to that effect; to find the number of the wheels and axles. Or, having the number of the wheels and axles given, to find the ratio of the radii of the wheels and axles. Here, putting

p = the power acting on the last wheel,
w = the weight to be raised,
r = the radius of the axles,
R = the radius of the wheels,
n = the number of the wheels and axles; then, by the general proportion, as $r^n : R^n = p : w$; therefore $p\,R^n = w\,r^n$ is a general theorem, whence may be found any one of these five letters or quantities, when the other four are given. Thus, to find n the number of wheels: we have first

$$\frac{R^n}{r^n} = \frac{w}{p}, \text{ then } n = \frac{\log. w - \log. p}{\log. R. - \log. r}.$$

And to find $\frac{R}{r}$, the ratio of the wheel to the axle; it is $\frac{R}{r} = \sqrt[n]{\frac{w}{p}}$.

Undershot Water-Wheels, are those in which the motion of the stream exceeds that of the wheel, so that the fluid impels them forward by its action against the float boards, which are immersed in it to a certain depth.

Overshot Water-Wheel, is that which consists of a frame of open buckets, so disposed round the rim of the wheel as to receive the water from its fall, in such a manner that one side is loaded with water while the other is empty, in consequence of which the loaded side descends, and empties its buckets, while those on the other side revolve round and are filled in their turn; and by means of which a uniform circular motion is produced.

WINCH, the bent handle employed for turning round wheels, and producing circular motion.

WIND, sensible current in the atmosphere. The motions of the atmosphere are subject in some degree to the same laws as those of the denser fluids; if we remove a portion of water in a large reservoir, we see the surrounding water flow in to restore the equilibrium; and if we impel, in any direction, a certain portion, an equal quantity moves in a contrary direction from the same cause; or if a portion being rarefied by heat, or condensed by cold, ascends in the one instance and descends in the other, a counter-current is the visible and natural result; and similar effects are found to follow the same causes in the atmospheric fluid.

The general causes may be stated as follow:

1. The ascent of the air over certain tracts, heated by the sun. 2. Evaporation causing an actual increase in the volume of the atmosphere. 3. Rain, snow, &c. causing an actual decrease in its volume by the destruction of the vapour. Currents thus produced may be *permanent* and general, extending over a large portion of the globe; *periodical*, as in the Indian ocean; or *variable*, and as it were occasional, or at least uncertain, as the winds in temperate climates.

General or permanent winds blow always nearly in the same direction. In the Atlantic and Pacific oceans, under the equator, the wind is almost always easterly; it blows, indeed, in this direction, on both sides of the equator to the latitude of 28°. More to the northward of the equator, the wind generally blows between the north and east; and the farther north we proceed, we find the wind to blow to a more northern direction; more to the southward of the equator it blows between the south and east; and the farther to the south, the more it comes in that direction.

Between the parallels of 28° and

40° south latitude, in that tract which extends from 30° west to 100° east longitude from London, the wind is variable, but it most frequently blows from between the N. W. and S. W. so that the outward-bound East-India ships generally run down their easting on the parallel of 36° south.

Periodical WINDS.—Those winds which blow in a certain direction for a time, and at certain stated seasons change, and blow for an equal space of time from the opposite point of the compass, are called monsoons. During the months of April, May, June, July, August, and September, the wind blows from southward over the whole length of the Indian ocean, *viz.* between the parallels of 28° N. and 28° S. latitude, and between the eastern coast of Africa and the meridian which passes through the western part of Japan; but in the other months, October, November, December, January, February, and March, the winds in all the northern parts of the Indian ocean shift round, and blow directly contrary to the course they held in the former six months. For some days before and after the change, there are calms, variable winds, and tremendous storms, with thunder, &c.

Causes of the WIND.—Philosophers differ in their opinions respecting the cause of these periodical winds; but a more probable theory of the general trade-winds is, that they are occasioned by the heat of the sun in the regions above the equator, where the air is heated to a greater degree, and consequently rarefied more than in the more northern parts of the globe. From this expansion of the air in these tropical regions, the denser air, in higher latitudes, rushes violently towards the equator from both sides of the globe. By this conflux of the denser air, without any other circumstances intervening, a direct northerly wind would be produced in the northern tropic, and a southern one in the other tropic; but as the earth's diurnal motion varies the direct influence of the sun over the surface of the earth, and as by that motion this influence is communicated from east to west, an easterly wind would be produced if this influence alone prevailed. On account of the cooperation of these two causes at the same time, the trade-winds blow naturally from the N. E. on the north, and from the S. E. on the south of the line, throughout the whole year; but, as the sun approaches nearer the tropic of Cancer in our summer season, the point towards which these winds are directed will not be invariably the same, but they will incline more towards the north in that season, and more towards the south in our winter.

The land and sea breezes in the tropical climates may be considered as partial interruptions of the general trade-winds; and the cause of these it is not very difficult to explain. From water being a better conductor of heat than earth, the water is always of a more even temperature. During the day, therefore, the land becomes considerably heated, the air rarefied, and consequently in the afternoon a breeze sets in from the sea, which is less heated at that time than the land. On the other hand, during the night the earth loses its surplus heat, while the sea continues more even in its temperature. Towards morning, therefore, a breeze regularly proceeds from the land towards the ocean, where the air is warmer, and consequently more rarefied, than on shore.

The cause of the monsoons is not so well understood as that of the general trade-winds; but what has been just remarked suggests, at least, a probable theory on the subject. It is well known, that at the equator the changes of heat and cold are occasioned by the diurnal motion of the earth, and that the difference between the heat of the day and the night is almost all that is perceived in those tropical regions; whereas in the polar regions the great vicissitudes of heat and cold are occasioned by the annual motion of the globe, which produces the sensible changes of winter and summer; consequently, if the heat of the sun was the only cause of the variation of the winds, the changes, if any, that would be produced by those means in equatorial regions, ought to be diurnal only, but the changes about the pole should be experienced only once in six months. As the effects arising from the heat of the sun upon the air must be greater at the equator than at the poles, the changes of the wind arising from the expansion of the air by the sun's rays must be more steady in equatorial than in polar regions. The incontrovertible evidence of navigators proves this truth, that winds are more variable towards the poles, and more constant towards the equator. But in summer, the continual heat, even in high latitudes, comes to be sensibly felt, and produces changes in the wind, which are distinctly perceptible. In our own cold region, the effects of the sun on the wind are felt during the summer months; for while the weather in that season of the year is fine, the wind generally becomes stronger as the time of the day advances, and dies away towards the evening, and assumes that pleasing serenity so delightful to our feelings. Such are the diurnal changes of the wind in northern climates. The annual revolution of the sun produces still more sensible effects. The prevalence of the western winds during summer we may attribute to this cause, which is still more perceptible in France and Spain; because the continent of land to the eastward, being heated more than the waters of the Atlantic ocean, the air is drawn, during that season, towards the east, and consequently produces a western wind.

But these effects are much more perceptible in countries near the tropics than with us. For when the sun approaches the tropic of Cancer, the soil of Persia, Bengal, China, and the adjoining countries, becomes so much more heated than the sea to the southward of those countries, that the current of the general trade-wind is interrupted, so as to blow, at that season, from the south to the north, contrary to what it would do if no land was there. But as the high mountains of Africa, during all the year, are extremely cold, the low countries of India, to the eastward of it, become hotter than Africa in summer, and the air is naturally drawn thence to the eastward. From the same cause it follows, that the trade-wind in the Indian ocean, from April till October, blows in a north-east direction, contrary to that of the general trade-wind in open seas in the same latitude; but when the sun retires towards the tropic of Capricorn, these northern parts become cooler, and the general trade-wind assumes its natural direction.

Having given the most obvious causes of the periodical monsoons in the Indian seas, it is necessary to observe, that no monsoon takes place to the southward of the equator, except in that part of the ocean adjoining to New Holland. There the same causes concur to produce a monsoon as in the northern tropic, and similar appearances take place. From October till April, the monsoon sets in from the N. W. to S. E. opposite to the general course of the trade-wind on the other side of the line; and here also the general trade-wind resumes its usual course during the other months, which constitute the winter season in these regions. It may not be improper to conclude this account of the tropical winds, by enumerating some of the principal inflections of the monsoons.

Between the months of April and October, the winds blow constantly from W. S. W. in all that part of the Indian ocean which lies between Madagascar and Cape Comorin, and in the contrary direction from October till April, with some small variation in different places; but in the Bay of Bengal these winds are neither so strong nor so constant as in the Indian ocean. It must also be remarked, that the S. W. winds in those seas are more southerly on the African side, and more westerly on the side of India; but these variations are not so great as to be repugnant to the general theory.

The cause of this variation is, as was before intimated, that the mountainous lands of Africa are colder than the flatter regions of Arabia and India; consequently, the wind naturally blows from these cold mountains, in the summer season, towards the warmer lands of Asia, which occasions those inflections of the wind to the eastward during the summer months. The peninsula of India lying so much farther to the south than the kingdoms of Arabia and Persia, adds greatly to this effect; because the wind naturally draws towards them, and produces that easterly variation of the monsoon which takes place in this part of the ocean, while the sandy deserts of Arabia draw the winds more directly northward, near the African coast.

Variable WINDS.—In the temperate zones the direction of the winds is by no means so regular as between the tropics. Even in the same degree of latitude we find them often blowing in different directions at the same time; while their changes are frequently so sudden and so capricious, that to account for them has hitherto been found impossible. When winds are violent, and continue long, they generally extend over a large tract of country; and this is more certainly the case when they blow from the north or east than from any other points. By the multiplication and comparison of meteorological tables, some regular connection between the changes of the atmosphere in different places may, in time, be observed, which may at last lead to a satisfactory theory of the winds. It is from such tables chiefly, that the following facts have been collected:

In Virginia, the prevailing winds are between the south-west, west, north, and north-west; the most frequent is the south-west, which blows more constantly in June, July, and August, than at any other season. The north-west winds blow most constantly in November, January, and February. At Ipswich, in New England, the prevailing winds are also between the south-west, west, north, and north-east; the most frequent is the north-west. But at Cambridge, in the same province, the most frequent wind is the south-east. The predominant winds at New York are the north and west; and in Nova Scotia north-west winds blow for three-fourths of the year. The same wind blows most frequently at Montreal in Canada; but at Quebec the wind generally follows the direction of the river St. Lawrence, blowing either from the north-east or south-west. At Hudson's Bay westerly winds blow for three-fourths of the year; the north-west wind occasions the greatest cold, but the north and north-east are the vehicles of snow.

It appears from these facts, that westerly winds are most frequent over the whole eastern coast of North America; that, in the southern provinces, south-west winds predominate; and that the north-west become gradually more frequent as we approach the frigid zone.

In Egypt, during part of May, and during June, July, August, and September, the wind blows almost constantly from the north, varying sometimes in June to the west, and in July to the west and the east; during part of September, and in October and November, the winds are variable, but blow more regularly from the east than any other quarter; in December, January, and February, they blow from the north, north-west, and west; towards the end of February they change to the south, in which quarter they continue till near the end of March; during the last days in March and in April, they blow from the south-east, south, and south-west, and at last from the east; and in this direction they continue during a part of May.

In the Mediterranean the wind blows nearly three-fourths of the year from the north; about the equinoxes there is always an easterly wind in that sea, which is generally more constant in spring than in autumn. These observations do not apply to the gut of Gibraltar, where there are seldom any winds except the east and the west. At Bastia, in the island of

Corsica, the prevailing wind is the south-west.

In Syria the north wind blows from the autumnal equinox to November; during December, January, and February, the winds blow from the west and south-west; in March they blow from the south; in May from the east; and in June from the north. From this month to the autumnal equinox, the wind changes gradually as the sun approaches the equator; first to the east, then to the south, and, lastly, to the west. At Bagdad, the most frequent winds are the south-west and north-west; at Pekin, the north and the south; at Kamtschatka, on the north east coast of Asia, the prevailing winds blow from the west.

In Italy, the prevailing winds differ considerably according to the situation of the places where the observations have been made: at Rome and Padua they are northerly, at Milan easterly. All that we have been able to learn concerning Spain and Portugal is, that, on the west coast of these countries, the west is by far the most common wind, particularly in summer; and that at Madrid the wind is north-east for the greatest part of the summer, blowing almost constantly from the Pyrenean mountains. At Berne, in Switzerland, the prevailing winds are the north and west; at St. Gothard, the north-east; at Lausanne, the north-west and south-west.

In the interior parts of France, the south-west wind blows most frequently in 18 places; the west wind in 14; the north in 13; the south in 6; the north-east in 4; the south-east in 2; the east and north-west each of them 1. On the west coast of the Netherlands, as far as Rotterdam, the prevailing winds are probably the south-west, at least this is the case at Dunkirk and Rotterdam. It is probable also that along the rest of this coast, from the Hague to Hamburgh, the prevailing winds are the north-west, at least these winds are most frequent at the Hague and at Franeker. The prevailing wind at Delft is the south-east; and, at Breda, the north and the east.

In Germany, the east wind is most frequent at Gottingen, Munich, Weissenfels, Dusseldorf, Saganum, Erford, and at Buda, in Hungary; the south-east at Prague and Wurtzburg; the north-east at Ratisbon; and the west at Manheim and Berlin.

From an average of ten years of the register kept by order of the Royal Society, it appears that at London the winds blow in the following order:

Winds.	*Days.*
South-west	112
North-east	58
North-west	50
West	53
South-east	32
East	26
South	18
North	16

It appears from the same register, that the south west wind blows at an average more frequently than any other wind during every month of the year, and that it blows longest in July and August; that the north-east blows most constantly during January, March, April, May, and June, and most seldom during February, July, September, and December; and that the north-west wind blows oftener from November to March, and more seldom during September and October than any other months. The south-west winds are also most frequent at Bristol, and next to them are the north-east.

The south-west wind is by far the most frequent all over Scotland, especially on the west coast. There it blows three-fourths of the year, and upon the north-east coast of Moray it blows two-thirds of the year.

The following table exhibits a view of the number of days during which the westerly and easterly winds blow in a year at different parts of the island. Under the term westerly are included the north-west, west, south.west, and south; the term easterly is taken in the same latitude.

Years of Observ.	Places.	Wind.	
		Westerly.	Easterly.
10	London	233	132
7	Lancaster	216	149
51	Liverpool	190	175
9	Dumfries	227.5	137.5
10	Branxholm, fifty-four miles south-west of Berwick	232	133
7	Cambuslang	214	151
8	Hawkhill, near Edinburgh	229.5	135.5
	Mean	220.3	144.7

In Ireland the south-west and west are the grand trade-winds, blowing most in summer, autumn, and winter, and least in spring. The north-east blows most in spring, and nearly double to what it does in autumn and winter. The south-east and north-west are nearly equal, and are most frequent after the south-west and west.

At Copenhagen the prevailing winds are the east and south-east; at Stockholm, the west and north. In Russia, from an average of a register of 16 years, the winds blow from November to April in the following order:

	W.	N. W.	E.	S. W.	S.
Days	45	26	23	22	20
	N. E.	N.	S. E.		
——	19	14	12		

And during the other six months,

	W.	N. W.	E.	S. W.	S.
Days	27	27	19	24	22
	N. E.	N.	S. E.		
——	15	32	18		

The west wind blows during the whole year 72 days; the north-west 58; the south-west and north 46 days each. During summer it is calm for 41 days, and during winter for 21. In Norway, the most frequent winds are the south, the south-west, and south-east. The wind at Bergen is seldom directly west, but generally south-west or south-east; a north-west, and especially a north-east wind, are but little known there.

From the whole of these facts, it appears that the most frequent winds on the south coasts of Europe are the north, the north-east, and north-west; and on the western coast, the south-west; that in the interior parts which lie most contiguous to the Atlantic ocean, south-west winds are also most frequent; but that easterly winds prevail in Germany. Westerly winds are also most frequent on the north-east coast of Asia.

It is probable that the winds are more constant in the south temperate zone, which is in a great measure covered with water, than in the north temperate zone, where their direction must be frequently interrupted and altered by mountains and other causes.

M. De la Caille, who was sent thither by the French king to make astronomical observations, informs us that, at the Cape of Good Hope, the principal winds are the south-east and north-west; that other winds seldom last longer than a few days; and that the east and north-east winds blow very seldom. The south-east wind blows in most months of the year, but chiefly from October to April; the north-west prevails during the other six months, bringing along with it rain, and tempests, and hurricanes. Between the Cape of Good Hope and New Holland, the winds are commonly westerly, and blow in the following order: north-west, south-west, west, north.

In the South Sea, from latitude 30° to 40° south, the south-east trade-wind blows most frequently, especially when the sun approaches the tropic of Capricorn; the wind next to it in frequency is the north-west, and next to that is the south-west. From south latitude 40° to 50°, the prevailing wind is the north-west, and next the south-west. From 50° to 60°, the most frequent wind is also the north-west, and next to it is the west.

Thus it appears that the trade-winds sometimes extend farther into the south temperate zone than their usual limits, particularly during summer; that beyond their influence the winds are commonly westerly, and that they blow in the following order, north-west, south-west, west.

Such is the present state of the history of the direction of the winds. In the torrid zone they blow constantly from the north-east on the north side of the equator, and from the south-east on the south side of it. In the north temperate zone they blow most frequently from the south-west; in the south temperate zone from the north-west, changing, however, frequently to all points of the compass; and in the north temperate zone blowing, particularly during spring, from the north-east.

Force and Velocity of the WIND.—As to the velocity of the wind, its variations are almost infinite; from the gentlest breeze, to the hurricane which tears up trees and blows down houses. It has been remarked, that our most violent winds take place when neither the heat nor the cold is greatest; that violent winds generally extend over a great tract of country, and that they are accompanied by sudden and great falls in the mercury of the barometer. The reason appears to be, that violent winds succeed the precipitation in rain of a large quantity of vapour, which previously constituted a part of the bulk of the atmosphere; and this precipitation cannot take place when the general temperature approaches to either extreme. The wind is sometimes very violent at a distance from the earth, while it is quite calm at its surface. On one occasion, Lunardi went at the rate of 70 miles an hour in his balloon, though it was quite calm at Edinburgh when he ascended, and continued so during his whole voyage. The same thing happened to Garnerin and his companion in their aerostatic voyage to Colchester; they having been carried from London to Colchester, a distance of at least 60 miles, in three-quarters of an hour, making the velocity of the wind, at that time, 80 miles per hour, or 1⅓ miles per minute. This again may be illustrated by the motions of dense fluid which are always impeded in the parts contiguous to the sides and bottom of the vessels; and the same thing happens in tide-rivers, where the boatman, when he wishes to proceed with the tide, commits himself to the middle of the stream: but when he has to strive against it, he keeps close to the shore. It is, therefore, not the upper parts of the atmosphere which are accelerated, but the lower are retarded by friction against the surface of the earth.

WINDLASS, or *Windlace*, a particular machine used for raising heavy weights, as guns, stones, anchors, &c.

This is a very simple machine, consisting only of an axis or roller, supported horizontally at the two ends by two pieces of wood and a pulley: the two pieces of wood meet at top, being placed diagonally so as to prop each other; and the axis or roller goes through the two pieces, and turns in them. The pulley is fastened at top, where the pieces join. Lastly, there are two staves, or handspikes, which go through the roller, to turn it by; and the rope, which comes over the pulley, is wound off and on the same.

WINDLASS, in a ship, is an instrument in small ships placed upon deck, just abaft the foremast. It is made of a piece of timber six or eight feet square, in form of an axle-tree, whose length is placed horizontally upon two pieces of wood at the ends, and upon which it is turned about by the help of handspikes put into holes made for that purpose. This instrument serves for weighing anchors, or hoisting of any weight in or out of the ship, and will purchase much more than any capstan, and that without any danger to those who heave: for if in heaving the windlass about, any of the handspikes should happen to break, the windlass would stop of itself, by means of a catch for that purpose.

WINDMILL, a well-known engine which receives its motion from the impulse of the wind.

We may observe, that were the sails set square upon their arms or yards, and perpendicular to the axle-tree, or to the wind, no motion would ensue, because the direct wind would keep them in an exact balance. But by setting them obliquely to the common axis, like sails of a smoke-jack, or inclined like the rudder of a ship; the wind, by striking the surface of them obliquely, turns them about. Now this angle which the sails are to make with their common axis, or the degree of weathering, as the mill-wrights call it, so as that the wind may have the greatest effect, is a matter of nice inquiry, and has occupied the thoughts of the mathematician and artist.

In examining the compound motions of the rudder of a ship, we find that the more it approaches to the direction of the keel, or to the course of the water, the more weakly this strikes it; but, on the other hand, the greater is the power of the lever to turn the vessel about. The obliquity of the rudder has, at the same time, both an advantage and a disadvantage. It has been a point of inquiry, therefore, to find the position of the rudder when the ratio of the advantage over the disadvantage is the greatest. And M. Renau, in his theory of the working of ships, has found, that the best situation of the rudder is when it makes an angle of about 55 degrees with the keel.

The obliquity of the sails, with regard to their axis, has precisely the same advantage and disadvantage, with the obliquity of the rudder to the keel. And M. Parent, seeking by the new analysis the most advantageous situation of the sails on the axis, finds it the same angle of about 55 degrees. This obliquity has been determined by many other mathematicians, and found to be more accurately 54° 44′.

This angle, however, is only that which gives the wind the greatest force to put the sail in motion, but not the angle which gives the force of the wind a maximum upon the sail when in motion: for when the sail has a certain degree of velocity, it yields to the wind; and then that angle must be increased, to give the wind its full effect.

It may be observed, that the increase of this angle should be different according to the different velocities from the axle-tree to the farther extremity of the sail. At the beginning, or axis, it should be 54° 44′; and thence continually increasing, giving the vane a twist, and so causing all the ribs of the vane to lie in different planes.

It is farther observed, that the ribs of the vane, or sail, ought to decrease in length from the axis to the extremity, giving the vane a curvilinear form; so that no part of the force of any one rib be spent upon the rest, but all move on independent of each other. The twist above-mentioned, and the diminution of the ribs, are exemplified in the wings of birds.

As the ends of the sail nearest the axis cannot move with the same velocity which the tips or farthest ends have, although the wind acts equally strong upon them both, Ferguson (Lectures on Mechanics, p. 54, by Brewster,) suggests, that, perhaps, a better position than that of stretching them along the arms directly from the centre of motion, might be to have them set perpendicularly across the farther ends of the arms, and there adjusted lengthwise to the proper angle: for in that case both ends of the sails would move with the same velocity; and being farther from the centre of motion they would have so much the more power, and then there would be no occasion for having them so large as they are generally made; which would render them lighter, and, consequently, there would be so much the less friction on the thick neck of the axle, when it turns in the wall. Mr. Smeaton, from his experiments with windmill sails, deduces several practical maxims.

That when the wind falls upon a concave surface, it is an advantage to the power of the whole, though every part, taken separately, should not be disposed to the best advantage.

That a broader sail requires a greater angle; and that when the sail is broader at the extremity than near the centre, this shape is more advantageous than that of a parallelogram.

When the sails, made like sectors of circles joining at the centre or axis, filled up about seven-eighths of the whole circular space, the effect was the greatest.

The velocity of windmill sails, whether unloaded or loaded, so as to produce a maximum of effect, is nearly as the velocity of the wind, their shape and position being the same.

The load at the maximum is nearly, but somewhat less than as the square of the velocity of the wind.

The effects of the same sails at a maximun, are nearly, but somewhat less, than as the cubes of the velocity of the wind.

In sails of a similar figure and position, the number of turns in a given time, are reciprocally as the radius or length of the sail.

The effects of sails of similar figure and position are as the square of their length.

The velocity of the extremity of Dutch mills, as well as of the enlarged sails, in all their usual positions, is considerably greater than the velocity of the wind.

M. Parent, in considering what figure the sails of a wind-mill should have, to receive the greatest impulse from the wind, finds it to be a sector of an ellipsis, whose centre is that of the axle-tree of the mill; and the less semi-axis the height of 32 feet; as for the greater, it follows necessarily from the rule that directs the sail to be inclined to the axis in the angle of 55 degrees.

On this foundation he assumes four such sails, each being a quarter of an ellipse; which he shows will receive all the wind and lose none, as the common ones do. These four surfaces, multiplied by the lever, with which the wind acts on one of them, express the whole power the wind has to move the machine, or the whole power the machine has when in motion.

A wind-mill with six elliptical sails, he shews, would still have more power than one with only four. It would only have the same surface with the four; since the four contain the whole space of the ellipses, as well as the six. But the force of the six would be greater than that of the four in the ratio of 245 to 231. If it were desired to have only two sails each, being a semi-ellipsis, the surface would be still the same: but the power would be diminished by near one-third of that with six sails; because the greatness of the sectors would much shorten the lever, with which the wind acts.

The same author has also considered which form, among the rectangular sails, will be most advantageous; *i. e.* that which shall have the product of the surface by the lever of the wind the greatest. The result of this inquiry is, that the width of the rectangular sail should be nearly double its length; whereas, usually, the length is made almost five times the width.

The power of the mill, with four of these new rectangular sails, M. Parent shows, will be to the power of four elliptic sails, nearly as 13 to 23; which leaves a considerable advantage on the side of the elliptic ones; and yet the force of the new rectangular sails will still be considerably greater than that of the common ones.

M. Parent also considers what number of the new sails will be most advantageous; and finds that the fewer the sails, the more surface there will be, but the power the less. Farther, the power of a wind-mill with six sails is denoted by 14; that of another with four will be as 13; and another with two sails will be denoted by 9.

That, as to the common windmill, its power still diminishes as the breadth of the sails is smaller, in proportion to the length; and therefore the usual proportion of 5 to 1 is exceedingly disadvantageous.

WINTER, one of the four seasons or quarters of the year. This quarter commences on the day when the sun's distance from the zenith of the place is the greatest, or when the sun has the greatest declination on the contrary side of the equator, *viz.* in northern latitudes, when he has the greatest southern declination; and in southern latitudes when he has the greatest northern declination; and it ends when he next crosses the equinoctial.

WITCH, in the *Higher Geometry*, a curve defined by the equation

$$y = \frac{a\sqrt{(ax - x^2)}}{x}.$$

Y.

YARD, an English measure of length, which contains 3 feet, and is equal

to 4·5th of the English ell,
to 7·9th of the Paris ell,
to 4·3d of the Flemish ell,
to 56·5th of the Spanish vasa or yard.

YEAR, the portion of time occupied by the sun in passing over the twelve signs of the zodiac, and in which is comprehended the several changes of the seasons.

The mean solar year, according to the observations of the best modern astronomers, contains 365 days, 5 hours, 48 minutes, 48 seconds; the quantity assumed by the authors of the Gregorian calendar, is 395 days, 5 hours, 49 minutes; but in the civil or popular account the year contains 365 days, 6 hours, or rather 365 days for three years in succession, and every fourth year 366 days.

The vicissitude of seasons seems to have given occasion to the first institution of the year. Man, naturally curious to know the cause of their diversity, soon conjectured that it depended upon the motion of the sun, and therefore gave the name year to the space of time in which that luminary seemed to perform his whole course, by returning again to the same point of its orbit.

According to the accuracy of their observations, the year of some nations was more perfect than that of others, but none of them quite exact, nor whose parts did not shift with regard to the parts of the sun's course.

According to Herodotus, it was the Egyptians who first formed the year, making it to consist of 360 days, which they subdivided into 12 months, of 30 days each. Mercury Trismegistus added five days more to the account; and which form of the year Thales is said to have instituted amongst the Greeks; and hence, with successive improvements, it has been handed down to the moderns.

The *Astronomical Solar* Year, is that which is precisely determined by astronomical observations, and is of two kinds, *tropical* and *sidereal*, or *astral*.

Tropical or *Natural* Year, is the time which the sun, or rather the earth, employs in passing through the 12 signs of the zodiac, and which, as stated above, contains 365 days, 5 hours, 48 minutes, 48 seconds, which is the only natural year, because it always keeps the same seasons in the same months.

Sidereal Year, or *Astral Year*, is the space of time the sun takes in passing from any fixed star, till his return to it again. This consists of 365 days, 6 hours, 9 minutes, 11 seconds, being 20 minutes, 29 seconds longer than the true solar year.

Anomalistic Year, is the interval which is occupied by the sun in passing from apogee to apogee, or from perigee to perigee: it is greater than the sidereal year by the time required to describe the annual progression of the apogee. The length of the anomalistic year is 365 days, 6 hours, 14 minutes, 1 second.

Lunar Year, is the space of 12 lunar months. Hence, from the two kinds of synodical lunar months, there arise two kinds of lunar years; the one astronomical, the other civil.

Lunar Astronomical Year, consists of 12 lunar synodical months; and therefore contains 354 days, 8 hours, 48 minutes, 38 seconds, and is therefore 10 days, 21 hours, 0 minutes, 10 seconds shorter than the solar year. A difference which is the foundation of the epact.

Lunar Civil YEAR, is either common or embolismic.

The *Common Lunar* YEAR, consists of 12 lunar civil months; and therefore contains 354 days. And

The *Embolismic*, or *Intercalary Lunar* YEAR, consists of 13 lunar civil months, and therefore contains 384 days.

Thus far we have considered years and months, with regard to astronomical principles, upon which the division is founded. By this, the various forms of civil years that have formerly obtained, or that do still obtain, in divers nations, are to be examined.

Civil YEAR, is that form of the year which every nation has contrived or adopted for computing their time by. Or the civil is the tropical year, considered as only consisting of a certain number of whole days: the odd hours and minutes being set aside, to render the computation of time, in the common occasions of life, more easy. As the tropical year is 365 days, 5 hours, 49 minutes, or almost 365 days, 6 hours, which is 365 days and a quarter; therefore, if the civil year be made 365 days, every 4th year it must be 366 days, to keep nearly to the course of the sun. And hence the civil year is either common or bissextile. The

Common Civil YEAR, is that consisting of 365 days; having seven months of 31 days each, four of 30 days, and one of 28 days; as indicated by the following well-known verses:

Thirty days hath September
April, June, and November;
February twenty-eight alone,
And all the rest have thirty-one,
Except in every fourth year, when
February hath twenty-nine.

Bissextile, or *Leap* YEAR, contains 366 days, having one day extraordinary, called the intercalary, or bissextile day, and takes place every 4th year. This additional day to every 4th year was first introduced by Julius Cæsar; who, to make the civil years keep pace with the tropical ones, contrived that the six hours which the latter exceeded the former should make one day in four years, and be added between the 24th and 23d of February, which was their 6th of the calends of March; and as they then counted this day twice over, or had *bis sexto calendas*, hence the year itself came to be called *bis sextus*, and *bissextile*.

However, among us, the intercalary day is not introduced by counting the 23d of February twice over, but by adding a day at the end of that month, which therefore in that year contains 29 days.

A farther reformation was made in the civil year by pope Gregory.

The civil, or legal year, in England, formerly commenced on the day of the Annunciation, or 25th of March; though the historical year began on the day of the circumcision, or 1st of January; on which day the German and Italian year also begins. The part of the year between these two terms was usually expressed both ways; as 1745-6, or 174$\frac{5}{6}$. But, by the act for altering the style, the civil year now commences with the 1st of January.

Ancient Roman YEAR. This was the lunar year, which, as first settled by Romulus, contained only ten months, of unequal numbers of days, in the following order: *viz.*

March 31; April 30; May 31; June 30; Quintilis 31; Sextilis 30; September 30; October 31; November 30; December 30; in all 304 days; which came short of the true lunar year by 50 days; and of the solar by 61 days. Hence, the beginning of Romulus's year was vague, and unfixed to any precise season; to remove which inconvenience, that prince ordered so many days to be added yearly as would make the state of the heavens correspond to the first month, without calling them by the name of any month.

Numa Pompilius corrected this irregular constitution of the year, composing two new months, January and February, of the days that were used to be added to the former year. Thus Numa's year consisted of 12 months, of different days, as follow; *viz.*

Januray	29;	February	28;
March	31;	April	29;
May	31;	June	29;
Quintilis	31;	Sextilis	29;
September	29;	October	31;
November	29;	December	29;

In all 355 days; therefore exceeding the quantity of a lunar civil year by one day; that of a lunar astronomical year by 15 hours, 11 minutes, 22 seconds; but falling short of the common solar year by 10 days; so that its beginning was still vague and unfixed.

Numa, however, desiring to have it begun in the winter solstice, ordered 22 days to be intercalated in February every 2d year, 23 every fourth, 22 every 6th, and 23 every 8th year.

But this rule failing to keep matters even, recourse was had to a new way of intercalating; and instead of 23 days every 8th year, only 15 were to be added. The care of the whole was committed to the pontifex maximus; who, however, neglecting the trust, let things run to great confusion. And thus the Roman year stood till Julius Cæsar reformed it.

The *Ancient Egyptian* YEAR, called also the year of Nabonassar, on account of the epocha of Nabonassar, is the solar year of 365 days, divided into 12 months of 30 days each, besides five intercalary days added at the end. The names, &c. of the months are as follow: 1. Troth. 2. Paophi. 3. Athyr. 4. Chojac. 5. Tybi. 6. Mecheir. 7. Phamenoth. 8. Pharmuthi. 9. Pachon. 10. Pauni. 11. Epiphi. 12. Mesori.

The *Ancient Greek* YEAR, was lunar; consisting of 12 months, which at first had 30 days a-piece, then alternately 30 and 29 days, computed from the first appearance of the new moon; with the addition of an embolismic month of 30 days every 3d, 5th, 8th, 11th, 14th, 16th, and 19th year of a cycle of 19 years; in order to keep the new and full moons to the same terms or seasons of the year. Their year commenced with that new moon, the full moon of which comes next after the summer solstice.

The *Ancient Jewish* YEAR, is a lunar year, consisting commonly of 11 months, which alternately contain 30 and 29 days. It was made to agree with the solar year, either by the adding of 11, and sometimes 12 days, at the end of the year, or by an embolismic month. The names and quantities of the months stand thus: 1. Nisan, or Abib. 30 days. 2. Jiar, or Zius, 29. 3. Sivan, or Siwan, 30. 4. Thammuz, or Tammuz, 29. 5. Ab, 30. 6. Elul, 29. 7. Tisri, or Ethanim, 30. 8. Marchesvam, or Bull, 29. 9. Cisleu, 30. 10. Tebeth, 29. 11. 3 Sabat, or Schebeth, 30. 12. Adar, in the embolismic year, 30. Adar, in the common year, was but 29. Note, in the defective year, Cisleu, was only 29 days; and in the redundant year, Marchesvam was 30.

The *Persian* YEAR, is a solar year of about 365 days; consisting of 12 months of 30 days each, with five intercalary days added at the end.

The *Arabic*, *Mahometan*, and *Turkish* YEARS, called also the year of the Hegira, is a lunar year, equal to 354 days, 8 hours, and 48 minutes, and consists of 12 months, which contain alternately 30 and 39 days.

The *Hindoo* YEAR differs from all these, and is indeed different in different provinces of India.

Z.

ZENITH, in astronomy, the vertical point of the heavens, or that point directly over our heads. Or, it is a point in the sphere through which if a right line were drawn through the eye of the spectator, it would pass through the centre of the earth; and, if produced, meet the *nadir*, forming thus a diameter of the earth.

The zenith is called the pole of the horizon, being 90° distant from every point of that circle; and the parallels of altitude are drawn round it as a centre.

ZENITH *Distance*, the arc intercepted between any celestial object and the zenith, being the same as the co-altitude.

ZETETIC *Method*, an old term for what we now call analytic method.

ZODIAC, in *Astronomy*, an imaginary broad circle in the heavens, in form of a belt or girdle, within which the planets all make their excursions. In the very middle of it runs the ecliptic, or path of the sun in his apparent annual course; and its breadth, comprehending the deviations or latitudes of the earlier known planets, is by some authors accounted 16, some 18, and others 20 degrees.

The zodiac, cutting the equator obliquely, makes with it the same angle as the ecliptic; this angle continually varying, is now nearly equal to 23° 28′; and is called the obliquity of the ecliptic, and constantly varies between certain limits which it can never exceed.

The zodiac is divided into 12 equal parts, of 30 degrees each, called the signs of the zodiac, being so named from the constellations which anciently occupied them. But the stars having a motion from west to east, those constellations do not now correspond to their proper signs; from whence arises what is called the precession of the equinoxes. And therefore when a star is said to be in such a sign of the zodiac, it is not to be understood of that constellation, but only of that dodecatemory or 12th part of it.

Cassini has also observed a tract in the heavens, within whose bounds most of the comets, though not all of them, are observed to keep, and which he therefore calls the zodiac of the comets. This he makes as broad as the other zodiac, and marks it with signs or constellations, like that; as Antinous, Pegasus, Andromeda, Taurus, Orion, the Lesser Dog, Hydra, the Centaur, Scorpion, and Sagittary.

It is a curious fact, that the solar division of the Indian zodiac is the same in substance with that of the Greeks, and yet that it has not been borrowed either from the Greeks or the Arabians. The identity, or at least striking similarity of the division, is universally known; and Montucla has endeavoured to prove that the Brahmins received it from the Arabs. His opinion, we believe, has been very generally admitted; but in the second volume of the Asiatic Researches, the accomplished president, Sir William Jones, has proved unanswerably, that neither of those nations borrowed that division from the other; that it has been known among the Hindoos from time immemorial; and that it was probably invented by the first progenitors of that race, whom he considers as the most ancient of mankind, before their dispersion.

The Greek zodiac originally contained only eleven signs: the Scorpion occupying the place of two.

ZODIACAL *Light*, a brightness sometimes observed in the zodiac, resembling that of the galaxy or milky way. It appears at certain seasons, *viz.* towards the end of winter and in spring, after sunset, or before its rising, in autumn and beginning of winter, resembling the form of a pyramid, lying lengthwise with its axis along the zodiac, its base being placed obliquely with respect to the horizon.

The zodiacal light, according to Mairan, is the solar atmosphere, a rare and subtile fluid, either luminous by itself, or made so by the rays of the sun surrounding its globe; but, in a greater quantity, and more extensively, about his equator, than any other part.

Mairan says, it may be proved, from many observations, that the sun's atmosphere sometimes reaches as far as the earth's orbit, and there meeting with our atmosphere, produces the appearance of an aurora borealis.

The length of the zodiacal light varies sometimes in reality, and sometimes in appearance only, from various causes.

Cassini often mentions the great resemblance between the zodiacal light and the tails of comets. The same observation has been made by Fatio; and Euler endeavoured to prove that they were owing to similar causes.

This light seems to have no other motion than that of the sun itself; and its extent from the sun to its point is seldom less than 50 or 60 degrees in length, and more than 20 degrees in breadth; but it has been known to extend to 100 or 103°, and from 8° to 9° broad.

It is now generally acknowledged, that the electric fluid is the cause of the aurora borealis, ascribed by Mairan to the solar atmosphere, which produces the zodiacal light, and which is thrown off chiefly and to the greatest distance from the equatorial parts of the sun, by means of the rotation on his axis, and extending visibly as far as the orbit of the earth, where it falls into the upper regions of our atmosphere, and is collected chiefly towards the polar parts of the earth, in consequence of the diurnal revolution, where it forms the aurora borealis. And hence it has been suggested, as a probable conjecture, that the sun may be the fountain of the electrical fluid, and that the zodiacal light, and the tails of comets, as well as the aurora borealis, the lightning, and artificial electricity, are its various and not very dissimilar modifications.

ZONE, a division of the earth's surface, by means of parallel circles, chiefly with respect to the degree of heat in the different parts of that surface.

The zones were commonly accounted five in number; one a broad belt round the middle of the earth, having the equator in the middle of it, and bounded towards the north and south by parallel circles passing through the tropics of Cancer and Capricorn. This they called the torrid zone, which they supposed not habitable, on account of its extreme heat. Though sometimes they divided this into two equal torrid zones, by the equator, one to the north, and the other south; and then the whole number of zones was accounted six.

The difference of zones is attended with a great diversity of phenomena. In the torrid zone, the sun passes through the zenith of every place in it twice a year; making as it were two summers in the year; and the inhabitants of this zone are called amphiscians, because they have their noon-day shadows projected different ways in different times of the year, northward at one season, and southward at the other. In the temperate and frigid zones, the sun rises and sets every natural day of 24 hours. Yet every where, but under the equator, the artificial days are of unequal lengths, and the inequality is the greater, as the place is farther from the equator. The inhabitants of the temperate zones are called heteroscians, because their noon-day shadow is cast the same way all the year round, *viz.* those in the north zone toward the north pole, and those in the south zone toward the south pole. Within the frigid zones, the inhabitants have their artificial days and nights extended out to a great length; the sun sometimes skirting round a little above the horizon for many days together; and at another season never rising above the horizon at all, but making continual night for a considerable space of time. The inhabitants of these zones are called periscians, because sometimes they have their shadows going quite round them in the space of 24 hours.

THE END.

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